Abelian 3-form gauge theory: Superfield Formalism

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References

1. R. P. Malik, *Eur. Phys. J. C* **60** (2009)

2. L. Bonora, R. P. Malik, *J. Phys. A: Math. Theor.* **43** (2010)

Superfield Approach:

Bonora, Pasti & Tonin (81)
Bonora & Tonin (82)

Ordinary fields of D-dimensional gauge theory $\longrightarrow$

Superfields on (D, 2)-dimensional super manifold

$$\phi(x) \longrightarrow \tilde{\phi}(x, \theta, \bar{\theta})$$: Superfield

(D) $\longleftrightarrow$ x^μ, (D, 2)-dimensions $\longleftrightarrow$ $(x^\mu, \theta, \bar{\theta}) = Z^M$

(i) $\theta^2 = \bar{\theta}^2 = 0, \quad \theta\bar{\theta} + \bar{\theta}\theta = 0$

(ii) $\partial_\theta^2 = \partial_{\bar{\theta}}^2 = 0, \quad \partial_\theta \partial_{\bar{\theta}} + \partial_{\bar{\theta}} \partial_\theta = 0$

} Grassmannian Variables and derivatives

BRST approach to Gauge Theory

BRST: Becchi-Rouet-Stora-Tyutin

Local gauge symmetry (First-class constraints) → BRST symmetry (s_b)

Local gauge symmetry (First-class constraints) → Anti-BRST symmetry (s_{ab})

$s_b^2 = 0, \quad s_{ab}^2 = 0$ Nilpotency property

Sacrosanct

$s_b s_{ab} + s_{ab} s_b = 0$ Absolute anticommutativity

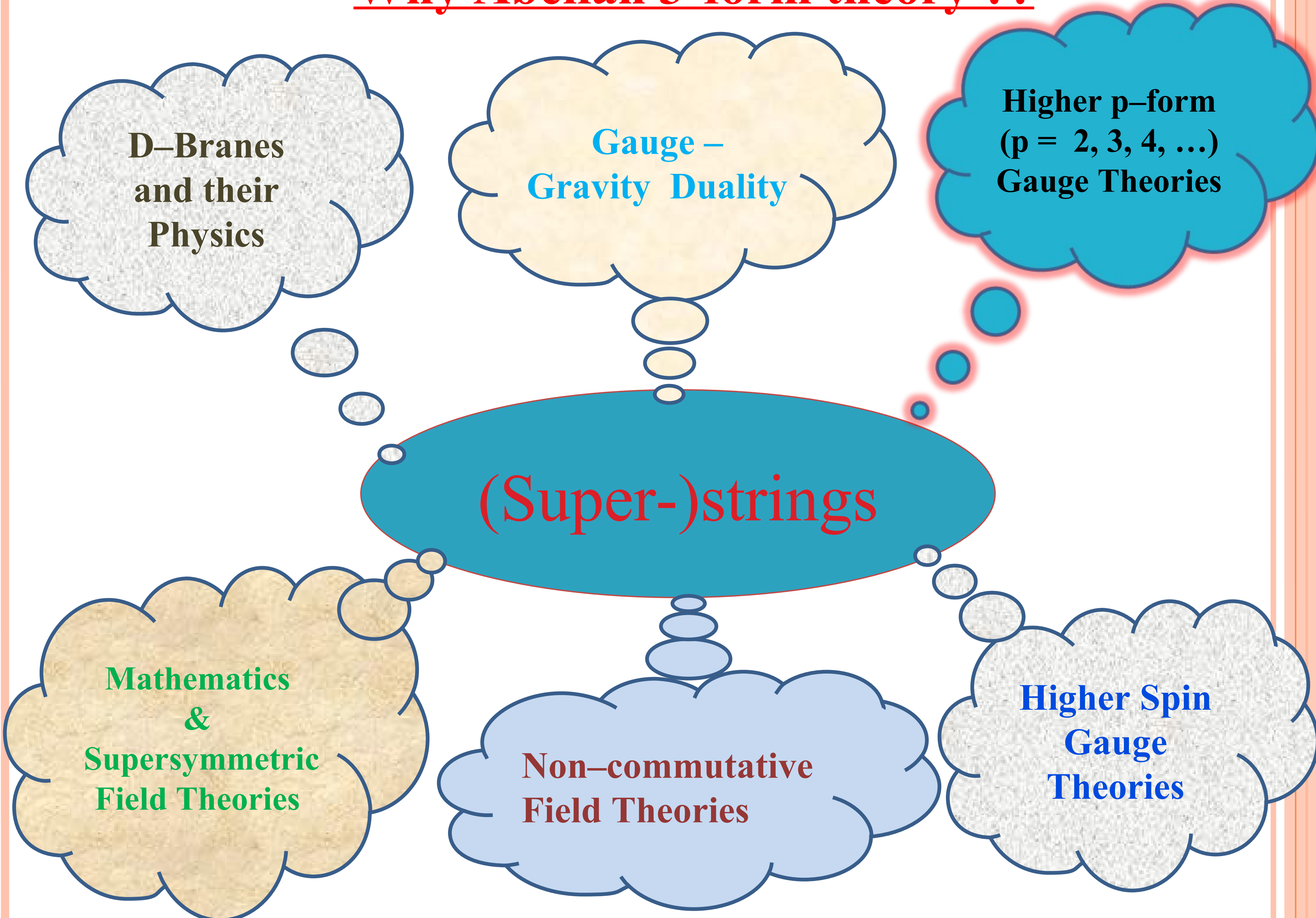
$$Q_b^2 = 0, \quad Q_{ab}^2 = 0, \quad Q_b Q_{ab} + Q_{ab} Q_b = 0$$

Q_b: BRST charge $\quad$ Q_{ab} : Anti-BRST charge

Plan of Talk

- Why Abelian 3-form theory?
- Gauge field ($B_{\mu\nu\eta}$) and Ghost fields
- Horizontality condition ⟶ symmetries
- Curci - Ferrari type restrictions
- Geometrical Aspects ⟶ Gerbs
- Conclusions

Why Abelian 3-form theory ??



VICTOR I. OGIEVETSKY

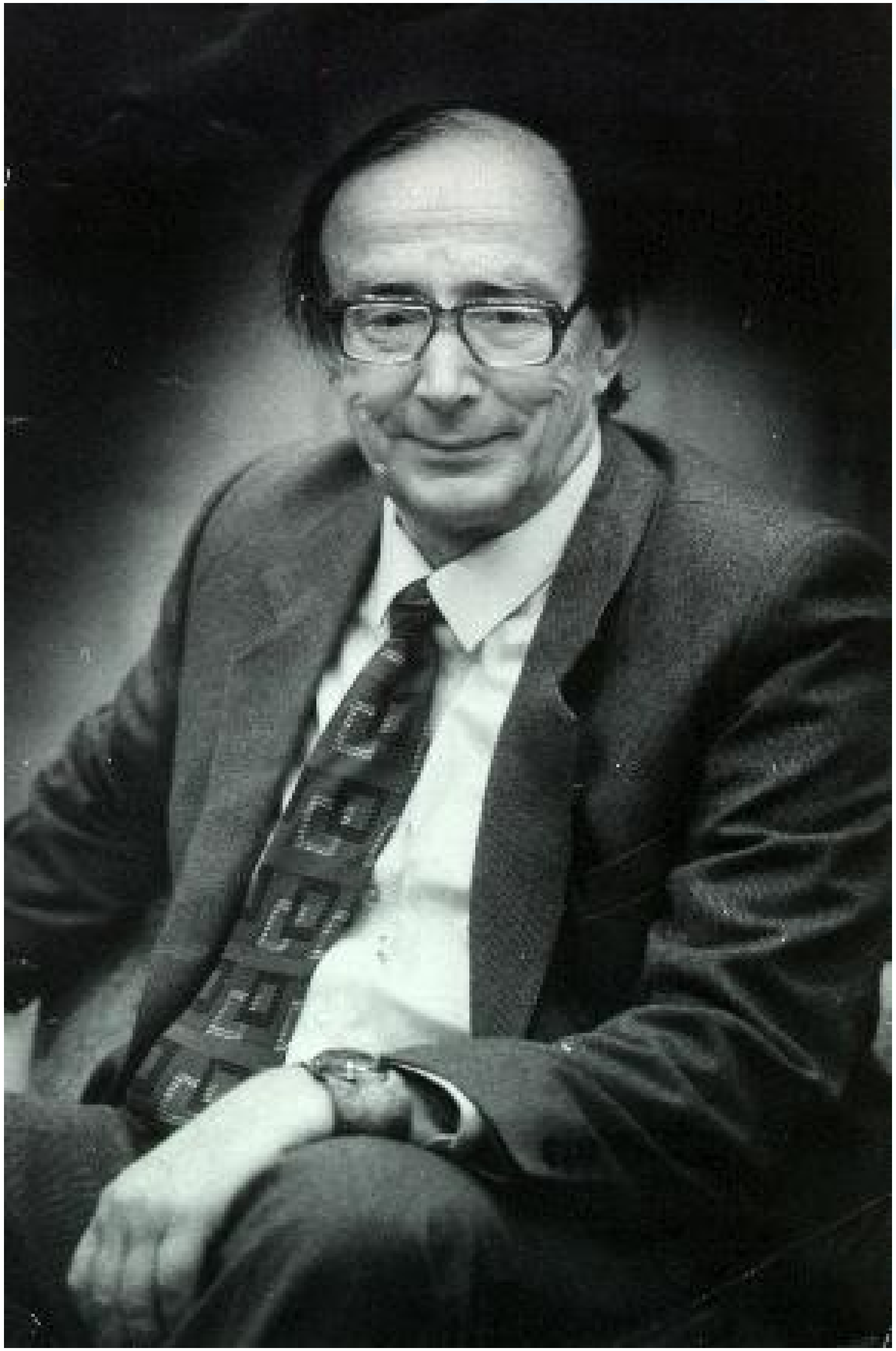
(1928—1996)

&

I. V. PALUBARINOV

COINED THE WORD

“NOTOPH”



NOTOPH ⟺ opposite of PHOTON

Nomenclature : Ogieveskty & Palubarinov

(1966-67)

Notoph gauge field = $B^{(2)} = \frac{1}{2!}(dx^{\mu} \wedge dx^{\nu}) B_{\mu\nu}$

[Abelian 2-form gauge field]

Antisymmetric tensor gauge field $B_{\mu\nu} = -B_{\nu\mu}$

Abelian 3-form $B^{(3)}$ defines $B_{\mu\nu\eta}$ as

$$B^{(3)}(x) = \frac{1}{3!}(dx^\mu \wedge dx^\nu \wedge dx^\eta) B_{\mu\nu\eta}(x)$$

which can be generalized to super 3-form as

$$\tilde{B}^{(3)}(x,\theta,\bar{\theta}) = \frac{1}{3!}\,(dZ^M \wedge dZ^N \wedge dZ^K)\tilde{B}_{MNK}$$

$$\begin{aligned}
&\equiv \frac{1}{3!}(dx^\mu \wedge dx^\nu \wedge dx^\eta)\tilde{B}_{\mu\nu\eta} + \frac{1}{2}(dx^\mu \wedge dx^\nu \wedge d\theta)\tilde{B}_{\mu\nu\theta} \\
&+ \frac{1}{2}(dx^\mu \wedge dx^\nu \wedge d\bar{\theta})\tilde{B}_{\mu\nu\bar{\theta}} + \frac{1}{3!}(d\theta \wedge d\theta \wedge d\theta)\tilde{B}_{\theta\theta\theta} \\
&+ \frac{1}{3!}\,(d\bar{\theta} \wedge d\bar{\theta} \wedge d\bar{\theta})\,\tilde{B}_{\bar{\theta}\bar{\theta}\bar{\theta}} + (dx^\mu \wedge d\theta \wedge d\bar{\theta})\tilde{B}_{\mu\theta\bar{\theta}} \\
&+ \frac{1}{2}(dx^\mu \wedge d\theta \wedge d\theta)\tilde{B}_{\mu\theta\theta} + \frac{1}{2}(dx^\mu \wedge d\bar{\theta} \wedge d\bar{\theta})\tilde{B}_{\mu\bar{\theta}\bar{\theta}} \\
&+ \frac{1}{2}(d\theta \wedge d\bar{\theta} \wedge d\bar{\theta})\,\tilde{B}_{\theta\bar{\theta}\bar{\theta}} + \frac{1}{2}(d\theta \wedge d\theta \wedge d\bar{\theta})\,\tilde{B}_{\theta\theta\bar{\theta}}
\end{aligned}$$

The above superfields provide hints for the existence of gauge fields and bosonic/fermionic (anti-) ghost fields of the theory.

Identifications: $Z^M = (x^\mu, \theta, \bar{\theta}), \quad \partial_M = (\partial_\mu, \partial_\theta, \partial_{\bar{\theta}})$

$$\tilde{\mathcal{B}}_{\mu\nu\eta} = \tilde{\mathcal{B}}_{\mu\nu\eta}(x,\theta,\bar{\theta}), \quad \tilde{\mathcal{B}}_{\mu\nu\theta} = \tilde{\bar{\mathcal{F}}}_{\mu\nu}(x,\theta,\bar{\theta}),$$
$$\tilde{\mathcal{B}}_{\mu\nu\bar{\theta}} = \tilde{\mathcal{F}}_{\mu\nu}(x,\theta,\bar{\theta}), \quad \tilde{\mathcal{B}}_{\mu\theta\bar{\theta}} = \tilde{\Phi}_\mu(x,\theta,\bar{\theta}),$$
$$\frac{1}{3!}\,\tilde{\mathcal{B}}_{\theta\theta\theta} = \tilde{\bar{\mathcal{F}}}_2(x,\theta,\bar{\theta}), \quad \frac{1}{3!}\,\tilde{\mathcal{B}}_{\bar{\theta}\bar{\theta}\bar{\theta}} = \tilde{\bar{\mathcal{F}}}_2(x,\theta,\bar{\theta}),$$
$$\frac{1}{2!}\,\tilde{\mathcal{B}}_{\mu\theta\theta} = \tilde{\bar{\beta}}_\mu(x,\theta,\bar{\theta}), \quad \frac{1}{2!}\,\tilde{\mathcal{B}}_{\mu\bar{\theta}\bar{\theta}} = \tilde{\beta}_\mu(x,\theta,\bar{\theta}),$$
$$\frac{1}{2!}\,\tilde{\mathcal{B}}_{\theta\bar{\theta}\bar{\theta}} = \tilde{\mathcal{F}}_1(x,\theta,\bar{\theta}), \quad \frac{1}{2!}\,\tilde{\mathcal{B}}_{\theta\theta\bar{\theta}} = \tilde{\bar{\mathcal{F}}}_1(x,\theta,\bar{\theta}),$$

The above superfields are the generalization of the D-dimensional local fields $(B_{\mu\nu\eta}, C_{\mu\nu}, \bar{C}_{\mu\nu}, \Phi_\mu, \bar{C}_2, C_2, \bar{C}_1, C_1, \beta_\mu, \bar{\beta}_\mu)$ of the BRST and anti-BRST invariant Lagrangian density for Abelian 3-form gauge theory.

We can now expand the above superfields in terms of the D-dimensional local fields and secondary fields, e.g.;

$$\tilde{\mathcal{B}}_{\mu\nu\eta}(x,\theta,\bar{\theta}) = B_{\mu\nu\eta}(x) + \theta\, \bar{R}_{\mu\nu\eta}(x) + \bar{\theta}\, R_{\mu\nu\eta}(x) + i\,\theta\,\bar{\theta}\, S_{\mu\nu\eta}(x),$$

$$\tilde{\mathcal{F}}_{\mu\nu}(x,\theta,\bar{\theta}) = C_{\mu\nu}(x) + \theta\, \bar{B}^{(1)}_{\mu\nu}(x) + \bar{\theta}\, B^{(1)}_{\mu\nu}(x) + i\,\theta\,\bar{\theta}\, S_{\mu\nu}(x),$$

$$\tilde{\bar{\mathcal{F}}}_{\mu\nu}(x,\theta,\bar{\theta}) = \bar{C}_{\mu\nu}(x) + \theta\, \bar{B}^{(2)}_{\mu\nu}(x) + \bar{\theta}\, B^{(2)}_{\mu\nu}(x) + i\,\theta\,\bar{\theta}\bar{S}_{\mu\nu}(x)$$

etc.

Where $R_{\mu\nu\eta}(x), \bar{R}_{\mu\nu\eta}(x), S_{\mu\nu\eta}(x), B^{(1)}_{\mu\nu}(x),$ etc., are secondary fields that are determined in the terms of local basic fields and auxiliary fields of the D-dimensional theory by exploiting the horizontality condition (HC)

Horizontality Condition [HC] (Soul-flatness condition)

$$\tilde{d}\tilde{B}^{(3)} = dB^{(3)}$$

(D, 2) ⟷ (D)

Where: $d = dx^\mu \partial_\mu, \quad \tilde{d} = dx^\mu \partial_\mu + d\theta \partial_\theta + d\bar{\theta}\partial_{\bar{\theta}}$

$$dB^{(3)} = \left(\frac{dx^\mu \wedge dx^\nu \wedge dx^\eta \wedge dx^\rho}{4!} \right) H_{\mu\nu\eta\rho}$$

$$H_{\mu\nu\eta\rho} = \partial_\mu B_{\nu\eta\rho} - \partial_\nu B_{\eta\rho\mu} + \partial_\eta B_{\rho\mu\nu} - \partial_\rho B_{\mu\nu\eta}$$

: Curvature tensor remains invariant under (anti-) BRST symmetry transformations

l.h.s. has spacetime differentials as well as Grassmannian differentials

The H C condition leads to, e.g. (setting Grassmannian components = 0)

$$R_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu},$$
$$\bar{R}_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu},$$
$$S_{\mu\nu\eta} = -\,i\,(\partial_\mu B^{(2)}_{\nu\eta} + \partial_\nu B^{(2)}_{\eta\mu} + \partial_\eta B^{(2)}_{\mu\nu})$$

The insertions of the secondary fields in terms of the basic and auxiliary fields leads to the derivation of the (anti-) BRST symmetry transformations; e.g.

$$\tilde{B}_{\mu\nu\eta}(x,\theta,\bar{\theta}) = B_{\mu\nu\eta}(x) + \theta\left[\partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}\right]$$

$$+ \bar{\theta}\left[\partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}\right]$$

$$+ \theta\bar{\theta}\left[\partial_\mu B^{(2)}_{\nu\eta} + \partial_\nu B^{(2)}_{\eta\mu} + \partial_\eta B^{(2)}_{\mu\nu}\right]$$

$$= B_{\mu\nu\eta}(x) + \theta(s_{ab} B_{\mu\nu\eta}) + \bar{\theta}(s_b B_{\mu\nu\eta})$$

$$+\theta\bar{\theta}(s_b s_{ab} B_{\mu\nu\eta})$$

This implies that (with $s_b \to \lim_{\theta\to 0} \partial_{\bar{\theta}}, \;\; s_{ab} \to \lim_{\bar{\theta}\to 0} \partial_\theta$)

$$s_b B_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu},$$

$$s_{ab} B_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}$$

HC leads to the following BRST symmetry transformations

$$s_b B_{\mu\nu\eta} = \partial_\mu C_{\nu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}, \quad s_b \bar{C}_{\mu\nu} = B_{\mu\nu},$$
$$s_b C_{\mu\nu} = \partial_\mu \beta_\nu - \partial_\nu \beta_\mu, \quad s_b \beta_\mu = \partial_\mu C_2, \quad s_b C_1 = -\bar{B},$$
$$s_b \bar{C}_1 = B_1, \quad s_b \bar{C}_2 = B_2, \quad s_b \bar{\beta}_\mu = F_\mu, \quad s_b \phi_\mu = f_\mu,$$
$$s_b \bar{F}_\mu = -\partial_\mu \bar{B}, \quad s_b \bar{B}_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu,$$
$$s_b \bar{f}_\mu = \partial_\mu B_1, \quad s_b [\bar{B}, B_1, B_2, C_2, F_\mu, f_\mu, B_{\mu\nu}] = 0$$

The above transformations are off-shell nilpotent $s_b^2 = 0$

The anti-BRST symmetry

$$s_{ab}B_{\mu\nu\eta} = \partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}, \; s_{ab}C_{\mu\nu} = \bar{B}_{\mu\nu},$$
$$s_{ab}\bar{C}_{\mu\nu} = \partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu, \; s_{ab}\bar{\beta}_\mu = \partial_\mu \bar{C}_2, \; s_{ab}\phi_\mu = \bar{f}_\mu,$$
$$s_{ab}C_1 = -B_1, \;\; s_{ab}\bar{C}_1 = -B_2, \;\; s_{ab}C_2 = \bar{B},$$
$$s_{ab}\beta_\mu = \bar{F}_\mu, \;\; s_{ab}\bar{F}_\mu = -\partial_\mu B_2, \;\; s_{ab}f_\mu = -\partial_\mu B_1,$$
$$s_{ab}B_{\mu\nu} = \partial_\mu \bar{f}_\nu - \partial_\nu \bar{f}_\mu,$$
$$s_{ab}[\bar{B}, B_1, B_2, \bar{C}_2, \bar{F}_\mu, \bar{f}_\mu, \bar{B}_{\mu\nu}] = 0$$

These transformations are off-shell nilpotent $(s_{ab}^2 = 0)$

Anticommutativity property

$$\{s_b, s_{ab}\}\, B_{\mu\nu\eta} \neq 0, \qquad \{s_b, s_{ab}\}\, C_{\mu\nu} \neq 0,$$

$$\{s_b, s_{ab}\}\, \bar{C}_{\mu\nu} \neq 0$$

Rest of the fields respect anticommutativity

$$\{s_b, s_{ab}\}\Psi = 0,$$

$$\Psi = \beta_\mu, \bar{\beta}_\mu, f_\mu, \bar{f}_\mu, F_\mu, \bar{F}_\mu, C_1, \bar{C}_1, C_2, \bar{C}_2, \ldots$$

Superfield formalism yields following Curci-Ferrari type restriction

$$f_\mu + F_\mu = \partial_\mu C_1, \qquad \bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1,$$

$$B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

$$\begin{aligned}
\{s_b, s_{ab}\} B_{\mu\nu\eta} &= s_b s_{ab} B_{\mu\nu\eta} + s_{ab} s_b B_{\mu\nu\eta} \\
&= s_b[\partial_\mu \bar{C}_{\mu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}] \\
&+ s_{ab}[\partial_\mu C_{\mu\eta} + \partial_\nu C_{\eta\mu} + \partial_\eta C_{\mu\nu}] \\
&= \partial_\mu [B_{\nu\eta} + \bar{B}_{\nu\eta}] + \partial_\nu [B_{\eta\mu} + \bar{B}_{\eta\mu}] \\
&+ \partial_\eta [B_{\mu\nu} + \bar{B}_{\mu\nu}] = 0
\end{aligned}$$

on $B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$

Similarly

$$\{s_b, s_{ab}\}\, C_{\mu\nu} = 0, \qquad \{s_b, s_{ab}\}\, \bar{C}_{\mu\nu} = 0$$

Thus, on the constrained surface, defined by the CF-type conditions, the (anti-)BRST symmetry transformations $s_{(a)b}$ are found to be off-shell nilpotent $(s^2_{(a)b} = 0)$ and absolutely anticommuting $(s_b s_{ab} + s_{ab} s_b = 0)$

Without knowledge of the Lagrangian density, we have derived the proper (anti-)BRST symmetry transformations

Remarks

- Off-shell nilpotency and Absolute anticommutativity $\longrightarrow$ Superfield formalism (Bonora & Tonin [81, 82])

- *Three* CF-type conditions $\longrightarrow$ 3-form Abelian theory

  One CF- type condition $\longrightarrow$ 2-form Abelian theory $B_\mu - \bar{B}_\mu = \partial_\mu \phi$

  One CF- condition $\longrightarrow$ 1-form non-Abelian theory $B + \bar{B} = -\, i\, (C \times \bar{C})$

  One CF- type condition $\longrightarrow$ 1-form Abelian theory (trivial $B + \bar{B} = 0$)

- Two CF-type conditions → Fermionic in nature

$$f_\mu + F_\mu = \partial_\mu C_1, \qquad \bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1$$

One CF-type condition → Bosonic

} Abelian 3-form theory

ONLY BOSONIC → 1-form (non-)Abelian/2-form Abelian gauge theories

- CF-type restriction is *ONE* of the key features of any arbitrary p-form gauge theory. Within the framework of BRST, a gauge theory is always *endowed* with CF-type restriction(s) ⟷ **HALLMARK**

- CF-type restrictions are (anti-)BRST invariant, e.g.

$$s_{(a)b}[f_\mu + F_\mu - \partial_\mu C_1] = 0,$$

$$s_{(a)b}[\bar{f}_\mu + \bar{F}_\mu - \partial_\mu \bar{C}_1] = 0,$$

$$s_{(a)b}[B_{\mu\nu} + \bar{B}_{\mu\nu} - (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)] = 0$$

- This is a key consequence of our superfield formulation

Lagrangian densities:

$$\mathcal{L}_B = \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} + s_b s_{ab} \Big[\frac{1}{2} \bar{C}_2 C_2 - \frac{1}{2} \bar{C}_1 C_1 + \frac{1}{2} \bar{C}^{\mu\nu} C_{\mu\nu} - \bar{\beta}^{\mu} \beta_{\mu} - \frac{1}{2} \phi^{\mu} \phi_{\mu} - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta} \Big]$$

$$\mathcal{L}_{\bar{B}} = \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} - s_{ab} s_b \Big[\frac{1}{2} \bar{C}_2 C_2 - \frac{1}{2} \bar{C}_1 C_1 + \frac{1}{2} \bar{C}^{\mu\nu} C_{\mu\nu} - \bar{\beta}^{\mu} \beta_{\mu} - \frac{1}{2} \phi^{\mu} \phi_{\mu} - \frac{1}{6} B^{\mu\nu\eta} B_{\mu\nu\eta} \Big]$$

where kinetic term is generated by

$$dB^{(3)} = H^{(4)}, \quad H^{(4)} = \Big(\frac{dx^{\mu} \wedge dx^{\nu} \wedge dx^{\eta} \wedge dx^{\zeta}}{4!} \Big) H_{\mu\nu\eta\zeta}$$

with

$$H_{\mu\nu\eta\zeta} = \partial_\mu B_{\nu\eta\zeta} - \partial_\nu B_{\eta\zeta\mu} + \partial_\eta B_{\zeta\mu\nu} - \partial_\zeta B_{\mu\nu\eta}$$

Finally, in an explicit form, we have

$$\begin{aligned}
\mathcal{L}_B &= \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} - \frac{1}{2} B^{\mu\nu} B_{\mu\nu} - B B_2 - \frac{1}{2} B_1^2 \\
&+ B^{\mu\nu}\Big[\partial^\eta B_{\eta\mu\nu} + \frac{1}{2}\Big(\partial_\mu \phi_\nu - \partial_\nu \phi_\mu\Big)\Big] + (\partial \cdot \phi) B_1 \\
&+ \Big(\partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}\Big)\Big(\partial^\mu C^{\nu\eta}\Big) - (\partial \cdot \bar{\beta}) B \\
&- \Big(\partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu\Big)\partial^\mu \beta^\nu + \partial^\mu \bar{C}_2 \partial_\mu C_2 + (\partial \cdot \beta) B_2 \\
&+ \Big(\partial_\mu \bar{C}^{\mu\nu} + \partial^\nu \bar{C}_1\Big) f_\nu - \Big(\partial_\mu C^{\mu\nu} + \partial^\nu C_1\Big) \bar{F}_\nu
\end{aligned}$$

It should be noted that, by using the CF-conditions, the above form has been obtained

Similarly we have

$$\begin{aligned}
\mathcal{L}_{\bar{B}} &= \frac{1}{24} H^{\mu\nu\eta\zeta} H_{\mu\nu\eta\zeta} - \frac{1}{2} \bar{B}^{\mu\nu} \bar{B}_{\mu\nu} - B B_2 - \frac{1}{2} B_1^2 \\
&- \bar{B}^{\mu\nu} \Big[\partial^\eta B_{\eta\mu\nu} - \frac{1}{2} \Big(\partial_\mu \phi_\nu - \partial_\nu \phi_\mu\Big)\Big] + (\partial \cdot \phi) B_1 \\
&+ \Big(\partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu}\Big)\Big(\partial^\mu C^{\nu\eta}\Big) - (\partial \cdot \bar{\beta}) B \\
&- \Big(\partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu\Big) \partial^\mu \beta^\nu + \partial^\mu \bar{C}_2 \partial_\mu C_2 + (\partial \cdot \beta) B_2 \\
&+ \Big(\partial_\mu \bar{C}^{\mu\nu} + \partial^\nu \bar{C}_1\Big) f_\nu - \Big(\partial_\mu C^{\mu\nu} + \partial^\nu C_1\Big) \bar{F}_\nu
\end{aligned}$$

The above Lagrangian densities ($\mathcal{L}_B$ and $\mathcal{L}_{\bar{B}}$) are *coupled* but *equivalent*

Under the BRST and anti-BRST transformations

$$s_b \mathcal{L}_B = \partial_\mu \Big[\Big(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu} \Big) B_{\nu\eta} + B^{\mu\nu} f_\nu - (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) \bar{F}_\nu + B_1 f^\mu - B \bar{F}^\mu + B_2 \partial^\mu C_2 \Big]$$

$$s_{ab} \mathcal{L}_{\bar{B}} = \partial_\mu \Big[\Big(\partial^\mu \bar{C}^{\nu\eta} + \partial^\nu \bar{C}^{\eta\mu} + \partial^\eta \bar{C}^{\mu\nu} \Big) \bar{B}_{\nu\eta} + \bar{B}^{\mu\nu} \bar{f}_\nu - (\partial^\mu \bar{\beta}^\nu - \partial^\nu \bar{\beta}^\mu) F_\nu + B_1 \bar{f}^\mu + B_2 F^\mu - B \partial^\mu \bar{C}_2 \Big]$$

This establishes (anti-)BRST invariance

To show the equivalence between the above Lagrangian densities and (anti-)BRST symmetries it can be checked that [LB & RPM, J. Phys. A (2010)]

$$s_b \mathcal{L}_{\bar{B}} = \partial_\mu [.........] +$$ Terms that are zero on *CF-type conditions*

$$s_{ab} \mathcal{L}_B = \partial_\mu [.........] +$$ Terms that are zero on *CF-type conditions*

Let us write one term explicitly [LB & RPM (2010)]

$$s_b \mathcal{L}_{\bar{B}} = -\partial_\mu \Big[\Big(\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu} \Big) \bar{B}_{\nu\eta} + B^{\mu\nu} F_\nu$$

$$- (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) \bar{f}_\nu - B_1 f^\mu + B \bar{F}^\mu - B_2 \partial^\mu C_2$$

$$+ B^{\mu\nu\eta} (\partial_\nu f_\eta - \partial_\eta f_\nu) + \bar{C}^{\mu\nu} \partial_\nu B + C^{\mu\nu} \partial_\nu B_1 \Big] + X$$

$$X = (\partial^\mu C^{\nu\eta} + \partial^\nu C^{\eta\mu} + \partial^\eta C^{\mu\nu}) \partial_\mu [\bar{B}_{\nu\eta} + B_{\nu\eta}$$

$$- (\partial_\nu \phi_\eta - \partial_\eta \phi_\nu)] - [f^\mu + F^\mu - \partial^\mu C_1] (\partial_\mu B_1)$$

$$- [\bar{B}_{\mu\nu} + B_{\mu\nu} - (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu)] (\partial^\mu f^\nu)$$

$$+ [\bar{f}^\mu + \bar{F}^\mu - \partial^\mu \bar{C}_1] (\partial_\mu B) + B^{\mu\nu} \partial_\mu [f_\nu + F_\nu - \partial_\nu C_1]$$

$$- (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu) \, \partial_\mu [\bar{f}_\nu + \bar{F}_\nu - \partial_\nu \bar{C}_1]$$

Which is zero on the constrained surface defined by CF-conditions

Ghost Symmetries:

$$\mathcal{L}_{(g)} = (\partial_\mu \bar{C}_{\nu\eta} + \partial_\nu \bar{C}_{\eta\mu} + \partial_\eta \bar{C}_{\mu\nu})(\partial^\mu C^{\nu\eta}) - (\partial \cdot \bar{\beta}) B$$
$$- (\partial_\mu \bar{\beta}_\nu - \partial_\nu \bar{\beta}_\mu)(\partial^\mu \beta^\nu) - B B_2 + (\partial_\mu \bar{C}^{\mu\nu} + \partial^\nu \bar{C}_1) f_\nu$$
$$- (\partial_\mu C^{\mu\nu} - \partial^\nu C_1) \bar{F}_\nu + \partial_\mu \bar{C}_2 \partial^\mu C_2 + (\partial \cdot \beta) B_2$$

The above Lagrangian density has the following symmetry transformations

$$C_{\mu\nu} \to e^{+\Omega} C_{\mu\nu}, \;\; \bar{C}_{\mu\nu} \to e^{-\Omega} \bar{C}_{\mu\nu}, \;\; C_1 \to e^{+\Omega} C_1,$$
$$\bar{C}_1 \to e^{-\Omega} \bar{C}_1, \;\; f_\mu \to e^{+\Omega} f_\mu, \;\; F_\mu \to e^{+\Omega} F_\mu,$$
$$\bar{f}_\mu \to e^{-\Omega} \bar{f}_\mu, \;\; \bar{F}_\mu \to e^{-\Omega} \bar{F}_\mu, \;\; \beta_\mu \to e^{+2\Omega} \beta_\mu,$$
$$\bar{\beta}_\mu \to e^{-2\Omega} \bar{\beta}_\mu, \; B \to e^{+2\Omega} B, \; B_2 \to e^{-2\Omega} B_2,$$
$$C_2 \to e^{+3\Omega} C_2, \;\; \bar{C}_2 \to e^{-3\Omega} \bar{C}_2$$

Conserved Charges by Noether's Theorem:

We obtain conserved currents and they lead to the following charges

$$
\begin{aligned}
Q_b &= \int d^3x \Big[H^{0ijk}(\partial_i C_{jk}) + (\partial^0 C^{\nu\eta} + \partial^\nu C^{\eta 0} + \partial^\eta C^{0\nu}) B_{\nu\eta} + B_1 f^0 \\
&- (\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0)\partial_i C_2 - (\partial^0 \beta^i - \partial^i \beta^0)\bar{F}_i + B^{0i} f_i + B_2 \dot{C}_2 \\
&- (\partial^0 \bar{C}^{\nu\eta} + \partial^\nu \bar{C}^{\eta 0} + \partial^\eta \bar{C}^{0\nu})(\partial_\nu \beta_\eta - \partial_\eta \beta_\nu) - B \bar{F}^0 \Big]
\end{aligned}
$$

$$
\begin{aligned}
Q_{ab} &= \int d^3x \Big[H^{0ijk}(\partial_i \bar{C}_{jk}) - (\partial^0 \bar{C}^{\nu\eta} + \partial^\nu \bar{C}^{\eta 0} + \partial^\eta \bar{C}^{0\nu}) \bar{B}_{\nu\eta} + B_1 \bar{f}^0 \\
&- (\partial^0 \beta^i - \partial^i \beta^0)\partial_i \bar{C}_2 - (\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0) F_i + \bar{B}^{0i} \bar{f}_i - B \dot{\bar{C}}_2 \\
&+ (\partial^0 C^{\nu\eta} + \partial^\nu C^{\eta 0} + \partial^\eta C^{0\nu})(\partial_\nu \bar{\beta}_\eta - \partial_\eta \bar{\beta}_\nu) + B_2 F^0 \Big]
\end{aligned}
$$

$$
\begin{aligned}
Q_g &= \int d^3x \Big[3\dot{\bar{C}}_2 C_2 - 3\bar{C}_2 \dot{C}_2 + (\partial^0 \bar{C}^{\nu\eta} + \partial^\nu \bar{C}^{\eta 0} + \partial^\eta \bar{C}^{0\nu}) C_{\nu\eta} - \bar{C}_1 f^0 \\
&+ 2(\partial^0 \beta^i - \partial^i \beta^0)\bar{\beta}_i - 2(\partial^0 \bar{\beta}^i - \partial^i \bar{\beta}^0)\beta_i - \bar{C}^{0i} f_i - C^{0i} \bar{F}_i \\
&+ 2B\bar{\beta}^0 + 2B_2 \beta^0 + C_1 \bar{F}^0 + (\partial^0 C^{\nu\eta} + \partial^\nu C^{\eta 0} + \partial^\eta C^{0\nu}) \bar{C}_{\nu\eta} \Big]
\end{aligned}
$$

The above charges are the generators of the nilpotent and continuous (anti-)BRST symmetries and continuous ghost scale transformations

They obey the *standard BRST algebra*

The application of the continuous symmetry transformations on the above charges produces the following algebra

$$s_b Q_b = -i\{Q_b, Q_b\} = 0 \;\;\Rightarrow\;\; Q_b^2 = 0,$$
$$s_{ab} Q_{ab} = -i\{Q_{ab}, Q_{ab}\} = 0 \;\;\Rightarrow\;\; Q_{ab}^2 = 0,$$

$$s_{ab} Q_b = -i\{Q_b, Q_{ab}\} = 0 \;\;\Rightarrow\;\; Q_b Q_{ab} + Q_{ab} Q_b = 0,$$

$$s_b Q_g = -i[Q_g, Q_b] = -Q_b \;\Rightarrow\; i[Q_g, Q_b] = +Q_b,$$
$$s_{ab} Q_g = -i[Q_g, Q_{ab}] = +Q_{ab} \Rightarrow\; i[Q_g, Q_{ab}] = -Q_{ab}$$

These are the standard algebra of BRST formalism. As it turns out

$$Q_{(a)b}|phys\rangle = 0 \Rightarrow \text{First-class constraints } |phys\rangle = 0$$

Thus, the BRST formalism gives standard results.

Superfield formulation: Any arbitrary p-form (p = 1, 2, 3, ….) Abelian gauge theory in any arbitrary D-dimensions can be described in the language of BRST approach

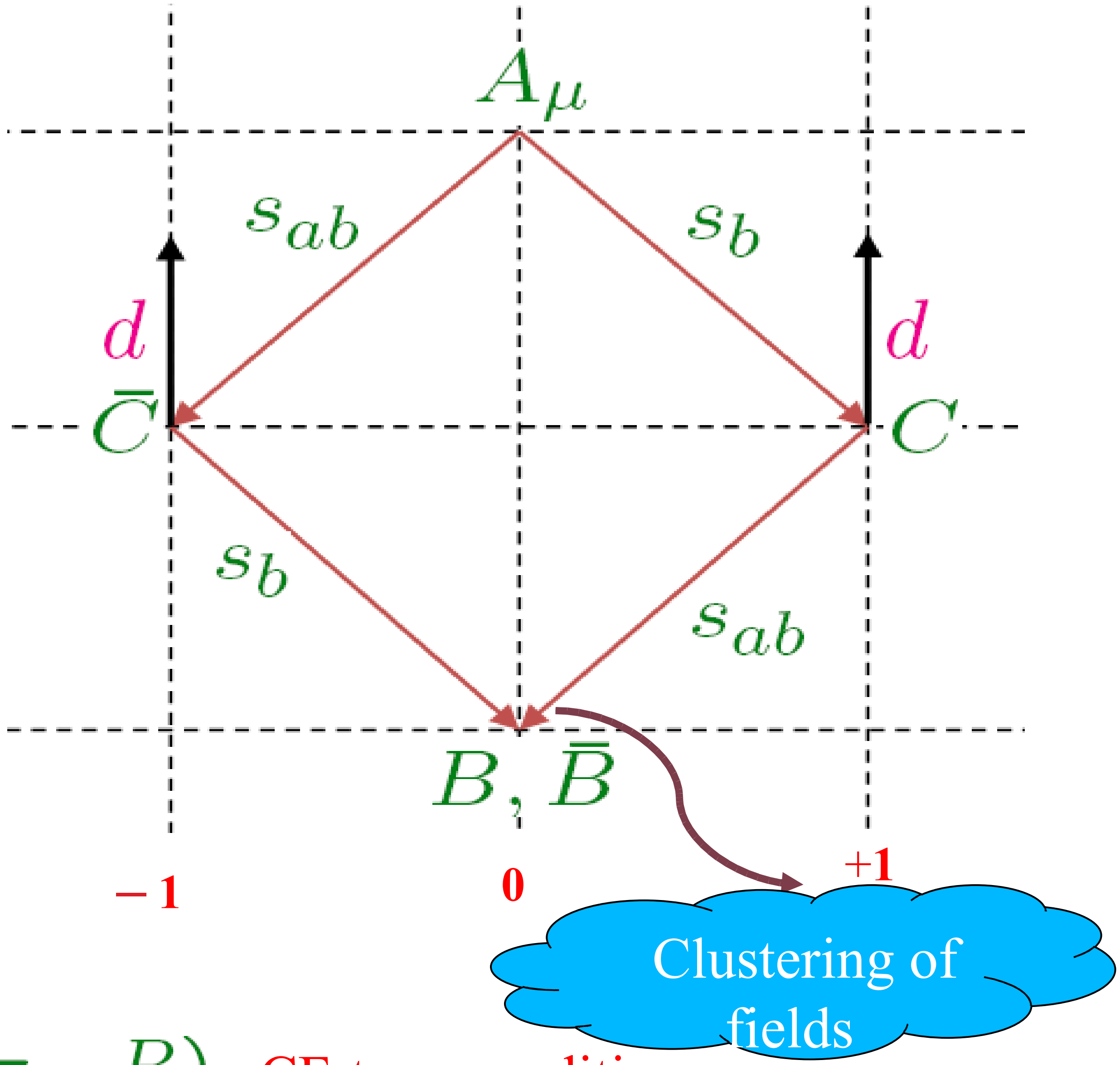
- Off-shell nilpotent & Absolutely Anticommuting (anti-)BRST symmetries are natural consequences!!

Abelian 1-form gauge theory

$$A^{(1)} = dx^{\mu}\partial_{\mu}$$

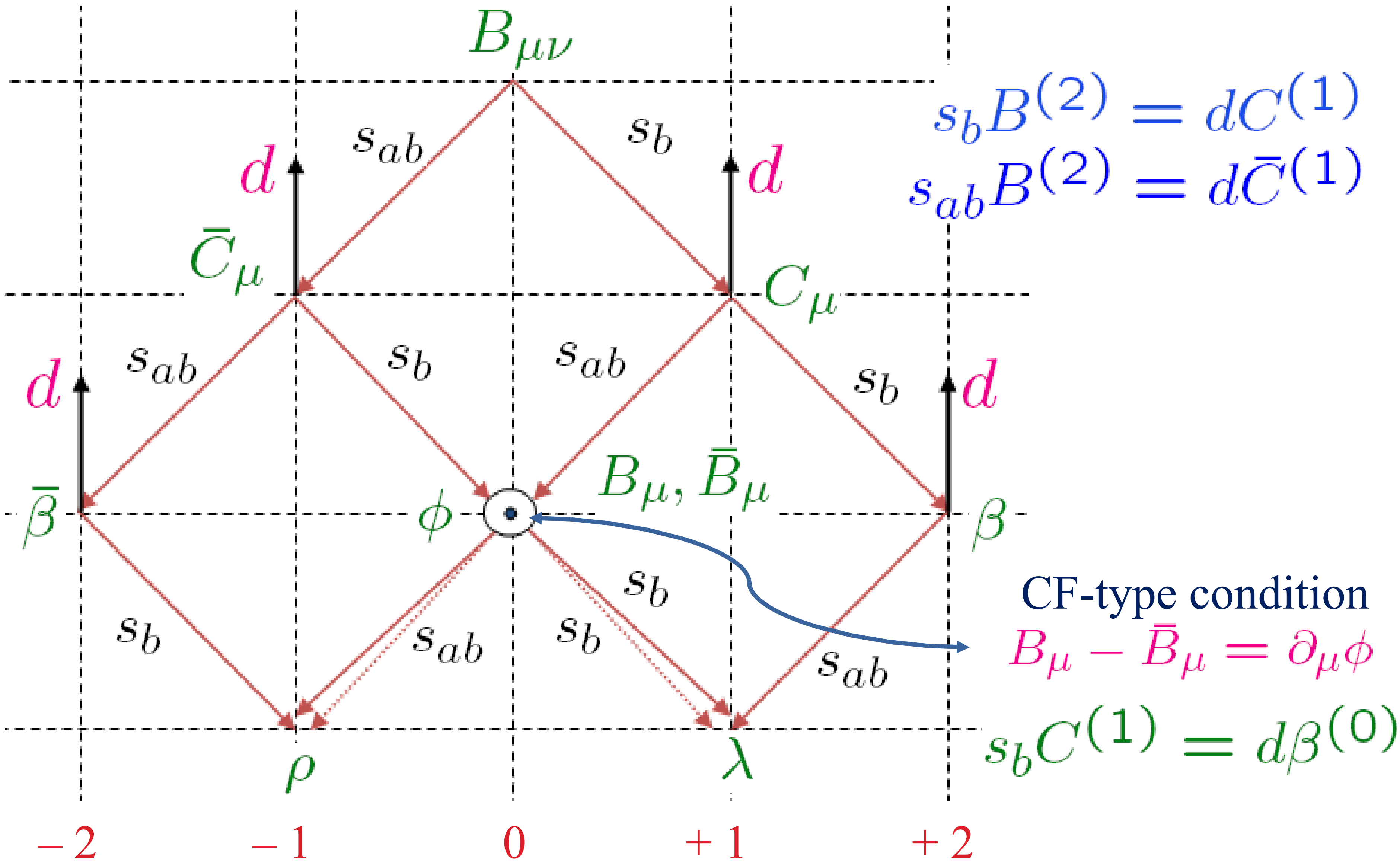
$$s_b A^{(1)} = dC^{(0)}$$

$$s_{ab} A^{(1)} = d\bar{C}^{(0)}$$



$(B + \bar{B} = 0, \bar{B} = -B)$ - CF-type condition

Abelian 2-form gauge theory

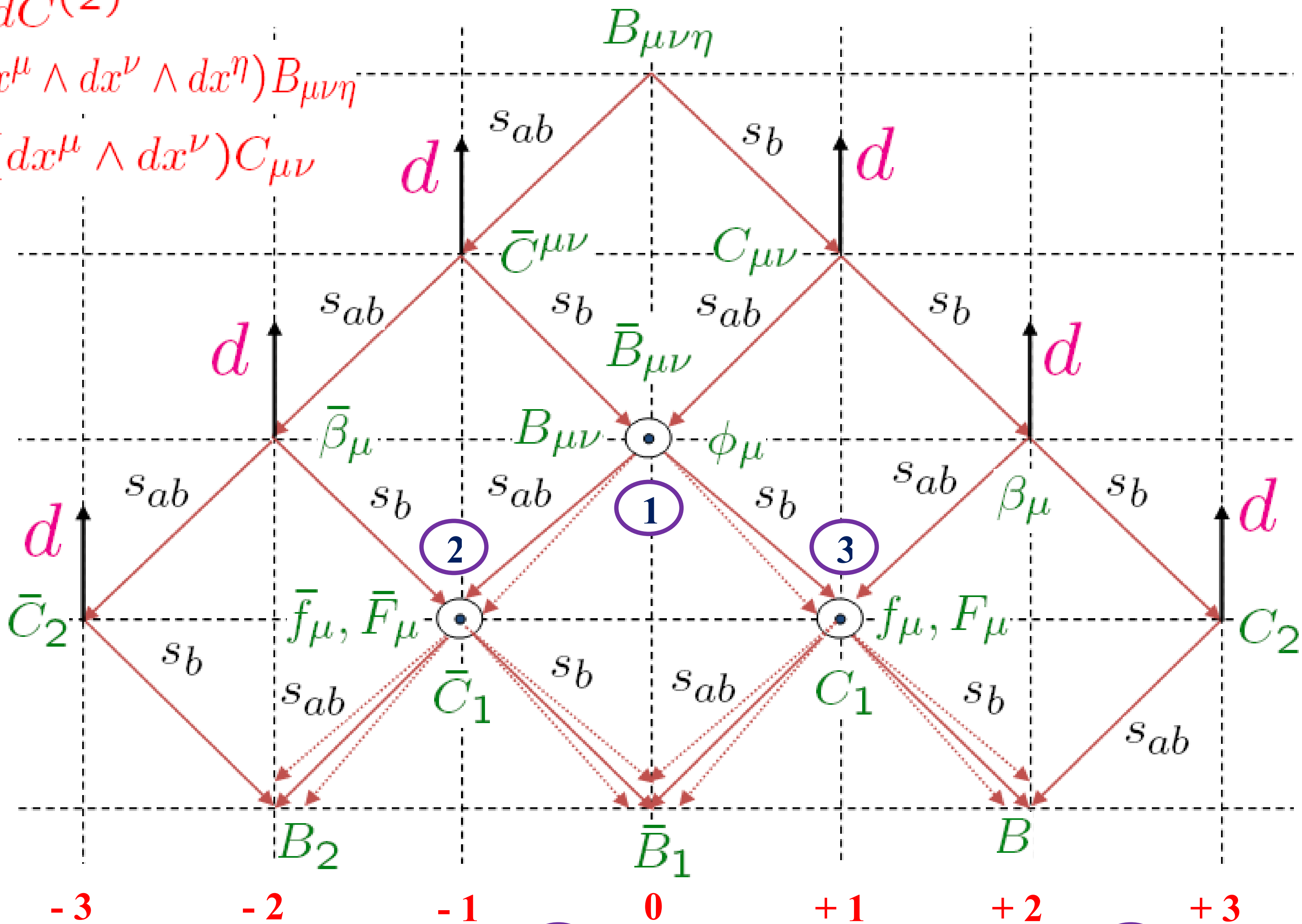


Abelian 3-form theory

$$s_b B^{(3)} = dC^{(2)}$$

$$B^{(3)} = \frac{1}{3!}\,(dx^\mu \wedge dx^\nu \wedge dx^\eta)\,B_{\mu\nu\eta}$$

$$C^{(2)} = \frac{1}{2!}\,(dx^\mu \wedge dx^\nu)\,C_{\mu\nu}$$



(1) $B_{\mu\nu} + \bar{B}_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu,$ (2) $\bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{C}_1,$ (3) $f_\mu + F_\mu = \partial_\mu C_1$

Future directions:

- ✓Non-Abelian Generalization
- ✓Still higher p-form ($p = 4, 5$) theories
- ✓Merging of 1-form and 3-form theories
- ✓Merging of 2-form and 3-form theories

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