

Black Holes Tits -Satake Universality classes and Nilpotent Orbits

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Based on common work with Aleksander S. Sorin
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A well defined mathematical problem

Our goal is just to find and classify all spherical symmetric solutions of Supergravity with a static metric of Black Hole type

The solution of this problem is found by reformulating it into the context of a very rich mathematical framework which involves:

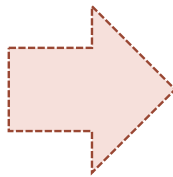
1. The Geometry of **COSET MANIFOLDS**
2. The theory of **Liouville Integrable systems** constructed on Borel-type subalgebras of **SEMISIMPLE LIE ALGEBRAS**
3. A very topical issue in contemporary **ADVANCED LIE ALGEBRA THEORY** namely:
 1. **THE CLASSIFICATION OF ORBITS OF NILPOTENT OPERATORS**

The N=2 Supergravity Theory



$$\begin{aligned}\mathcal{L}^{(4)} = & \sqrt{\det g} \left[-2R[g] - \frac{1}{6} \partial_{\hat{\mu}} \phi^a \partial^{\hat{\mu}} \phi^b h_{ab}(\phi) \right. \\ & \left. + \text{Im} \mathcal{N}_{\Lambda\Sigma} F_{\hat{\mu}\hat{\nu}}^{\Lambda} F^{\Sigma|\hat{\mu}\hat{\nu}} \right] \\ & + \frac{1}{2} \text{Re} \mathcal{N}_{\Lambda\Sigma} F_{\hat{\mu}\hat{\nu}}^{\Lambda} F_{\hat{\rho}\hat{\sigma}}^{\Sigma} \epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\end{aligned}$$

We have gravity
and
n vector multiplets



2 n scalars yielding n complex
scalars z^i

and n+1 vector fields \mathbf{A}^{Λ}

The matrix $\mathcal{N}_{\Lambda\Sigma}$ encodes together with the metric

h_{ab} Special Geometry

Special Kahler Geometry



Let $\mathcal{L} \rightarrow \mathcal{M}$ complex line bundle such that first Chern class equals Kähler form K . Let $\mathcal{SV} \rightarrow \mathcal{M}$ be a holomorphic flat vector bundle of rank $2n+2$ with structural group $\mathrm{Sp}(2n+2, \mathbb{R})$

$$\Omega = \begin{pmatrix} X^\Lambda \\ F_\Sigma \end{pmatrix} \quad \Lambda, \Sigma = 0, 1, \dots, n \quad \text{symplectic section}$$

$$i\langle \Omega | \bar{\Omega} \rangle \equiv i\Omega^T \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} \bar{\Omega}$$



$$K = \frac{i}{2\pi} \partial \bar{\partial} \log (i\langle \Omega | \bar{\Omega} \rangle)$$

Special Geometry identities



$$V = \begin{pmatrix} L^\Lambda \\ M_\Sigma \end{pmatrix} \equiv e^{\mathcal{K}/2} \Omega = e^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ F_\Sigma \end{pmatrix}$$

$$U_i = \nabla_i V = \left(\partial_i + \frac{1}{2} \partial_i \mathcal{K} \right) V \equiv \begin{pmatrix} f_i^\Lambda \\ h_{\Sigma|i} \end{pmatrix}$$

$$\bar{U}_{i^*} = \nabla_{i^*} \bar{V} = \left(\partial_{i^*} + \frac{1}{2} \partial_{i^*} \mathcal{K} \right) \bar{V} \equiv \begin{pmatrix} \bar{f}_{i^*}^\Lambda \\ \bar{h}_{\Sigma|i^*} \end{pmatrix}$$

$$\nabla_i V = U_i$$

$$\nabla_i U_j = i C_{ijk} g^{k\ell^*} U_{\ell^*}$$

$$\nabla_{i^*} U_j = g_{i^*j} V$$

$$\nabla_{i^*} V = 0$$

The matrix $N_{\Lambda\Sigma}$



the two $(n + 1) \times (n + 1)$ vectors

$$f_I^\Lambda = \begin{pmatrix} f_i^\Lambda \\ \bar{L}^\Lambda \end{pmatrix} \quad ; \quad h_{\Lambda|I} = \begin{pmatrix} h_{\Lambda|i} \\ \bar{M}_\Lambda \end{pmatrix}$$

$$\bar{N}_{\Lambda\Sigma} = h_{\Lambda|I} \circ \left(f^{-1} \right)_\Sigma^I$$

When the special manifold is a symmetric coset ..



$$SK_n = \frac{U_{D=4}}{H_{D=4}}$$

$$U_{D=4} \ni \mathbb{L}(\phi) \mapsto \left(\begin{array}{c|c} A(\phi) & B(\phi) \\ \hline C(\phi) & D(\phi) \end{array} \right) \in \text{Sp}(2n+2, \mathbb{R})$$

Symplectic embedding

$$\mathbf{f} = \frac{1}{\sqrt{2}} (A(\phi) - i B(f))$$

$$\mathbf{h} = \frac{1}{\sqrt{2}} (C(\phi) - i D(f))$$

$$\overline{\mathcal{N}}(\phi) = \mathbf{h} \mathbf{f}^{-1}$$



The main point

- 1) space-like p -branes as the cosmic billiards, or
- 2) time-like p -branes as several rotational invariant black-holes in $D = 4$ and more general solitonic branes in diverse dimensions

reduce to geodesic equations on coset manifolds of the type

$$\mathcal{M} = \frac{U}{H} \quad \text{or} \quad \mathcal{M}^* = \frac{U}{H^*} \simeq \exp [\text{Solv}_{\mathcal{M}}]$$

Dimensional Reduction to D=3



THE C-MAP

D=4 SUGRA with SK_n



D=3 σ -model on Q_{4n+4}

$$ds^2_Q = \frac{1}{4} \left[dU^2 + g_{i\bar{j}} dz^i d\bar{z}^{\bar{j}} + e^{-2U} (da + \mathbf{Z}^T \mathbb{C} d\mathbf{Z})^2 \mp 2 e^{-U} d\mathbf{Z}^T \mathcal{M}_4(z, \bar{z}) d\mathbf{Z} \right]$$

*Space red. / Time red.
Cosmol. / Black Holes*

$$\underbrace{\{U, a\}}_2 \cup \underbrace{\{z^i\}}_{2n} \cup \underbrace{\mathbf{Z} = \{Z^\Lambda, Z_\Sigma\}}_{2n+2} \quad 4n + 4 \text{ coordinates}$$

Gravity

scalars

From vector fields



$$\mathcal{M}_4 = \left(\begin{array}{c|c} \text{Im}\mathcal{N}^{-1} & \text{Im}\mathcal{N}^{-1} \text{Re}\mathcal{N} \\ \hline \text{Re}\mathcal{N} \text{Im}\mathcal{N}^{-1} & \text{Im}\mathcal{N} + \text{Re}\mathcal{N} \text{Im}\mathcal{N}^{-1} \text{Re}\mathcal{N} \end{array} \right)$$

SUGRA BH.s = one-dimensional Lagrangian model



Evolution parameter $\tau \sim \frac{1}{r}$ $\dot{f} \equiv \frac{d}{d\tau} f$

$$\mathcal{L} = \dot{U}^2 + h_{rs} \dot{\phi}^r \dot{\phi}^s + e^{-2U} (\dot{a} + \mathbf{Z}^T \mathbb{C} \dot{\mathbf{Z}})^2 + 2e^{-U} \dot{\mathbf{Z}}^T \mathcal{M}_4 \dot{\mathbf{Z}}$$

$$\mathcal{L} = \begin{cases} v^2 > 0 & \text{Time-like geodesic = non-extremal Black Hole} \\ v^2 = 0 & \text{Null-like geodesic = extremal Black Hole} \\ -v^2 < 0 & \text{Space-like geodesic = naked singularity} \end{cases}$$

A Lagrangian model can always be turned into a Hamiltonian one by means of standard procedures.

SO BLACK-HOLE PROBLEM = DYNAMICAL SYSTEM

FOR SK_n = symmetric coset space THIS DYNAMICAL SYSTEM is LIOUVILLE INTEGRABLE, always!

When homogeneous symmetric manifolds



$$\frac{U_{D=4}}{H_{D=4}} \rightarrow \frac{U_{D=3}}{H_{D=4}}$$

C-MAP

$$U_{D=3} \supset U_{D=4}$$

General Form of the Lie algebra decomposition

$$\text{adj}(U_{D=3}) = \text{adj}(U_{D=4}) \oplus \text{adj}(SL(2, \mathbb{R})_E) \oplus W_{(2, \mathbf{W})}$$

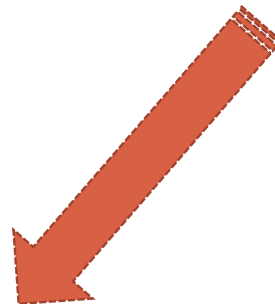
$$[T^a, T^b] = f^ab_c T^c$$

$$[L^x, L^y] = f^{xy}_z L^z,$$

$$[T^a, W^{iM}] = (\Lambda^a)^M_N W^{iN},$$

$$[L^x, W^{iM}] = (\lambda^x)^i_j W^{jM},$$

$$[W^{iM}, W^{jN}] = \epsilon^{ij} (K_a)^{MN} T^a + \mathbb{C}^{MN} k_x^{ij} L^x$$



Relation between

$$\frac{U}{H} \quad \text{and} \quad \frac{U}{H^*}$$

One just changes the sign of the scalars coming from $W_{(2,R)}$ part in:

$$\text{adj}(G_{D=3}) = \text{adj}(G_{D=4}) \oplus \text{adj}(SL(2, \mathbb{R})) \oplus W_{(2,R)}$$

where R is a **symplectic** representation of $G_{D=4}$

Examples

$$\begin{aligned} \frac{E_{8(8)}}{SO(16)} &\rightarrow \frac{E_{8(8)}}{SO(16)^*} \\ \frac{SO(4,4)}{SO(4) \times SO(4)} &\rightarrow \frac{SO(4,4)}{SO(2,2) \times SO(2,2)} \\ \frac{G_{(2,2)}}{SU(2) \times SU(2)} &\rightarrow \frac{G_{(2,2)}}{SU(1,1) \times SU(1,1)} \end{aligned}$$

The solvable parametrization

There is a fascinating theorem which provides an identification of the **geometry of moduli spaces** with **Lie algebras** for (almost) all supergravity theories.

THEOREM: All **non compact** (symmetric) coset manifolds are **metrically equivalent** to a **solvable group manifold**

$$U/H \cong \exp [Solv (U/H)]$$

Splitting the Lie algebra U into the maximal compact subalgebra H plus the orthogonal complement K

$$U = H \oplus K$$

- There are precise rules to construct **Solv(U/H)**
- Essentially **Solv(U/H)** is made by
 - the non-compact Cartan generators $H_i \in \mathbf{CSA} \cap \mathbf{K}$ and
 - those positive root step operators E^α which are not orthogonal to the non compact Cartan subalgebra $\mathbf{CSA} \cap \mathbf{K}$

The simplest example $G_{2(2)}$

One vector multiplet

$$\text{adj } \mathfrak{g}_{2(2)} = (\text{adj } [\mathfrak{sl}(2, \mathbb{R})_E] \ 1) \oplus (1, \text{adj } [\mathfrak{sl}(2, \mathbb{R})]) \oplus (2, 4)$$

$$g_{z\bar{z}} dz d\bar{z} = \frac{3}{4} \frac{1}{(\text{Im}z)^2} \partial^\mu z \partial_\mu \bar{z} \quad \text{Poincaré metric}$$

$$\Omega(z) = \begin{pmatrix} -\sqrt{3}z^2 \\ z^3 \\ \sqrt{3}z \\ 1 \end{pmatrix} \quad \text{Symplectic section}$$

$$\bar{N}_{\Lambda\Sigma}(z) = \begin{pmatrix} -\frac{3z+\bar{z}}{2z\bar{z}} & -\frac{\sqrt{3}(z+\bar{z})}{2z\bar{z}^2} \\ -\frac{\sqrt{3}(z+\bar{z})}{2z\bar{z}^2} & -\frac{z+3\bar{z}}{2z\bar{z}^3} \end{pmatrix} \quad \text{Matrix } N_{\Lambda\Sigma}$$

OXIDATION 1

The metric

$$ds_{(4)}^2 = -e^{U(\tau)} (dt + A_{KK})^2 + e^{-U(\tau)} \left[e^{4A(\tau)} d\tau^2 + e^{2A(\tau)} (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

where $A_{KK} = 2n \cos \theta d\varphi$

Taub-NUT charge

$$\underbrace{\left[e^{-2U} (\dot{a} + Z^\Lambda \dot{Z}_\Lambda - Z_\Sigma \dot{Z}^\Sigma) \right]}_{n = \text{Taub NUT charge}}$$

The electromagnetic charges

$$Q^M = \sqrt{2} \left[e^{-U} \mathcal{M}_4 \dot{Z} - n \mathbb{C} Z \right]^M = \begin{pmatrix} p^\Lambda \\ e_\Sigma \end{pmatrix}$$

From the σ -model viewpoint all these first integrals of the motion

$$e^{2A(\tau)} = \begin{cases} \frac{v^2}{\sinh^2(v\tau)} & \text{if } v^2 > 0 \\ \frac{1}{\tau^2} & \text{if } v^2 = 0 \end{cases}$$

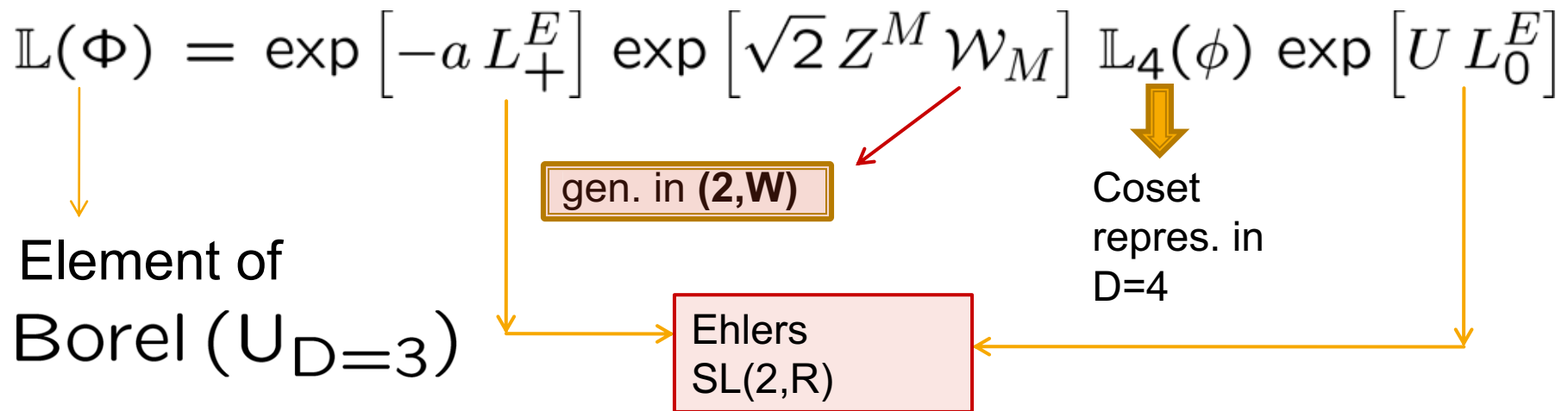
Extremality parameter

OXIDATION 2

The electromagnetic field-strengths

$$F^\wedge = 2 p^\wedge \sin \theta d\theta \wedge d\varphi + \dot{Z}^\wedge d\tau \wedge (dt + 2n \cos \theta d\varphi)$$

$U, a, \phi \sim z, Z^A$ parameterize in the G/H case the coset representative



From coset rep. to Lax equation

$$\Sigma(\tau) \equiv \mathbb{L}^{-1}(\tau) \frac{d}{d\tau} \mathbb{L}(\tau) \quad \text{From coset representative}$$

$$\Sigma(\tau) = L(\tau) \oplus W(\tau)$$

$$W(\tau) \in \mathbb{H}^* \Rightarrow \eta W^T(\tau) + W(\tau)\eta = 0 \quad \text{decomposition}$$

$$L(\tau) \in \mathbb{K} \Rightarrow \eta L^T(\tau) - L(\tau)\eta = 0$$

$$W(\tau) = L_{>}(\tau) - L_{<}(\tau) \quad \text{R-matrix}$$

$$\frac{d}{d\tau} L(\tau) = [W(\tau), L(\tau)] \quad \text{Lax equation}$$

Integration algorithm

Initial conditions $L_0 = L(0)$, $\mathbb{L}_0 = \mathbb{L}(0)$

Building block $\mathcal{C}(\tau) := \exp[-2\tau L_0]$

$$\mathfrak{D}_i(\mathcal{C}) := \text{Det} \begin{pmatrix} \mathcal{C}_{1,1}(\tau) & \dots & \mathcal{C}_{1,i}(\tau) \\ \vdots & \vdots & \vdots \\ \mathcal{C}_{i,1}(\tau) & \dots & \mathcal{C}_{i,i}(\tau) \end{pmatrix}, \quad \mathfrak{D}_0(\tau) := 1.$$

$$\left(\mathbb{L}(\tau)^{-1}\right)_{ij} \equiv \frac{1}{\sqrt{\mathfrak{D}_i(\mathcal{C})\mathfrak{D}_{i-1}(\mathcal{C})}} \text{Det} \begin{pmatrix} \mathcal{C}_{1,1}(\tau) & \dots & \mathcal{C}_{1,i-1}(\tau) & (\mathcal{C}(\tau)\mathbb{L}(0)^{-1})_{1,j} \\ \vdots & \vdots & \vdots & \vdots \\ \mathcal{C}_{i,1}(\tau) & \dots & \mathcal{C}_{i,i-1}(\tau) & (\mathcal{C}(\tau)\mathbb{L}(0)^{-1})_{i,j} \end{pmatrix}$$

Found by Fre & Sorin 2009 - 2010

Key property of integration algorithm

$$L(\tau) = Q(\mathcal{C}) L_0 (Q(\mathcal{C}))^{-1}$$

$$Q(\mathcal{C}) \in H^*$$

Hence all LAX evolutions occur within distinct orbits of H^*

Fundamental Problem: classification of ORBITS

The role of H^*

$$U_{D=3} \supset \left\{ \begin{array}{ll} H & \text{Max. comp. subgroup} \\ \text{and} & \\ H^* & \text{Different real form of } H \end{array} \right. \begin{array}{l} \text{COSMOL.} \\ \\ \text{BLACK} \\ \text{HOLES} \end{array}$$

In our simple $G_{2(2)}$ model

$$\mathbb{H}^* = \mathfrak{sl}(2, R) \oplus \mathfrak{sl}(2, R)$$

The algebraic structure of Lax

For the simplest model ,the Lax operator, is in the representation

$$\left(j = \frac{1}{2}\right) \times \left(j = \frac{3}{2}\right)$$

of $\mathfrak{sl}(2, R) \times \mathfrak{sl}(2, R)$

$$L \sim \Delta^\alpha | A$$

We can construct invariants and tensors with powers of L

Invariants & Tensors

$$\mathfrak{h}_6 = \frac{1}{6} \text{Tr} L^6 + \frac{1}{96} (\text{Tr} L^2)^3$$

$$\mathfrak{h}_2 = \frac{1}{4} \text{Tr} L^2$$

$$\left[\left(\mathbf{j} = \frac{3}{2} \right) \otimes \left(\mathbf{j} = \frac{3}{2} \right) \right]_{\text{symm}} = \underbrace{(\mathbf{j} = 3)}_7 \oplus \underbrace{(\mathbf{j} = 1)}_1$$

$$\left[\left(\mathbf{j} = \frac{3}{2} \right) \otimes \left(\mathbf{j} = \frac{3}{2} \right) \right]_{\text{antisym}} = \underbrace{(\mathbf{j} = 2)}_5 \oplus \underbrace{(\mathbf{j} = 0)}_1$$

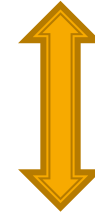
Quadratic Tensor

$$\mathcal{T}^{xy} \equiv \frac{128}{\sqrt{3}} t^{xy}_{AB} \Delta^{\alpha|A} \Delta^{\beta|B} \epsilon_{\alpha\beta}$$


Tensors 2

QUADRATIC
BIVECTOR

$$\mathcal{W}^{a|x} \equiv 1280 \Pi_{\alpha\beta}^a \Sigma_{AB}^z \Delta^{\alpha|A} \Delta^{\beta|B}$$



$$\left[\left(\mathbf{j} = \frac{3}{2} \right) \otimes \left(\mathbf{j} = \frac{3}{2} \right) \right]_{symm} = \underbrace{(\mathbf{j} = 3)}_7 \oplus \underbrace{(\mathbf{j} = 1)}_1$$

$$\left[\left(\mathbf{j} = \frac{3}{2} \right) \otimes \left(\mathbf{j} = \frac{3}{2} \right) \right]_{antisym} = \underbrace{(\mathbf{j} = 2)}_5 \oplus \underbrace{(\mathbf{j} = 0)}_1$$

Tensors 3

Hence we are able to construct
quartic tensors

$$\mathcal{Z}^{xy} = \mathcal{W}^{a|x} \mathcal{W}^{b|y} \eta_{ab}$$
$$\mathbb{T}^{ab} = \mathcal{W}^{a|x} \mathcal{W}^{b|y} \eta_{xy}$$

**ALL TENSORS, QUADRATIC and QUARTIC
are symmetric**

Their signatures classify orbits, both regular and nilpotent!

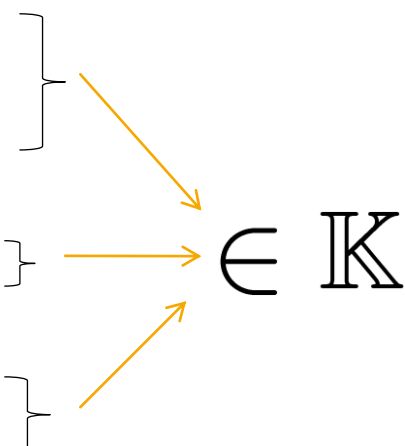
Tensor classification of orbits

Orbit	Order Nilp.	Stand. Repr.	Stab. subg.	Sign. \mathcal{T}^{xy}	Sign. \mathcal{I}^{xy}	Sign. \mathbb{T}^{ab}	Bivect. $W^{a x}$	\mathfrak{J}_4 at $\mathfrak{n} = 0$	Dim. $\mathfrak{n} = 0$ shell
Schw.	∞	\mathfrak{G}	$O(2)$	$\{+, +, +\}$	$\{+, 0, 0\}$	$\{+, 0, 0\}$	$\neq 0$	$\neq 0$	4
Dil.	∞	\mathfrak{D}	$O(1, 1)$	$\{-, -, +\}$	$\{-, 0, 0\}$	$\{-, 0, 0\}$	$\neq 0$	$\neq 0$	4
N01	2	\mathcal{L}_{NO_1}	$O(1, 1) \ltimes \mathbb{R}^2$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	0	0	2
N02	3	\mathcal{L}_{NO_2}	$O(1, 1) \ltimes \mathbb{R}$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\neq 0$	0	3
N03	3	\mathcal{L}_{NO_3}	\mathbb{R}	$\{0, 0, +\}$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\neq 0$	< 0	4
N03'	3	\mathcal{L}'_{NO_3}	\mathbb{R}	$\{0, 0, -\}$	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\neq 0$	< 0	4
N04	3	\mathcal{L}_{NO_4}	\mathbb{R}	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\{0, 0, +\}$	$\neq 0$	> 0	4
N04'	3	\mathcal{L}'_{NO_4}	\mathbb{R}	$\{0, 0, 0\}$	$\{0, 0, 0\}$	$\{0, 0, -\}$	$\neq 0$	> 0	4
N05	7	\mathcal{L}_{NO_5}	0	$\{0, +, -\}$	$\{0, 0, -\}$	$\{0, +, -\}$	$\neq 0$	< 0	5

How do we get to this classification? The answer is the following: by choosing a new Cartan subalgebra inside \mathfrak{H}^* and recalculating the step operators associated with roots in the new Cartan Weyl basis! Then applying the technique of **standard triples...!**

Relation between old and new Cartan Weyl bases

New Cartan Weyl generators	their form in the HK -basis
H_1	h_3
H_2	h_5
E_1	$\frac{1}{2\sqrt{2}}(k_3 - 3\mathcal{H}_1 - \mathcal{H}_2)$
E_2	$-\frac{1}{4}\sqrt{\frac{3}{2}}(k_1 + k_2 + k_4 - k_6)$
E_3	$\frac{1}{4\sqrt{2}}(-3h_1 + h_2 - h_4 + 3h_6)$
E_4	$\frac{1}{4\sqrt{2}}(-3k_1 + k_2 + k_4 + 3k_6)$
E_5	$\frac{1}{4}\sqrt{\frac{3}{2}}(-h_1 - h_2 + h_4 + h_6)$
E_6	$\frac{1}{2}\sqrt{\frac{3}{2}}(k_5 - \mathcal{H}_1 - \mathcal{H}_2)$



$$\begin{array}{ll}
 h_1 = e_2 + f_2 & k_1 = e_2 - f_2 \\
 h_2 = e_1 - f_1 & k_2 = e_1 + f_1 \\
 h_3 = e_3 + f_3 & k_3 = e_3 - f_3 \\
 h_4 = e_4 + f_4 & k_4 = e_4 - f_4 \\
 h_5 = e_5 + f_5 & k_5 = e_5 - f_5 \\
 h_6 = e_6 - f_6 & k_6 = e_6 + f_6
 \end{array}$$

The method of standard triplets

The basic theorem proved by mathematicians is that any nilpotent element of a Lie algebra $X \in \mathfrak{g}$ can be regarded as belonging to a triplet of elements $\{x, y, h\}$ satisfying the standard commutation relations of the $\mathfrak{sl}(2)$ Lie algebra, namely:

$$[h, x] = x \quad ; \quad [h, y] = -y \quad ; \quad [x, y] = 2h$$

Hence the classification of nilpotent orbits is just the classification of embeddings of an $\mathfrak{sl}(2)$ Lie algebra in the ambient one, modulo conjugation by the full group $G_{\mathbb{R}}$ or by one of its subgroups. In our case the relevant subgroup is $H^* \subset G_{\mathbb{R}}$.

Angular momenta

Embeddings of subalgebras $\mathfrak{h} \subset \mathfrak{g}$ are characterized by the branching law of any representation of \mathfrak{g} into irreducible representations of \mathfrak{h} . In the case of the $\mathfrak{sl}(2) \sim \mathfrak{so}(1, 2)$ algebra the branching law is expressed by listing the angular momenta $\{j_1, j_2, \dots, j_n\}$ of the irreducible blocks into which the fundamental representations decomposes.

$$\sum_{i=1}^n (2j_i + 1) = N$$

Partitions

- $(j=3)$ \Longrightarrow The largest orbit NO_5
- $(j=1, j=1/2, j=1/2)$ \Longrightarrow The orbit NO_2
- $(j=1, j=1, j=0)$ \Longrightarrow Splits into NO_3 and NO_4 orbits and their primed versions
- $(j=1/2, j=1/2, j=0, j=0, j=0)$ \Longrightarrow The smallest orbit NO_1

Results to appear in short

- Mario Trigiante will describe the classification of nilpotent orbits by means of α , β and γ – labels in the next talk.
- In a paper to appear in a couple of weeks we (Fre, Trigiante & Sorin) will:
 - Show that the classification of nilpotent orbits is a universality property, namely depends only on the **Tits Satake universality class** of the considered homogeneous special geometry.
 - Describe a computational algorithm of H^* orbits based on the **Weyl group** and certain **appropriate subgroups** thereof so far not yet introduced in the math literature

Спасибо за внимание

Thank you for your attention