Branes, Duality and Doubled Geometry

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based on a collaboration with Fabio Riccioni,

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Outline

Introduction

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Question

- Branes are massive objects with a number of worldvolume and transverse directions; they play an important role in string theory

- Question : What can we learn about branes using supergravity as a low-energy approximation to string theory ?

- New insight : supergravity can be extended with many p-form potentials not describing physical degrees of freedom

Brane Criterion: Gauge-invariant WZ Term

- Target space potentials form non-trivial "p-form algebra"

- Construction of gauge-invariant WZ term therefore requires on top of the embedding scalars extra worldvolume potentials

- Criterium: the worldvolume fields must fit into a worldvolume supermultiplet

This Talk

In this talk I will discuss solitonic branes in $D \leq 10$ dimensions

Outline

The Fundamental String

The fundamental string couples to the background metric via a Nambu-Goto term and to the NS-NS 2-form via a WZ term :

$$\mathcal{L}_{\mathrm{F1}}(D=10) \;=\; \mathrm{T}\sqrt{-g} \;+\; B_2$$

fundamental brane : Tension $T \;\sim (g_s)^0$, $g_s =< e^{\phi} >$

Classifying Branes

according to brane tension $T \sim (g_s)^{\alpha}$:

$\alpha = 0$: Fundamental ; $\alpha = -1$: D-brane; $\alpha = -2$: Solitonic

according to $\#$ transverse directions :

"standard" branes (D0 – D6) , "defect branes" (D7),

domain walls (D8) and space-filling branes (D9)

IIA p-forms

field	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$
1-form		$\mathbf{1}_{-1}$	
2-form	$\mathbf{1}_2$		
3-form		$\mathbf{1}_1$	
5-form		$\mathbf{1}_3$	
6-form			$\mathbf{1}_2$
7-form		$\mathbf{1}_5$	
9-form		$\mathbf{1}_7$	

α is determined by $\mathbb{R}^+$-symmetry : $\alpha = -\frac{1}{2}(n - w)$

IIB $SL(2, \mathbb{R})$-duality

scalars transform nonlinear under $SL(2, \mathbb{R})$

they parametrize coset $SL(2, \mathbb{R})/SO(2)$ via matrix $\mathcal{M}$

three dual 8-forms with one nonlinear constraint on curvatures

IIB p-forms

field	U repr	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha = -3$	$\alpha = -4$
2-form	$\mathbf{2}$	$\mathbf{1}_1$	$\mathbf{1}_{-1}$			
4-form	$\mathbf{1}$		$\mathbf{1}_0$			
6-form	$\mathbf{2}$		$\mathbf{1}_1$	$\mathbf{1}_{-1}$		
8-form	$2 \subset \mathbf{3}$		$\mathbf{1}_2$		$\mathbf{1}_{-2}$	
10-form	$2 \subset \mathbf{4}$		$\mathbf{1}_3$			$\mathbf{1}_{-3}$

$$\mathrm{SL}(2,\mathbb{R}) \supset \mathbb{R}^+ \;:\quad \alpha = \frac{1}{2}\left(-\frac{n}{2} + w\right)$$

$$(\text{WZ term})_i \sim \overline{(\text{WV curvature})}\,\Gamma_i\,(\text{TS gauge field}) \quad \rightarrow \quad i = \pm\,, i \neq 3$$

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Fundamental Branes

For every compactified direction we have two fundamental 0-branes coming from a wrapped fundamental string and a pp-wave

The corresponding 1-forms transform as a vector $B_{1,A}$ under the T-duality group $\mathrm{SO}(10-\mathrm{D}, 10-\mathrm{D})$

Wess-Zumino term $\quad \mathcal{L}_{\mathrm{WZ}}(D<10) = B_2 + \eta^{AB}\mathcal{F}_{1,A}B_{1,B}$

contains "extra scalars" $b_{0,A}$ via $\mathcal{F}_{1,A} = db_{0,A} + B_{1,A}$

Counting Worldvolume Degrees of Freedom

$$D = 10 \quad : \qquad (10-2) = 8\,,$$

$$D < 10 \quad : \qquad (D-2) + 2(10-D) \neq 8\,!$$

Twice too many "extra scalars" $b_{0,A}$ $\rightarrow$ "doubled geometry"

Hull, Reid-Edwards (2006-2008)

Self-duality conditions on the extra scalars $b_{0,A}$ give correct counting

Wrapping rule

wrapped	$\longrightarrow$	doubled
unwrapped	$\longrightarrow$	undoubled

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	10	12	14
1	1/1	1	1	1	1	1	1	1

$(\mathrm{F0})_A$ and F1

D-branes

D-branes transform as spinors under T-duality

$$\boxed{\mathcal{L}_{\mathrm{WZ}}(D \leq 10) = e^{\mathcal{F}_2} e^{\mathcal{F}_{1,A}\Gamma^A} C}$$

Riccioni + E.B. (2010)

$$D = 10 \quad : \qquad (p-1) + (10-p-1) = 8$$

$$D < 10 \quad : \qquad (p-1) + (D-p-1) + 2(10-D) \neq 8\,!$$

only half of the $\mathcal{F}_{1,A}\Gamma^A$ contributes to a particular spinor component !

Wrapping rule

wrapped	$\longrightarrow$	undoubled
unwrapped	$\longrightarrow$	undoubled

Dp-brane	IIA/IIB	9	8	7	6	5	4	3
0	1/0	1	2	4	8	16	32	64
1	0/1	1	2	4	8	16	32	64
2	1/0	1	2	4	8	16	32	64
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
8	1/0	1						
9	0/1							

spinors $(\mathrm{Dp})_\alpha$, $\qquad \alpha = 1 \cdots 2^{9-D}$

Outline

“Standard” Solitons

we first restrict to solitons with more than 2 transvere directions

For every compactified direction we have two solitonic (D-4)-branes coming from an unwrapped solitonic brane and a KK-monopole

fundamental brane $\overset{\text{duality}}{\longleftrightarrow}$ solitonic brane $\Rightarrow$ “dual” wrapping rule

wrapped	$\longrightarrow$	undoubled
unwrapped	$\longrightarrow$	doubled

Reduced Solitons

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	
1					1	10		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

S(D-5)-brane and S(D-4)-brane$_A$

"Non-standard" Solitons

there are also solitons with less than or equal to 2 transverse directions

U-duality representations of target space gauge fields are known

$$\text{U-duality} \supset \text{SO(10-D,10-D)} \times \mathbb{R}^+$$

S(D-5)-brane
$[\text{S(D-4)-brane}]_A$
$[\text{S(D-3)-brane}]_{AB}$
$[\text{S(D-2)-brane}]_{ABC}$
$[\text{S(D-1)-brane}]_{ABCD}$

Supersymmetric Solitons

use lightcone basis : $A = (1\pm\,, 2\pm\,, \cdots\,, d\pm)$

only $[ABC\cdots] = [m\pm n\pm p\pm\cdots]$ with $m\neq n\neq p\cdots$ are susy

Example : $[\text{S4-brane}]_{ABC}$ in D=6 dimensions :

only 32 $\subset$ **56** of SO(4,4) T-duality are supersymmetric

144 of SO(5, 5) $\supset$ **56** of SO(4, 4) $\supset$ 32

Summary D=6

field	U repr	$\alpha = 0$	$\alpha = -1$	$\alpha = -2$	$\alpha \leq -3$
1-form	$\mathbf{16}$	$(\mathbf{8}_{\mathrm{C}})_1$	$(\mathbf{8}_{\mathrm{S}})_{-1}$		
2-form	$\mathbf{10}$	$\mathbf{1}_2$	$(\mathbf{8}_{\mathrm{V}})_0$	$\mathbf{1}_{-2}$	
3-form	$\overline{\mathbf{16}}$		$(\mathbf{8}_{\mathrm{S}})_1$	$(\mathbf{8}_{\mathrm{C}})_{-1}$	
4-form	$\mathbf{45}$		$(\mathbf{8}_{\mathrm{V}})_2$	$24 \subset \mathbf{28}_0$	–
5-form	$\mathbf{144}$		$(\mathbf{8}_{\mathrm{S}})_3$	$32 \subset (\mathbf{56}_{\mathrm{C}})_1$	–
6-form	$\mathbf{320}$		$(\mathbf{8}_{\mathrm{V}})_4$	$8 \subset (\mathbf{35}_{\mathrm{V}})_2$	–
	$\overline{\mathbf{126}}$			$8 \subset (35_{\mathrm{S}})_2$	–

U-duality is SO(5,5) T-duality is SO(4,4)

Dual Doubled Geometry

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

dual wrapping rule gives correct number of susy solitons !

Question

where do the extra solitons come from ?

Outline

Standard KK Monopole

the standard monopole has (D-4) WV, 1 isometry and 3 transverse directions

D=10 KK monopole $\leftrightarrow$ dual graviton $D_{7,1}$

$$D_{7,1} \quad \to D_{6\sharp,\sharp} \qquad \text{but no} \qquad D_{7,\sharp}$$

$D = 7$: $\quad D_6 \to D_{3ijk}$ (1) $+$ D_{4ij} (3) and $D_{7,1} \to D_{4ijk,i}$ (3) , $\quad i = 1, 2, 3$

Mixed-symmetry Fields

all solitons come from $D_{6+n,n}$, $n = 0, 1, 2, 3, 4$

Example : D=6

$D_6 \longrightarrow D_{2ijkl}$ (1) D_{3ijk} (4) D_{4ij} (6) D_{5i} (4) D_6 (1)

$D_{7,1} \longrightarrow D_{3ijkl,i}$ (4) $D_{4ijk,i}$ (12) $D_{5ij,i}$ (12) $D_{6i,i}$ (4)

$D_{8,2} \longrightarrow D_{4ijkl,ij}$ (6) $D_{5ijk,ij}$ (12) $D_{6ij,ij}$ (6)

$D_{9,3} \longrightarrow D_{5ijkl,ijk}$ (4) $D_{6ijk,ijk}$ (4)

$D_{10,4} \longrightarrow D_{6ijkl,ijkl}$ (1)

$1 + 8 + 24 + 32 + 16$ solitons

Suggestion

string theory contains "generalized" KK monopoles with

6 WV, n isometry and 4-n transverse directions for n=0,1,2,3,4

Outline

What is left ?

The only branes left are non-standard branes with $\alpha \leq -3$

What I did not discuss

- "E-branes", i.e. branes with $\alpha = -3$
  - satisfy new wrapping rule !
- Defect branes
  - Defect branes and central charges
- Domain Walls
  - Domain walls and the embedding tensor
- Space-filling branes

Summary

- supergravity predicts a number of branes
- some of these are understood by simple wrapping rules
- there are many non-standard and highly non-perturbative branes that are not well understood

Open Issues

- Can the ten-dimensional origin be clarified by the very extended Kac-Moody algebra E_{11} ?

- Is there any relation with non-geometric compactifications ?

- Applications to phenomenology ?