GALILEAN CONFORMAL SYMMETRIES AND SUPERSYMMETRIES

1. **Three versions of NR conformal symmetries**
   - Schrödinger algebra $Schr$(d) and its generalization $Schr_N(d)$
   - Galilean conformal algebra (GCA) - C(d)
   - ∞ - dim algebra of NR conformal isometries
2. **Galilean conformal symmetries - arbitrary d**
   - two contractions: "physical" $c \to \infty$ and "geometric" Inonu-Wigner
   - realizations of GCA (σ-model type, NR twistors)
3. **Galilean conformal supersymmetries (SUSY GCA)**
   - two contractions and fermionic new charges: algebraic structure
   - quaternionic structure and Galilean supertwistors
4. **Final remarks**

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Related: Galilean N-extended SUSY

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1. Three Versions of NR Conformal Symmetries

"Physical" contraction $c \to \infty$:
relativistic (super)symmetries $\to$ Galilean (super)symmetries
Usually performed on the level of Lie algebras:
Example: Poincaré algebra $\mathcal{P}(D = d+1) \longrightarrow$ Galilean algebra $G(d)$

$$(M_{\mu\nu} = (M_{rs}, M_{or}), P_\mu = (P_r, P_0)) \xrightarrow{\ c\,\to\,\infty\ } M_{rs}, B_r, P_r, H$$
$$\mu, \nu = 0, 1 \ldots d \quad r, s = 1, 2 \ldots d$$

Rescaling: $M_{r0} = cB_r \quad P_0 = m_0 c + \frac{H}{c} \quad B_i$ - Galilean boosts

$$\mathcal{P}(d) = O(d,1) \Subset T^{d+1} \xrightarrow[c\to\infty]{} G(d) = (O(d) \oplus R) \Subset T^{2d}$$

One can also study the $c \to \infty$ limit of space-time and of space-time differential realizations

$$x_\mu = (x_r, x_0 = ct) \quad \xrightarrow[c\to\infty]{} \quad x_\mu^{NR} = (x_r, t)$$

Minkowski space — NR space-time

Second way of getting NR symmetries - from isometries of NR space-time $(\vec{x}, t) \in (R^d \otimes R)$

Relativistic space-time: conformal Killing vectors $\Rightarrow$

$\Rightarrow$ finite-dimensional relativistic conformal algebra $O(d+1,2) = O(D,2)$

Newton-Cartan structure of NR space-time $(R^d \otimes R)$

NR conformal Killing vectors $\Rightarrow$

$\Rightarrow$ ∞-dimensional Galilei-conformal Lie algebra

In flat space: vector fields are parametrized by $2 + \frac{d(d+1)}{2}$ functions of time

$$X = h(t)\frac{\partial}{\partial t} + (\omega_{rs}(t)x_s + \beta_r(t) + \kappa_r(t)x_s x_s + 2\kappa_s(t)x_r x_s + \chi(t)x_r)\frac{\partial}{\partial x_r}$$

Special choice: generalized Schrödinger-Virasoro Lie algebra

$$X_{\mathrm{sv}} = zh(t)\frac{\partial}{\partial t} + (\omega_{rs}(t)x_s + \beta_r(t) + h'(t)x_r)\frac{\partial}{\partial x_r}$$

z is a dynamical exponent entering the solutions of Killing eqs.

$$\partial_r X_s + \partial_s X_r = \delta_{rs} f \quad \partial_t X_t = g \quad f + zg = 0 \quad X = X_r \partial_r + X_t \partial_t$$

In general z can be rational. If $z = \frac{2}{N}$ (N integer) one gets finite-dimensional generalized Schrödinger algebra $Schr_N(d)$ (Duval, Horvathy 2011)

$$X_S^{(N)} = \frac{2}{N}(\alpha_0 + \alpha_1 t + \alpha_2 t^2)\frac{\partial}{\partial t} + [\omega_{rs}x_s + \sum_{n=0}^{N} \beta_n t^n + (\alpha_1 + 2\alpha_2 t)x_r]\frac{\partial}{\partial x_r}$$

α_0	– time translations	$H = \frac{2}{N}\frac{\partial}{\partial t}$
α_1	– space and time rescalings	$D = x_r\frac{\partial}{\partial x_r} + \frac{2}{N}t\frac{\partial}{\partial t}$
α_2	– space and time expansions	$K = 2tx_r\frac{\partial}{\partial x_r} + \frac{2}{N}t^2\frac{\partial}{\partial t}$
ω_{ij}	– O(d) rotations	$M_{rs} = x_{[r}\frac{\partial}{\partial x_{s]}}$
β_0	– space translations	$P_r = \frac{\partial}{\partial x_r}$
β_1	– Galilean boosts	$B_r = t\frac{\partial}{\partial x_r}$
β_2	– constant accelerations	$F_r = t^2\frac{\partial}{\partial x_r}$
β_l	– $(l \geq 3)$ higher accelerations	$F_r^{(l)} = t^l\frac{\partial}{\partial x_r}$

a) Schrödinger algebra - $Schr(d) \equiv Schr_1(d)$ $N = 1,\ z = 2$

$$(\underbrace{M_{rs}, P_r, H, B_r}_{\text{Galilean algebra}}, D, K)$$

Galilean algebra extra 2 generators D,K

b) Galilean conformal algebra $GCA : C(g) \equiv Schr_2(d)$

$$(\underbrace{M_{rs}, P_r, H, B_r}_{\text{Galilean algebra}}, F_r, D, K) \qquad z = 1, N = 2$$

Galilean algebra extra 2+d generators

Galilean algebra $G(d) = (O(d) \oplus R) \Subset T^{2d}$

Schrödinger algebra $Schr(d) = (O(d) \oplus O(2,1)) \Subset T^{2d}$

Galilean conf. algebra $C(g) = (O(d) \oplus O(2,1)) \Subset T^{3d}$

Generalized Schr. algebras $Schr_N(d) = (O(d) \oplus O(2,1)) \Subset T^{(N+1)d}$

(Newton-Hooke (NR) algebras - $(O(d) \oplus O(2)) \Subset T^{2d}$, $(O(d) \oplus O(1,1)) \in T^{2d}$) nonlinear change of Galilean time: $R \to O(2)$ or $R \to O(1,1)$

If $d = 0$ one gets

$$Schr(0) = C(0) = O(2,1) \qquad C(0) = (H, D, K)$$

$$[D, H] = -H \qquad [K, H] = -2D \qquad [D, K] = K$$

Unique conformal algebra in $D = 0 + 1 \Rightarrow$ conformal classical and quantum mechanics (de Alvaro, Fubini, Furlan, 1976)

If $d > 0$ difference between $Schr(d)$ and $C(d)$

$$Schr(d) : [D, B_r] = -B_r \qquad C(d) : [D, B_r] = 0$$

For $Schr_N(d)$: $\qquad [D, B_r] = (1 - \frac{2}{N})B_r$

$$D = x_r \frac{\partial}{\partial x_r} + \frac{2}{N} t \frac{\partial}{\partial t} \longrightarrow x'_r = \lambda x_r \qquad t' = \lambda^{\frac{2}{N}} t$$

Only for CGA (N=2) space and time rescales identically.

In Horava approach to gravity $N = \frac{1}{6}$ $(z = 3)$ - no finite-dimensional z=3 CGA.

$Schr_N(d)$ studied also by Henkel (1994), under the name of $\mathrm{alt}_{\frac{N}{2}}(d)$

2. Galilean Conformal Symmetries

$$[M_{\mu\nu}, M_{\rho\tau}] = \eta_{\mu\tau} M_{\nu\rho} - \eta_{\mu\rho} M_{\nu\tau} + \eta_{\nu\rho} M_{\mu\tau} - \eta_{\nu\tau} M_{\mu\rho}$$

$$[M_{\mu\nu}, P_\rho] = \eta_{\nu\rho} P_\mu - \eta_{\mu\rho} P_\nu \qquad [P_\mu, P_\nu] = 0$$

$$[M_{\mu\nu}, K_\rho] = \eta_{\nu\rho} K_\mu - \eta_{\mu\rho} K_\nu \qquad [K_\mu, K_\nu] = 0$$

$$[P_\mu, K_\nu] = 2(\eta_{\mu\nu} D - M_{\mu\nu})$$

$$[D, M_{\mu\nu}] = 0 \qquad [D, P_\mu] = -P_\mu \qquad [D, K_\mu] = K_\mu$$

"Physical" contraction of relativistic conformal algebra:

$$C^{\mathrm{rel}}(d) = (M_{\mu\nu} = (M_{rs}, M_{r0}),\ P_\mu = (P_r, P_0), D, K_\mu = (K_r, K_0))$$

Rescaling before taking the limit $c \rightarrow \infty$

$$P_0 = \frac{H}{c} \quad M_{0r} = cB_r \qquad K_0 = cK \qquad K_r = c^2 F_r$$

M_{rs}, D, P_r – not rescaled; $K_0 = cK$ ↑ expansion; $K_r = c^2 F_r$ ↑ accelerations

Galilean conformal algebra (GCA) - C(d):

d-dimensional Euclidean Weyl algebra (M_{rs}, P_r, D) +
+ additional generators (H, K, B_r, F_r)

$$C(d) = (O(d) \oplus O(2,1) \in T^{3d}$$

$$\begin{array}{ccc} \nearrow & \uparrow & \Uparrow \\ M_{rs} & T_a = (H, D, K) & \textbf{Abelian}(P_r, B_r, F_r) = A_{r,a} \end{array}$$

$A_{r,a}$: O(d) index r=1 . . . d; O(2,1) index a=1,2,3

Covariance: $[M_{rs}, A_{t,a}] = \delta_{st} A_{r,a} - \delta_{rt} A_{s,a}$

$(\eta_{ab} = (1, 1, -1))$ $\quad [T_a, A_{r,b}] = \widetilde{\varepsilon}_{ab}{}^{c} A_{r,b}$

GCA has no central charges except if d=2 (D=2+1).
One obtains so-called "exotic" central charge θ

$$[B_r, B_s] = \theta \varepsilon_{rs} \qquad (r, s = 1, 2)$$

The mass m_0 occurring as central charge in G(d) is not possible in GCA. For $Schr_N(d)$ $m_0 \neq 0$ as central charge only for N=1, z=2, i.e. for Schrödinger algebra.

Second "geometric" WI contraction:

One can rescale P_μ and K_μ without changing relativistic conformal algebra

$$\widetilde{P}_\mu = \xi P_\mu \qquad \widetilde{K}_\mu = \xi^{-1} K_\mu$$

Composing this rescaling with "physical" one and putting $\xi = c$ we get unchanged P_0, K_0, D and M_{ij} and

$$\widetilde{P}_r = \xi P_r \qquad \widetilde{K}_r = \xi K_r \qquad \widetilde{M}_{r0} = \xi B_r$$

One can decompose $O(d+1,2)$ as symmetric Riemannian pair:

$$O(d+1,2) = (O(d) \oplus O(2,1)) \oplus \frac{O(d+1,2)}{O(2,1)\otimes O(d)} = h \oplus k$$

$$\begin{array}{ccc} \uparrow & \Uparrow & \Uparrow \\ M_{rs} & (P_0, K_0, D) & (M_{r0}, P_r, K_r) \end{array}$$

$$h'_A = h_A, \quad k'_l = \xi k_l \quad \longrightarrow \quad O(d+1,2) \xrightarrow[\xi\to\infty]{} C(d)$$

The coset generators k_l are becoming Abelian (Barut 1973). This is standard Wigner-Inönü contraction.

Realization of GCA symmetry ⇒ σ-model constructions

α) d=0 - AFF conformal mechanics (Ivanov, Krivonos, Leviant, 1985)

$$G_0 = e^{itH}\, e^{izk}\, e^{ikD} \qquad z = z(t) \quad u = u(t)$$

$$\Omega_0 = G_0^{-1} dG_0 = i(\omega_H\, H + \omega_K\, K + \omega_D\, D) \quad dz = \dot{z}dt \quad du = \dot{u}dt$$

One can write $\omega_D = du - 2zdt = D\omega_D dt = (\dot{u} - 2z)dt$ etc.

Constraints - "inverse Higgs mechanism":
$$\begin{cases} \omega_D = 0 \quad \Rightarrow \quad z = \frac{1}{2}\dot{u} \\ \omega_- = \gamma^{-1}\omega_K - \gamma\omega_H = 0 \end{cases}$$

One gets field equations

$$\ddot{\rho} = \gamma^2 \rho^{-3} \qquad \rho = e^{\frac{u}{2}}$$

The constraints and e.o.m. can be obtained from the action:

$$S_0 = -\gamma \int \omega_+ = -\int dt[e^u(\dot{z} + z^2) + \gamma^2 e^{-u}] \quad \omega_+ = \gamma^{-1}\omega_K + \gamma\omega_H$$

β) d>0 - extensions of AFF conformal mechanics model (Fedoruk, Ivanov, J.L., PRD 2011)

$$K = \frac{G_d}{O(d)} = G_0 \cdot e^{ix_r P_r}\, e^{iv_r B_r}\, e^{if_r F_r} \qquad \begin{aligned} x_r &= x_r(t) \\ v_r &= v_r(t) \\ f_r &= f_r(t) \end{aligned}$$

$$\Omega = \Omega_0 + i(\omega_{P,r} P_r + \omega_{B,r} B_r + \omega_{F,r} F_r) \qquad \omega_{a,r} = D\omega_{a,r} dt$$

Two actions:

$$i) \qquad S_d^{(1)} = \int m_{ab} \frac{\omega_{a,r}\omega_{b,r}}{\omega_H} = \int dt e^{-u} m_{ab}\, D\omega_{a,r} D\omega_{b,r}$$

where $\omega_{+,\,r} = \gamma^{-1}\omega_{Fr} - \gamma\omega_{Pr}$ one gets GCA-invariant model

$$S_d^{(1)} = \frac{1}{2}\int dt \rho^2 [\dot{x}_r^+ + \gamma^{-1} v_r (\rho\ddot{\rho} - \gamma^2\rho^{-2})]^2 \qquad x_r^+ = \gamma^{-1} f_r - \gamma x_r$$

$$ii) \qquad S_d^{(2)} = m \int (m_{ab}\omega_{a,r}\omega_{b,r})^{1/2} \xrightarrow[\text{choice}]{\text{special}} m \int dt (D\omega_{+,r} D\omega_{+,r})^{1/2}$$

From $S_d^{(2)}$ one gets the same equations as from $S_d^{(1)}$ but supplemented with the mass-shell condition

$$(\mathcal{P}_r^+ \mathcal{P}_r^+ - m^2) = 0 \qquad \mathcal{P}_r^+ = e^u \, \dot{X}_r^+$$

iii) $d = 2$

In the presence of central charge θ one gets the extension of AFF model which is a decoupled sum of two models

$$S^{(3)} = \int dt(\dot{\rho}^2 - \frac{\gamma^2}{\rho^2} + \frac{\theta}{2}\varepsilon_{rs} y_{\dot{r}} \ddot{y}_s) \qquad y_r = e^u x_r$$

Second term leads to higher order equation as firstly appeared in (J.L., Stichel, Zakrzewski, 1997). It was linked with classical mechanics on noncommutative 2+1 space-time with higher order Chern-Simmons type action.

Realizations of GCA (for d=3) $\Rightarrow$ NR counterpart of twistors

(Fedoruk, Kosiński, J.L., Maślanka, PRB 2011)

D=4 relativistic twistors $\sim$spinorial realization of $O(4,2)\simeq SU(2,2)$

For NR conformal algebra some modification:

d=3 NR counterpart of twistors = Galilean twistors	$\sim$	spinorial realization of semi-simple part of GCA: $O(3)\oplus O(2,1)\simeq SU(2)\oplus SU(1,1)$

Penrose twistors $t_A=(\lambda_\alpha,\omega^{\dot\alpha})\subset C^4$ $\xrightarrow{\text{NR contraction}}$ Galilean twistors $t_{\alpha,i}\in M_2(c)$

t_A and $t_{\alpha,i}$ are the same variables - only in NR contraction do disappear the transformations extending $O(3)\oplus O(2,1)\rightarrow O(4,2)$ and they transform under GCA as 2×2 complex matrices

Quantized Galilean twistors (N copies: k,l=1 ... N)

$$[t^{(k)}_{\alpha,i},\bar t^{(l)}_{\dot\beta,j}]=\delta^{kl}\delta_{\alpha\dot\beta}\omega_{ij}$$

$\omega_{ij}-SU(1,1)$ –Hermitean metric

$\delta_{\dot\alpha\beta}-SU(2)$ –invariant metric

Basic question: can one introduce the bilinear N-twistor realization of GCA using quantized Galilean twistors?

Relativistic case: Yes, even if N=1 one gets one-twistor realization describing e.g. massless relativistic particle.

For N Penrose twistors $t_A^{(k)} = (\lambda_\alpha^{(k)}, \omega^{(k)\dot\beta})$ $(k = 1, \ldots N)$ we get e.g.

$$P_\mu = \sum_{k=1}^{N} \bar\lambda_{\dot\alpha}^{(k)} (\sigma_\mu)^{\dot\alpha\beta} \lambda_\beta^{(k)} \Rightarrow P_0 = \sum_{k=1}^{N} \bar\lambda_{\dot\alpha}^{(k)} \lambda_\alpha^{(k)} \qquad P_0 \geq 0!$$

Galilean case:
- one-twistor realization is not possible
- for N$\geq$ 2 one can introduce a twistor realization of GCA but for any N the generator of time translation H is indefinite i.e. H$\geq$ 0 not valid.

Galilean twistors require nonstandard QM!

3. Galilean Superconformal Symmetry (SUSY GCA)

Mathematical problem: supersymmetrization of non-semisimple GCA.

In d=3 (D=3+1) the semisimple part of GCA is supersymmetrized via quaternionic supergroups:

$$O(3)\oplus O(2,1)\simeq O^{\star}(4)\simeq U_{\alpha}(2|H)\xrightarrow{\text{SUSY}} U_{\alpha}U(2,N|H)\simeq OSp(4^{\star}|2N)$$

Bosonic sector of

$$U_{\alpha}U(2;N|H): \quad U_{\alpha}(2|H)\oplus U(N|H)=O^{\star}(4)\oplus USp(2N)$$

$$N=1,2: \quad U(1;H)\simeq SU(2)\simeq O(3) \quad U(2;H)\simeq USp(4)\simeq O(5)$$

Fermionic sector of $U_{\alpha}U(2;N|H)$:

2N quaternionic supercharges $\leftrightarrow$ 8N real supercharges = 8N complex supercharges with SU(2)-Majorana subsidiary condition.

What about supersymmetrization of Abelian sector?

$$(P_r, B_r, F_r) = A_{r;a} \qquad \begin{matrix} r = 1,2,3 \\ a = 1,2,3 \end{matrix} \quad (d=3)$$

We supersymmetrize the coset decomposition of relativistic D=4 CA with stability group given by semisimple part of GCA

$$\overline{O^\star(4)} \oplus \frac{SU(2,2)}{O^\star(4)} \stackrel{\text{SUSY}}{\longrightarrow} U_\alpha U(2,N|H) \oplus \frac{SU(2,2:2N)}{U_\alpha U(2;N|H)} = \widetilde{h} \oplus \widetilde{k}$$

B : 6	9	B: 6+N(2N+1)	9+N(2N−1)	U(2N)− $4N^2$ generators
		F : 8N	8N	

$N(2N+1)$ - the number of generators of $USp(2N) = U(N|H)$
$N(2N-1) = 4N^2 - N(2N+1)$ real generators of $\frac{U(2N)}{USp(2N)}$
If we rescale $\widetilde{k} = \xi k$ and perform WI contraction $\xi \to 0$ of symmetric supercoset decomposition for $SU(2,2;2N)$, the generators $\widetilde{k}$ become the following Abelian superalgebra:

- **Fermionic: 8N real graded - commutative charges obtained from fermionic generators in $\widetilde{k}$ (will denote by $\widetilde{Q}^-$)**
- **Bosonic: 9 real generators $(P_r, B_r, F_r) \subset$ CGA and $N(2N-1)$ internal tensorial central charges from $\widetilde{k}$.**

The WI contraction of the supercoset leads to the following structure of N-extended d=3 SUSY GCA (Sakaguchi, JMP 2010, Fedoruk + J.L., PRD 2011)

i) Semisimple superalgebra $OSp(4^\star, 2; 2N) \equiv U_\alpha U(2; N|H)$

$$\{\widetilde{\mathbb{Q}}^+, \widetilde{\mathbb{Q}}^+\} \subset O^\star(4) \oplus USp(2N)$$

$\widetilde{\mathbb{O}}^+$ – $8N$ real supercharges

ii) Graded Abelian algebra of fermionic charges

$$\{\widetilde{\mathbb{Q}}^-, \widetilde{\mathbb{Q}}^-\} = 0$$

iii) Supersymmetrization of bosonic tensorial central charges

$$\{\widetilde{\mathbb{Q}}^+, \widetilde{\mathbb{Q}}^-\} \subset \{(P_r, B_r, F_r) \oplus N(2N-1) \text{ internal charges}\}$$

Interesting: N-extended SUSY GCA can be obtained explicitly by "physical" contraction $c \to \infty$ (de Azcárraga, J.L., PLB 2009, Fedoruk, J.L., PRD 2011)

In order to obtain the supercharges $(\widetilde{\mathbb{Q}}^{+}, \widetilde{\mathbb{Q}}^{-})$ we project 8N Weyl $SU(2,2|2N)$ supercharges $(Q_{\alpha,i}, S_{\alpha,i})$ $(\alpha = 1,2,$ $i = 1 \ldots 2N)$ as follows

$$Q^{\pm}_{\alpha,i} = \tfrac{1}{2}(Q_{\alpha,i} \pm \varepsilon_{\alpha\beta}\Omega_{ij}\bar{Q}_{\dot{\beta};j}) \qquad \begin{pmatrix} \text{breaks } O(3,1) \to \\ \to O(3) \end{pmatrix}$$

$$S^{\pm}_{\alpha,i} = \tfrac{1}{2}(S_{\alpha,i} \pm \varepsilon_{\alpha\beta}\Omega_{ij}\bar{S}_{\dot{\beta};j}) \qquad \Omega_{ij} = \begin{pmatrix} 0 & 1\!\!1_N \\ -1\!\!1_N & 0 \end{pmatrix}$$

$$\Rightarrow \ \bar{Q}^{\pm}_{\alpha,i} = \pm\varepsilon_{\dot{\alpha}\dot{\beta}}\Omega_{ij}Q^{\pm}_{\beta;j}$$

$$\Rightarrow \ \bar{S}^{\pm}_{\alpha,i} = \pm\varepsilon_{\dot{\alpha}\dot{\beta}}\Omega_{ij}S^{\pm}_{\beta;j}$$

$\Rightarrow$ **SU(2)–Majorana or Majorana-symplectic subsidiary conditions**

and define $\mathbb{Q}^{+} = (Q^{+}_{\alpha,i}, S^{+}_{\alpha,i}), \ \ \mathbb{Q}^{-} = (Q^{-}_{\alpha,i}, S^{-}_{\alpha,i})$

The "physical" rescaling is given by (rescaled denoted by "$\wedge$")

$$Q^+_{\alpha,i} = \frac{1}{\sqrt{c}}\widehat{Q}^+_{\alpha,i} \qquad Q^-_{\alpha,i} = \sqrt{c}\,\widehat{Q}_{\alpha,i} \qquad \widehat{Q}^\pm_{\alpha,i} = \stackrel{c\to\infty}{\longrightarrow} \widetilde{Q}^\pm_{\alpha,i}$$

$$S^+_{\alpha,i} = \sqrt{c}\;\widehat{S}^+_{\alpha,i} \qquad S^-_{\alpha,i} = (\sqrt{c})^{3/2}\,\widehat{S}_{\alpha,i} \qquad \widehat{S}^\pm_{\alpha,i} = \stackrel{c\to\infty}{\longrightarrow} \widetilde{S}^\pm_{\alpha,i}$$

as well as by

$$\widetilde{h}^B_{ij} = \widehat{h}^B_{ij} \qquad \text{in } c\to\infty \text{ limit:} \qquad h^G_{ij} \in USp(2N)$$

$$\widetilde{k}^B_{ij} = c\widehat{k}^B_{ij} \qquad \text{in } c\to\infty \text{ limit:} \qquad k^G_{ij} \in T^{N(2N-1)}$$

Performing the "physical" contraction in all $SU(2,2;2N)$ relations one obtains explicitly N-extended d=3 SUSY GCA.

For odd N the possible contractions of $SU(2,2;N)$ have pathological properties - do not "supersymmetrize" all generators of GCA. The model by Baghi, Mandal (2009) for N=1 provides ($F = (\widetilde{Q}^\pm, \widetilde{S}^\pm)$)

$$\{F,F\} \subset (P_i, B_i, F_i) \quad (\text{no } H, K, M_{ij}!)$$

Most unwanted feature: H is not a bilinear in F

Galilean supertwistors $q_A = (q_1, q_2; \theta_1 \ldots \theta_N)$ $\leftrightarrow$ fundamental realizations of maximal compact sub -superalgebra $U_\alpha U(2; N|H) \simeq OSp(4^\star|2N)$ of SUSY GCA

We see that Galilean twistors and supertwistors are endowed with quaternionic structure:

(q_1, q_2) – fundamental representation of $U_\alpha(2; H) \equiv O^\star(4) \simeq \simeq O(2,1) \oplus O(3)$ (space-time sector)

(θ_1, θ_N) – fundamental representation of $U_\alpha(N; H) = USp(2N)$ (internal sector)

Remark: We presented d=3 (D=3+1) case. One can describe as well Galilean conformal algebra for d=2,4,5 (D=3,5,6) and their supersymmetrization (Fedoruk, J.L., 2011). In D=3,6 one gets infinite sequences of superalgebras. For D=5 unique relativistic CA is obtained as exceptional superalgebra F(4) with real form providing bosonic sector $O(5,2) \oplus O(3)$

$$F(4) = OSp(4^\star|2) \oplus \frac{F(4)}{OSp(4^\star|2)} \xrightarrow{\text{contraction}} \text{D=5 SUSY } GCA$$

4. Final Remarks

The table for D=4 simple Galilean (super) conformal symmetries

Poincaré $O(3,1) \Subset T^4$	$\xrightarrow{c\to\infty}$	Galilei $(O(3)\oplus R) \Subset T^6$
$\downarrow$		$\downarrow$
conformal $O(4,2)$	$\xrightarrow{c\to\infty}$	Galilean conformal $(O(3)\oplus O(2,1)) \Subset T^9$
$\downarrow$		$\downarrow$
superconformal N=2 (Wess-Zumino) $SU(2,2;2)$	$\xrightarrow{c\to\infty}$	Galilean superconformal $OSp(4^\star;2) \Subset T^{10;8} \equiv$ $\equiv U_\alpha(2;1\vert H) \Subset T^{10;8}$

$T^{10;8}$ is graded Abelian superalgebra $(Q^-_{\alpha,i}, S^-_{\alpha i}; P_r, B_r, F_r, A)$ with 8 real fermionic and 10 bosonic generators

Question: how looks **general Galilean N-extended SUSY?**

First step - without Galilean central charges:
Galilean N-extended SUSY is a sub-superalgebra of Galilean superconformal algebra $\Rightarrow$ Galilean SUSY inherits the quaternionic structure and internal symmetry USp(2N).

Two ways of supplementing Galilean SUSY with the most general Galilean central charges (N(2N-1) real ones):

a) By making general ansatz and studying Jacobi identities

b) By considering "physical" contraction $c \to \infty$ of 2N-extended Poincaré superalgebra with complex $\frac{2N(2N-1)}{2}$ central charges - one gets after suitable rescaling by c and $\frac{1}{c}$ in limit $c \to \infty$ the same number of Galilean central charges. One of Galilean central charges is the mass parameter m_0 (J.L., PLB 2011)

Presented algebraic set-up should have link with various applications, e.g.

- **N - extended SUSY of superconformal mechanics models, eg.**
  - **for d=3 with N=4 $OSp(4^{\star}|2)$ model**
  - **for d=5 with N=8 $OSp(4^{\star}|4)$ model**
- **Nonrelativistic p-branes (Gomis, Kamimura, Townsend, JHEP 2004)**
- **Galilean (super) conformal field theory**
- **Nonrelativistic AdS/CFT (e.g. Bagchi, Gopakumar, JHEP 2009)**

! THANK YOU !