

# Contraction of Lepton Sector of Electroweak Model Can Explain the Neutrino-Matter Interactions

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**Supersymmetries and Quantum Symmetries**  
**Dubna, Russia, July 18-23, 2011**

# Outline

- ▶ **Neutrino: main experimental properties.**
- ▶ Semi-Riemannian geometry and group contractions.
- ▶ The Standard Electroweak Model
- ▶ Modified Model
- ▶ Conclusion

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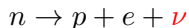
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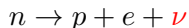
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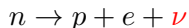


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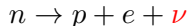
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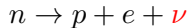
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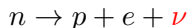
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- ▶ All neutrinos are stable. They are **weak** and **gravity** interacted particles.
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- ▶ The **vanishingly small interaction with matter** especially for low energies distinguishes neutrinos from other elementary particles. The attenuation length of neutrino in water is about 100 light years.
- ▶ The second remarkable experimental fact is that **neutrinos cross-section increases with the energy**. For energies greater than  $1\text{ GeV}$  this dependence is linear.
- ▶ We want to explain both facts with the help of group contraction.
- ▶ More precisely, the modification of the Standard Electroweak Model shall be suggested, where the boson and quark sectors are the same as in the Standard Model, but the lepton sector is invariant with respect to the gauge group  $SU(2; j) \times U(1)$ , where  $j$  is a small dimensionless parameter.

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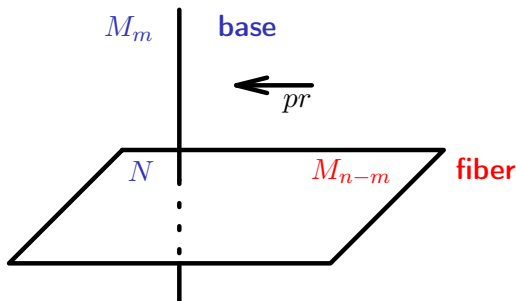


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## Semi-Riemannian geometry and group contractions

**Semi-Riemannian geometry**  $V_n^m$  is defined by a smooth fibration  $pr : M_n \mapsto M_m$ , with the **base**  $M_m$  and the **fiber**  $M_{n-m} = pr^{-1}(N)$ ,  $N \in M_m$ .



- ▶ Only those geometrical objects which are consistent with this projection can be regarded, in particular, only the following coordinate transformations

$$\begin{cases} x^{\mu'} = f^{\mu'}(x^1, \dots, x^m), \\ x^{i'} = f^{i'}(x^1, \dots, x^m, x^{m+1}, \dots, x^n) \end{cases} ,$$

with the standard conditions on the smoothness and Jacobian. Here and below  $\alpha, \beta = 1, \dots, n$ , for all coordinates,  $\mu, \nu = 1, \dots, m$  for coordinates **in the base** and  $i, k = m + 1, \dots, n$  for coordinates **in the fiber**.

- ▶ In the base  $M_m$  a nondegenerate (pseudo)Riemannian metrics is defined whose components depend only on base coordinates  $g_{\mu\nu}(x^1, \dots, x^m)$ .
- ▶ In the fibers  $M_{n-m}$  also a nondegenerate (pseudo)Riemannian metrics  $g_{ik}(x^1, \dots, x^n)$  is defined but its components depend on all coordinates  $M_n$ .
- ▶ In order to equalize the number of metrics components of the fiber geometry and the (pseudo)Riemannian one the additional metrics components are introduced

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- ▶ Denoting  $D_{\beta}^{\alpha'} = \partial_{\beta} x^{\alpha'}$ , we can write down the matrix of coordinate transformations as

$$(D_{\beta}^{\alpha'}) = \begin{pmatrix} (D_{\nu}^{\mu'}) & 0 \\ (D_{\nu}^{i'}) & (D_k^{i'}) \end{pmatrix}, \quad \det(D_{\nu}^{\mu'}) \neq 0, \quad \det(D_k^{i'}) \neq 0. \quad (1)$$

- ▶ The metric (flag) tensor

$$(g_{\alpha\beta}) = \begin{pmatrix} g_{\mu\nu} & g_{\mu k} \\ g_{i\nu} & g_{ik} \end{pmatrix}$$

of semi-Riemannian space  $V_n^m$  is transformed under admissible transformations (1) by the formulas

$$g_{\mu'\nu'} = g_{\mu\nu} D_{\mu'}^{\mu} D_{\nu'}^{\nu}, \quad g_{i'k'} = g_{ik} D_{i'}^i D_{k'}^k,$$

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- ▶ Thus, semi-Riemannian geometry  $V_n^m$  is defined by the set of objects with appropriate transformation properties.
- ▶ For us the most important and interesting feature of the semi-Riemannian geometry is that **properties of the base do not depend on the points of the fiber.**

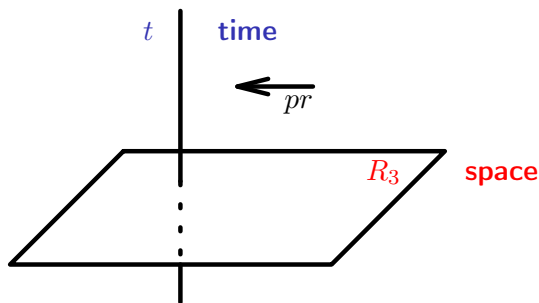
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- ▶ In order to avoid terminological misunderstanding let us stress that the fibering  $pr$  has nothing to do with the principal bundle, where some group acts on the fiber.
- ▶ In the last approach something similar to a tangent space  $(dx^1, \dots, dx^n)$  is built on the space  $(x^1, \dots, x^n)$  and then unified object  $(x^1, \dots, x^n, dx^1, \dots, dx^n)$  is regarded as having base  $(x^1, \dots, x^n)$  and fiber  $(dx^1, \dots, dx^n)$ .
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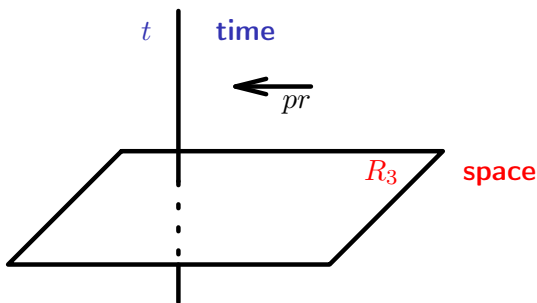
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- ▶ Very simple and most known example of the semi-Riemannian (or more exactly semi-Euclidean) geometry is the Galilei space-time of classical physics with 1D base, which is interpreted as time axis  $t$ , and 3D fiber, which is interpreted as space  $R_3$ .



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## Semi-Riemannian geometry and group contractions

- ▶ In the previous sections semi-Riemannian geometry was constructed for **real objects** as the structure consistent with fiberig.
- ▶ At the same time geometry of this sort can be realized with the help of **nilpotent objects**.
- ▶ In particular, semi-Riemannian space can be obtained from Riemannian space by multiplying fiber coordinates by nilpotent unit

$$\iota x^{m+1}, \dots, \iota x^n,$$

where  $\iota \neq 0$ ,  $\iota^2 = 0$ , division  $\frac{a}{\iota}$  is defined only for  $a = 0$ , but  $\frac{\iota}{\iota} = 1$ .

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- ▶ The space  $C_2(\iota)$  has two hermitian forms: one in the base  $\bar{z}_2 z_2 = |z_2|^2$  and one in the fiber  $\bar{z}_1 z_1 = |z_1|^2$ .
- ▶ Both forms can be unified in a single expression

$$z^\dagger z(j) = j^2 |z_1|^2 + |z_2|^2,$$

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- ▶ The equivalent and more traditional way of group contraction in physics is to tend contraction parameter to zero  $j \rightarrow 0$ .
- ▶ But in this case the deep mathematical matter connected with the conception of fiber space and semi-Riemannian geometry is hidden.
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- ▶ This model is a gauge theory with the gauge group  $SU(2) \times U(1)$ , which act in the boson, lepton and quark sectors.
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- ▶ At the same time the grave disadvantage of the Standard Model is the presence more then fifteen free parameters.
- ▶ But among these there is not such parameter, which a priori can be regarded as a **small one** and can be connected with the very rare interaction neutrinos with matter.



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► Elementary particles content of the Standard Model.

Gauge bosons:

$\gamma$  (photon),  $W^\pm$  (charged weak bosons),

$Z^0$  (neutral weak boson).

Quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \in C_2.$$

Leptons:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \in C_2.$$

- ▶ The Lagrangian of the model is given by the sum:

$$L = L_B + L_Q + L_L$$

of the boson  $L_B$ , of the quark  $L_Q$  and of the lepton  $L_L$  Lagrangians

- ▶ and is invariant under the action of the gauge group  $SU(2) \times U(1)$  in  $C_2$

$$\begin{pmatrix} z'_1 \\ z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1.$$

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$$L_{L,e} = L_l^\dagger i \tilde{\tau}_\mu D_\mu L_l + e_r^\dagger i \tau_\mu D_\mu e_r - h_e [e_r^\dagger (\phi^\dagger L_l) + (L_l^\dagger \phi) e_r],$$

- ▶ where  $L_l = \begin{pmatrix} \nu_l \\ e_l \end{pmatrix} \in C_2$  is the  $SU(2)$ -doublet,  $e_r$  is the  $SU(2)$ -singlet,  $h_e$  is constant,  $\tau_0 = \tilde{\tau}_0 = \mathbf{1}$ ,  $\tilde{\tau}_k = -\tau_k$ ,  $\tau_\mu$  are the Pauli matrices,  $\phi \in C_2$  are the matter fields and  $e_r, e_l, \nu_l$  are the two component Lorentzian spinors.
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## Modified Model

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- ▶ We consider a model where the group  $SU(2) \times U(1)$  acts only in the boson and quark sectors, whereas **the group  $SU(2; j) \times U(1)$  acts in the lepton sector.**
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- ▶ The main idea is that **lepton fields belong to the fiber space**  $C_2(j)$  and are as follows

$$\begin{pmatrix} j\nu_e \\ e \end{pmatrix}, \begin{pmatrix} j\nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} j\nu_\tau \\ \tau \end{pmatrix} \in C_2(j).$$

- ▶ They are obtained from those of the Standard Model by transformation of the neutrino fields

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$$A_\mu^1 \rightarrow j A_\mu^1, \quad A_\mu^2 \rightarrow j A_\mu^2, \quad A_\mu^3 \rightarrow A_\mu^3, \quad B_\mu \rightarrow B_\mu.$$

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$$\begin{aligned} SU(2; j) \ni g(j) &= \exp \{ A_\mu^1(jT_1) + A_\mu^2(jT_2) + A_\mu^3 T_3 \} = \\ &= \exp \{ (j A_\mu^1) T_1 + (j A_\mu^2) T_2 + A_\mu^3 T_3 \}. \end{aligned}$$

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- ▶ For the new gauge fields these substitutions are as follows:

$$W_{\mu}^{\pm} \rightarrow jW_{\mu}^{\pm}, \quad Z_{\mu} \rightarrow Z_{\mu}, \quad A_{\mu} \rightarrow A_{\mu}.$$

- ▶ Let us stress that we can substitute the transformation of the generators by the transformation of the gauge fields **only in the lepton Lagrangian**, which is built with the help of  $SU(2; j) \times U(1)$  group. In the boson and quark Lagrangians gauge fields are not changed.

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$$\begin{aligned}
 L_{L,e}(j) = & e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l + e_r^\dagger i \tau_\mu \partial_\mu e_r - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l + \\
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- ▶ Lagrangian  $j^2 L_{e,f}$  describe free neutrino and its interactions with electron and gauge fields.
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- ▶ We put  $\phi = \phi^{vac} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$  in  $L_{L,e}$  and denote the electron mass as  $m_e = \frac{h_e v}{\sqrt{2}}$ .
- ▶ Next lepton generation fields are transformed in a similar way:

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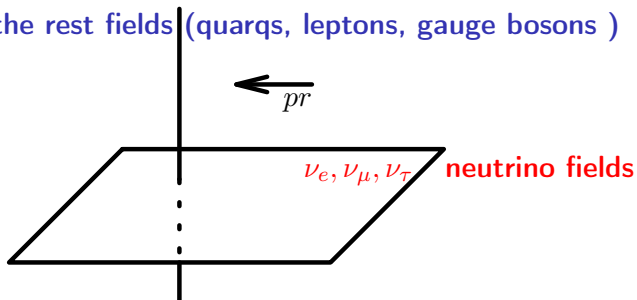
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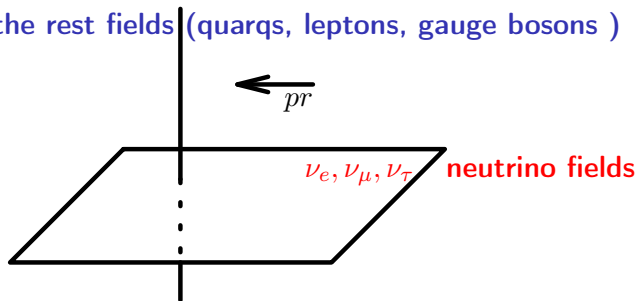
the rest fields (quarks, leptons, gauge bosons )



- ▶ From algebraical point of view the contribution of neutrino fields as well as their interactions with electron, muon and tau-lepton fields to the Lagrangian  $L_L(j)$  will be **vanishingly small** in comparison with the contribution of the rest fields .

- ▶ From geometrical point of view in the limit  $j \rightarrow 0$  or for  $j = \iota$  the **neutrino fields** are **in the fiber** and **does not act on the rest fields**, which are **in the base**.

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- ▶ From algebraical point of view the contribution of neutrino fields as well as their interactions with electron, muon and tau-lepton fields to the Lagrangian  $L_L(j)$  will be **vanishingly small** in comparison with the contribution of the rest fields .

- ▶ In general, ideal mathematical constructions are physically realized approximately with some errors.
- ▶ Suppose that contraction parameter  $j$  is small, but different from zero. The full Lagrangian of the model

$$L(j) = L_B + L_Q + L_L(j) = L_r + j^2 L_\nu$$

is split in two parts:

- ▶ the Lagrangian  $L_\nu$ , which includes neutrino fields along with their interactions with gauge and lepton fields and
- ▶ Lagrangian  $L_r$ , which includes all other fields.
- ▶ The neutrino fields part  $j^2 L_\nu$  turns out very small with respect to all other fields  $L_r$  due to the small parameter  $j$ .

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- ▶ It is well known, that the space-time of the special relativity is transformed to the nonrelativistic space-time when dimensionfull contraction parameter — velocity of light  $c$  — tends to the infinity and dimensionless parameter  $\frac{v}{c} \rightarrow 0$ .
- ▶ Weak interactions for low energies are characterized by the Fermi constant  $G_F$ . This constant is determined by experimental measurements and turn out to be very small  $G_F = 10^{-5} \frac{1}{m_p^2} = 1,17 \cdot 10^{-5} GeV^{-2}$ .
- ▶ Probability amplitude for weak current interactions which include two neutrino fields for example

$$\nu_\mu e^- \rightarrow \nu_e \mu^- \quad \text{or} \quad \nu_\mu e^- \rightarrow \nu_\mu e^- \quad (\text{elastic scattering})$$

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- ▶ If one introduces the dimensionfull constant  $G_0 = \frac{1}{m_p^2}$ , where  $m_p \approx 1 \text{ GeV}$  is the proton mass, then

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and **one can approximate the minimal value of the dimensionless parameter as  $j^2_{min} \approx 10^{-5}$ , when energy is measured in  $\text{GeV}$ .**

- ▶ **The linear dependence** of the neutrinos cross-section for energies  $\approx 1 \text{ GeV}$  **is directly coupled with the multiplication** of the probability amplitude for weak current interactions **by the factor  $j^2$ .**

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- ▶ **So the suggested model explain both experimental facts in the neutrino-matter interactions.**
- ▶ Let my remained that limit transition  $c \rightarrow \infty$  in special relativity was resulted in the notion of group contraction by E.P. Wigner and E. Inönü (1953).
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## Conclusion

In the Standard Electroweak Model all fields (gauge, quarks, leptons) belong to the same space  $C_2$ .

The replacement of  $C_2$  by the 3D spherical space  $S_3$  in boson sector eliminates Higgs boson.

(NAG, arXiv:0705.4575v1, arXiv:1009.1456v2, J. Phys.: Conf. Ser., 2008, v.128, 012005; 2011, v.284, 012033.)

The replacement of  $C_2$  by the space  $C_2(j)$  in lepton sector explain neutrino properties for low energies already at classical level.

(NAG, Komi SC Scientific Reports №512, Syktyvkar, 2010; arXiv:1010.5512v3.)

What about the replacement of  $C_2$  in quark sector

$$C_2 \rightarrow ?$$

This is the open question.

**Thank you for your attention.**