# Kerr-Schild way to quantum gravity: a heterotic string beyond the Dirac electron

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**Outline:**

**i– Kerr-Newman background of the electron contains a closed heterotic string,**

**ii– Closed heterotic string as a source of the Dirac equation.**

**based on:**

**A.B.,** *Gravity vs. Quantum theory: Is electron really pointlike?* **arXiv:1104.0573;**

**A.B.,** *The Dirac–Kerr-Newman electron,* **Grav. Cosmol. 14, 109 (2008);**

**A.B.,** *Regularized KN Solution as Gravitating Soliton,* **J.Phys.A, 43, 392001 (2010).**

# KERR-SCHILD WAY TO QUANTUM GRAVITY.

**Resolution of the conflict between Quantum theory and Gravity is a way to their Unification. Against common conjectures**
**GRAVITY IS NOT WEAK AND VERY ESSENTIAL ON ALL LEVELS!**

**General covariance is main merit of General Relativity and main reason of the conflict: Quantum theory works in the local momentum space – Gravity requires global configuration space.**
**NO USUAL PLANE WAVES IN GRAVITY ! No Fourier transform!**

**PP-WAVES are consistent with Superstrings and Quantum Theory. No Quantum and Stringy corrections!**
**The Kerr-Schild form of metrics $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$ is rigidly linked with auxiliary Minkowski background $\eta_{\mu\nu}$, and $k_\mu$ is a null Killing direction. SINGULAR RAY – pole in the orthogonal to $k_\mu$ plane.**

**KERR–SCHILD GEOMETRY RESOLVES PRINCIPAL CONFLICT!**
**In spice of the rigidity, the KS metrics describe practically all the physically interesting solutions of General Relativity. Vector field $k^\mu$ is determined by the KERR THEOREM in twistor terms.**

**The curved KS spacetimes are foliating onto twistor null planes. Twistor version of the Fourier transform ! [E.Witten, 2003]**

**On the level of cosmic black holes Gravity turns** *elementary excitations of black-holes into twistor-beams* – **Singular beams supported by twistor lines of the Kerr congruence. Twistor-beams tend asymptotically into pp-waves, analogs of HETEROTIC STRINGS of the superstring theory.**

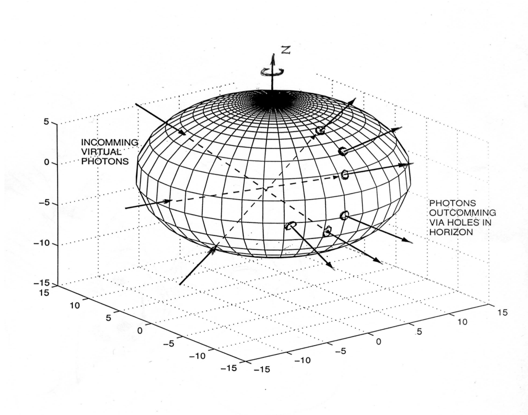



Figure 1: **Excitations of a black hole by weak electromagneticfield yields twistor-beams creating a horizon covered by fluctuating micro-holes.**

# KERR-NEWMAN ELECTRON.

**Quantum theory states that electron is pointlike and structureless.**
**"...There's no evidence that electrons have internal structure (and a lot of evidence against it)" (Frank Wilczek),**
**electron radius is "...most probably not much bigger and not much smaller than the Planck length.."(Leonard Susskind).**

**Kerr-Newman solution gives NEW DIMENSIONAL PARAMETER**

$$a = J/m.$$

**The "KN electron" is consistent with gravity and the low energy string theory!**

**Parameters of electron: mass, spin, charge and magnetic moment determine unambiguously parameters of the Kerr-Newman background!**

**Large spin of electron, $a = J/m >> m \Rightarrow$ black hole horizons disappear:**
**NAKED SINGULAR RING IS A SOURCE OF SPINNING PARTICLE.**
**The Kerr-Newman gravity: the metric is almost flat, but the spacetime has a topological peculiarity at the Compton distance $r_c = a = \frac{\hbar}{2m}$.**
**The Kerr ring forms a closed GRAVITATIONAL STRING of the Compton size. D.Ivanenko & AB (Izv. Vuzov. 1975), Excitations of the Kerr string are TRAVELING WAVES (AB, ZETP 1974).**

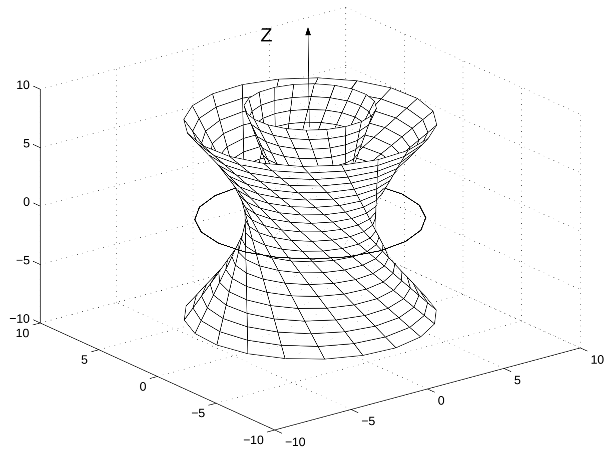
## The Kerr-Newman solution: Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \qquad H = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}, \tag{1}$$

**and electromagnetic (EM) vector potential is**

$$A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu. \tag{2}$$

**Gravitational and EM fields are concentrated near the Kerr singular ring.**




**The Kerr ring forms a sort of waveguide, or a closed string. It is a branch line of the TWOSHEETED Kerr-Schild geometry.**
**Vector field $k_\mu(x)$ is tangent to Principal Null Congruence (PNC).**

$$k_\mu dx^\mu = P^{-1}(du + \bar{Y}d\zeta + Yd\bar{\zeta} - Y\bar{Y}dv), \tag{3}$$

**where** $Y(x) = e^{i\phi} \tan \frac{\theta}{2}$ **is a projective angular coordinate, and**

$$\zeta = (x + iy)/\sqrt{2}, \ \bar{\zeta} = (x - iy)/\sqrt{2}, \ u = (z - t)/\sqrt{2}, \ v = (z + t)/\sqrt{2}$$

*are the null Cartesian coordinates.*

**Kerr congruence (PNC) is controlled by**

**KERR THEOREM:**

**The geodesic and shear-free Principal null congruences (type D metrics) are determined by holomorphic function** $Y(x)$ **which is analytic solution of the equation**

$$F(T^a) = 0 \ , \tag{4}$$

**where** $F$ **is an arbitrary analytic function of the**

***projective twistor coordinates***

$$T^a = \{Y, \quad \zeta - Yv, \quad u + Y\bar{\zeta}\}. \tag{5}$$

**The Kerr theorem is a practical tool for obtaining exact solutions:**

$$F(T^a) = 0 \Rightarrow F(Y, x^\mu) = 0 \Rightarrow Y(x^\mu) \Rightarrow k^\mu(x)$$

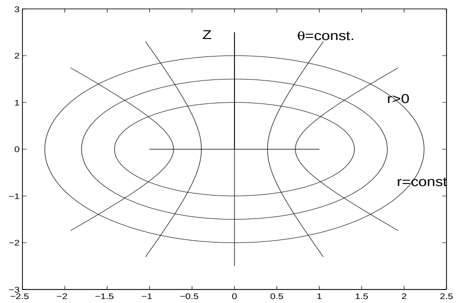
**For the Kerr-Newman solution function** $F$ **is quadratic in** $Y$**, which yields TWO roots** $Y^\pm(x) \Rightarrow$ **two congruences!**

The Kerr singular ring $r = \cos\theta = 0$ is a branch line of space on two sheets: "negative (–)" and "positive (+)" where the fields change their directions. In particular,

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \quad g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}. \tag{6}$$

**Twosheetedness! Mystery of the Kerr source!**

Kerr's oblate spheroidal coordinates $x + iy = (r + ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta$, cover spacetime twice: disk $r = 0$ separates the 'out'-sheet $r > 0$, from the 'in'-sheet $r < 0$.




*a) Stringy source:* AB 1974, D.Ivanenko & AB 1975, W.Israel 1977, Gravitational strings. Kerr's ring as 'Alice' string: a 'mirror gate' to 'Alice world'. STRINGS AS SOLITONS of the low energy SFT.

*b) Rotating superconducting disk (bubble):* W.Israel (1969), Hamity, I.Tiomno (1973), C.A. L'opez (1983)9; A.B. (1989,2000-2004) Regularized KN solution – GRAVITATING SOLITON (chiral Higgs model, AB 2010).

FUNDAMENTAL STRINGS as soliton like classical solutions in the effective field theory. Dabholkar at.al (NPB 1990).

Classical solutions in the effective string field theory may correspond to fields around a HETEROTIC STRING E. Witten (Phys.Lett.B 1985).

MACROSCOPIC CHARGED HETEROTIC STRING, A. Sen (NPB 1992-1993): bosonic zero modes of the four dimensional solutions in the effective field theory are in one to one correspondence to the bosonic degrees of freedom of heterotic string moving in four dimensions. PP-wave solutions. In particular, *critical heterotic string theory* in four dimensions with the extra six dimensions compactified.

Solutions to Einstein's eqs. are solutions of (super)string theory. PP-WAVES, Horowitz & Steif (PRL 1990), A. Tseytlin (PRD 1993).
Strings as Solitons & Black Holes as Strings Dabholkar at.al (NPB 1995).

Traveling waves as modes of string excitations, D.Garfinkel (PRD 1992).

Kerr-Sen solution to low energy string theory: The Kerr solution with axion and dilaton, A. Sen (PRL 1992).

The Kerr SINGULAR RING is a 'closed' heterotic string. The field around Kerr-Sen solution to low energy string theory is similar to the Sen solution for HETEROTIC STRING. AB (PRD 1995)

## How can it be related with the Dirac theory of electron?

### ZITTERBEWEGUNG

**The KN gravity indicates Compton radius of the Kerr string. Compton area plays also peculiar role in QED and in the Dirac theory as a limit of localization of the wave packet leading to "zitterbewegung".**

***"The variables $\alpha$ (velocity operators, AB) also give rise to some rather unexpected phenomena concerning the motion of the electron. .. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light"*, (P.A.M.Dirac in his Nobel Prize Lecture.)**

### CORPUSCULAR ASPECT OF TRAVELING WAVES

### Relation with the DIRAC EQUATION: Mass without mass

**Lightlike character of "zitterbewegung" suggests a corpuscular analogue with the string traveling waves and with Wheeler's model of "mass without mass": MASSLESS PARTICLE CIRCULATES IN (x,y)-PLANE.**

**The local 4-momentum is lightlike, $p_x^2 + p_y^2 + p_z^2 = E^2$, while the effective mass-energy was created by an averaged orbital motion,**

$$< p_x^2 > + < p_y^2 >= \tilde{m}^2. \tag{7}$$

**Averaging four-momentum $p^\mu$ under condition (7) yields**

$$< p_x^2 + p_y^2 + p_z^2 >= \tilde{m}^2 + p_z^2 = E^2. \tag{8}$$

**Quantum analog of this model corresponds to a wave function $\phi(\vec{x}, t)$ and operators, $\vec{p} \rightarrow \hat{\vec{p}} = -i\hbar\nabla, \quad \hat{E} = i\hbar\partial_t$. From () and (7) one obtains two wave equations:**

$$(\partial_x^2 + \partial_y^2)\phi = \tilde{m}^2\phi = (\partial_t^2 - \partial_z^2)\phi, \tag{9}$$

**which may be separated by the ansatz**

$$\phi = \mathcal{M}(x, y)\Phi_0(z, t). \tag{10}$$

**The RHS of (9) yields the usual equation for a massive particle, $(\partial_t^2 - \partial_z^2)\Phi_0 = \tilde{m}^2\Phi_0$, and the corresponding (de Broglie) plane wave solution**

$$\Phi_0(z, t) = \exp\frac{i}{\hbar}(zp_z - Et), \tag{11}$$

**while the LHS determines the "internal" structure factor**

$$\mathcal{M}_\nu = \mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho)\exp\{i\nu\varphi\}, \tag{12}$$

**in polar coordinates $\rho, \phi$, where $\mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho)$ are the Hankel functions of index $\nu$. $\mathcal{M}_\nu$ are eigenfunctions of operator $\hat{J}_z = \frac{\hbar}{i}\partial_\varphi$ with eigenvalues $J_z = \nu\hbar$. For electron we have $J_z = \pm\hbar/2, \quad \nu = \pm 1/2$, and the factor**

$$\mathcal{M}_{\pm 1/2} = \rho^{-1/2} \exp\{i(\frac{\tilde{m}}{\hbar}\rho \pm \frac{1}{2}\varphi)\} \tag{13}$$

**creates a singular ray along $z$-axis, which forms a branch line, and the wave function is twovalued.**

**Principal peculiarity of the obtained solution is that the de Broglie plane wave $\Psi_0$ appears as a MODULATION of a FUNDAMENTAL STRING formed by the singular vortex solution $\mathcal{M}$.**

**There are diverse generalizations of this solution.**

**THE CORRESPONDING SPINOR MODEL.**

**The massless spinor equation $i\gamma^\mu\partial_\mu\psi = 0$ has the solutions**

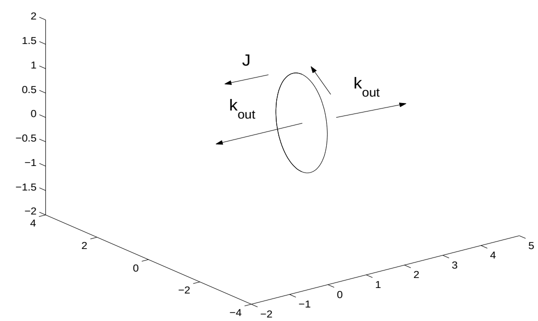
**$\psi = \gamma^\mu\partial_\mu\phi = \psi_0\mathcal{M}(x,y) + \Phi_0(z,t)\mathcal{M}_D$ where the spinor plane wave $\psi_0 = \gamma^\mu\partial_\mu\Phi_0(z,t)$ satisfies the usual massive Dirac equation $i\gamma^\mu\partial_\mu\psi_0 = m\psi_0$, while the spinor 'formfactor' $\mathcal{M}_D = \gamma^\mu\partial_\mu\mathcal{M}(x,y)$ satisfies the spinor constraints $(\gamma^x\partial_x + \gamma^y\partial_y)\mathcal{M}_D = m\mathcal{M}_D$, similar to LHS of (9).**

## Physical picture and exact solutions for excitations:
## THE KERR RING AS A CLOSED LIGHTLIKE STRING

Kerr string forms a waveguide for the lightlike traveling waves – a wave analogue of the dual corpuscular picture of the circulating light-like particle. Analogue of the Wheeler 'geon' model of 'mass without mass' [”microgeon with spin”, AB (1969 – 1974)].

Gravitational strings [D. Ivanenko and AB (1975), AB (1995)] .

Kerr's string carries the LIGHTLIKE current and the lightlike excitations: TRAVELING waves [AB, ZETP (1974)]. The field structure of the Kerr ring is close to that of the Sen heterotic string [AB, Phys.Rev. D (1995)]. Exact KS solutions for the electromagneticexcitations [AB (2009)] show that there appear two mutually antipodal 'axial' heterotic half-strings which are coupled topologically with the Kerr closed heterotic string.



**ELECTRON AS A GRAVITATING SOLITON (BUBBLE MODEL).**
**Alternate resolution of Kerr's twosheetedness is to cover the Kerr singular ring by a bubble with flat interior. Evolution of the Israel-Hamity-Lopez models for four decades [AB, 2010].**

**REGULARIZATION of the KN solution by phase transition from the external KN 'vacuum state', $\mathbf{V_{ext} = 0}$, to a flat internal 'pseudovacuum' state, $\mathbf{V_{int} = 0}$. The $\mathbf{U(1) \times \tilde{U}(1)}$ chiral field model of phase transition.**
**Flat interior of the bubble determines unambiguously form of the bubble boundary, $H = 0 \Rightarrow \mathbf{r = r_e = e^2/(2m)}$ – a relativistically rotating oblate disk of the Compton radius $\mathbf{r_c \approx a = \hbar/(2m)}$ with the thickness $r_e$.**

**Interior of the bubble is formed by the Higgs condensate $\Phi = |\Phi| \exp\{i\chi\}$, and electromagnetic Kerr-Newman field $A_\mu$ is regularized by the Higgs mechanism of broken symmetry, similarly to other solitonic models of the electroweak theory. Main equations $\Box A_\mu = I_\mu = e|\Phi|^2(\chi,_\mu + eA_\mu)$.**
**Inside the bubble, $|\Phi| > 0, \quad I_\mu = 0$, we have**

$$\Box A_\mu = 0, \quad \chi,_\mu + eA_\mu = 0, \tag{14}$$

**which shows that gradient of the phase of the Higgs field $\chi,_\mu$ "eats up" the KN electromagnetic(EM) field $A_\mu$, expelling the field strength and currents to the string-like boundary of the bubble.**

**The Kerr-Newman gravitating soliton exhibits essential peculiarities:**

**(i) the Kerr ring is regularized, forming a closed relativistic string of the Compton radius $r_c$ on the border of disklike bubble,**

**(ii) the KN electromagnetic potential forms on the perimeter a quantized loop $\oint eA_\varphi d\varphi = -4\pi ma$, which results in quantization of the soliton spin, $J = ma = n\hbar/2, \ n = 1, 2, 3, ...$,**

**(iii) the Higgs condensate forms a coherent vacuum state oscillating with the frequency $\omega = 2m$ – oscillon,**

**The Kerr-Newman gravitating soliton forms a background for excitations of the Kerr string.**

**EXPERIMENT: In spite of the very large Compton size of the bubble, the inner false vacuum will not interact with the particles of high energy. The high energy scattering will have only a pointlike contact interaction with the string. Neither shape of the string nor its extension can be recognized. To recognize the string as a whole, the experiment should be based on the wavelengths comparable with the extension of the string. It is necessary a relatively low-energy, resonance scattering.**

## THANK YOU FOR ATTENTION!

## Consequence of the twosheetedness.

$$k^{\mu(+)} \neq k^{\mu(-)} \quad \Rightarrow \quad g^{(+)}_{\mu\nu} \neq g^{(-)}_{\mu\nu}, \quad A^{\mu(+)} \neq A^{\mu(-)}$$

**Exact solutions demand ALIGNMENT of the electromagnetic field to the Kerr congruences,** $A_\mu k^\mu = 0$ !

**It does not allow to mix the** $A^{\mu(+)}$ **and** $A^{\mu(-)}$ **solutions, which is ignored in perturbative approach.**

**As a result, the exact solutions differ drastically from perturbative ones!**

**Algebraically degenerated solutions of the Einstein-Maxwell equations.**

**The the Einstein-Maxwell equations for the metrics of the KS class** $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, **were integrated out and reduced to the system of the ordinary differential equations (Debney, Kerr and Schild 1969).**

**Notations: the null tetrad $e^a, a = 1,2,3,4$, where $e^{3\mu} = Pk^\mu$ is real and aligned with the PNC, $\partial_a \equiv (.),_a \equiv e^\mu_a \partial_\mu$ - tetrad derivatives.**

**Electromagnetic field is determined by functions $A$ and $\gamma$,**

$$A,_2 - 2Z^{-1}\bar{Z}Y,_3 A = 0, \quad A,_4 = 0, \tag{15}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma,_2 - Z^{-1}Y,_3 \gamma = 0. \tag{16}$$

**Tetrad components of the electromagnetic strength (autodual) $\mathcal{F}_{ab} = e^\mu_a e^\nu_b \mathcal{F}_{\mu\nu}$, $\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ),_1$ , where $PZ^{-1} = dF/dY$.**

**Gravitational sector: has two equations for function $M$, which take into account the action of electromagnetic field**

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = A\bar{\gamma}\bar{Z}, \tag{17}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}. \tag{18}$$

**where $\mathcal{D} = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2$ .**

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \quad H = \frac{P^2}{2}[M(Z + \bar{Z}) - A\bar{A}Z\bar{Z}], \quad T^{rad}_{\mu\nu} = \frac{1}{2}\bar{\gamma}\gamma k_\mu k_\nu.$$

**In DKS-work integration was restricted by the case $\gamma = 0$, corresponding to the non-radiative, stationary solutions, in fact, restricted by the Kerr-Newman solution.**

**Stationary case $\gamma = 0$** $\Rightarrow$ **no radiation** $T^{rad}_{\mu\nu} = \frac{1}{2}\bar{\gamma}\gamma k_\mu k_\nu = 0.$

**The black-hole is at rest** $\Rightarrow P = 2^{-1/2}(1 + Y\bar{Y}), \quad Y_{,3} = -ZP_Y/P, \quad \mathcal{D}A = \mathcal{D}M = 0,$
$-PZ^{-1} = r + ia\cos\theta$ **is a complex radial distance.**

**Strong reduction of the equations.**

**Explicit solution:** $A = \psi(Y)/P^2, \quad M = m/P^3$, **vector potential** $A^\mu = -\frac{1}{2}Re[\frac{\psi}{r+ia\cos\theta}]k^\mu$, **and metric**

$$H = \frac{mr - \psi\bar{\psi}/2}{r^2 + a^2\cos^2\theta}, \quad g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu,$$

**KERR-NEWMAN SOLUTION:** $\psi(Y) = constant = e$ **– the charge of black hole. One sees that the Kerr-Newman solution is not unique.**

**Integration shows that** $\psi(Y)$ **may be an arbitrary holomorphic function of** $Y(x)$.

$Y = e^{i\phi}\tan\frac{\theta}{2}$ **is a projective coordinate on celestial sphere** $S^2$**, and there is infinite set of the exact solutions, in which** $\psi(Y)$ **is singular at the set of points** $\{Y_i = e^{i\phi_i}\tan\frac{\theta_i}{2}\}$,

$$\psi(Y) = \sum_i \frac{q_i}{Y(x) - Y_i},$$

**corresponding to angular directions** $\phi_i, \ \theta_i$.

**Twistor-beams.**

**Poles at** $Y_i$ **produce semi-infinite singular electromagnetic beams, supported by twistor rays of the Kerr congruence. One sees that the electromagnetic twistor-beams have very strong back-reaction on metric.**

**Integration of the non-stationary DKS case $\gamma \neq 0$,** **[A.B. (2004-2009)]** *shows that typical time-dependent (type D) solutions contain outgoing twistor-beam pulses which have very strong back reaction on metric and perforate horizon.*

**There appears a semiclassical Kerr-Schild geometry formed by fluctuating twistor-beams. It should still be averaged to get the usual classical gravity, and therefore it takes an intermediate position between Classical and Quantum gravity.**

**Any interaction of the black hole with external,** *even very weak,* **electromagnetic field results in the creation of a twistor-beams pulse.**

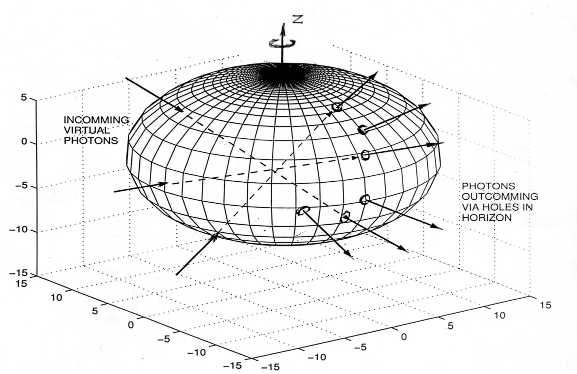



Figure 2: **The horizon covered by fluctuating micro-holes.**

## Complex Shift and Complex Structure of the Kerr geometry.

**Appel solution 1887!**

**A point-like charge $e$, placed on the complex z-axis $(x_0, y_0, z_0) = (0, 0, -ia)$, gives a real potential**

$$\phi_a = Re\ e/\tilde{r}, \tag{19}$$

**where $\tilde{r}$ turns out to be a complex radial coordinate $\tilde{r} = r + i\xi$, and we obtain $\tilde{r}^2 = r^2 - \xi^2 + 2ir\xi = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$, which leads to two equations**

$$r\xi = az, \quad r^2 - \xi^2 = x^2 + y^2 + z^2 - a^2, \tag{20}$$

**corresponding to the Kerr oblate spheroidal coordinates $r$ and $\theta$.**

**Starting from the usual system of angular coordinates, we would like to retain after complex shift the relation $z = r\cos\theta$, and obtain $\xi = a\cos\theta$,**

$$\tilde{r} = r + ia\cos\theta, \tag{21}$$

**and the equation $(r^2 + a^2)\sin^2\theta = x^2 + y^2$, which may be split into two complex conjugate equations. This splitting**

$$(x \pm iy) = (r \pm ia)e^{\pm i\phi}\sin\theta \tag{22}$$

**yields the Kerr-Schild Cartesian coordinate system.**

## Complex Light Cone. Twistors as Null Planes.

**Complex Kerr-Schild null tetrad $e^a$, $\quad (e^a)^2=0$: real directions**

$$e^3 = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv$$

**(PNC) and**

$e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv, \quad e^4 = dv + h e^3.$

**The complex light cone with the vertex at some complex point $x_0^\mu \in CM^4$: $(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0$, can be split into two families of null planes: "left" planes**

$$x_L^\mu = x_0^\mu(\tau) + \alpha e^{1\mu} + \beta e^{3\mu} \tag{23}$$

**spanned by null vectors $e^1$ and $e^3$, and"right"planes**

$$x_R^\mu = x_0^\mu(\tau) + \alpha e^{2\mu} + \beta e^{3\mu}, \tag{24}$$

**spanned by null vectors $e^2$ and $e^3$.**

**The Kerr congruence $\mathcal{K}$ arises as a real slice of the family of the "left" null planes ($Y = const.$) of the complex light cones whose vertices lie at a complex source $x_0(\tau)$.**

## Complex Source of the Kerr geometry and Retarded Time

**Appel's complex shift** $(x_o, y_o, z_o) \rightarrow (0, 0, -ia)$ **[Appel 1887!].**

**Kerr's source can be considered as a mysterious "particle" propagating along a** *complex world-line* $x_0^\mu(\tau)$ **in** $CM^4$, **parametrized by a complex time** $\tau$. **There appears a complex retarded-time construction (Newman, Lind.) In the complex case there are two different ways for obtaining retarded time. For a given real point** $x \in M^4$ **one considers the past light cone to obtain the root of its intersection with a given complex world-line** $x_0(\tau)$. **It is known that a LIGHT CONE splits into**

**the left:**

$\swarrow^{x}$

**and right:** ${}^{x}\searrow$ **complex null planes, which are spanned, correspondingly, on the null forms** $e^3 \wedge e^1$ **and** $e^3 \wedge e^2$.

**Correspondingly, there are two roots:**

${}_{x_0}\swarrow^{x}$ **and** ${}^{x}\searrow_{x_0}$ ,

**and two different (in general case) retarded times**

${}_{\tau_0}\swarrow^{t}$ **and** ${}^{t}\searrow_{\tau_0}$

**for the same complex world-line: the 'Left' and 'Right' projections.**

**The real Kerr-Schild geometry appears as a real slice of this complex structure.** **This construction may be super-generalized by 'super-complex translation', leading to super Kerr-Newman solution to broken N=2 supergravity [AB, Clas.Q. Grav.,2000].**

**Along with the considered complex world-line (say 'Left'), there is a complex conjugate world-line, $X_L(\tau_L)$ and $X_R(\tau_R)$.**

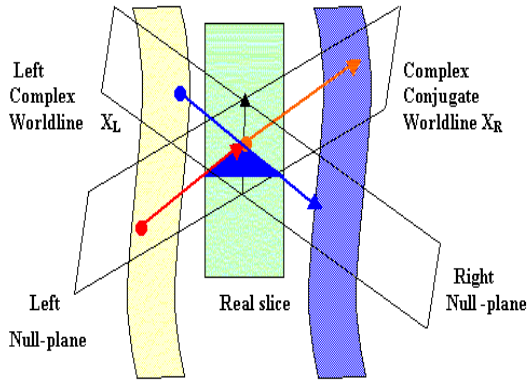



Figure 3: Complex light cone at a real point $x$. The adjoined to congruence Left and Right complex null planes. Four roots: $X_L^{adv}$, $X_L^{ret}$ and $X_R^{adv}$, $X_R^{ret}$ which are related by crossing symmetry.

**Complex world-line forms a world-sheet of an open Euclidean string $X_L(\tau_L) \equiv X_L^\mu(t_L + i\sigma_L)$ with the ends at $\sigma = \pm a$. Left and Right structures form an Orientifold ($\Omega = Antip.map + CC + Revers\ of\ time$).**

**Antipodal map: $Y \to -1/\bar{Y}$.**

## Twistor-Beams. The exact time-dependent KS solutions.

**Debney, Kerr and Schild (1969). The black-hole at rest:** $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, $P = 2^{-1/2}(1 + Y\bar{Y})$.

**Tetrad components of electromagnetic field** $\mathcal{F}_{ab} = e^\mu_a e^\nu_b \mathcal{F}_{\mu\nu}$,

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}\,, \tag{25}$$

**here** $Z = -P/(r + ia\cos\theta)$ **is a complex expansion of the congruence. Stationarity** $\Rightarrow \gamma = 0$.

**Kerr-Newman solution is exclusive:** $\psi(Y) = const.$

**In general case** $\psi(Y)$ **is an arbitrary holomorphic function of** $Y(x) = e^{i\phi}\tan\frac{\theta}{2}$, **which is a projective coordinate on celestial sphere** $S^2$,

$$A = \psi(Y)/P^2, \tag{26}$$

**and there is** *infinite set of the exact solutions*, **in which** $\psi(Y)$ **is singular at the set of points** $\{Y_i\}$, $\psi(Y) = \sum_i \frac{q_i}{Y(x) - Y_i}$, **corresponding to angular directions** $\phi_i,\ \theta_i$.

**Twistor-beams. Poles at** $Y_i$ **produce semi-infinite singular** *lightlike beams*, **supported by twistor rays of the Kerr congruence. The twistor-beams have very strong backreaction to metric** $g^{\mu\nu} = \eta^{\mu\nu} - 2Hk^\mu k^\nu$, **where**

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2\cos^2\theta}. \tag{27}$$

**How act such beams on the BH horizon?**

*Black holes with holes in the horizon*, **A.B., E.Elizalde, S.R.Hildebrandt and G.Magli, Phys. Rev. D74 (2006) 021502(R)**

**Singular beams lead to formation of the holes in the black hole horizon, which opens up the interior of the "black hole" to external space.**

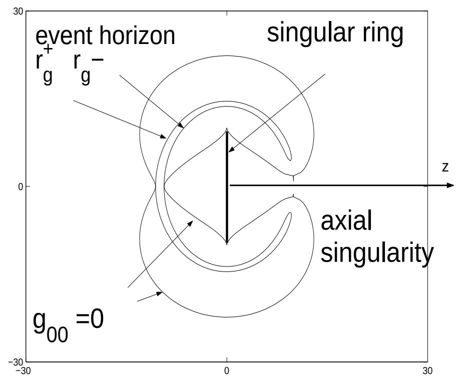



Figure 4: Near extremal black hole with a hole in the horizon. The event horizon is a closed surface surrounded by surface $g_{00} = 0$.

***Twistor-beams*** **are exact stationary and time-dependent Kerr-Schild solutions (of type D) which show that 'elementary' electromagneticexcitations have generally** ***singular beams supported by twistor null lines.*** **Interaction of a black-hole with external,** ***even very weak,*** **electromagnetic field resulted in appearance of the beams, which have very strong back reaction to metric and horizon and form a fine-grained structure of the horizon pierced by fluctuating microholes.** [**A.B., E. Elizalde, S.R. Hildebrandt and G. Magli, Phys.Lett. B 671 486 (2009), arXiv:0705.3551[hep-th]; A.B., arXiv:gr-qc/0612186.**]

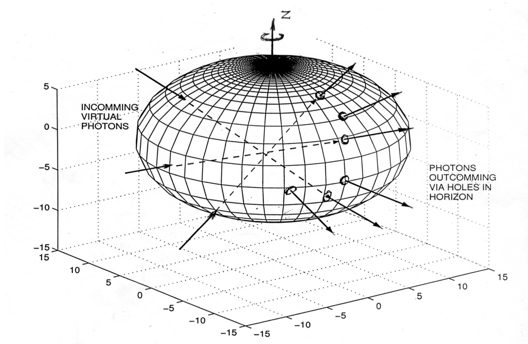



Figure 5: Excitations of a black hole by weak electromagneticfield yields twistor-beams creating a horizon covered by fluctuating micro-holes.

## CONCLUSIONS:

**– As a consequence of the Einstein-Maxwell field equations, the experimentally observable, mass, spin, charge and magnetic momentum of the electron lead unambiguously to the conclusion that the electron background has to be described by the Kerr-Newman gravitational field.**

**– Topology of the Minkowski background should be broken in the Compton zone of the electron by the Kerr singular ring with the appearance of the twosheeted Kerr spacetime.**

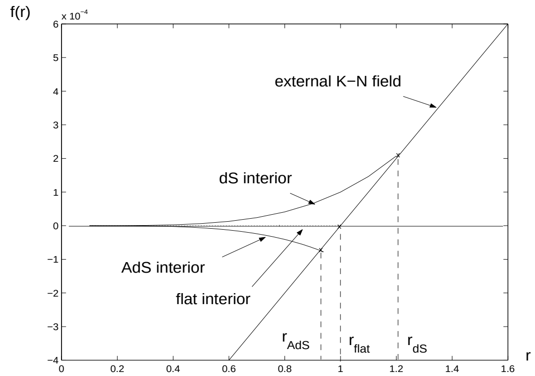
**– The regularized Kerr-Newman solution forms a GRAVITATING SOLITON, a thin rotating disk (membrane) spanned by the Kerr ring. 'Material' of the disk represents a COHERENT pseudovacuum state of the chiral Higgs field with a closed relativistic string on the perimeter of the Compton region.**

**– Wave function of the electron corresponds to the oscillating Higgs field forming a superconducting false-vacuum state inside the disklike bubble.**

**– It is assumed that the Kerr string may be detected by the novel experimental approach – the "non-forward Compton scattering."**

## THANK YOU FOR YOUR ATTENTION!

**Gravitational sector of the model is described by the Gürses–Gürsey form of metric, $g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu$, where $H = f(r)/(r^2 + a^2\cos^2\theta)$. It allows one to match smoothly the rotating metrics of different types! For $f_{int} = \alpha r^4$, the Kerr singularity is suppressed: a regular rotating internal space-time with the constant curvature $R = -24\alpha$,(A.B. 2002).**




**The KN source is formed by a rotating oblate BUBBLE ($r$ is the oblate spheroidal coordinate). We use the bubble with flat interior, $\alpha = 0$, which corresponds to parameters of the López model, $r = r_e = e^2/2m$, however the interior of the bubble is not empty, but filled by Higgs field interaction with the Kerr-Newman EM field.**

# Chiral sector.

**Phase transition from external KN solution to internal vacuum state is determined by a supersymmetric chiral model with three chiral fields $\Phi^i$.**

**The potential $V$ is determined via superpotential $W$ by the relation**
$V(r) = \sum_i |\partial_i W|^2$, **where** $\partial_1 = \partial_\Phi,\ \partial_2 = \partial_Z,\ \partial_3 = \partial_\Sigma.$

**Superpotential**

$$W = \lambda Z(\Sigma\bar{\Sigma} - \eta^2) + (cZ + \mu)\Phi\bar{\Phi}, \tag{28}$$

**produces a domain wall interpolating between the internal $V^{(int)} = 0$ and external vacuum states $V^{(ext)} = 0$, determined by the condition**
$\partial_i W = 0.$

**We will use the thin wall approximation, assuming that the depth of wall $\xi$, is much smaller than its position $r_0$, which yields**

**Int vacuum state, for $r < r_0$,** $\quad V^{(int)} = 0$ **:** $\quad Z = -\mu/c;\ \Sigma = 0;\ |\Phi| = \eta\sqrt{\lambda/c}.$

**Ext vacuum state, for $r > r_0$,** $\quad V^{(ext)} = 0$ **:** $\quad Z = 0;\ \Phi = 0;\ \Sigma = \eta.$

**The EM – Higgs sector is described by the usual Higgs Lagrangian,**

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\mathcal{D}_\mu\Phi\bar{\mathcal{D}}^\mu\bar{\Phi} + V, \tag{29}$$

**where** $\mathcal{D}_\mu = \nabla_\mu + ieA_\mu$; $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$ ; $\Phi = \Phi_0 \exp\{i\chi\}$, **leading to equations**

$$\Box A_\mu = I_\mu = e|\Phi|^2(\chi,_\mu + eA_\mu). \tag{30}$$

**Potential** $V$ **provides the phase transition from external KN solution, where** $\Phi = 0$, **to superconducting internal state** $I_\mu = 0$, **but** $|\Phi| = \Phi_0 > 0$.

**The vector-potential**

$$A_\mu dx^\mu|_r = \frac{-er}{r^2 + a^2\cos^2\theta}[dt + a\sin^2\theta d\phi] + \frac{2erdr}{(r^2 + a^2)} \tag{31}$$

**increases, approaching the boundary of bubble** $r = r_0 = e^2/2m$, **and in the equatorial plane,** $\cos\theta = 0$, **it reaches the magnitude** $A_\mu^{(edge)}dx^\mu = -\frac{2m}{e}[dt + ad\phi] + \frac{2er_0dr}{(r_0^2+a^2)}$. **The directions** $A_\mu^{(edge)}$ **in the equatorial plane are tangent to the Kerr singular ring and form a closed loop at the edge. The Wilson loop integral**

$$S^{(edge)} = \oint_{(edge)} A_\mu(x)dx^\mu = \oint eA_\phi^{(edge)}d\phi = -4\pi ma = -4\pi J \tag{32}$$

**turns out to be proportional to the KN angular momentum.**

**The Higgs field $\Phi(x) = \Phi_0 e^{i\chi(x)}$ expels the electromagnetic field and current from the bulk of the superconducting bubble, and we should set $I_\mu = 0$ for $r < r_0$. It gives the internal solution**

$$\chi_{,\mu} = -eA_\mu^{(in)}, \tag{33}$$

**as a full differential, and the second equation $\Box A_\mu^{(in)} = 0$ is satisfied automatically. Taking the Higgs phase in general form $\chi = \omega t + n\phi + \chi_1(r)$, one obtains from (33) the internal solution**

$$A_0^{(in)} = -\frac{\omega}{e}; \;\; A_\phi^{(in)} = -\frac{n}{e}; \;\; A_r^{(in)} = \chi_1'(r)/e. \tag{34}$$

**Matching the edge field with internal one, we obtain**

$$\omega = 2m; \;\; J = ma = n/2; \;\; \chi_1(r) = -\ln(r^2 + a^2), \tag{35}$$

**and therefore**

$$\Phi(x) = \Phi_0 \exp\{i\chi\} = \Phi_0 \exp\{i2mt - i\ln(r^2 + a^2) + in\phi\}. \tag{36}$$

# THANK YOU FOR ATTENTION!

**Singular pp-wave solutions (A.Peres)**

**Self-consistent solution of the Einstein-Maxwell equations: singular plane-fronted waves (pp-waves). Kerr-Schild form of metric** $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$ **with a constant vector** $k_\mu = \sqrt{2}du = dz - dt$.
**Function** $h$ **determines the Ricci tensor**

$$R^{\mu\nu} = -k^\mu k^\nu \Box h, \tag{37}$$

**where** $\Box$ **is a flat D'Alembertian**

$$\Box = 2\partial_\zeta \partial_{\bar\zeta} + 2\partial_u \partial_v \ . \tag{38}$$

**The Maxwell equations take the form** $\Box\mathcal{A} = J = 0$, **and can easily be integrated leading to the solutions**

$$\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar\zeta)] f^+(u) du, \tag{39}$$

**where** $\Phi^\pm$ **are arbitrary analytic functions, and function** $f^+$ **describes retarded waves.**

**The poles in** $\Phi^+(\zeta)$ **and** $\Phi^-(\bar\zeta)$ **lead to the appearance of singular lightlike beams (pp-waves) which propagate along the** $z^+$ **semi-axis.**

**PP-waves have very important quantum properties, being exact solutions in string theory with vanishing all quantum corrections [ G.T. Horowitz, A.R. Steif, PRL 64 (1990) 260; A.A. Coley, PRL 89 (2002) 281601.]**

**Quadratic generating function F(Y) and interpretation of parameters. [A.B. and G. Magli, Phys.Rev.D 61044017 (2000)].**
**The considered in DKS function $F$ is quadratic in $Y$,**

$$F \equiv a_0 + a_1 Y + a_2 Y^2 + (qY + c)\lambda_1 - (pY + \bar{q})\lambda_2, \tag{40}$$

**where the coefficients $c$ and $p$ are real constants and $a_0, a_1, a_2, q, \bar{q}$, are complex constants. The Killing vector of the solution is determined as**

$$\hat{K} = c\partial_u + \bar{q}\partial_\zeta + q\partial_{\bar{\zeta}} - p\partial_v. \tag{41}$$

**Writing the function F in the form**

$$F = AY^2 + BY + C, \tag{42}$$

**one can find two solutions of the equation $F = 0$ for the function $Y(x)$**

$$Y_{1,2} = (-B \pm \Delta)/2A, \tag{43}$$

**where $\Delta = (B^2 - 4AC)^{1/2}$.**

**We have also**

$$\tilde{r} = -\partial F/\partial Y = -2AY - B, \tag{44}$$

**and consequently**

$$\tilde{r} = PZ^{-1} = \mp\Delta. \tag{45}$$

**These two roots reflect the known twofoldedness of the Kerr geometry. They correspond to two different directions of congruence on positive and negative sheets of the Kerr space-time. In the stationary case**

$$P = pY\bar{Y} + \bar{q}\bar{Y} + qY + c \ . \tag{46}$$

*Link to the complex world line of the source.* **The stationary and boosted Kerr geometries are described by a straight complex world line with a real 3-velocity $\vec{v}$ in $CM^4$:**

$$x_0^\mu(\tau) = x_0^\mu(0) + \xi^\mu \tau; \qquad \xi^\mu = (1, \vec{v}) \ . \tag{47}$$

**The gauge of the complex parameter $\tau$ is chosen in such a way that $Re\ \tau$ corresponds to the real time $t$.**

$\hat{K}$ **is a Killing vector of the solution**

$$\hat{K} = \partial_\tau x_0^\mu(\tau)\partial_\mu = \xi^\mu \partial_\mu \ . \tag{48}$$

$$P = \hat{K}\rho = \partial_\tau x_0^\mu(\tau) e^3_\mu \ , \tag{49}$$

**where**

$$\rho = \lambda_2 + \bar{Y}\lambda_1 = x^\mu e^3_\mu . \tag{50}$$

**It allows one to set the relation between the parameters $p, c, q, \bar{q}$, and $\xi^\mu$, showing that these parameters are connected with the boost of the source.**

**The complex initial position of the complex world line $x_0^\mu(0)$ in Eq. (47) gives six parameters for the solution, which are connected to the coefficients $a_0, \ a_1 \ a_2$ . It can be decomposed as $\vec{x}_0(0) = \vec{c} + i\vec{d}$, where $\vec{c}$ and $\vec{d}$ are real 3-vectors with respect to the space O(3)-rotation. The real part $\vec{c}$ defines the initial position of the source, and the imaginary part $\vec{d}$ defines the value and direction of the angular momentum (or the size and orientation of a singular ring).**

**It can be easily shown that in the rest frame, when $\vec{v} = 0, \quad \vec{d} = \vec{d}_0$, the singular ring lies in the orthogonal to $\vec{d}$ plane and has a radius $a = |\vec{d}_0|$. The corresponding angular momentum is $\vec{J} = m\vec{d}_0$.**

## Wonderful Consequences of the Kerr Theorem

**Kerr's multi-particle solution is obtained on the base of the Kerr theorem. Choosing generating function of the Kerr theorem $F$ as a product of partial functions $F_i$ for spinning particles i=1,...k, we obtain a multi-sheeted, multi-twistorial space-time over $M^4$ possessing unusual properties. Twistorial structures of the i-th and j-th particles do not feel each other, forming a type of its internal space. Gravitation and electromagnetic interaction of the particles occurs via a singular twistor line which is common for twistorial structures of interacting particles.**

## Action-at-the-distance.

**The obtained multi-particle Kerr-Newman solution turns out to be 'dressed' by singular twistor lines linked to surrounding particles. We conjecture that this structure of space-time has the relation to a stringy structure of vacuum and opens a geometrical way to quantum gravity.**

## THE KERR THEOREM.

**One-particle generating function $F(T^a)$ is to be quadratic in $Y$, which corresponds to the Kerr PNC up to the Lorentz boosts, orientations of angular momenta and the shifts of origin.**

**Function $F(Y)$ can be expressed via the set of parameters $q$ which determine the boost and orientation of the Kerr spinning particle**

$$F(Y|q) = A(x|q)Y^2 + B(x|q)Y + C(x|q). \tag{51}$$

**This equations can be resolved explicitly, leading to two roots $Y = Y^{\pm}(x|q)$ which correspond to two sheets of the Kerr space-time.**

**The root $Y^+(x)$ determines out-going congruence on the $(+)$-sheet, while the root $Y^-(x)$ gives in-going congruence on the $(-)$-sheet.**

**Therefore, function $F$ may be represented in the form**

$$F(Y|q) = A(x|q)(Y - Y^+)(Y - Y^-),$$

**which allows one to obtain all the required functions of the Kerr solution in explicit form. The detailed form of $Y^{\pm}(x|q)$ is not important for further treatment.**

## Multi-twistorial space-time.

**Selecting an isolated i-th particle with parameters $q_i$, one can obtain the roots $Y_i^{\pm}(x)$ of the equation $F_i(Y|q_i)=0$ and express $F_i$ in the form**

$$F_i(Y)=A_i(x)(Y-Y_i^{+})(Y-Y_i^{-}). \tag{52}$$

**Then, the $(+)$ or $(-)$ root $Y_i^{\pm}(x)$ determines congruence $k_\mu^{(i)}(x)$ and consequently, the Kerr-Schild metric**

$$g_{\mu\nu}^{(i)}=\eta_{\mu\nu}+2h^{(i)}k_\mu^{(i)}k_\nu^{(i)}, \tag{53}$$

**and finally, the function $h^{(i)}(x)$ may be expressed in terms of $\tilde r_i=-d_Y F_i$, (??), as follows**

$$h^{(i)}=\frac{m}{2}\left(\frac{1}{\tilde r_i}+\frac{1}{\tilde r_i^*}\right)+\frac{e^2}{2|\tilde r_i|^2}. \tag{54}$$

**Electromagnetic field is given by the vector potential**

$$A_\mu^{(i)}=\Re e(e/\tilde r_i)k_\mu^{(i)}. \tag{55}$$

**For a system of $k$ particles we form the function $F$ as a product of the known blocks $F_i(Y)$,**

$$F(Y)\equiv\prod_{i=1}^{k}F_i(Y). \tag{56}$$

**The solution of the equation $F=0$ acquires $2k$ roots $Y_i^{\pm}$, and the twistorial space turns out to be multi-sheeted.**

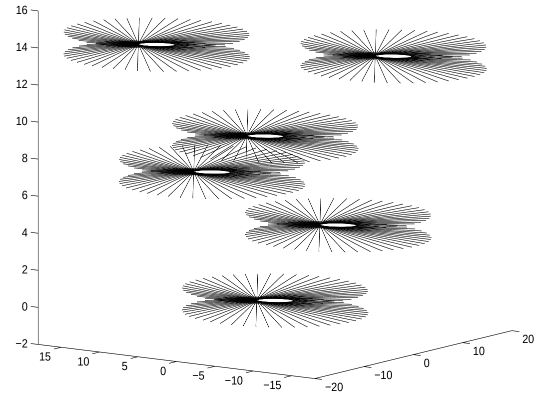



Figure 6: Multi-sheeted twistor space over the auxiliary Minkowski space-time of the multi-particle Kerr-Schild solution. Each particle has twofold structure.

**The twistorial structure on the i-th $(+)$ or $(-)$ sheet is determined by the equation $F_i = 0$ and does not depend on the other functions $F_j, \quad j \neq i$. Therefore, the particle $i$ does not feel the twistorial structures of other particles. Similar, the condition for singular lines $F = 0,\ d_Y F = 0$ acquires the form**

$$\prod_{l=1}^{k} F_l = 0, \qquad \sum_{i=1}^{k} \prod_{l \neq i}^{k} F_l d_Y F_i = 0 \tag{57}$$

**and splits into k independent relations**

$$F_i = 0, \quad \prod_{l \neq i}^{k} F_l d_Y F_i = 0. \tag{58}$$

**One sees, that i-th particle does not feel also singular lines of other particles. The space-time splits on the independent twistorial sheets, and therefore, the twistorial structure related to the i-th particle plays the role of its "internal space".**

**It looks wonderful. However, it is a direct generalization of the well known twofoldedness of the Kerr space-time which remains one of the mysteries of the Kerr solution for the very long time.**

**Multi-particle Kerr-Schild solution.** Using the Kerr-Schild formalism with the considered above generating functions $\prod_{i=1}^{k} F_i(Y) = 0$, one can obtain the exact asymptotically flat multi-particle solutions of the Einstein-Maxwell field equations. Since congruences are independent on the different sheets, the congruence on the i-th sheet retains to be geodesic and shear-free, and one can use the standard Kerr-Schild algorithm of the paper [?]. One could expect that result for the i-th sheet will be in this case the same as the known solution for isolated particle. Unexpectedly, there appears a new feature having a very important consequence.

Formally, we have only to replace $F_i$ by $F = \prod_{i=1}^{k} F_i(Y) = \mu_i F_i(Y)$, where $\mu_i = \prod_{j\neq i}^{k} F_j(Y)$ is a normalizing factor which takes into account the external particles. However, in accordance with (??) this factor $\mu_i$ will appear also in the function $\tilde{r} = -d_Y F = -\mu_i d_Y F_i$, and in the function $P = \mu_i P_i$.

So, we obtain the different result

$$h_i = \frac{m_i(Y)}{2\mu_i^3}(\frac{1}{\tilde{r}_i} + \frac{1}{\tilde{r}_i^*}) + \frac{(e/\mu_i)^2}{2|\tilde{r}_i|^2}, \tag{59}$$

$$A_\mu^{(i)} = \Re e \frac{e}{\mu_i \tilde{r}_i} k_\mu^{(i)} \tag{60}$$

which looks like a renormalization of the mass $m$ and charge $e$.[1]

This fact turns out to be still more intriguing if we note that $\mu_i$ is not constant, but a function of $Y_i$. We can specify its form by using the known structure of blocks $F_i$

$$\mu_i(Y_i) = \prod_{j\neq i} A_j(x)(Y_i - Y_j^+)(Y_i - Y_j^-). \tag{61}$$

[1]Function $m_i(Y)$ is free and satisfies the condition $(m_i)_{,\bar{Y}} = 0$. It and has to be chosen in the form $m_i(Y) = m_0 \mu_i^3$ to provide reality of metric.

**The roots $Y_i$ and $Y_j^\pm$ may coincide for some values of $Y_i$, which selects a common twistor for the sheets $i$ and $j$. Assuming that we are on the i-th $(+)$-sheet, where congruence is out-going, this twistor line will also belong to the in-going $(-)$-sheet of the particle $j$ . The metric and electromagnetic field will be singular along this twistor line, because of the pole $\mu_i \sim A(x)(Y_i^+ - Y_j^-)$. Therefore, interaction occurs along a light-like Schild string which is common for twistorial structures of both particles. The field structure of this string is similar to the well known structure of pp-wave solutions.**

**These equations give the** *exact multi-particle solution of the Einstein-Maxwell field equations.* **It follows from the fact that the equations were fully integrated out in [?] and expressed via functions $P$ and $Z$ before (without) concretization of the form of congruence, under the only condition that it is geodesic and shear free. In the same time the Kerr theorem determines the functions $P$ and $Z$ via generating function $F$, eq.(??), and the condition of reality for metric may be provided by a special choice of the free function $m(Y)$.**

**The obtained multi-particle solutions show us that, in addition to the usual Kerr-Newman solution for an isolated spinning particle, there is a series of the exact 'dressed' Kerr-Newman solutions which take into account surrounding particles and differ by the appearance of singular twistor strings connecting the selected particle to external particles. This is a new gravitational phenomena which points out on a probable stringy (twistorial) texture of vacuum and may open a geometrical way to quantum gravity.**

**The number of surrounding particles and number of blocks in the generating function $F$ may be assumed countable. In this case the multi-sheeted twistorial space-time will possess the properties of the multi-particle Fock space.**

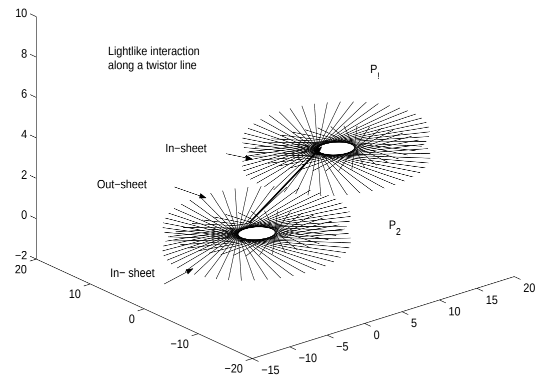



Figure 7: Schematic representation of the lightlike interaction via a common twistor line connecting out-sheet of one particle to in-sheet of another.