# Reduction of N=1 E8 SYM over SU(3)/U(1)xU(1)xZ3 and its 4d effective action

**N. I. & G. Zoupanos, PLB 698 (2011) 146**

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SQS 2011, Dubna, Russia

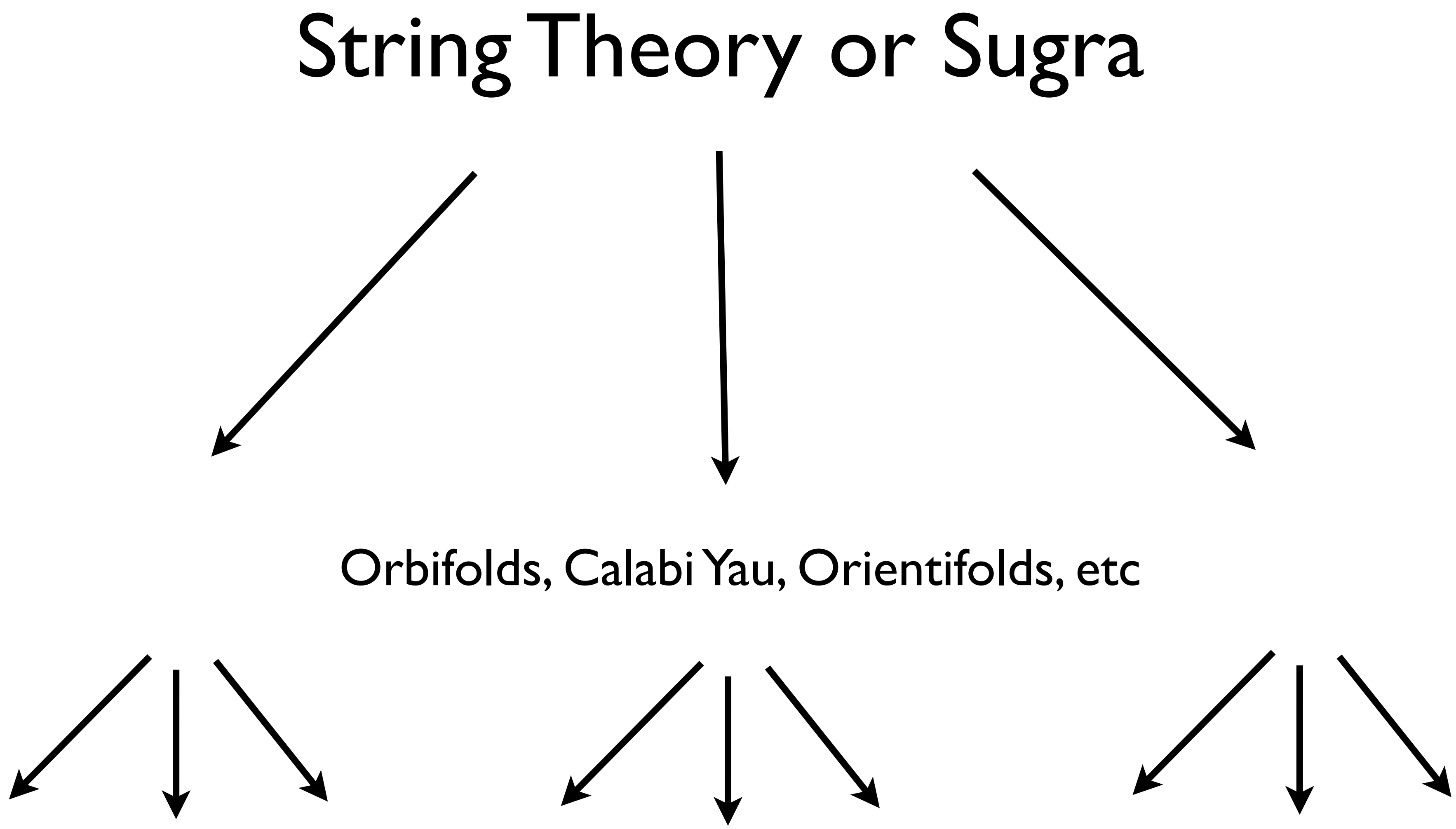
# Two approaches in BSM model building

I. Bottom-up

| | | |
|---|---|---|
| susy | extra dim | GUT |
| Strong BSM | massive neutr. | extra matter |
| extra gauge symm | hidden sectors | flavor symm |

Too many possibilities, each with loose theoretical constraints, no clear experimental input yet to select (apart from the neutrino masses) without any doubt the right combination. It is relatively easy to evade experimental bounds depending on how ambitious the model is.

# II. Top-down




Enormous number of possibilities, each very constrained but the resulting effective theories are far from the SM. It is ridiculously easy to exclude them using experimental data.

So what do we do?

One possibility is to just sit and wait and until the LHC data is out, or else do either SM phenomenology or rigorous mathematical physics.

Another is to try to combine the two approaches in order to construct a simple enough, hopefully viable extension of the Standard Model with new and interesting properties, however more constrained than its generic counterpart.

How do we do that:

1. select a string
2. truncate away everything but the gauge sector (with one noteable exception)
3. choose a compact manifold
4. compute the four-dimensional effective action
5. adjust so that it is consistent at the quantum level (that is, take care of anomalies)
6. patch holes by introducing extra structure (discrete symmetries etc.)

# Reduction of N=1, E8 SYM on SU(3)/U(1)xU(1)

D. Kapetanakis and G. Zoupanos, Phys. Rept. C219 (1992)
P. Forgacs and N. S. Manton, Commun. Math. Phys. 72 (1992)

Consider a D-dimensional Yang-Mills-Dirac theory with gauge group G defined on a manifold $M^D$ compactified on

$$M^4 \times S/R$$

$$A = \int d^4x d^d y \sqrt{-g} \Big[ -\frac{1}{4} Tr \left( F_{MN} F_{K\Lambda} \right) g^{MK} g^{N\Lambda} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \Big],$$

D=4+d, d=dimS-dimR.
S and R are Lie groups.

- The CSDR scheme demands that an S-transformation of the extra d coordinates is a gauge transformation of the fields that are defined on the coset, thus a gauge invariant Lagrangian written on this space is independent of the extra coordinates.

- The four-dimensional gauge group H is the centralizer of R in G.

- The extra dimensional components of the gauge field (which are scalars) transform under R as a vector and since the adjoint of R can be embedded in the adjoint of H, there will be $h_i$ multiplets of $r_i$

$$S \supset R \qquad adjS = adjR + \sum_i r_i$$

$$G \supset H \qquad adjG = (adjR, 1) + (1, adjH) + \sum (r_i, h_i)$$

- If the 10-dimensional fermions are Majorana-Weyl then there will be fermions in identical irreps with the scalars.

- There is a fixed scalar potential originating from the 10-dim kinetic term.

# Reduction of $\mathcal{N}=1$ $E_8$ SYM on $S/R = SU(3)/(U(1)\times U(1))$

The unbroken symmetry: $H = E_6 \times U(1)_A \times U(1)_B$

Matter:

$$\alpha_i \sim \mathbf{27}_{(3,\frac{1}{2})}$$

$$\beta_i \sim \mathbf{27}_{(-3,\frac{1}{2})}$$

$$\gamma_i \sim \mathbf{27}_{(0,-1)}$$

$$\alpha \sim \mathbf{1}_{(3,\frac{1}{2})}$$

$$\beta \sim \mathbf{1}_{(-3,\frac{1}{2})}$$

$$\gamma \sim \mathbf{1}_{(0,-1)}$$

P. Manousselis and G. Zoupanos, JHEP 0203 (2002) 002

The effective action can be written as a softly broken susy action

$$\mathcal{W}(A^i, B^j, C^k, A, B, C) = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC$$

$$V_D = \frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2$$

$$D^\alpha = \frac{1}{\sqrt{3}} \left( \alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)$$

$$D_1 = \sqrt{\frac{10}{3}} \left( \alpha^i (3\delta_i^j) \alpha_j + \overline{\alpha}(3)\alpha + \beta^i(-3\delta_i^j)\beta_j + \overline{\beta}(-3)\beta \right)$$

$$D_2 = \sqrt{\frac{40}{3}} \left( \alpha^i (\frac{1}{2}\delta_i^j) \alpha_j + \overline{\alpha}(\frac{1}{2})\alpha + \beta^i(\frac{1}{2}\delta_i^j)\beta_j + \overline{\beta}(\frac{1}{2})\beta + \gamma^i(-1\delta_i^j)\gamma_j + \overline{\gamma}(-1)\gamma \right)$$

$$V_{\rm soft} =$$

$$\left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left( \frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \overline{\alpha}\alpha + \left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \left( \frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \overline{\beta}\beta$$

$$+ \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i + \left( \frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \overline{\gamma}\gamma + \left[ \sqrt{2} 280 \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k \right.$$

$$\left. + \sqrt{2} 280 \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha\beta\gamma + h.c \right]$$

$$V = const. + V_F + V_D + V_{\rm soft}$$

# The Wilson flux breaking mechanism

$$M^4 \times B_0 \longrightarrow M^4 \times B \qquad B = B_0/F^{S/R}$$

$F^{S/R}$ : a freely acting symmetry of $B_0$

1. B becomes multiply connected
2. For every element $g \in F^{S/R}$ $\rightarrow$ $U_g = \mathcal{P}exp\left(-i \int_{\gamma_g} T^a A^a_M(x) dx^M\right) \in H$
3. If the contour is non-contractible then $U_g \neq 1$ and then $f(g(x)) = U_g f(x)$

(so, it is basically like a freely acting orbifold)

$$F^{S/R} = Z_3 \subseteq \mathrm{W} \text{ , } \mathrm{W} = \mathrm{W}_S/\mathrm{W}_R$$

$\mathrm{W}_S$ : Weyl group of S

$\mathrm{W}_R$ : Weyl group of R

$$\gamma_3 = diag(\mathbf{1}, \omega\mathbf{1}, \omega^2\mathbf{1}) \quad \omega = e^{2i\pi/3} \in Z_3$$

$$A_\mu = \gamma_3 A_\mu \gamma_3^{-1}$$

$$\vec{\phi}_i = \omega^i \gamma_3 \vec{\phi}_i$$

$$\phi_i = \omega^i \phi_i$$

$$A_\mu^A, \qquad A \in SU(3)_c \times SU(3)_L \times SU(3)_R$$

$$H_1 \sim (\bar{3}, 1, 3)_{(3,1/2)}$$

$$H_2 \sim (3, \bar{3}, 1)_{(0,-1)}$$

$$H_3 \sim (1, 3, \bar{3})_{(-3,1/2)}$$

$$\theta_{(0,-1)}$$

$$V_{\mathrm{sc}} = (3\Lambda^2 + \Lambda'^2)\left(\frac{1}{R_1^4} + \frac{1}{R_2^4}\right) + \frac{4\Lambda'^2}{R_3^4} + \ V_{\mathrm{susy}} + V_{\mathrm{soft}}$$

$$V_D = \frac{1}{2}\sum_A D^A D^A + \frac{1}{2}D^1 D^1 + \frac{1}{2}D^2 D^2$$

$$D^1 = 3\sqrt{\frac{10}{3}}\left(\langle H_1|H_1\rangle - \langle H_2|H_2\rangle\right)$$

$$D^2 = \sqrt{\frac{10}{3}}\left(\langle H_1|H_1\rangle + \langle H_2|H_2\rangle - 2\langle H_3|H_3\rangle - 2|\theta|^2\right)$$

$$D^A = \frac{1}{\sqrt{3}}\langle H_i|G^A|H_i\rangle$$

$$\langle H_i|G^A|H_i\rangle = \sum_{i=1,2,3} H_i^a (G^A)_a^b H_{ib}$$

$$\langle H_i|H_i\rangle = \sum_{i=1,2,3} H_i^a \delta_a^b H_{ib}$$

A trick: $$H_1 \sim (\bar{3}, 1, 3) \longrightarrow N_p^\alpha$$

T.W. Kephart and M.T. Vaughn, Ann. Phys. 145 (1983)

$$H_2 \sim (3, \bar{3}, 1) \longrightarrow L_\alpha^a$$

$$H_3 \sim (1, 3, \bar{3}) \longrightarrow M_a^p$$

$$\hat{N}_\alpha^p = \frac{1}{3} \frac{\partial I_3}{\partial N_p^\alpha}$$

$$\hat{M}_p^a = \frac{1}{3} \frac{\partial I_3}{\partial M_a^p}$$

$$\hat{L}_a^\alpha = \frac{1}{3} \frac{\partial I_3}{\partial L_\alpha^a}$$

$$I_3 = \mathrm{detN} + \mathrm{detM} + \mathrm{detL} - \mathrm{tr}(NML)$$

then:

$$\langle H_1|H_1\rangle = \mathrm{tr}(N^\dagger N)$$

$$\langle H_2|H_2\rangle = \mathrm{tr}(L^\dagger L)$$

$$\langle H_3|H_3\rangle = \mathrm{tr}(M^\dagger M)$$

$$d_{abc}H_1^a H_2^b H_3^c = \mathrm{detN}^\dagger + \mathrm{detM}^\dagger + \mathrm{detL}^\dagger - \mathrm{tr}(N^\dagger M^\dagger L^\dagger)$$

$$V_F = 40\mathrm{tr}(\hat{N}^\dagger \hat{N} + \hat{M}^\dagger \hat{M} + \hat{L}^\dagger \hat{L})$$

and the d-symbols have 'disappeared'

finally, re-introduce 3 flavors by having 3 windings in R

# Supersymmetry and gauge symmetry breaking

K.S. Babu, X.-G. He and S. Pakvasa, Phys. Rev. D33 (1986)
J. Sayre, S. Wiesenfeld and S. Willenbrock, Phys. Rev. D73 (2006)

consider the vevs:

$$M_0^{(1)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix} \qquad M_0^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

$$M_0^{(1)} : \;\; SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \to SU(2)_L \times SU(2)_R \times U(1)$$

$$M_0^{(2)} : \;\; SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \to SU(2)_L \times SU(2)'_R \times U(1)'$$

their combination gives:

$$SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \longrightarrow SU(2)_L \times U(1)_Y$$

electroweak symmetry breaking then proceeds by:

$$M_0^{(1)} = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & V \end{pmatrix}$$

do these vevs minimize V?

$$V_{D_1} = 15(V^2 + 2v^2)^2$$

$$V_{D_2} = \frac{5}{9}(V^2 + 2v^2 - \theta_0^2)^2$$

$$V_F = \frac{40}{9} v^2 (2V^2 + v^2)$$

$$V_{\rm soft} = \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2}\right)(V^2 + 2v^2) + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2}\right)(\theta_0^{(1)})^2 + 160\sqrt{3}\left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2}\right) V v^2$$

for R1=R2=R3,V=0 if

$$(\theta_0^{(1)})^2 = \frac{1}{10R^2}\Big[5R^2V^2 + 10R^2v^2 + 9 + \big(-675V^4R^4 - 3100V^2v^2R^4 + 270V^2R^2 - 2900v^4R^4 + 540v^2R^2 + 27 - 21600\sqrt{3}Vv^2R^3\big)^{1/2}\Big]$$

for $V = 1$ and $R \sim \mathcal{O}(1/2) \quad \longrightarrow \quad v \sim \mathcal{O}(0.1)$

the representations involved are

$$(\overline{3}, 1, 3)_{(3,1/2)} \longrightarrow (\overline{3}, 1, 1+1+1)_{(3,1/2)} \quad \longrightarrow \quad \overline{\mathbf{u}}, \overline{\mathbf{d}}, \overline{\mathbf{g}}$$

$$(3, \overline{3}, 1)_{(-3,1/2)} \longrightarrow (3, 2+1, 1)_{(-3,1/2)} \quad \longrightarrow \quad \mathbf{Q}, \mathbf{g}$$

$$(1, 3, \overline{3})_{(0,-1)} \longrightarrow (1, 2+1, 1+1+1)_{(0,-1)} \longrightarrow L, \overline{e}, \overline{N}_1, \overline{N}_1, H_u, H_d$$

the full spectrum:

| $SU(3)_c \times SU(2)_L$ | $U(1)_Y$ | $U(1)_A$ | $U(1)_B$ |
|---|---|---|---|
| $\mathbf{Q} \sim (3,2)$ | $1/6$ | $-3$ | $1/2$ |
| $\overline{\mathbf{u}} \sim (\bar{3},1)$ | $-2/3$ | $3$ | $1/2$ |
| $\overline{\mathbf{d}} \sim (\bar{3},1)$ | $1/3$ | $3$ | $1/2$ |
| $L \sim (1,2)$ | $-1/2$ | $0$ | $-1$ |
| $\bar{e} \sim (1,1)$ | $1$ | $0$ | $-1$ |
| $H_u \sim (1,2)$ | $1/2$ | $0$ | $-1$ |
| $H_d \sim (1,2)$ | $-1/2$ | $0$ | $-1$ |
| $\mathbf{g} \sim (3,1)$ | $-1/3$ | $-3$ | $1/2$ |
| $\overline{\mathbf{g}} \sim (\bar{3},1)$ | $1/3$ | $3$ | $1/2$ |
| $\overline{N}_1 \sim (1,1)$ | $0$ | $0$ | $-1$ |
| $\overline{N}_2 \sim (1,1)$ | $0$ | $0$ | $-1$ |
| $\Theta^{(1)} \sim (1,1)$ | $0$ | $0$ | $-1$ |

1. Notice that $U(1)_A = -9B$

2. at least one of the U(1)'s is anomalous. One needs to add by hand (not really)

C. Coriano, N. Irges and E. Kiritsis, Nucl. Phys. B746 (2006)

$$\mathcal{L}_{\mathrm{St-WZ}} = \frac{1}{2}(\partial_\mu a + M A_\mu)^2 + c\frac{a}{M} F_A \wedge F_A + \mathcal{L}_{\mathrm{an}}$$

3. the coset structure eventually breaks:

$$\Theta^{(1)}\bar{\mathbf{g}}\mathbf{g}$$

4. the susy structure will also be violated and fermion masses and mixing can be generated

$$L\bar{e}H_d \left(\frac{\Theta^{(1)*}}{M}\right)^3 \qquad LH_u\overline{N}\left(\frac{\Theta^{(1)*}}{M}\right)^3 \qquad M\overline{NN}\left(\frac{\Theta^{(1)*}}{M}\right)^2$$

# Conclusions

I. we have constructed an extension of the SM that could originate from the heterotic string

II. the 4D effective action can be written in softly broken form
(without any special susy breaking mechanism)

III. the superpotential and the scalar potential are completely fixed

IV. there is no proton decay

V. the Froggatt-Nielsen mechanism is naturally relaized