Instantons and 2d Superconformal field theory

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29 June 2011

Introduction

During last years AGT correspondence between $2d$ Conformal field theories and $\mathcal{N} = 2$ SUSY $4d$ Gauge theories was extended for cases of CFT with such symmetries, as affine Lie algebras, $\mathcal{W}$ algebras et cet.

The instanton calculus in the gauge theories on $\mathbb{R}^4/\mathbb{Z}_2$ give rise to the super-Virasoro conformal blocks. The idea to use the $\mathbb{Z}_2$ symmetric instanton moduli $\mathcal{M}_{\text{sym}}$ is based on its relation to the coset $\widehat{gl}(n)_2/\widehat{gl}(n-2)_2$ which is isomorphic to $\mathcal{A} = \widehat{gl}(2)_2 \times \mathcal{NSR}$.

Then, instead of ADHM moduli space of the instantons $\mathcal{M}$, the equivariant integration is restricted by its $\mathbb{Z}_2$ invariant subspace $\mathcal{M}_{\text{sym}}$.

Whittaker limit of the Super conformal block is equal to the $\mathbb{Z}_2$ restricted instanton partition function of $SU(2)$ $\mathcal{N} = 2$ supersummetric pure gauge theory.

We generalize this construction . Firstly we confirm the correspondence by comparing the number of states of some set of states in the representations of $\mathcal{A} = \widehat{gl}(2)_2 \times \mathcal{NSR}$ and the number of the pairs of Young diagrams $\vec{Y}$ corresponding to those fixed points of the vector field which belong to the needed connected component of $\mathcal{M}_{\text{sym}}$.

Then we use the proposed relation to construct explicit representation of the four-point in generic case super-Liouville conformal block function for Neveu-Schwatz sector. The integral over the moduli space involves also zero modes of the matter fermions.

The instanton partition function coincide with the four-point conformal block up to additional factor related to the algebra $\widehat{gl}(2)_2$.

The talk is organized as follows.Firstly we describe the structure of $\mathbb{Z}_2$ symmetric moduli space $\mathcal{M}_{\mathsf{sym}}$. Only two of its connected components of $\mathcal{M}_{\mathsf{sym}}$ are relevant for constructing Neveu–Schwarz conformal blocks.

In the next section we compare fixed points on such components with the number of states in some subspace of NS representation of $\widehat{gl}(2)_2 \times \mathcal{NSR}$.

The instanton moduli integral is evaluatied in the next section , where also the contribution of the matter fields in the equivariant integral is found .

Then we recall the definition of the four-point conformal block function in super-Liouville theory and give its new representation for them in terms of Young diagrams with two sorts of boxes .

N=2 Super Yang-Mills theory

$\mathcal{N} = 2$ extended $d = 4$ Super Yang-Mills theory

$$L_{N=2} = tr\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + 2\mathcal{D}_\mu\Phi^*\mathcal{D}^\mu\Phi + \right.$$

$$\left. + \sum_a (i\bar{\lambda}_a\bar{\sigma}_\mu\mathcal{D}^\mu\lambda^a + g\Phi^*[\lambda_a, \lambda^a] + g\Phi[\bar{\lambda}_a, \bar{\lambda}^a]) + 2g^2[\Phi^*, \Phi]^2)\right\}$$

Using non-renormalization theorem,holomophicity and electric-magnetic duality Seiberg-Witten (1994) proposed an exact expression for the low energy effective action in this theory with spontaneous breakdown of the gauge symmetry $SU(2) - - > U(1)$ in terms of so-called prepotential $\mathcal{F}$.

S-W used also some additional physical assumptions like the conjecture about the connection the analytic properties of $\mathcal{F}$ and vanishing masses of dyons.

$$\mathcal{F}(\Psi) = \frac{i}{2\pi}\Psi^2 \log\frac{\Psi^2}{\Lambda^2} - \frac{i}{\pi}\sum_{n=1}^{\infty}\mathcal{F}_N\left(\frac{\Lambda}{\Psi}\right)^{4N}\Psi^2$$

To verify thi proposal Dorey-Khoze-Mattis started the direct computation coefficients $\mathcal{F}_N$ quasiclassicaly.They get that N-instanton contribution is

$$\mathcal{F}_N = \int d\mu^{(n)} e^{-S_{ind}^{(N)}}$$

$$S_{ind}^{(N)} = \int d^4 x tr \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2\mathcal{D}_\mu \Phi^* \mathcal{D}^\mu \phi + g\Phi^* [\lambda_a, \lambda^a] \right\}$$

The fields satisfy a reduced set of eq-s of motion

$$F_{\mu\nu} = \widetilde{F}_{\mu\nu};$$

$$\bar{\sigma}_\mu \mathcal{D}^\mu \lambda^a = 0;$$

$$\mathcal{D}_\mu \mathcal{D}^\mu \Phi = -g[\lambda_a, \lambda^a]$$

All solution of the eq-s depend on $ADHM$ data ,matrixes (B_1, B_2, I, J) which can be conveniently organizied to matix Δ

$$\Delta = a + bz = \begin{pmatrix} J & I^\dagger \\ B_1 & -B_2^\dagger \\ B_2 & B_1^\dagger \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ z_1 & -\bar{z}_2 \\ z_2 & \bar{z}_1 \end{pmatrix}$$

and obey

$$[B_1, B_2] + IJ = 0$$

$$[B_1 B_1^\dagger] + [B_2, B_2^\dagger] + II^\dagger - J^\dagger J = 0$$

$$A_\mu = \bar{U}(x)\partial_\mu U(x), \quad \bar{\Delta} U = 0$$

Due to appearence Weyl zero modes of positive chirality in selfdual Y-M background $S_{ind}^{(N)}$ depends also on grassmanian matrixes (M_1, M_2, μ, ν) which obey

$$[M_1, B_2] + [B_1, M_2] + \mu J + I\nu = 0$$
$$[M_1, B_1^\dagger] + [M_2, B_2^\dagger] + \mu I^\dagger - J^\dagger \nu = 0$$

Taking into account that elements of cotangent bundle $(DB_1, DB_2, D\mu, D\nu)$ satisfy to the same eq-s after the change

$$(M_1, M_2, \mu, \nu) -> (DB_1, DB_2, D\mu, D\nu)$$

DKM transform the integral over super moduli space to integral of the exponential of an mixed differential form.

$$\mathcal{F}_N = \int_{\mathcal{M}_N} e^{-S_{ind}^{(N)}(A, \mathcal{D}A)}$$

Flume-Poghossian proved that $S_{ind}^{(N)}(A, \mathcal{D}A)$ is an exact equivariant form

$$S_{ind}^{(N)}(A, \mathcal{D}A) = d_v \omega$$

and obtain

$$\mathcal{F}_N = \int_{\mathcal{M}_N} e^{-d_v \omega}$$

Using the localization technique the computation of the integral is reduced to finding fixed points of the vector field and its determinants in these fixed points and gives Nekrasov formula for coefficients of S-W prepotential

$$\mathcal{F}_N = \sum_P \frac{1}{\det v(P)}$$

Moduli space

ADHM date consist of $N \times N$ matrices B_1, B_2, a $N \times 2$ matrix I and a $2 \times N$ matrix J, which are subject of the following set of conditions:

$$[B_1, B_2] + IJ = 0,$$

The solutions related by $GL(N)$ transformations

$$B_i' = gB_ig^{-1}, \;\; I' = gI, \;\; J' = Jg^{-1}; \quad g \in GL(N)$$

are equivalent.

The vectors obtained by the repeated action of B_1 and B_2 on $I_{1,2}$,columns of the matrix I ,span N-dimensional vector space V ,a fiber of the N-dimensional fiber bundle ,whose base is the moduli space $\mathcal{M}_N$ itself.

Modified moduli space

The subspace of the Moduli space $\mathcal{M}_{\text{sym}}$ for $SU(2)$ gauge group is defined by the following additional restriction of $\mathbb{Z}_2$ symmetry

$$B_1 = -PB_1P^{-1}; B_2 = -PB_2P^{-1}; \qquad I = PI; \qquad J = JP^{-1}.$$

where $P \in GL(N)$ is some gauge transformation, obviously $P^2 = 1$.

New manifold $\mathcal{M}_{\text{sym}}$ is a disjoint union of connected components $\mathcal{M}_{\text{sym}}(N_+, N_-)$, where N_+ and N_- are integers which denote the dimensions of V_+ and V_- (*i.e.* even and odd subspaces of the fiber V) , $N_+ + N_- = N$. These numbers are fixed inside a given connected component of $\mathcal{M}_{\text{sym}}$. Each component is connected and can be considered separately.

The vector field and its fixed points.

The construction of the instanton partition function involves the determinants of the vector field v on $\mathcal{M}_N$, defined by

$$B_l \to t_l B_l; \quad I \to I t_v; \quad J \to t_1 t_2 t_v^{-1} J,$$

where parameters $t_l \equiv \exp \epsilon_l \tau$, $l = 1, 2$ and $t_v = \exp a\sigma_3 \tau$.

Fixed points, which are relevant for the determinants evaluation, are found from the conditions:

$$t_l B_l = g^{-1} B_l g; \quad I t_v = g^{-1} I; \quad t_1 t_2 t_v^{-1} J = J g.$$

The solutions of this system can be parameterized by pairs of Young diagrams $\vec{Y} = (Y_1, Y_2)$ such that the total number of boxes $|Y_1| + |Y_2| = N$. The cells $(i_1, j_1) \in Y_1$ and $(i_2, j_2) \in Y_2$ correspond to vectors $B_1^{i_1} B_2^{j_1} I_1$ and $B_1^{i_2} B_2^{j_2} I_2$ respectively. It is convenient to use these vectors as a basis in the fiber V attached to some fixed point.

Then the explicit form of the ADHM data for the given fixed point is defined straightaway

$$
\begin{aligned}
&g_{ss'} = \delta_{ss'} t_1^{i_s-1} t_2^{j_s-1}, \\
&(B_1)_{ss'} = \delta_{i_s+1, i_{s'}} \delta_{j_s, j_{s'}}, \\
&(B_2)_{ss'} = \delta_{i_s, i_{s'}} \delta_{j_{s+1}, j_{s'}}, \\
&(I_1)_s = \delta_{s,1}, \\
&(I_2)_s = \delta_{s,|Y_1|+1}, \\
&J = 0,
\end{aligned}
$$

where $s = (i_s, j_s)$.

Returning to to $\mathcal{M}_{\mathrm{sym}}$ we note that it contains all fixed points of the vector field found above. For the operator P in the fixed point $\vec{Y}$ we get

$$P(B_1^{i-1}B_2^{j-1}I_\alpha) = (-1)^{i+j}B_1^{i-1}B_2^{j-1}I_\alpha,$$

so its matrix elements can be found explicitly, $P_{ss'} = (-1)^{i_s+j_s}\delta_{ss'}$ The parity characteristic $P(s) = (-1)^{i_s+j_s}$ is assigned to each box in the Young diagrams related to the fixed point. We use convenient notation that each box is white or black. If $P(s) = 1$, the box is white, and if $P(s) = -1$,it is black.

Therefore the fixed points can be classified by the numbers of white and black boxes, N_+ and N_-. These numbers are equal to the dimensions of the subspaces V_+ and V_- of the fiber attached to those points of $\mathcal{M}_{\mathrm{sym}}$ which belong to the same component as the fixed point itself.

Modified moduli space and $\widehat{gl}(2)_2 \times \mathcal{NSR}$ algebra

The Whittaker vector found used not the whole space $\mathcal{M}_{\text{sym}}$, but only its connected components $\mathcal{M}_{\text{sym}}(N, N)$ and $\mathcal{M}_{\text{sym}}(N, N-1)$. The norm of the Whittaker vector is equal to the sum of contributions of fixed points. In this section we calculate the number of fixed points on such components and discuss the result from the $\widehat{gl}(2)_2 \times \mathcal{NSR}$ point of view.

We introduce the generating function

$$\chi(q) = \sum_N |\mathcal{M}_{\text{sym}}(N, N)| q^N + \sum_N |\mathcal{M}_{\text{sym}}(N, N-1)| q^{N-1/2},$$

where $|\mathcal{M}_{\text{sym}}(N_+, N_-)|$ is a number of fixed points on $\mathcal{M}_{\text{sym}}(N_+, N_-)$. This number equal to the number of pairs of Young diagrams with N_+ white boxes and N_- black boxes.

Denote by $d(Y) = N_+(Y) - N_-(Y)$ the difference between number of white and black boxes in Young diagram Y. For any integer k we denote by

$$\chi_k(q) = \sum_{d(Y)=k} q^{\frac{|Y|}{2}},$$

the generating function of Young diagrams of given difference $d(Y)$. This function has the form:

$$\chi_k(q) = q^{\frac{2k^2-k}{2}} \prod_{m\geq 0} \frac{1}{(1-q^{m+1})^2}.$$

for $k = 0$. The factor $q^{\frac{2k^2+k}{2}}$ corresponds to the smallest Young diagram with $d(Y) = k$. For $k > 0$ this diagram consist of $2k-1$ rows of length $2k-1, 2k-2, \ldots, 1$. For $k < 0$ this diagram consist of $2|k|$ rows of length $2|k|, 2|k|-1, \ldots, 1$.

The generating function of pairs Young diagrams with $N_+ - N_- = k$ reads

$$\chi_k^{(2)} = \sum_{k_1+k_2=k} \chi_{k_1}\chi_{k_2},$$

Using () and Jacobi triple product identity

$$\sum_{n\in\mathbb{Z}} (-1)^n t^n q^{n^2} = \prod_{m\geq 0} (1-q^{2m+2})(1-q^{2m+1}t)(1-q^{2m+1}t^{-1})$$

we get

$$\chi(q) = \chi_0^{(2)}(q) + \chi_1^{(2)}(q) = \prod_{m\geq 0} \frac{(1-q^{2m+1})^2}{(1-q^{2m+2})^3} = \chi_B(q)^3 \chi_F(q)^2,$$

where

$$\chi_B(q) = \prod_{n\in\mathbb{Z},\, n>0} \frac{1}{(1-q^n)}$$

$$\chi_F(q) = \prod_{r\in\mathbb{Z}+\frac{1}{2},\, r>0} (1+q^r).$$

The first terms of the series for $\chi(q)$ looks as follows

$$\chi(q) = 1 + 2q^{1/2} + 4q + 8q^{3/2} + 16q^2 + 28q^{5/2} + \ldots$$

The $\chi_B(q)\chi_F(q)$ equals to the character of standard representation of the $\mathcal{NSR}$ algebra with generators L_n, G_r. $\chi_B(q)$ equals to the character of the Fock representation of the Heisenberg algebra. The term $\chi_B(q)\chi_F(q)$ should be related to the fact that $\widehat{sl}(2)$ representation of level 2 can be realized by one bosonic and one fermionic field .

The the generating function the whole space $\mathcal{M}_{\mathrm{sym}}$ has the form

$$\chi(q) = \sum_N |\mathcal{M}_{\mathrm{sym}}(N)| q^{\frac{N}{2}} = \prod_{n\in\mathbb{Z},\, n>0} \frac{1}{\left(1 - q^{\frac{n}{2}}\right)^2}$$

The result equals to the character of the simple representation of $\widehat{gl}(2)_2 \times \mathcal{NSR}$ namely the tensor product of Fock representation of Heisenberg algebra, vacuum representation of $\widehat{gl}(2)_2$ and NS representation of $\mathcal{NSR}$.

Determinants of the vector field

The form of $\mathcal{N} = 2$ $SU(2)$ instanton partition function was derived to be equal an integral of the equivariantly form, defined in terms of the vector field v acting on the moduli space $\mathcal{M}_N$.

By localization technique , the moduli integral is reduced to the determinants of the vector field v in the vicinity of its fixed points

$$\mathcal{Z}_N\,(a,\epsilon_1,\epsilon_2) = \sum_n \frac{1}{\mathrm{det}_n\, v}.$$

We need to find all eigenvectors of the vector field on the tangent space passing through the fixed points

$$t_i \delta B_i = \wedge\, g\delta B_i g^{-1},$$
$$\delta I t = \wedge\, g\delta I,$$
$$t_1 t_2 t^{-1} \delta J = \wedge\, \delta J g^{-1}.$$

This is equivalent to the following set of equations

$$\lambda\,(\delta B_i)_{ss'} = (\epsilon_i + \phi_{s'} - \phi_s)\,(\delta B_i)_{ss'},$$
$$\lambda\,(\delta I)_{sp} = (a_p - \phi_s)\,(\delta I)_{sp},$$
$$\lambda\,(\delta J)_{ps} = (\epsilon_1 + \epsilon_2 - a_p + \phi_s)\,(\delta J)_{ps},$$

where $\Lambda = \exp \lambda\tau$, $g_{ss} = \exp \phi_s\tau$ and

$$\phi_s = (i_s - 1)\epsilon_1 + (j_s - 1)\epsilon_2 + a_{p(s)}.$$

We should keep only those eigenvectors which belong to the tangent space. This means excluding variations breaking ADHM constraints. On the Moduli space

$$[\delta B_1, B_2] + [B_1, \delta B_2] + \delta I J + I \delta J = 0.$$

Gauge symmetry can be taken into account in the following way. We fix a gauge in which $\delta B_{1,2}, \delta I, \delta J$ are orthogonal to any gauge transformation of $B_{1,2}, I, J$. This gives additional constraint

$$\left[\delta B_l, B_l^\dagger\right] + \delta I I^\dagger - J^\dagger \delta J = 0.$$

The variations in the LHS of the eq-ns above should be excluded.

The corresponding eigenvalues are defined from the equations

$$t_1 t_2([\delta B_1, B_2] + [B_1, \delta B_2] + \delta I J + I \delta J) =$$
$$= \Lambda\, g \Big([\delta B_1, B_2] + [B_1, \delta B_2] + \delta I J + I \delta J \Big) g^{-1},$$
$$\left[\delta B_l, B_l^{\dagger}\right] + \delta I I^{\dagger} - J^{\dagger} \delta J = \Lambda\, g \Big(\left[\delta B_l, B_l^{\dagger}\right] + \delta I I^{\dagger} - J^{\dagger} \delta J \Big) g^{-1}.$$

One finds the following eigenvalues, which should be excluded :

$$\lambda = (\epsilon_1 + \epsilon_2 + \phi_s - \phi_{s'}),$$
$$\lambda = (\phi_s - \phi_{s'}).$$

Thus, the determinant of the vector field is given by

$$\det v =$$
$$\frac{\prod\limits_{s,s' \in \vec{Y}} (\epsilon_1 + \phi_{s'} - \phi_s)(\epsilon_2 + \phi_{s'} - \phi_s) \prod\limits_{l=1,2; s \in \vec{Y}} (a_l - \phi_s)(\epsilon_1 + \epsilon_2 - a_l + \phi_s)}{\prod\limits_{s,s' \in \vec{Y}} (\phi_{s'} - \phi_s)(\epsilon_1 + \epsilon_2 - \phi_{s'} + \phi_s)}$$

Determinants of the vector field for $\mathcal{M}_{\text{sym}}$

The tangent space for this case is reduced by the additional requirement

$$-\delta B_{1,2} = P\delta B_{1,2}P^{-1}; \qquad \delta I = P\delta I; \qquad \delta J = \delta J P^{-1},$$

or, on the level of the matrix elements,

$$-(\delta B_{1,2})_{ss'} = P(s)(\delta B_{1,2})_{ss'}P(s'); \quad (\delta I)_{sp} = P(s)(\delta I)_{sp};$$

$$(\delta J)_{ps} = (\delta J)_{ps}P(s),$$

The first relation means that only eigenvectors $(\delta B_{1,2})_{ss'}$ with the different colors of s and s' belong to $\mathcal{M}_{\text{sym}}$. Similarly, the second and third leave $(\delta J)_{ps}$and $(\delta J)_{ps}$ only if s is white. Thus, we get the new determinant $\det{}' v$

$$\frac{\prod\limits_{\substack{s,s'\in\vec{Y}\\ P(s)\neq P(s')}} (\epsilon_1+\phi_{s'}-\phi_s)(\epsilon_2+\phi_{s'}-\phi_s) \prod\limits_{\substack{\alpha=1,2;s\in\vec{Y}\\ P(s)=1}} (a_\alpha-\phi_s)(\epsilon_1+\epsilon_2-a_\alpha+\phi_s)}{\prod\limits_{\substack{s,s'\in\vec{Y}\\ P(s)=P(s')}} (\phi_{s'}-\phi_s)(\epsilon_1+\epsilon_2-\phi_{s'}+\phi_s)}$$

Re-expressed in terms of arm-length and leg-length this expression gives

$$\text{det}\,'v = \prod_{\alpha,\beta=1}^{2} \prod_{s\in {}^{\diamondsuit}Y_\alpha(\beta)} E\Big(a_\alpha - a_\beta, Y_\alpha, Y_\beta\Big| s\Big)(Q - E\Big(a_\alpha - a_\beta, Y_\alpha, Y_\beta\Big| s\Big)),$$

here $E\Big(a, Y_1, Y_2\Big| s\Big)$ are defined as follows

$$E\Big(a, Y_1, Y_2\Big| s\Big) = a + b(L_{Y_1}(s) + 1) - b^{-1}A_{Y_2}(s)\ ,$$

where $A_Y(s)$ and $L_Y(s)$ are respectively the arm-length and the leg-length for a cell s in Y. The region ${}^{\diamondsuit}Y_\alpha(\beta)$ is defined as

$${}^{\diamondsuit}Y_\alpha(\beta) = \Big\{(i,j) \in Y_\alpha \Big| P\Big(k'_j(Y_\alpha)\Big) \neq P\Big(k_i(Y_\beta)\Big)\Big\},$$

or, in other words, the boxes having different parity of the leg- and arm-factors. So the contribution of the vector multiplet reads

$$Z^{\text{sym}}_{\text{vec}}(\vec{a}, \vec{Y}) \equiv \frac{1}{\text{det}\,'v}$$

Matter multiplets

The hypermultiplets with masses μ give some additional contribution because of appearence of the N fermionic null-modes . The amplitudes ψ of the null-modes can be considered as of the fiber V attached to one of the fixed point $\vec{Y}$. The eigenvalues of the vector field are defined from the equation

$$\lambda\,\psi_s = (\mu + \phi_s)\,\psi_s,$$

The corresponding contribution of the fundamental hypermultiplets with masses μ_i looks as follows

$$Z_{\mathsf{f}}(\mu_i, \vec{a}, \vec{Y}) = \prod_{i=1}^{4} \prod_{\alpha=1}^{2} \prod_{s \in Y_\alpha} \Big(\phi(a_\alpha, s) + \mu_i\Big),$$

Considering the case of $\mathcal{M}_{\mathsf{sym}}$ we impose some restrictions on the set of eigenvectors for the fundamental multiplets. $\psi \in V_+$,if N-even and $\psi \in V_+$, if N-odd.

The above consideration suggests the following form of the contributions of the fundamental hyper multiplets

$$Z_{\mathsf{f}}^{\mathsf{sym}(0)}(\mu_i,\vec{a},\vec{Y}) = \prod_{i=1}^{4}\prod_{\alpha=1}^{2}\prod_{s\in Y_\alpha, s-\mathsf{white}}\Big(\phi(a_\alpha,s)+\mu_i\Big),$$

$$Z_{\mathsf{f}}^{\mathsf{sym}(1)}(\mu_i,\vec{a},\vec{Y}) = \prod_{i=1}^{4}\prod_{\alpha=1}^{2}\prod_{s\in Y_\alpha, s-\mathsf{black}}\Big(\phi(a_\alpha,s)+\mu_i\Big),$$

The first expression correspond to the case with even number of instantons ,the second one correspond to the case with odd number of instantons.

Four-point Super Liouville conformal block

Two-dimensional super conformal Liouville field theory arises in non-critical String theory. The Lagrangian of the theory reads

$$\mathcal{L}_{\text{SLFT}} = \frac{1}{8\pi}(\partial_a\phi)^2 + \frac{1}{2\pi}\left(\psi\bar{\partial}\psi + \bar{\psi}\partial\bar{\psi}\right) + 2i\mu b^2\bar{\psi}\psi e^{b\phi} + 2\pi b^2\mu^2 e^{2b\phi} .$$

Here μ is the cosmological constant and parameter b is related to the central charge c of the super-Virasoro algebra

$$c = 1 + 2Q^2 , \;\; Q = b + \frac{1}{b} .$$

We are interested in the Neveu-Schwarz sector of the super-Virasoro algebra

$$[L_m, L_n] = (n-m)L_{n+m} + \frac{c}{8}(n^3 - n)\delta_{n+m} ,$$
$$\{G_r, G_s\} = 2L_{r+s} + \frac{1}{2}c(r^2 - \frac{1}{4})\delta_{r+s} ,$$
$$[L_n, G_r] = (\frac{1}{2}n - r)G_{n+r} .$$

where the subscripts m, n –integers and r, s – half-integers. The NS fields belong to highest weight representations of super-Virasoro algebra.

The central problems in CFT is the computation of the correlation functions of the primary fields Φ_Δ and Ψ_Δ has the conformal dimension Δ defined by $L_0|\Delta\rangle = \Delta|\Delta\rangle$, while $\Psi_\Delta \equiv G_{-1/2}\Phi_\Delta$. Together fields Φ_Δ and Ψ_Δ form primary super doublet. The standart parametrization of the conformal dimensions

$$\Delta(\lambda) = \frac{Q^2}{8} - \frac{\lambda^2}{2}.$$

4-point correlation function of bosonic primaries Φ_i is expressed in terms superconformal blocks

$$\langle \Phi_1(q)\Phi_2(0)\Phi_3(1)\Phi_4(\infty)\rangle =$$

$$(q\bar{q})^{\Delta-\Delta_1-\Delta_2} \sum\nolimits_\Delta \Big(C_{12}^{\Delta} C_{34}^{\Delta} F_0(\Delta_i|\Delta|q) F_0(\Delta_i|\Delta|\bar{q}) + \widetilde{C}_{12}^{\Delta} \widetilde{C}_{34}^{\Delta} F_1(\Delta_i|\Delta|q) F_1(\Delta_i|\Delta|\bar{q}) \Big) .$$

The first few coefficients of the superconformal blocks $F_{0,1}$

$$F_0(\Delta_i|\Delta|q) = \sum_{N=0,1,\dots} q^N F^{(N)}(\Delta_i|\Delta) \ ,$$

$$F_1(\Delta_i|\Delta|q) = \sum_{N=1/2,3/2,\dots} q^N F^{(N)}(\Delta_i|\Delta) \ ,$$

$$F^{(0)} = 1,$$

$$F^{(\frac{1}{2})} = \frac{1}{2\Delta},$$

$$F^{(1)} = \frac{(\Delta + \Delta_1 - \Delta_2)(\Delta + \Delta_3 - \Delta_4)}{2\Delta}$$

$$F^{(\frac{3}{2})} = \frac{(1 + 2\Delta + 2\Delta_1 - 2\Delta_2)(1 + 2\Delta + 2\Delta_3 - 2\Delta_4)}{8\Delta(1 + 2\Delta)}$$

$$+ \frac{4(\Delta_1 - \Delta_2)(\Delta_3 - \Delta_4)}{(1 + 2\Delta)(c + 2(-3 + c)\Delta + 4\Delta^2)},$$

We suggest the new representation for the NS four-point conformal blocks :

$$\sum_{N=0,1,\dots} q^N \sum_{\vec{Y},\, \substack{N_+(\vec{Y})=N \\ N_-(\vec{Y})=N}} \frac{Z_{\mathrm{f}}^{\mathrm{sym}(0)}(\mu_i, \vec{a}, \vec{Y})}{Z_{\mathrm{vec}}^{\mathrm{sym}}(\vec{a}, \vec{Y})} = (1-q)^A F_0(\Delta(\lambda_i)|\Delta(a)|q)$$

$$\sum_{N=\frac{1}{2},\frac{3}{2},\dots} q^N \sum_{\vec{Y},\, \substack{N_+(\vec{Y})=N+\frac{1}{2} \\ N_-(\vec{Y})=N-\frac{1}{2}}} \frac{Z_{\mathrm{f}}^{\mathrm{sym}(1)}(\mu_i, \vec{a}, \vec{Y})}{Z_{\mathrm{vec}}^{\mathrm{sym}}(\vec{a}, \vec{Y})} = (1-q)^A F_1(\Delta(\lambda_i)|\Delta(a)|q) .$$

The formula is the main result of this talk. The parameters of the conformal block are related to those of the instanton partition function as

$$\mu_1 = \frac{Q}{2} - (\lambda_1 + \lambda_2), \qquad \mu_2 = \frac{Q}{2} - (\lambda_1 - \lambda_2),$$
$$\mu_3 = \frac{Q}{2} - (\lambda_3 + \lambda_4), \qquad \mu_4 = \frac{Q}{2} - (\lambda_3 - \lambda_4),$$

and

$$A = \left(\frac{Q}{2} - \lambda_1\right)\left(\frac{Q}{2} - \lambda_3\right).$$