SO(32) Heterotic 5–brane from superembedding approach

Progress report

Igor A. Bandos

[†] Department of Theoretical Physics, University of the Basque Country, Bilbao, Spain,
[♣] IKERBASQUE, the Basque Foundation for Science, Bilbao, Spain

SQS11, BLTP, JINR, Dubna, July 18 - 23, 2011

July 21, 2011

Outline

1. Introduction
   - SUSY extended objects
   - 'Simple' D=10, N=1 5-brane and heterotic 5–branes
2. Superembedding approach for 'simple' N=1, D=10 5-brane
   - Worldvolume superspace and superembedding equation
   - Moving, and spinor moving frame and geometry induced by superembedding
3. 'Simple' 5-brane equations of motion from superembedding approach
4. Superembedding description of the *SO*(32) heterotic 5-brane
   - Basic superfield equations of the *SO*(32) heterotic 5-brane
   - From basic superfield equations for *SO*(32) heterotic 5-brane to equations of motion.
5. Conclusions and outlook

Supersymmetric extended objects, **super–*p*-branes** and their description

- Supersymmetric extended objects- especially 10D and 11D super–p–branes, play an important role in String/M-theory and ADS/CFT.

Supersymmetric extended objects, **super–*p*-branes** and their description

- Supersymmetric extended objects- especially 10D and 11D super–p–branes, play an important role in String/M-theory and ADS/CFT.
- They can be described by worldvolume actions, by supersymmetric solutions of supergravity and also in the frame of superembedding approach (which we will be using in this talk).

Supersymmetric extended objects, **super–*p*-branes** and their description

- Supersymmetric extended objects- especially 10D and 11D super–p–branes, play an important role in String/M-theory and ADS/CFT.
- They can be described by worldvolume actions, by supersymmetric solutions of supergravity and also in the frame of superembedding approach (which we will be using in this talk).
- The worldvolume actions are presently known for majority of super-*p*-branes, including fundamental strings
  D=10, string =**F1**-brane ← Green, Schwarz, 1984
  all the M-brane **M2**← 1987: Bergshoeff, Sezgin, Townsend 1987,
  M0← 1996: **BT**:= Bergshoeff, Townsend,
  M5← 1997: BPSTV:= *I.B.*, Lechner, Nurmagambetov, Pasti, Sorokin, Tonin; APSch :=Aganagic, Popescu, Schwarz)
  and D=10 Dirichlet *p*–branes, **D*p*-branes** ← 1996 CNWSG:= Cederwall, Nilsson, Westengerg, Sundell, Gussich; 1996 APSch; 1996 BT

Supersymmetric extended objects, **super–*p*-branes** and their description

- Supersymmetric extended objects- especially 10D and 11D super–p–branes, play an important role in String/M-theory and ADS/CFT.
- They can be described by worldvolume actions, by supersymmetric solutions of supergravity and also in the frame of superembedding approach (which we will be using in this talk).
- The worldvolume actions are presently known for majority of super-*p*-branes, including fundamental strings
  D=10, string =**F1**-brane ← Green, Schwarz, 1984
  all the M-brane **M2**← 1987: Bergshoeff, Sezgin, Townsend 1987,
  M0← 1996: **BT**:= Bergshoeff, Townsend,
  M5← 1997: BPSTV:= *I.B.*, Lechner, Nurmagambetov, Pasti, Sorokin, Tonin; APSch :=Aganagic, Popescu, Schwarz)
  and D=10 Dirichlet *p*–branes, **D*p*-branes** ← 1996 CNWSG:= Cederwall, Nilsson, Westengerg, Sundell, Gussich; 1996 APSch; 1996 BT
- The superembedding approach was proposed and developed for 10D F1 and M2 in [1995 BPSTV:= *I.B.*, Pasti, Sorokin, Tonin, Volkov].
  It uses the worldvolume superfields, developing the STV:= Sorokin, Tkach, Volkov [1988] to D=3,4 particles and strings
  [STV formalism was further developed in 90-94 by Delduc, Galperin, Ivanov, Sokatchev, Howe, Pasti, Tonin, Bergshoeff, Sezgin, Townsend ...]
  related approach: VZ=Volkov, Zheltukhin 1988; Uvarov 2000-08
  Superembedding approach to M5-brane: 1996 HS:= Howe and Sezgin
  S-emb. app. to D*p*-branes: 1996 HS; 1997 BST:=*I.B.*, Sorokin, Tonin,

super-*p*-branes and their description

super-p-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).

super-p-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]

super-p-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996
- Still there exist some BPS solutions of SUGRA equations describing some *p*-branes for which neither worldvolume action nor eqs. of motion are known.

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996
- Still there exist some BPS solutions of SUGRA equations describing some *p*-branes for which neither worldvolume action nor eqs. of motion are known.
- In particular these are D=10 Heterotic 5-branes:

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996
- Still there exist some BPS solutions of SUGRA equations describing some *p*-branes for which neither worldvolume action nor eqs. of motion are known.
- In particular these are D=10 Heterotic 5-branes:
- SO(32) Heterotic 5-brane

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996
- Still there exist some BPS solutions of SUGRA equations describing some *p*-branes for which neither worldvolume action nor eqs. of motion are known.
- In particular these are D=10 Heterotic 5-branes:
- SO(32) Heterotic 5-brane
- $E_8 \times E_8$ heterotic 5-brane

super-*p*-branes and their description

- Notice that the M5-brane equations of motion were obtained in [HS 1996] in the frame of superembedding approach some months before the covariant action was found in [1997 BLNPST, 1997 APSch]
- When the action for a p–brane is known, the superembedding approach can be deduced from that (through GAP:= generalized action principle [1995 BSV:= *I.B.*, Sorokin, Volkov]).
- The way from superembedding approach to the covariant action also exists [1998 Howe, Raetzel, Sezgin] [this can be considered as bootom-up version of GAP and is also related with *Ectoplasm* method by Gates et al]
- The way from BPS solution of supergravity equations to the worldvolume actions is not so straightforward. Examples:
  M5-solution: 1992 Güven (equations 1996, action 1997)
  Dp-branes: solutions are known from early 90th and action in 1996
- Still there exist some BPS solutions of SUGRA equations describing some *p*-branes for which neither worldvolume action nor eqs. of motion are known.
- In particular these are D=10 Heterotic 5-branes:
- SO(32) Heterotic 5-brane
- $E_8 \times E_8$ heterotic 5-brane
- This talk is devoted to the search for $SO(32)$ Heterotic 5-brane equation in the frame of superembedding approach.

string — 5-brane duality and heterotic 5-brane

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?
- Is it the 'simple' 5-brane from the first 'brane scan' by Achucarro, Evan, Townsend and Wiltshire [AETW 1987]?

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?
- Is it the 'simple' 5-brane from the first 'brane scan' by Achucarro, Evan, Townsend and Wiltshire [AETW 1987]?
- **No.** This one is anomalous.

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?
- Is it the 'simple' 5-brane from the first 'brane scan' by Achucarro, Evan, Townsend and Wiltshire [AETW 1987]?
- **No.** This one is anomalous.
- The non-anomolous five branes should be dual to the consistent $N = 1$, $D = 10$ heterotic strings.

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?
- Is it the 'simple' 5-brane from the first 'brane scan' by Achucarro, Evan, Townsend and Wiltshire [AETW 1987]?
- **No.** This one is anomalous.
- The non-anomolous five branes should be dual to the consistent $N = 1$, $D = 10$ heterotic strings.
- As far as there are two anomaly–free heterotic strings, carrying charges of $SO(32)$ and of $E_8 \times E_8$ gauge theories, respectively,

string — 5-brane duality and heterotic 5-brane

- In $D = 10$ supergravity there exists a (BPS) string solution
- and also 5-brane solution which is dual to the string in the same sense as the magnetic monopole is dual to electric charge in D=4.
- What is the worldvolume action for this 5-brane?
- Is it the 'simple' 5-brane from the first 'brane scan' by Achucarro, Evan, Townsend and Wiltshire [AETW 1987]?
- **No.** This one is anomalous.
- The non-anomolous five branes should be dual to the consistent $N = 1$, $D = 10$ heterotic strings.
- As far as there are two anomaly–free heterotic strings, carrying charges of $SO(32)$ and of $E_8 \times E_8$ gauge theories, respectively,
- there should be two anomaly-free 5-branes:
  $SO(32)$ 5-brane and
  $E_8 \times E_8$ 5-branes

'Simple' 5-brane [AETW 1987]

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi\sqrt{g} + \int \hat{B}_6 \ ,$$

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi\sqrt{g} + \int \hat{B}_6 \ ,$$

- $$g = det(g_{mn}) \ , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \ , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi \sqrt{g} + \int \hat{B}_6 \, ,$$

- $$g = det(g_{mn}) \, , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \, , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

- $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$ are supercoordinate functions describing embedding of the worldvolume to the target superspace

$$W^6 \subset \Sigma^{(10|16)} \quad : \qquad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$$

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi \sqrt{g} + \int \hat{B}_6 \; ,$$

- $$g = det(g_{mn}) \; , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \; , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

- $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$ are supercoordinate functions describing embedding of the worldvolume to the target superspace

$$W^6 \subset \Sigma^{(10|16)} \; : \qquad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$$

- $E^a(Z) = dZ^M E_M^a(Z)$ is bosonic vielbein of the 10D $\mathcal{N} = 1$ SUGRA

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi \sqrt{g} + \int \hat{B}_6 \ ,$$

- $$g = det(g_{mn}) \ , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \ , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

- $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$ are supercoordinate functions describing embedding of the worldvolume to the target superspace

$$W^6 \subset \Sigma^{(10|16)} \ : \qquad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$$

- $E^a(Z) = dZ^M E_M^a(Z)$ is bosonic vielbein of the 10D $\mathcal{N} = 1$ SUGRA
- $\hat{E}^a$ is its pull–back to W^6, i.e. $\hat{E}^a = E^a(\hat{Z}) = d\hat{Z}^M E_M^a(\hat{Z})$.

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi \sqrt{g} + \int \hat{B}_6 \; ,$$

- $$g = det(g_{mn}) \; , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \; , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

- $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi) \, , \hat{\theta}^{\underline{\mu}}(\xi))$ are supercoordinate functions describing embedding of the worldvolume to the target superspace

$$W^6 \subset \Sigma^{(10|16)} \; : \qquad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi) \, , \hat{\theta}^{\underline{\mu}}(\xi))$$

- $E^a(Z) = dZ^M E_M^a(Z)$ is bosonic vielbein of the 10D $\mathcal{N} = 1$ SUGRA
- $\hat{E}^a$ is its pull–back to W^6, i.e. $\hat{E}^a = E^a(\hat{Z}) = d\hat{Z}^M E_M^a(\hat{Z})$.
- $\hat{B}_6 = B_6(\hat{Z})$ is the pull–back to W^6 of $B_6 = \frac{1}{6!} dZ^{M_6} \wedge \ldots \wedge dZ^{M_1} B_{M_1 \ldots M_6}(Z)$.

'Simple' 5-brane [AETW 1987]

- The 'simple' (and anomalous) $D = 10$ $N = 1$ 5-brane from [AETW 1987] is described by

$$S = S^{DNG} + S^{WZ} = \int d^6\xi \sqrt{g} + \int \hat{B}_6 \ ,$$

-

$$g = det(g_{mn}) \ , \quad g_{mn} = \hat{E}_m^{\underline{a}} \hat{E}_{n\underline{a}} \ , \quad \hat{E}_m^{\underline{a}} = \partial_m \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z})$$

- $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\, , \hat{\theta}^{\underline{\mu}}(\xi))$ are supercoordinate functions describing embedding of the worldvolume to the target superspace

$$W^6 \subset \Sigma^{(10|16)} \ : \qquad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\, , \hat{\theta}^{\underline{\mu}}(\xi))$$

- $E^a(Z) = dZ^M E_M^a(Z)$ is bosonic vielbein of the 10D $\mathcal{N} = 1$ SUGRA
- $\hat{E}^a$ is its pull–back to W^6, i.e. $\hat{E}^a = E^a(\hat{Z}) = d\hat{Z}^M E_M^a(\hat{Z})$.
- $\hat{B}_6 = B_6(\hat{Z})$ is the pull–back to W^6 of $B_6 = \frac{1}{6!} dZ^{M_6} \wedge \ldots \wedge dZ^{M_1} B_{M_1 \ldots M_6}(Z)$.
- $E^a(Z)$ and $B_6(Z)$ obey the superspace supergravity constraints $\Rightarrow$ the action possesses local fermionic κ–symmetry.

SO(32) heterotic 5-brane

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\,, \hat{\theta}^{\underline{\mu}}(\xi))$,

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\,,\hat{\theta}^{\underline{\mu}}(\xi))$,
- $d = 6$, $N = 2$ $SU(2)$ SYM multiplet: a traceless 2×2 matrix connection $A_{\tilde{B}}{}^{\tilde{A}} = d\xi^m A_{m\tilde{B}}{}^{\tilde{A}}$ ($A_{\tilde{B}}{}^{\tilde{B}} = 0$, $\tilde{A}, \tilde{B} = 1, 2$) and its superpartner $(W^{\beta}_{B})_{\tilde{B}}{}^{\tilde{A}}$

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\, , \hat{\theta}^{\underline{\mu}}(\xi))$,
- $d = 6$, $N = 2$ $SU(2)$ SYM multiplet: a traceless 2×2 matrix connection $A_{\tilde{B}}{}^{\tilde{A}} = d\xi^m A_{m\tilde{B}}{}^{\tilde{A}}$ ($A_{\tilde{B}}{}^{\tilde{B}} = 0$, $\tilde{A}, \tilde{B} = 1, 2$) and its superpartner $(W_B^{\beta})_{\tilde{B}}{}^{\tilde{A}}$
- hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$:
- bosonic and fermionic fields $(H^{A\tilde{B}J}(\xi), \psi_\alpha^{\tilde{B}J}(\xi))$ related by susy

$$\delta_{susy} H^{A\tilde{B}J} = 4i\epsilon^{\alpha A} \psi_\alpha^{\tilde{B}J} \; ,$$

$$A, B = 1, 2 \; , \quad J = 1, ..., 32 \qquad \tilde{A}, \tilde{B} = 1, 2 \; , \quad \alpha = 1, 2, 3, 4 \; .$$

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\,, \hat{\theta}^{\underline{\mu}}(\xi))$,
- $d = 6$, $N = 2$ $SU(2)$ SYM multiplet: a traceless 2×2 matrix connection $A_{\tilde{B}}{}^{\tilde{A}} = d\xi^m A_{m\tilde{B}}{}^{\tilde{A}}$ ($A_{\tilde{B}}{}^{\tilde{B}} = 0$, $\tilde{A}, \tilde{B} = 1, 2$) and its superpartner $(W^{\beta}_{B})_{\tilde{B}}{}^{\tilde{A}}$
- hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$:
- bosonic and fermionic fields $(H^{A\tilde{B}J}(\xi), \psi_{\alpha}^{\tilde{B}J}(\xi))$ related by susy

$$\delta_{susy} H^{A\tilde{B}J} = 4i\epsilon^{\alpha A}\psi_{\alpha}^{\tilde{B}J} \; ,$$

$$A, B = 1, 2 \; , \quad J = 1, ..., 32 \qquad \tilde{A}, \tilde{B} = 1, 2 \; , \quad \alpha = 1, 2, 3, 4 \; .$$

- Neither action nor equations of motion of the $SO(32)$ heterotic 5-brane are known.

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi), \hat{\theta}^{\underline{\mu}}(\xi))$,
- $d = 6$, $N = 2$ $SU(2)$ SYM multiplet: a traceless 2×2 matrix connection $A_{\tilde{B}}{}^{\tilde{A}} = d\xi^m A_{m\tilde{B}}{}^{\tilde{A}}$ $(A_{\tilde{B}}{}^{\tilde{B}} = 0,\ \tilde{A}, \tilde{B} = 1, 2)$ and its superpartner $(W^{\beta}_{B})_{\tilde{B}}{}^{\tilde{A}}$
- hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$:
- bosonic and fermionic fields $(H^{A\tilde{B}J}(\xi), \psi^{\tilde{B}J}_{\alpha}(\xi))$ related by susy

$$\delta_{susy} H^{A\tilde{B}J} = 4i\epsilon^{\alpha A}\psi^{\tilde{B}J}_{\alpha}\ ,$$

$$A, B = 1, 2\ , \quad J = 1, ..., 32 \quad \tilde{A}, \tilde{B} = 1, 2\ , \quad \alpha = 1, 2, 3, 4\ .$$

- Neither action nor equations of motion of the $SO(32)$ heterotic 5-brane are known.
- For $E_8 \times E_8$ even the field content is not clear.

Then the natural proposition is

- Then the natural proposition is to use superembedding approach to search for the $SO(32)$ heterotic 5-brane equations of motion.

SO(32) heterotic 5-brane

- In 1995 Witten argued that the spectrum of SO(32) heterotic five-brane contains, besides the 'geometrical sector' $\hat{Z}^{\underline{\mathcal{M}}}(\xi) = (\hat{x}^{\underline{m}}(\xi)\,, \hat{\theta}^{\underline{\mu}}(\xi))$,
- $d = 6$, $N = 2$ $SU(2)$ SYM multiplet: a traceless 2×2 matrix connection ${A_{\tilde{B}}}^{\tilde{A}} = d\xi^m {A_{m\tilde{B}}}^{\tilde{A}}$ (${A_{\tilde{B}}}^{\tilde{B}} = 0$, $\tilde{A}, \tilde{B} = 1, 2$) and its superpartner ${(W_B^\beta)_{\tilde{B}}}^{\tilde{A}}$
- hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$:
- bosonic and fermionic fields $(H^{A\tilde{B}J}(\xi), \psi_\alpha^{\tilde{B}J}(\xi))$ related by susy

$$\delta_{susy} H^{A\tilde{B}J} = 4i\epsilon^{\alpha A}\psi_\alpha^{\tilde{B}J}\ ,$$

$$A, B = 1, 2\ , \quad J = 1, ..., 32 \quad \tilde{A}, \tilde{B} = 1, 2\ , \quad \alpha = 1, 2, 3, 4\ .$$

- Neither action nor equations of motion of the $SO(32)$ heterotic 5-brane are known.
- For $E_8 \times E_8$ even the field content is not clear.

Then the natural proposition is

- Then the natural proposition is to use superembedding approach to search for the $SO(32)$ heterotic 5-brane equations of motion.
- This talk is a progress report on elaboration of this program.

Then the natural proposition is

- The presence of non–geometrical sector makes the heterotic 5-brane similar to multiple (D)p–brane systems (mD*p*) the superembedding approach for which was proposed and elaborated for the case of mD0 and mM0 system in [*I.B.* 2009, *I.B.* 2010].

Then the natural proposition is

- The presence of non–geometrical sector makes the heterotic 5-brane similar to multiple (D)p–brane systems (mD*p*) the superembedding approach for which was proposed and elaborated for the case of mD0 and mM0 system in [*I.B.* 2009, *I.B.* 2010].
- The basic proposition is similar to the one in [*I.B.* 2009, *I.B.* 2010]. Schematically it is: to describe the heterotic 5-brane by the superspace constraints of $SU(2)$ SYM and of the $(2, 32)$ hypermultiplet on the curved superspace $W^{(6|8)}$ of a 'simple' 5-brane.

Then the natural proposition is

- The presence of non–geometrical sector makes the heterotic 5-brane similar to multiple (D)p–brane systems (mD*p*) the superembedding approach for which was proposed and elaborated for the case of mD0 and mM0 system in [*I.B.* 2009, *I.B.* 2010].
- The basic proposition is similar to the one in [*I.B.* 2009, *I.B.* 2010]. Schematically it is: to describe the heterotic 5-brane by the superspace constraints of $SU(2)$ SYM and of the $(2, 32)$ hypermultiplet on the curved superspace $W^{(6|8)}$ of a 'simple' 5-brane.
- We can consider a more general framework, *e.g.* trying to make the basic superspace $W^{(6|8)}$ different from the worldvolume superspace of the 'simple' 5-brane.

Then the natural proposition is

- The presence of non–geometrical sector makes the heterotic 5-brane similar to multiple (D)p–brane systems (mDp) the superembedding approach for which was proposed and elaborated for the case of mD0 and mM0 system in [*I.B.* 2009, *I.B.* 2010].
- The basic proposition is similar to the one in [*I.B.* 2009, *I.B.* 2010]. Schematically it is: to describe the heterotic 5-brane by the superspace constraints of $SU(2)$ SYM and of the $(2, 32)$ hypermultiplet on the curved superspace $W^{(6|8)}$ of a 'simple' 5-brane.
- We can consider a more general framework, *e.g.* trying to make the basic superspace $W^{(6|8)}$ different from the worldvolume superspace of the 'simple' 5-brane.
- But anyway, the natural first step is to discuss the superembedding approach on the relatively simple example of 'simple' 5-brane.

Outline

1. Introduction
   - SUSY extended objects
   - 'Simple' D=10, N=1 5-brane and heterotic 5–branes
2. Superembedding approach for 'simple' N=1, D=10 5-brane
   - Worldvolume superspace and superembedding equation
   - Moving, and spinor moving frame and geometry induced by superembedding
3. 'Simple' 5-brane equations of motion from superembedding approach
4. Superembedding description of the *SO*(32) heterotic 5-brane
   - Basic superfield equations of the *SO*(32) heterotic 5-brane
   - From basic superfield equations for *SO*(32) heterotic 5-brane to equations of motion.
5. Conclusions and outlook

Worldvolume superspace

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, *p*-branes.

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, p-branes.
- The worldvolume W^{p+1} is extended in it till the worldvolume superspace $\mathcal{W}^{(p+1|\frac{n}{2})}$ with $\frac{n}{2}$ fermionic dimensions, where the target superspace is $\Sigma^{(D|n)}$ ($n = 32$ for 11D and type II 10D, $n = 16$ for 10D, $\mathcal{N} = 1$ branes).

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, p-branes.
- The worldvolume W^{p+1} is extended in it till the worldvolume superspace $\mathcal{W}^{(p+1|\frac{n}{2})}$ with $\frac{n}{2}$ fermionic dimensions, where the target superspace is $\Sigma^{(D|n)}$ ($n = 32$ for 11D and type II 10D, $n = 16$ for 10D, $\mathcal{N} = 1$ branes).
- Hence for simple and heterotic $D = 10$, $\mathcal{N} = 1$ five-brane, we have to consider $\mathcal{W}^{(6|8)}$ with local coordinates

$$\zeta^{\mathcal{M}} = (\xi^m \, , \, \eta^\mu) \, , \qquad \eta^\mu \eta^\nu = -\eta^\nu \eta^\mu \, , \qquad \begin{cases} m = 0, 1, ..., 5 \, , \\ \mu = 1, ..., 8 \, , \end{cases}$$

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, *p*-branes.
- The worldvolume W^{p+1} is extended in it till the worldvolume superspace $\mathcal{W}^{(p+1|\frac{n}{2})}$ with $\frac{n}{2}$ fermionic dimensions, where the target superspace is $\Sigma^{(D|n)}$ ($n=32$ for 11D and type II 10D, $n=16$ for 10D, $\mathcal{N}=1$ branes).
- Hence for simple and heterotic $D=10$, $\mathcal{N}=1$ five-brane, we have to consider $\mathcal{W}^{(6|8)}$ with local coordinates

$$\zeta^{\mathcal{M}} = (\xi^m \, , \, \eta^\mu) \, , \qquad \eta^\mu \eta^\nu = -\eta^\nu \eta^\mu \, , \qquad \begin{cases} m = 0, 1, ..., 5 \, , \\ \mu = 1, ..., 8 \, , \end{cases}$$

- The embedding of $\mathcal{W}^{(6|8)}$ into $\Sigma^{(10|16)}$ can be described in terms of coordinate functions $\hat{Z}^{\underline{\mathcal{M}}}(\zeta) = (\hat{x}^{\underline{m}}(\zeta) \, , \hat{\theta}^{\underline{\mu}}(\zeta))$, ($\underline{m} = 0, 1, ..., 9$, $\underline{\mu} = 1, ..., 16$) which are *worldvolume superfields*

$$\mathcal{W}^{(6|8)} \in \Sigma^{(10|16)} : \quad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\zeta) \qquad \Leftrightarrow \qquad \begin{cases} x^{\underline{m}} = \hat{x}^{\underline{m}}(\zeta) \, , \\ \theta^{\underline{\mu}} = \hat{\theta}^{\underline{\mu}}(\zeta) \, . \end{cases}$$

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, *p*-branes.
- The worldvolume W^{p+1} is extended in it till the worldvolume superspace $\mathcal{W}^{(p+1|\frac{n}{2})}$ with $\frac{n}{2}$ fermionic dimensions, where the target superspace is $\Sigma^{(D|n)}$ ($n = 32$ for 11D and type II 10D, $n = 16$ for 10D, $\mathcal{N} = 1$ branes).
- Hence for simple and heterotic $D = 10$, $\mathcal{N} = 1$ five-brane, we have to consider $\mathcal{W}^{(6|8)}$ with local coordinates

$$\zeta^{\mathcal{M}} = (\xi^m \, , \, \eta^\mu) \, , \qquad \eta^\mu \eta^\nu = -\eta^\nu \eta^\mu \, , \qquad \begin{cases} m = 0, 1, ..., 5 \, , \\ \mu = 1, ..., 8 \, , \end{cases}$$

- The embedding of $\mathcal{W}^{(6|8)}$ into $\Sigma^{(10|16)}$ can be described in terms of coordinate functions $\hat{Z}^{\underline{\mathcal{M}}}(\zeta) = (\hat{x}^{\underline{m}}(\zeta) \, , \hat{\theta}^{\underline{\mu}}(\zeta))$, ($\underline{m} = 0, 1, ..., 9$, $\underline{\mu} = 1, ..., 16$) which are *worldvolume superfields*

$$\mathcal{W}^{(6|8)} \in \Sigma^{(10|16)} : \quad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\zeta) \qquad \Leftrightarrow \qquad \begin{cases} x^{\underline{m}} = \hat{x}^{\underline{m}}(\zeta) \, , \\ \theta^{\underline{\mu}} = \hat{\theta}^{\underline{\mu}}(\zeta) \, . \end{cases}$$

- In the case of 'simple' five–brane **(and all known p-branes!)**, $\hat{Z}^{\underline{\mathcal{M}}}(\zeta)$ is determined by the superembedding equation

Worldvolume superspace

- Superembedding approach provides the superfield description of the worldvolume dynamics of supersymmetric extended objects, *p*-branes.
- The worldvolume W^{p+1} is extended in it till the worldvolume superspace $\mathcal{W}^{(p+1|\frac{n}{2})}$ with $\frac{n}{2}$ fermionic dimensions, where the target superspace is $\Sigma^{(D|n)}$ ($n=32$ for 11D and type II 10D, $n=16$ for 10D, $\mathcal{N}=1$ branes).
- Hence for simple and heterotic $D=10$, $\mathcal{N}=1$ five-brane, we have to consider $\mathcal{W}^{(6|8)}$ with local coordinates

$$\zeta^{\mathcal{M}} = (\xi^m\ ,\ \eta^\mu)\ , \qquad \eta^\mu\eta^\nu = -\eta^\nu\eta^\mu\ , \qquad \begin{cases} m = 0, 1, ..., 5\ , \\ \mu = 1, ..., 8\ , \end{cases}$$

- The embedding of $\mathcal{W}^{(6|8)}$ into $\Sigma^{(10|16)}$ can be described in terms of coordinate functions $\hat{Z}^{\underline{\mathcal{M}}}(\zeta) = (\hat{x}^{\underline{m}}(\zeta)\, , \hat{\theta}^{\underline{\mu}}(\zeta))$, ($\underline{m} = 0, 1, ..., 9\ ,\ \underline{\mu} = 1, ..., 16$) which are *worldvolume superfields*

$$\mathcal{W}^{(6|8)} \in \Sigma^{(10|16)}:\quad Z^{\underline{\mathcal{M}}} = \hat{Z}^{\underline{\mathcal{M}}}(\zeta) \qquad \Leftrightarrow \qquad \begin{cases} x^{\underline{m}} = \hat{x}^{\underline{m}}(\zeta)\ , \\ \theta^{\underline{\mu}} = \hat{\theta}^{\underline{\mu}}(\zeta)\ . \end{cases}$$

- In the case of 'simple' five–brane **(and all known p-branes!)**, $\hat{Z}^{\underline{\mathcal{M}}}(\zeta)$ is determined by the superembedding equation
- (= are the solutions of the superembedding equations)

Superembedding equation

Superembedding equation

- Let us introduce the supervielbein forms of $\mathcal{W}^{(6|8)}$

$$e^{\mathcal{A}} := (e^a\ ,\ e^{\alpha A}) := d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^{\mathcal{A}}(\zeta)\ , \quad a = 0, 1, ..., 5\ , \quad \begin{cases} \alpha = 1, 2, 3, 4\ , \\ A = 1, 2\ \ : \end{cases}$$

6-vector one-form $e^a = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^a(\zeta)$ and the $SU(2)$ doublet of $SO(1,5)$-spinor fermionic forms $e^{\alpha A}$.

Superembedding equation

- Let us introduce the supervielbein forms of $\mathcal{W}^{(6|8)}$

$$e^{\mathcal{A}} := (e^a\ ,\ e^{\alpha A}) := d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^{\mathcal{A}}(\zeta)\ , \quad a = 0, 1, ..., 5\ , \quad \begin{cases} \alpha = 1, 2, 3, 4\ , \\ A = 1, 2\ \ : \end{cases}$$

  6-vector one-form $e^a = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^a(\zeta)$ and the $SU(2)$ doublet of $SO(1,5)$-spinor fermionic forms $e^{\alpha A}$.
- The pull–back $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z})$ of the supervielbein forms of the target superspace $\Sigma^{(10|16)}$

$$E^{\underline{\mathcal{A}}} := dZ^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(Z) = (E^{\underline{a}}, E^{\underline{\alpha}})\ , \qquad \underline{a} = 0, 1, ..., 9\ , \qquad \underline{\alpha} = 1, 2, ..., 16\ .$$

  can be decomposed on the basis of $e^{\mathcal{A}}$

Superembedding equation

- Let us introduce the supervielbein forms of $\mathcal{W}^{(6|8)}$

$$e^{\mathcal{A}} := (e^{a}\ ,\ e^{\alpha A}) := d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^{\mathcal{A}}(\zeta)\ , \quad a = 0, 1, ..., 5\ , \quad \begin{cases} \alpha = 1, 2, 3, 4\ , \\ A = 1, 2\ \ : \end{cases}$$

6-vector one-form $e^{a} = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^{a}(\zeta)$ and the $SU(2)$ doublet of $SO(1,5)$-spinor fermionic forms $e^{\alpha A}$.

- The pull–back $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z})$ of the supervielbein forms of the target superspace $\Sigma^{(10|16)}$

$$E^{\underline{\mathcal{A}}} := dZ^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(Z) = (E^{\underline{a}}, E^{\underline{\alpha}})\ , \qquad \underline{a} = 0, 1, ..., 9\ , \qquad \underline{\alpha} = 1, 2, ..., 16\ .$$

can be decomposed on the basis of $e^{\mathcal{A}}$

- $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^{b} \hat{E}_{b}{}^{\underline{\mathcal{A}}}\ .$

Superembedding equation

- Let us introduce the supervielbein forms of $\mathcal{W}^{(6|8)}$

$$e^{\mathcal{A}} := (e^a \, , \, e^{\alpha A}) := d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^{\mathcal{A}}(\zeta) \, , \quad a = 0, 1, ..., 5 \, , \quad \begin{cases} \alpha = 1, 2, 3, 4 \, , \\ A = 1, 2 \quad : \end{cases}$$

  6-vector one-form $e^a = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^a(\zeta)$ and the $SU(2)$ doublet of $SO(1,5)$-spinor fermionic forms $e^{\alpha A}$.

- The pull–back $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z})$ of the supervielbein forms of the target superspace $\Sigma^{(10|16)}$

$$E^{\underline{\mathcal{A}}} := dZ^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(Z) = (E^{\underline{a}}, E^{\underline{\alpha}}) \, , \qquad \underline{a} = 0, 1, ..., 9 \, , \qquad \underline{\alpha} = 1, 2, ..., 16 \, .$$

  can be decomposed on the basis of $e^{\mathcal{A}}$

- $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^b \hat{E}_b{}^{\underline{\mathcal{A}}} \, .$

- **The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection**

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B} \hat{Z}^{\underline{\mathcal{M}}} \, E_{\underline{\mathcal{M}}}{}^{\underline{a}}(\hat{Z}) = 0 \, .$$

Superembedding equation and moving frame

- $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^{b} \hat{E}_{b}{}^{\underline{\mathcal{A}}}$.
- The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B} \hat{Z}^{\underline{\mathcal{M}}} \, E_{\underline{\mathcal{M}}}{}^{\underline{a}}(\hat{Z}) = 0 \ .$$

Superembedding equation and moving frame

- $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^{b} \hat{E}_{b}{}^{\underline{\mathcal{A}}}$.
- The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B} \hat{Z}^{\underline{\mathcal{M}}} \, E_{\underline{\mathcal{M}}}{}^{\underline{a}}(\hat{Z}) = 0 \ .$$

- Equivalently we can write the superembedding equation as $\hat{E}^{\underline{a}} = e^{b} \hat{E}_{b}^{\underline{a}}$.

Superembedding equation and moving frame

- $\hat{E}^{\underline{\mathcal{A}}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}}\mathcal{D}_{\mathcal{B}}\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B}\hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^{b}\hat{E}_{b}{}^{\underline{\mathcal{A}}}$.
- The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B}\hat{Z}^{\underline{\mathcal{M}}}\, E_{\underline{\mathcal{M}}}{}^{\underline{a}}(\hat{Z}) = 0 \ .$$

- Equivalently we can write the superembedding equation as $\hat{E}^{\underline{a}} = e^{b}\hat{E}_{b}^{\underline{a}}$.
- 6 ten-vectors $u_{b}^{\underline{a}} = \hat{E}_{b}^{\underline{a}}$ are linearly independent and can be chosen orthogonal and normalized,

$$\hat{E}^{\underline{a}} = e^{b} u_{b}^{\underline{a}} \ , \qquad u_{a\,\underline{a}} u_{b}^{\underline{a}} = \eta_{ab} = diag(+,-,-,-,-,-) \ .$$

Superembedding equation and moving frame

- $\hat{E}^{\mathcal{A}} := d\hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\mathcal{A}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\mathcal{M}} E_{\mathcal{M}}{}^{\mathcal{A}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\mathcal{A}} + e^{b} \hat{E}_{b}{}^{\mathcal{A}}$.
- The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B} \hat{Z}^{\mathcal{M}}\, E_{\mathcal{M}}{}^{\underline{a}}(\hat{Z}) = 0 \ .$$

- Equivalently we can write the superembedding equation as $\hat{E}^{\underline{a}} = e^{b} \hat{E}_{b}^{\underline{a}}$.
- 6 ten-vectors $u_{b}^{\underline{a}} = \hat{E}_{b}^{\underline{a}}$ are linearly independent and can be chosen orthogonal and normalized,

$$\hat{E}^{\underline{a}} = e^{b} u_{b}^{\underline{a}} \ , \qquad u_{a\,\underline{a}} u_{b}^{\underline{a}} = \eta_{ab} = diag(+,-,-,-,-,-) \ .$$

- $\Rightarrow$ the worldvolume vielbein is induced by (super)embedding

$$e^{a} = \hat{E}^{\underline{a}} u_{\underline{a}}^{a} \ .$$

Superembedding equation and moving frame

- $\hat{E}^{\mathcal{A}} := d\hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\mathcal{B}} \mathcal{D}_{\mathcal{B}} \hat{Z}^{\underline{\mathcal{M}}} E_{\underline{\mathcal{M}}}{}^{\underline{\mathcal{A}}}(\hat{Z}) = e^{\beta B} \hat{E}_{\beta B}{}^{\underline{\mathcal{A}}} + e^{b} \hat{E}_{b}{}^{\underline{\mathcal{A}}}$.
- The superembedding equation states that the pull–back of the bosonic supervielbein of $\Sigma^{(10|16)}$ to $\mathcal{W}^{(6|8)}$ has no fermionic projection

$$\hat{E}_{\beta B}{}^{\underline{a}} := \mathcal{D}_{\beta B} \hat{Z}^{\underline{\mathcal{M}}}\, E_{\underline{\mathcal{M}}}{}^{\underline{a}}(\hat{Z}) = 0 \ .$$

- Equivalently we can write the superembedding equation as $\hat{E}^{\underline{a}} = e^{b} \hat{E}_{b}^{\underline{a}}$.
- 6 ten-vectors $u_{b}^{\underline{a}} = \hat{E}_{b}^{\underline{a}}$ are linearly independent and can be chosen orthogonal and normalized,

$$\hat{E}^{\underline{a}} = e^{b} u_{b}^{\underline{a}} \ , \qquad u_{a\,\underline{a}} u_{b}^{\underline{a}} = \eta_{ab} = diag(+,-,-,-,-,-) \ .$$

- $\Rightarrow$ the worldvolume vielbein is induced by (super)embedding

$$e^{a} = \hat{E}^{\underline{a}} u_{\underline{a}}^{a} \ .$$

- $\Rightarrow$ 6 vectors $u_{b}^{\underline{a}}$ are tangential to the worldvolume superspace $\mathcal{W}^{(6|8)}$.

Moving frame and superembedding equation

- Equivalent form of the superembedding equation

$$\hat{E}^{\underline{a}} = e^{b} u_{b}^{\underline{a}} \, , \qquad u_{a\,\underline{a}} u_{b}^{\underline{a}} = \eta_{ab} = diag(+,-,-,-,-,-) \, .$$

- $\Rightarrow$ 6 vectors $u_{b}^{\underline{a}}$ are tangential to the worldvolume superspace $\mathcal{W}^{(6|8)}$.

Moving frame and superembedding equation

- Equivalent form of the superembedding equation

$$\hat{E}^{\underline{a}} = e^b u_b^{\underline{a}} , \qquad u_{a\,\underline{a}} u_b^{\underline{a}} = \eta_{ab} = diag(+,-,-,-,-,-) \; .$$

- $\Rightarrow$ 6 vectors $u_b^{\underline{a}}$ are tangential to the worldvolume superspace $\mathcal{W}^{(6|8)}$.
- Actually, it is convenient to complete their set till *moving frame* by introducing four spatial 10-vectors $u_{B\check{B}}^{\underline{a}}$ orthogonal to them and normalized ($SO(4) = SU(2) \times SU(2)$),

$$\delta_{\underline{b}}{}^{\underline{a}} = u_{\underline{b}}{}^{c} u_{c}{}^{\underline{a}} - \frac{1}{2} u_{\underline{b}}^{A\check{B}} u_{A\check{B}}{}^{\underline{a}} \, , \qquad u_{\underline{a}}^{c} u^{B\check{B}\underline{a}} = 0 \, , \qquad u_{\underline{a}}^{A\check{A}} u^{B\check{B}\underline{a}} = -2\epsilon^{AB}\epsilon^{\check{A}\check{B}} \; .$$

These vectors can be used to write one more equivalent form of the superembedding equation,

$$\hat{E}^{A\check{A}} := \hat{E}^{\underline{a}} u_{\underline{a}}^{A\check{A}} = 0 \; .$$

Spinor moving frame and fermionic superveilbein

Spinor moving frame and fermionic superveilbein

- We can also define the fermionic supervielbein $e^{\alpha A}$ induced by superembedding,

$$e^{\alpha A} = \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\alpha A} \; .$$

  Then consistency requires to identify $v_{\underline{\alpha}}{}^{\alpha A}$ with one of the auxiliary *spinor moving frame superfields* (or spinorial Lorentz harmonics).

Spinor moving frame and fermionic superveilbein

- We can also define the fermionic supervielbein $e^{\alpha A}$ induced by superembedding,

$$e^{\alpha A} = \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\alpha A} \ .$$

  Then consistency requires to identify $v_{\underline{\alpha}}{}^{\alpha A}$ with one of the auxiliary *spinor moving frame superfields* (or spinorial Lorentz harmonics).

- These are two rectangular blocks of a $Spin(1,9)$ valued matrix (*spinor moving frame matrix*)

$$V_{\underline{\alpha}}{}^{(\beta)} = (v_{\underline{\alpha}}{}^{\beta B}, v_{\underline{\alpha}\beta}{}^{\check{B}}) \in Spin(1,9) \ , \quad \beta = 1, ..., 4 \ , \quad B = 1,2 \ , \quad \check{B} = 1,2$$

Spinor moving frame and fermionic superveilbein

- We can also define the fermionic supervielbein $e^{\alpha A}$ induced by superembedding,

$$e^{\alpha A} = \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\alpha A} \; .$$

  Then consistency requires to identify $v_{\underline{\alpha}}{}^{\alpha A}$ with one of the auxiliary *spinor moving frame superfields* (or spinorial Lorentz harmonics).

- These are two rectangular blocks of a $Spin(1,9)$ valued matrix (*spinor moving frame matrix*)

$$V_{\underline{\alpha}}{}^{(\beta)} = (v_{\underline{\alpha}}{}^{\beta B}, v_{\underline{\alpha}\beta}{}^{\check{B}}) \in Spin(1,9) \; , \quad \beta = 1, ..., 4 \; , \quad B = 1,2 \; , \quad \check{B} = 1,2$$

- which are related to the moving frame vectors by the following square–root–type relations

$$v^{\alpha A} \tilde{\sigma}_{\underline{a}} v^{\beta B} = \epsilon^{AB} \tilde{\gamma}_b^{\alpha\beta} u_{\underline{a}}{}^b \; , \qquad v_{\alpha}^{\check{A}} \tilde{\sigma}_{\underline{a}} v_{\beta}^{\check{B}} = -\epsilon^{\check{A}\check{B}} \gamma_{b\alpha\beta} u_{\underline{a}}{}^b \; ,$$

$$v^{\alpha A} \tilde{\sigma}_{\underline{a}} v_{\beta}^{\check{B}} = \delta_{\beta}^{\alpha} u_{\underline{a}}^{A\check{B}} \; , \qquad etc. \; .$$

  where $\gamma^a_{\gamma\delta} = -\gamma^a_{\delta\gamma}$ and $\tilde{\gamma}^{b\gamma\delta} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\gamma_{a\gamma\delta}$ are $d=6$ Pauli matrices, while $\sigma^{\underline{a}}_{\underline{\alpha\beta}} = \sigma^{\underline{a}}_{\underline{\beta\alpha}}$, $\tilde{\sigma}^{\underline{a}\underline{\alpha\beta}} = \tilde{\sigma}^{\underline{a}\underline{\beta\alpha}}$ are $D=10$ Pauli matrices, $\sigma^{(\underline{a}}\tilde{\sigma}^{\underline{b})} = \eta^{(\underline{ab})}$.

Connections induced by superembedding.

Connections induced by superembedding.

- We can define the $SO(1,5)$ and $SO(4)$ connections on $\mathcal{W}^{(6|8)}$:

$$\mathcal{D}u_{\underline{b}}^{\ a} = \frac{1}{2} u_{\underline{b}A\check{A}} \Omega^{a\,A\check{A}} \,, \qquad \mathcal{D}u_{\underline{b}}^{A\check{A}} = \frac{1}{2} u_{\underline{b}a} \Omega^{a\,A\check{A}} \,. \qquad (*)$$

$\Omega^{a\,A\check{A}}$ is the generalization of the $\frac{SO(1,9)}{SO(1,5)\otimes SO(4)}$ Cartan forms.

Connections induced by superembedding.

- We can define the $SO(1,5)$ and $SO(4)$ connections on $\mathcal{W}^{(6|8)}$:

$$\mathcal{D}u_{\underline{b}}^{\;a} = \frac{1}{2} u_{\underline{b}A\check{A}} \Omega^{a\,A\check{A}} \,, \qquad \mathcal{D}u_{\underline{b}}^{A\check{A}} = \frac{1}{2} u_{\underline{b}a} \Omega^{a\,A\check{A}} \,. \qquad (*)$$

$\Omega^{a\,A\check{A}}$ is the generalization of the $\frac{SO(1,9)}{SO(1,5)\otimes SO(4)}$ Cartan forms.

- The derivatives of spinor moving frame variables read

$$\mathcal{D}v_{\underline{\alpha}}^{\;\beta B} = \frac{1}{2} v_{\underline{\alpha}\gamma}^{\;\;\check{A}} \tilde{\gamma}_a^{\gamma\beta} \epsilon_{\check{A}\check{B}} \Omega^{a\,B\check{B}} \,, \qquad \mathcal{D}v_{\underline{\alpha}\beta}^{\;\;\check{B}} = \frac{1}{2} v_{\underline{\alpha}}^{\;\gamma A} \gamma^{a}_{\gamma\beta} \epsilon_{AB} \Omega^{a\,B\check{B}} \,.$$

Connections induced by superembedding.

- We can define the $SO(1,5)$ and $SO(4)$ connections on $\mathcal{W}^{(6|8)}$:

$$\mathcal{D}u_{\underline{b}}{}^{a} = \frac{1}{2} u_{\underline{b}A\check{A}} \Omega^{a\,A\check{A}} , \qquad \mathcal{D}u_{\underline{b}}^{A\check{A}} = \frac{1}{2} u_{\underline{b}a} \Omega^{a\,A\check{A}} . \qquad (*)$$

$\Omega^{a\,A\check{A}}$ is the generalization of the $\frac{SO(1,9)}{SO(1,5)\otimes SO(4)}$ Cartan forms.

- The derivatives of spinor moving frame variables read

$$\mathcal{D}v_{\underline{\alpha}}^{\beta B} = \frac{1}{2} v_{\underline{\alpha}\gamma}{}^{\check{A}} \tilde{\gamma}_a^{\gamma\beta} \epsilon_{\check{A}\check{B}} \Omega^{a\,B\check{B}} , \qquad \mathcal{D}v_{\underline{\alpha}\beta}{}^{\check{B}} = \frac{1}{2} v_{\underline{\alpha}}^{\gamma A} \gamma^{a}_{\gamma\beta} \epsilon_{AB} \Omega^{a\,B\check{B}} .$$

- The worldvolume curvature two form, $r^{ab} = -r^{ba}$ and the curvature of normal bundle $\mathcal{F}_B{}^A$ and $\mathcal{F}_{\check{B}}{}^{\check{A}}$ ($SO(4) = SU(2) \otimes SU(2)$), can be now defined by Ricci identities

$$\mathcal{D}\mathcal{D}u_{\underline{b}}{}^{a} = \hat{R}_{\underline{b}}{}^{\underline{a}} u_{\underline{a}}{}^{a} - u_{\underline{a}}{}^{b} r_b{}^{a} , \qquad \mathcal{D}\mathcal{D}u_{\underline{b}}^{A\check{A}} = \hat{R}_{\underline{b}}{}^{\underline{a}} u_{\underline{a}}^{A\check{A}} - u_{\underline{a}}^{B\check{A}} \mathcal{F}_B{}^A - u_{\underline{a}}^{A\check{B}} \mathcal{F}_{\check{B}}{}^{\check{A}} ,$$

where $\hat{R}_{\underline{b}}{}^{\underline{a}}$ is the pull–back of the SO(1,9) curvature of $\Sigma^{(10|16)}$.

Curvatures of the worldvolume superspace and of the normal bundle

- The worldvolume curvature two form, $r^{ab} = -r^{ba}$ and the curvature of normal bundle $\mathcal{F}_B{}^A$ and $\mathcal{F}_{\check{B}}{}^{\check{A}}$ ($SO(4) = SU(2) \otimes SU(2)$), can be now defined by Ricci identities

$$\mathcal{D}\mathcal{D}u_{\underline{b}}{}^{a} = \hat{R}_{\underline{b}}{}^{\underline{a}}u_{\underline{a}}{}^{a} - u_{\underline{a}}{}^{b}r_{b}{}^{a}\,, \qquad \mathcal{D}\mathcal{D}u_{\underline{b}}^{A\check{A}} = \hat{R}_{\underline{b}}{}^{\underline{a}}u_{\underline{a}}^{A\check{A}} - u_{\underline{a}}^{B\check{A}}\mathcal{F}_B{}^A - u_{\underline{a}}^{A\check{B}}\mathcal{F}_{\check{B}}{}^{\check{A}}\,,$$

where $\hat{R}_{\underline{b}}{}^{\underline{a}}$ is the pull–back of the SO(1,9) curvature of $\Sigma^{(10|16)}$.

Curvatures of the worldvolume superspace and of the normal bundle

- The worldvolume curvature two form, $r^{ab} = -r^{ba}$ and the curvature of normal bundle $\mathcal{F}_B{}^A$ and $\mathcal{F}_{\check{B}}{}^{\check{A}}$ ($SO(4) = SU(2) \otimes SU(2)$), can be now defined by Ricci identities

$$\mathcal{D}\mathcal{D}u_{\underline{b}}{}^a = \hat{R}_{\underline{b}}{}^{\underline{a}} u_{\underline{a}}{}^a - u_{\underline{a}}{}^b r_b{}^a \, , \qquad \mathcal{D}\mathcal{D}u_{\underline{b}}^{A\check{A}} = \hat{R}_{\underline{b}}{}^{\underline{a}} u_{\underline{a}}^{A\check{A}} - u_{\underline{a}}^{B\check{A}} \mathcal{F}_B{}^A - u_{\underline{a}}^{A\check{B}} \mathcal{F}_{\check{B}}{}^{\check{A}} \, ,$$

  where $\hat{R}_{\underline{b}}{}^{\underline{a}}$ is the pull–back of the SO(1,9) curvature of $\Sigma^{(10|16)}$.

- Substituting $\mathcal{D}u_{\underline{b}}{}^a = \frac{1}{2} u_{\underline{b}A\check{A}} \Omega^{a\,A\check{A}}$ and $\mathcal{D}u_{\underline{b}}^{A\check{A}} = \frac{1}{2} u_{\underline{b}a} \Omega^{a\,A\check{A}}$, we find the following superfield generalization of the Peterson–Codazzi, Gauss and Ricci equations [BPSTV:= *I.B.*, Pasti, Sorokin, Tonin, Volkov, 1995]

$$D\Omega^{a\,A\check{A}} = \hat{R}^{a\,A\check{A}} \, , \qquad r^{ab} = \hat{R}^{ab} + \frac{1}{2} \Omega^{a}_{A\check{A}} \wedge \Omega^{b\,A\check{A}} \, ,$$

$$\mathcal{F}_B{}^A = \frac{1}{4} \hat{R}_{B\check{B}}^{A\check{B}} + \frac{1}{4} \Omega_{B\check{B}} \wedge \Omega^{b\,A\check{B}} \, , \qquad \mathcal{F}_{\check{B}}{}^{\check{A}} = \frac{1}{4} \hat{R}_{B\check{B}}^{B\check{A}} + \frac{1}{4} \Omega_{bB\check{B}} \wedge \Omega^{b\,B\check{A}} \, ,$$

  where $\hat{R}^{a\,A\check{A}} := \hat{R}^{\underline{ab}} u_{\underline{a}}^a u_{\underline{b}}^{A\check{A}}$, $\hat{R}^{a\,b} := \hat{R}^{\underline{ab}} u_{\underline{a}}^a u_{\underline{b}}^b$ and $\hat{R}_{B\check{B}}^{A\check{A}} := \hat{R}^{\underline{ab}} u_{\underline{a}B\check{B}} u_{\underline{b}}^{A\check{A}}$.

Outline

1. Introduction
   - SUSY extended objects
   - 'Simple' D=10, N=1 5-brane and heterotic 5–branes
2. Superembedding approach for 'simple' N=1, D=10 5-brane
   - Worldvolume superspace and superembedding equation
   - Moving, and spinor moving frame and geometry induced by superembedding
3. 'Simple' 5-brane equations of motion from superembedding approach
4. Superembedding description of the $SO(32)$ heterotic 5-brane
   - Basic superfield equations of the $SO(32)$ heterotic 5-brane
   - From basic superfield equations for $SO(32)$ heterotic 5-brane to equations of motion.
5. Conclusions and outlook

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} \ ,$$

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} \, ,$$

- where $\hat{T}^{\underline{a}}$ is the pull–back to $\mathcal{W}^{(6|8)}$ of $T^{\underline{a}} := DE^{\underline{a}} := dE^{\underline{a}} - E^{\underline{b}} \wedge \omega_{\underline{b}}{}^{\underline{a}}$

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} \; ,$$

- where $\hat{T}^{\underline{a}}$ is the pull–back to $\mathcal{W}^{(6|8)}$ of $T^{\underline{a}} := DE^{\underline{a}} := dE^{\underline{a}} - E^{\underline{b}} \wedge \omega_{\underline{b}}{}^{\underline{a}}$
- The $D = 10$, $\mathcal{N} = 1$ supergravity constraints imply that

$$T^{\underline{a}} := DE^{\underline{a}} = -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}} \; ,$$

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} \; ,$$

- where $\hat{T}^{\underline{a}}$ is the pull–back to $\mathcal{W}^{(6|8)}$ of $T^{\underline{a}} := DE^{\underline{a}} := dE^{\underline{a}} - E^{\underline{b}} \wedge \omega_{\underline{b}}{}^{\underline{a}}$
- The $D = 10$, $\mathcal{N} = 1$ supergravity constraints imply that

$$T^{\underline{a}} := DE^{\underline{a}} = -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}} \; ,$$

- and also [Nilsson, Tollsen 86, ... , Tonin, Lechner, Bonora, ... 1988]

$$T^{\underline{\alpha}} := DE^{\underline{\alpha}} = \frac{i}{4} E^{\underline{b}} \wedge E^{\underline{\beta}} (\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3} \sigma_{\underline{b}})_{\underline{\beta}}{}^{\underline{\alpha}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} + \frac{1}{2} E^{\underline{b}} \wedge E^{\underline{a}} T_{\underline{ab}}{}^{\underline{\alpha}} \; ,$$

$$R^{\underline{ab}} := d\omega^{\underline{ab}} - \omega^{[\underline{a}|\underline{c}} \wedge \omega_{\underline{c}}{}^{|\underline{b}]} = \frac{1}{2} E^{\underline{\alpha}} \wedge E^{\underline{\beta}} \left(\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3\underline{ab}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} - 6h^{\underline{abc}} \sigma_{\underline{c}}\right)_{\underline{\alpha}\underline{\beta}} +$$

$$+ E^{\underline{c}} \wedge E^{\underline{\beta}} \left[- iT^{\underline{ab}\underline{\beta}} \sigma_{\underline{c}\underline{\beta}\underline{\alpha}} + 2iT_{\underline{c}}{}^{[\underline{a}\,\underline{\beta}} \sigma^{\underline{b}]}{}_{\underline{\beta}\underline{\alpha}} \right] + \frac{1}{2} E^{\underline{d}} \wedge E^{\underline{c}} R_{\underline{cd}}{}^{\underline{ab}}$$

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D} u_{\underline{a}}^{A\check{A}} \; ,$$

- where $\hat{T}^{\underline{a}}$ is the pull–back to $\mathcal{W}^{(6|8)}$ of $T^{\underline{a}} := DE^{\underline{a}} := dE^{\underline{a}} - E^{\underline{b}} \wedge \omega_{\underline{b}}{}^{\underline{a}}$
- The $D = 10$, $\mathcal{N} = 1$ supergravity constraints imply that

$$T^{\underline{a}} := DE^{\underline{a}} = -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha\beta}} \; ,$$

- and also [Nilsson, Tollsen 86, ... , Tonin, Lechner, Bonora, ... 1988]

$$T^{\underline{\alpha}} := DE^{\underline{\alpha}} = \frac{i}{4} E^{\underline{b}} \wedge E^{\underline{\beta}} (\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3}\sigma_{\underline{b}})_{\underline{\beta}}{}^{\underline{\alpha}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} + \frac{1}{2} E^{\underline{b}} \wedge E^{\underline{a}} T_{\underline{ab}}{}^{\underline{\alpha}} \; ,$$

$$R^{\underline{ab}} := d\omega^{\underline{ab}} - \omega^{[\underline{a}|\underline{c}} \wedge \omega_{\underline{c}}{}^{|\underline{b}]} = \frac{1}{2} E^{\underline{\alpha}} \wedge E^{\underline{\beta}} \left(\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3\underline{ab}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} - 6h^{\underline{abc}}\sigma_{\underline{c}}\right)_{\underline{\alpha\beta}} +$$

$$+ E^{\underline{c}} \wedge E^{\underline{\beta}} \left[-iT^{\underline{ab}\underline{\beta}} \sigma_{\underline{c}\underline{\beta\alpha}} + 2iT_{\underline{c}}{}^{[\underline{a}\underline{\beta}} \sigma^{\underline{b}]}{}_{\underline{\beta\alpha}} \right] + \frac{1}{2} E^{\underline{d}} \wedge E^{\underline{c}} R_{\underline{cd}}{}^{\underline{ab}}$$

- $h_{\underline{a}_1\underline{a}_2\underline{a}_3} = h_{[\underline{a}_1\underline{a}_2\underline{a}_3]}$ is related to the field strength of the 2-form (Ogievetsky–Polubarinov—Kalb-Ramond) gauge field $B_{\underline{ab}} = B_{[\underline{ab}]}$.

The selfconsistency conditions for the superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- can be collected in the differential form equation

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} \; ,$$

- where $\hat{T}^{\underline{a}}$ is the pull–back to $\mathcal{W}^{(6|8)}$ of $T^{\underline{a}} := DE^{\underline{a}} := dE^{\underline{a}} - E^{\underline{b}} \wedge \omega_{\underline{b}}{}^{\underline{a}}$
- The $D = 10$, $\mathcal{N} = 1$ supergravity constraints imply that

$$T^{\underline{a}} := DE^{\underline{a}} = -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}} \; ,$$

- and also [Nilsson, Tollsen 86, ... , Tonin, Lechner, Bonora, ... 1988]

$$T^{\underline{\alpha}} := DE^{\underline{\alpha}} = \frac{i}{4} E^{\underline{b}} \wedge E^{\underline{\beta}} (\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3} \sigma_{\underline{b}})_{\underline{\beta}}{}^{\underline{\alpha}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} + \frac{1}{2} E^{\underline{b}} \wedge E^{\underline{a}} T_{\underline{ab}}{}^{\underline{\alpha}} \; ,$$

$$R^{\underline{ab}} := d\omega^{\underline{ab}} - \omega^{[\underline{a}|\underline{c}} \wedge \omega_{\underline{c}}{}^{|\underline{b}]} = \frac{1}{2} E^{\underline{\alpha}} \wedge E^{\underline{\beta}} \left(\sigma^{\underline{a}_1\underline{a}_2\underline{a}_3\underline{ab}} h_{\underline{a}_1\underline{a}_2\underline{a}_3} - 6h^{\underline{abc}} \sigma_{\underline{c}}\right)_{\underline{\alpha}\underline{\beta}} +$$
$$+ E^{\underline{c}} \wedge E^{\underline{\beta}} \left[-iT^{\underline{ab}\underline{\beta}} \sigma_{\underline{c}\underline{\beta}\underline{\alpha}} + 2iT_{\underline{c}}{}^{[\underline{a}\,\underline{\beta}} \sigma^{\underline{b}]}{}_{\underline{\beta}\underline{\alpha}} \right] + \frac{1}{2} E^{\underline{d}} \wedge E^{\underline{c}} R_{\underline{cd}}{}^{\underline{ab}}$$

- $h_{\underline{a}_1\underline{a}_2\underline{a}_3} = h_{[\underline{a}_1\underline{a}_2\underline{a}_3]}$ is related to the field strength of the 2-form (Ogievetsky–Polubarinov—Kalb-Ramond) gauge field $B_{ab} = B_{[ab]}$.
- The modifications of the constraints to account for anomalies/ modifications of the BIs for H_3 and H_7 were studied during 25 years by many groups [B.E.W. Nilsson 86, ... Tonin, Lechner 2008, Howe 2008].

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- Studying

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} =$$
$$= -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha}\underline{\beta}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} u_{\underline{a}b} \wedge \Omega^{bA\check{A}} =$$
$$= -4ie^{\alpha A} \wedge \hat{E}_{\alpha}^{\check{A}} + e_b \wedge \Omega^{b\, A\dot{A}} = 0 \, ,$$

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- Studying

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} =$$
$$= -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha\beta}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} u_{\underline{a}b} \wedge \Omega^{bA\check{A}} =$$
$$= -4ie^{\alpha A} \wedge \hat{E}_{\alpha}^{\check{A}} + e_b \wedge \Omega^{b\,A\dot{A}} = 0\ ,$$

- we find ($e^{\alpha A} = \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\alpha A}$)

$$\hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} = e^a \chi_{a\alpha}^{\check{A}}\ ,$$
$$\Omega^{b\,A\dot{A}} = 4ie^{\alpha A} \chi_{a\alpha}^{\check{A}} + e^b K_b{}^{a\,A\check{A}}\ ,$$

with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$ generalizing the second fundamental form of the Surface Theory.

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- Studying

$$0 = \mathcal{D}\hat{E}^{A\check{A}} = \hat{T}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} \wedge \mathcal{D}u_{\underline{a}}^{A\check{A}} =$$
$$= -iE^{\underline{\alpha}} \wedge E^{\underline{\beta}} \sigma^{\underline{a}}_{\underline{\alpha\beta}} u_{\underline{a}}^{A\dot{A}} + \hat{E}^{\underline{a}} u_{\underline{a}b} \wedge \Omega^{bA\check{A}} =$$
$$= -4ie^{\alpha A} \wedge \hat{E}_{\alpha}^{\check{A}} + e_b \wedge \Omega^{b\, A\dot{A}} = 0\ ,$$

- we find ($e^{\alpha A} = \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\alpha A}$)

$$\hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}{}^{\check{A}} = e^a \chi_{a\alpha}{}^{\check{A}}\ ,$$
$$\Omega^{b\, A\dot{A}} = 4ie^{\alpha A}\chi_{a\alpha}{}^{\check{A}} + e^b K_b{}^{a\, A\check{A}}\ ,$$

with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$ generalizing the second fundamental form of the Surface Theory.

- Linearized and gauge fixed version $E_b^{\underline{a}} \mapsto \partial_b \hat{x}^{\underline{a}}$, $K_a{}^{a\, A\check{A}} \mapsto \partial_a\partial_b \hat{x}^{A\check{A}}$ indicates that the dynamical bosonic equations for the super-5-brane can be formulated as an expression for the trace of $K_{ab}^{A\check{A}}$, *mean curvature*, $\mathcal{H}^{A\check{A}} := K_a^{aA\check{A}} \mapsto \partial_a\partial^a \hat{x}^{A\check{A}}$.

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- $$0 = \mathcal{D}\hat{E}^{A\check{A}} \quad \Rightarrow , \begin{cases} \hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} = e^{a}\chi_{a\alpha}^{\check{A}} , \\ \Omega^{b\, A\dot{A}} = 4ie^{\alpha A}\chi_{a\alpha}^{\check{A}} + e^{b}K_{b}{}^{a\, A\check{A}} , \end{cases}$$

  with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$.

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- $$0 = \mathcal{D}\hat{E}^{A\check{A}} \quad \Rightarrow , \begin{cases} \hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} = e^a \chi_{a\alpha}^{\check{A}} , \\ \Omega^{b\,A\dot{A}} = 4ie^{\alpha A} \chi_{a\alpha}^{\check{A}} + e^b K_b{}^{a\,A\check{A}} , \end{cases}$$

  with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$.
- $0 = \mathcal{D}(\hat{E}_{\alpha}^{\check{A}} - e^a \chi_{a\alpha}^{\check{A}}) = \mathcal{D}(\hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} - e^a \chi_{a\alpha}^{\check{A}}) = 0 \Rightarrow$

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- $$0 = \mathcal{D}\hat{E}^{A\check{A}} \quad \Rightarrow , \begin{cases} \hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\;\check{A}} = e^a \chi_{a\alpha}^{\;\check{A}} , \\ \Omega^{b\,A\dot{A}} = 4ie^{\alpha A}\chi_{a\alpha}^{\;\check{A}} + e^b K_b{}^{a\;A\check{A}} , \end{cases}$$

  with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$.
- $0 = \mathcal{D}(\hat{E}_{\alpha}^{\check{A}} - e^a \chi_{a\alpha}^{\;\check{A}}) \; = \mathcal{D}(\hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\;\check{A}} - e^a \chi_{a\alpha}^{\;\check{A}}) = 0 \Rightarrow$
- $\Rightarrow$ fermionic equations of motion (free linearized limit: $\tilde{\gamma}^{a\alpha\beta}\partial_a \hat{\theta}_{\beta}^{\check{A}} = 0$)

$$\tilde{\gamma}^{a\alpha\beta}\chi_{a\beta}^{\;\check{A}} = 0 \qquad \Leftrightarrow \qquad \tilde{\gamma}^{a\alpha\beta}\hat{E}_a{}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\;\check{A}} = 0$$

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\check{A}} = 0$

- $$0 = \mathcal{D}\hat{E}^{A\check{A}} \quad \Rightarrow , \begin{cases} \hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}{}^{\check{A}} = e^a \chi_{a\alpha}{}^{\check{A}} \,, \\ \Omega^{b\,A\dot{A}} = 4ie^{\alpha A}\chi_{a\alpha}{}^{\check{A}} + e^b K_b{}^{a\,A\check{A}} \,, \end{cases}$$

  with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}}\, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$.
- $0 = \mathcal{D}(\hat{E}_{\alpha}^{\check{A}} - e^a \chi_{a\alpha}{}^{\check{A}}) = \mathcal{D}(\hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}{}^{\check{A}} - e^a \chi_{a\alpha}{}^{\check{A}}) = 0 \Rightarrow$
- $\Rightarrow$ fermionic equations of motion (free linearized limit: $\tilde{\gamma}^{a\alpha\beta}\partial_a \hat{\theta}_{\beta}^{\check{A}} = 0$)

$$\tilde{\gamma}^{a\alpha\beta}\chi_{a\beta}{}^{\check{A}} = 0 \qquad \Leftrightarrow \qquad \tilde{\gamma}^{a\alpha\beta}\hat{E}_a{}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}{}^{\check{A}} = 0$$

- $\Rightarrow$ bosonic equation of motion (free lin. limit: $\partial_a \partial^a \hat{x}^{A\check{A}} = 0$)

$$\eta^{bc} K_{bc\;B\check{A}} := -D^c \hat{E}_c{}^{\underline{a}} u_{\underline{a}B\check{A}} = \frac{3i}{2} h_{\underline{abc}}(\hat{Z})\, u^{\underline{a}}_{B\check{C}} u^{\underline{b}C\check{C}} u^{\underline{c}}_{C\check{A}} \,,$$

Simple 5-brane equations from superembedding equation $\hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

- $$0 = \mathcal{D}\hat{E}^{A\check{A}} \quad \Rightarrow , \begin{cases} \hat{E}_{\alpha}^{\check{A}} := \hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} = e^a \chi_{a\alpha}^{\check{A}} \, , \\ \Omega^{b\, A\dot{A}} = 4ie^{\alpha A} \chi_{a\alpha}^{\check{A}} + e^b K_b{}^{a\, A\check{A}} \, , \end{cases}$$

  with symmetric $K_{ab}{}^{A\check{A}} := -\mathcal{D}_a E_b^{\underline{a}} \, u_{\underline{a}}^{A\check{A}} = K_{ba}{}^{A\check{A}}$.
- $0 = \mathcal{D}(\hat{E}_{\alpha}^{\check{A}} - e^a \chi_{a\alpha}^{\check{A}}) \ = \mathcal{D}(\hat{E}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} - e^a \chi_{a\alpha}^{\check{A}}) = 0 \Rightarrow$
- $\Rightarrow$ fermionic equations of motion (free linearized limit: $\tilde{\gamma}^{a\alpha\beta} \partial_a \hat{\theta}_{\beta}^{\check{A}} = 0$)

  $$\tilde{\gamma}^{a\alpha\beta} \chi_{a\beta}^{\check{A}} = 0 \qquad \Leftrightarrow \qquad \tilde{\gamma}^{a\alpha\beta} \hat{E}_a{}^{\underline{\alpha}} v_{\underline{\alpha}\alpha}^{\check{A}} = 0$$
- $\Rightarrow$ bosonic equation of motion (free lin. limit: $\partial_a \partial^a \hat{x}^{A\check{A}} = 0$)

  $$\eta^{bc} K_{bc\ B\check{A}} := -D^c \hat{E}_c{}^{\underline{a}} u_{\underline{a}B\check{A}} = \tfrac{3i}{2} h_{\underline{abc}}(\hat{Z}) \, u_{B\check{C}}^{\underline{a}} u^{\underline{b}C\check{C}} u_{C\check{A}}^{\underline{c}} \, ,$$
- $\Rightarrow$ the restriction on the Ogievetsky–Polubarinov–Kalb–Ramond flux,

  $$h_{\underline{abc}}(\hat{Z}) u_a^{\underline{a}} u_b^{\underline{b}} u_{A\check{A}}^{\underline{c}} = 0 \, .$$

Outline

1. Introduction
   - SUSY extended objects
   - 'Simple' D=10, N=1 5-brane and heterotic 5–branes
2. Superembedding approach for 'simple' N=1, D=10 5-brane
   - Worldvolume superspace and superembedding equation
   - Moving, and spinor moving frame and geometry induced by superembedding
3. 'Simple' 5-brane equations of motion from superembedding approach
4. Superembedding description of the $SO(32)$ heterotic 5-brane
   - Basic superfield equations of the $SO(32)$ heterotic 5-brane
   - From basic superfield equations for $SO(32)$ heterotic 5-brane to equations of motion.
5. Conclusions and outlook

Our basic proposition is

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.
- This latter assumption in practical term implies that the embedding of $\mathcal{W}^{(6|8)}$ into the curved superspace of SUGRA+SO(32) SYM theory is defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \Leftrightarrow \hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$

Our basic proposition is

- Our basic proposition is to describe the *SO*(32) heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.
- This latter assumption in practical term implies that the embedding of $\mathcal{W}^{(6|8)}$ into the curved superspace of SUGRA+SO(32) SYM theory is defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \Leftrightarrow \hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$
- This item can be modified $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \;\mapsto\; \hat{E}_{\alpha A}{}^{\underline{a}} = ...$ (although it is not easy to modify superembedding eq. and to get a manageable system)

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.
- This latter assumption in practical term implies that the embedding of $\mathcal{W}^{(6|8)}$ into the curved superspace of SUGRA+SO(32) SYM theory is defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \Leftrightarrow \hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$
- This item can be modified $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \;\mapsto\; \hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (although it is not easy to modify superembedding eq. and to get a manageable system)
- But it is natural to begin from superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ obeying $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$, at least as an approximation

Our basic proposition is

- Our basic proposition is to describe the *SO*(32) heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.
- This latter assumption in practical term implies that the embedding of $\mathcal{W}^{(6|8)}$ into the curved superspace of SUGRA+SO(32) SYM theory is defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \Leftrightarrow \hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\dot{A}} = 0$
- This item can be modified $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \;\mapsto\; \hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (although it is not easy to modify superembedding eq. and to get a manageable system)
- But it is natural to begin from superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ obeying $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$, at least as an approximation
- The geometry of such a superspace has been completely described by the above study:

Our basic proposition is

- Our basic proposition is to describe the $SO(32)$ heterotic 5-brane by
- superfield equations of supermultiplet in **(2,32)** of $SU(2) \times SO(32)$
- and by the constraints of the d=6, $SU(2)$ SYM
- on the worldvolume superspace of 'simple' 5-brane $\mathcal{W}^{(6|8)}$.
- This latter assumption in practical term implies that the embedding of $\mathcal{W}^{(6|8)}$ into the curved superspace of SUGRA+SO(32) SYM theory is defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \Leftrightarrow \hat{E}^{A\check{A}} = \hat{E}^{\underline{a}} u_{\underline{a}}^{A\check{A}} = 0$
- This item can be modified $\hat{E}_{\alpha A}{}^{\underline{a}} = 0 \ \mapsto \ \hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (although it is not easy to modify superembedding eq. and to get a manageable system)
- But it is natural to begin from superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ obeying $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$, at least as an approximation
- The geometry of such a superspace has been completely described by the above study:

$$\mathcal{D}e^{a} = T^{\underline{a}} u_{\underline{a}}^{a} = -ie^{\alpha A} \wedge e^{\beta B} \epsilon_{AB} \gamma^{a}_{\alpha\beta} + ie^{c} \wedge e^{b} \epsilon_{\check{A}\check{B}} \chi_{b}^{\check{A}} \tilde{\gamma}^{a} \chi_{c}^{\check{B}} \ .$$

Our basic proposition is

- ...
- But it is natural to begin from superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ obeying $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$, at least as an approximation.
- The geometry of such a superspace has been completely described by the above study:

$$\mathcal{D}e^{a} = T^{\underline{a}}u_{\underline{a}}^{a} = -ie^{\alpha A} \wedge e^{\beta B}\epsilon_{AB}\gamma^{a}_{\alpha\beta} + ie^{c} \wedge e^{b}\epsilon_{\check{A}\check{B}}\chi_{b}^{\check{A}}\tilde{\gamma}^{a}\chi_{c}^{\check{B}} ,$$

Our basic proposition is

- ...
- But it is natural to begin from superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ obeying $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$, at least as an approximation.
- The geometry of such a superspace has been completely described by the above study:

$$\mathcal{D}e^a = T^{\underline{a}} u_{\underline{a}}^a = -ie^{\alpha A} \wedge e^{\beta B} \epsilon_{AB} \gamma^a_{\alpha\beta} + ie^c \wedge e^b \epsilon_{\check{A}\check{B}} \chi_b^{\check{A}} \tilde{\gamma}^a \chi_c^{\check{B}} \; ,$$

$$\mathcal{D}e^{\alpha A} = e^b \wedge e^{\beta B} t_{\beta B\; b}{}^{\alpha A} + \tfrac{1}{2} e^b \wedge e^a t_{ab}{}^{\alpha A} \; ,$$

$$t_{\beta B\; b}{}^{\alpha A} = 2i \chi_{a\beta\check{B}} \chi_{b\gamma}{}^{\check{B}} \tilde{\gamma}^{a\gamma\alpha} - \tfrac{i}{4} \hat{h}_{c_1c_2c_3} (\gamma^{c_1c_2c_3}\gamma_b)_\beta{}^\alpha \delta_B{}^A - \tfrac{3i}{4} \hat{h}_{b\, B\check{B}}{}^{A\check{B}} (\gamma^a\gamma_b)_\beta{}^\alpha \delta_B{}^A \; ,$$

$$t_{ab}{}^{\alpha A} = \epsilon_{\check{B}\check{C}} (\chi_{[a|}{}^{\check{C}} \tilde{\gamma}^c)^\alpha K_{|b]c}{}^{A\check{B}} + \tfrac{i}{2} \hat{h}_{D\check{C}}{}^{D\check{A}\; A\check{C}} \epsilon_{\check{A}\check{B}} (\chi^{\check{B}}_{[a} \tilde{\gamma}_{b]})^\alpha +$$
$$+ \tfrac{3i}{2} \epsilon^{AB} \hat{h}_{cd\; B\check{B}} (\tilde{\gamma}_{[a} \gamma^{cd} \chi^{\check{B}}_{b]})^\alpha + \hat{T}_{ab}{}^{\alpha A} \qquad ,$$

$$r^{ab} = \hat{R}^{ab} + 8 e^{\alpha A} \wedge e^{\beta B} \epsilon_{AB} \epsilon_{\check{A}\check{B}} \chi_\alpha^{a\check{A}} \chi_\beta^{b\check{B}} - 4ie^c \wedge e^{\alpha A} \chi_\alpha^{[a|\check{B}} K_c{}^{|b]}{}_{A\check{A}} +$$
$$+ \tfrac{1}{2} e^c \wedge e^d K_c{}^a{}_{A\check{A}} K_d{}^{b\; A\check{A}} \; ,$$

$$\mathcal{F}_B{}^A = \hat{R}^{ab} + 8 e^{\alpha A} \wedge e^{\beta B} \epsilon_{AB} \epsilon_{\check{A}\check{B}} \chi_\alpha^{a\check{A}} \chi_\beta^{b\check{B}} - 4ie^c \wedge e^{\alpha A} \chi_\alpha^{[a|\check{B}} K_c{}^{|b]}{}_{A\check{A}} +$$
$$+ \tfrac{1}{2} e^c \wedge e^d K_c{}^a{}_{A\check{A}} K_d{}^{b\; A\check{A}} \; .$$

Superfield description of heterotic d.o.f.s

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the $SO(32)$ heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the $SO(32)$ heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$
- either defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (good point to begin)
- or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (next stage)

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the *SO*(32) heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$
- either defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (good point to begin)
- or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (next stage)

Superfield description of heterotic degrees of freedom

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the $SO(32)$ heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$
- either defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (good point to begin)
- or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (next stage)

Superfield description of heterotic degrees of freedom

- The SU(2) SYM is described by $SU(2)$ connection 1-form on $\mathcal{W}^{(6|8)}$

$$A_{\tilde{B}}{}^{\tilde{A}} = e^{\alpha C} A_{\alpha C\, \tilde{B}}{}^{\tilde{A}}(\zeta) + e^{a} A_{a\, \tilde{B}}{}^{\tilde{A}}(\zeta)\, , \qquad (A_{\tilde{B}}{}^{\tilde{A}})^{*} = -A_{\tilde{A}}{}^{\tilde{B}} \quad (\Rightarrow\ A_{\tilde{A}}{}^{\tilde{A}} = 0)\, ,$$

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the $SO(32)$ heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$
- either defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (good point to begin)
- or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (next stage)

Superfield description of heterotic degrees of freedom

- The SU(2) SYM is described by $SU(2)$ connection 1-form on $\mathcal{W}^{(6|8)}$

$$A_{\tilde{B}}{}^{\tilde{A}} = e^{\alpha C} A_{\alpha C\, \tilde{B}}{}^{\tilde{A}}(\zeta) + e^{a} A_{a\, \tilde{B}}{}^{\tilde{A}}(\zeta) \, , \qquad (A_{\tilde{B}}{}^{\tilde{A}})^* = -A_{\tilde{A}}{}^{\tilde{B}} \quad (\Rightarrow \; A_{\tilde{A}}{}^{\tilde{A}} = 0) \, ,$$

- which obeys the constraints $F_{\alpha A \beta B \tilde{B}}{}^{\tilde{A}} = 0 \;\Rightarrow$

$$F_{\tilde{B}}{}^{\tilde{A}} := (dA - A \wedge A)_{\tilde{B}}{}^{\tilde{A}} = \frac{i}{2} e^{a} \wedge e^{\alpha A} \gamma_{b\alpha\beta} (W^{\beta}_{A})_{\tilde{B}}{}^{\tilde{A}} + \frac{1}{2} e^{b} \wedge e^{a} (F_{ab})_{\tilde{B}}{}^{\tilde{A}} \, ,$$

Superfield description of heterotic d.o.f.s

- The heterotic degrees of freedom of the $SO(32)$ heterotic 5-branes are described by superfields on the superspace $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$
- either defined by the superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (good point to begin)
- or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = \ldots$ (next stage)

Superfield description of heterotic degrees of freedom

- The SU(2) SYM is described by $SU(2)$ connection 1-form on $\mathcal{W}^{(6|8)}$

$$A_{\tilde{B}}{}^{\tilde{A}} = e^{\alpha C} A_{\alpha C\, \tilde{B}}{}^{\tilde{A}}(\zeta) + e^{a} A_{a\, \tilde{B}}{}^{\tilde{A}}(\zeta)\, , \qquad (A_{\tilde{B}}{}^{\tilde{A}})^* = -A_{\tilde{A}}{}^{\tilde{B}} \quad (\Rightarrow \; A_{\tilde{A}}{}^{\tilde{A}} = 0)\, ,$$

- which obeys the constraints $F_{\alpha A \beta B \tilde{B}}{}^{\tilde{A}} = 0 \;\Rightarrow$

$$F_{\tilde{B}}{}^{\tilde{A}} := (dA - A \wedge A)_{\tilde{B}}{}^{\tilde{A}} = \frac{i}{2} e^{a} \wedge e^{\alpha A} \gamma_{b\alpha\beta} (W^{\beta}_{A})_{\tilde{B}}{}^{\tilde{A}} + \frac{1}{2} e^{b} \wedge e^{a} (F_{ab})_{\tilde{B}}{}^{\tilde{A}}\, ,$$

- heterotic hypermultiplet(s) are defined by superfield $H^{A\tilde{B}J}(\zeta)$ in **(2,32)** representation of $SU(2) \times SO(32)$ which obeys

$$\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i \delta_{C}{}^{A} \psi_{\gamma}^{\tilde{B}J}$$

Superfield description of heterotic degrees of freedom: basic superfield eqs.

- The SU(2) SYM is described by $SU(2)$ connection one form on $\mathcal{W}^{(6|8)}$

$$A_{\tilde{B}}{}^{\tilde{A}} = e^{\alpha C} A_{\alpha C\,\tilde{B}}{}^{\tilde{A}}(\zeta) + e^{a} A_{a\,\tilde{B}}{}^{\tilde{A}}(\zeta)\,, \qquad (A_{\tilde{B}}{}^{\tilde{A}})^{*} = -A_{\tilde{A}}{}^{\tilde{B}} \quad (\Rightarrow\ A_{\tilde{A}}{}^{\tilde{A}} = 0)\,,$$

- which obeys the constraints

$$F_{\tilde{B}}{}^{\tilde{A}} := (dA - A\wedge A)_{\tilde{B}}{}^{\tilde{A}} = \frac{i}{2} e^{a}\wedge e^{\alpha A}\gamma_{b\alpha\beta}(W^{\beta}_{A})_{\tilde{B}}{}^{\tilde{A}} + \frac{1}{2} e^{b}\wedge e^{a}(F_{ab})_{\tilde{B}}{}^{\tilde{A}}\,,$$

- heterotic hypermultiplet(s) are defined by superfield $H^{A\tilde{B}J}(\zeta)$ in **(2,32)** representation of $SU(2)\times SO(32)$which obeys

$$\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_{C}{}^{A}\psi_{\gamma}^{\tilde{B}J}$$

Superfield description of heterotic degrees of freedom: basic superfield eqs.

- The SU(2) SYM is described by $SU(2)$ connection one form on $\mathcal{W}^{(6|8)}$

$$A_{\tilde{B}}{}^{\tilde{A}} = e^{\alpha C} A_{\alpha C\, \tilde{B}}{}^{\tilde{A}}(\zeta) + e^{a} A_{a\, \tilde{B}}{}^{\tilde{A}}(\zeta)\,, \qquad (A_{\tilde{B}}{}^{\tilde{A}})^* = -A_{\tilde{A}}{}^{\tilde{B}} \quad (\Rightarrow\ A_{\tilde{A}}{}^{\tilde{A}} = 0)\,,$$

- which obeys the constraints

$$F_{\tilde{B}}{}^{\tilde{A}} := (dA - A\wedge A)_{\tilde{B}}{}^{\tilde{A}} = \frac{i}{2} e^{a} \wedge e^{\alpha A} \gamma_{b\alpha\beta} (W^{\beta}_{A})_{\tilde{B}}{}^{\tilde{A}} + \frac{1}{2} e^{b} \wedge e^{a} (F_{ab})_{\tilde{B}}{}^{\tilde{A}}\,,$$

- heterotic hypermultiplet(s) are defined by superfield $H^{A\tilde{B}J}(\zeta)$ in **(2,32)** representation of $SU(2)\times SO(32)$which obeys

$$\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_C{}^{A} \psi_{\gamma}^{\tilde{B}J}$$

$\mathcal{D}_{\gamma C}$ is $SO(1,5)\otimes SO(4)\otimes SU(2) = SU(4)^*\otimes SU(2)\otimes SU(2)\otimes SU(2)$ covariant derivative on $\mathcal{W}^{(6|8)} \subset \Sigma^{(10|16)}$ defined by superembedding equation $\hat{E}_{\alpha A}{}^{\underline{a}} = 0$ (or by some its generalization $\hat{E}_{\alpha A}{}^{\underline{a}} = ...$) and by the constraints on the SUGRA+SYM background.

SO(32) H5-brane equations of motion: SU(2) SYM

SO(32) H5-brane equations of motion: SU(2) SYM

- The SYM constraints are on-shell: they result in equations of motion.

SO(32) H5-brane equations of motion: SU(2) SYM

- The SYM constraints are on-shell: they result in equations of motion.
- their consistency result in the fermionic equation

$$\gamma^a_{\alpha\beta}\mathcal{D}_a W^{\beta A} = W^{\gamma C} J_{\gamma C\ \alpha}{}^{A} \ .$$

SO(32) H5-brane equations of motion: SU(2) SYM

- The SYM constraints are on-shell: they result in equations of motion.
- their consistency result in the fermionic equation

$$\gamma^a_{\alpha\beta}\mathcal{D}_a W^{\beta A} = W^{\gamma C} J_{\gamma C\ \alpha}{}^A \ .$$

- and bosonic equations plus Binachi identities

$$\mathcal{D}^b F_{bc}\gamma^c_{\alpha\beta}\delta_B{}^A + \frac{1}{2}\mathcal{D}_{[a}F_{bc]}\gamma^{abc}_{\alpha\beta}\delta_B{}^A - i\epsilon_{\alpha\beta\gamma\delta}\{W^\gamma_C\ ,\ W^{\delta C}\}\delta_B{}^A - \\ -\frac{1}{2}F_{ab}J^{ab\ A}_{\beta\alpha B} + iW^{\gamma C}J_{\beta\gamma\alpha\ BC}{}^A\ ,$$

SO(32) H5-brane equations of motion: SU(2) SYM

- The SYM constraints are on-shell: they result in equations of motion.
- their consistency result in the fermionic equation

$$\gamma^a_{\alpha\beta}\mathcal{D}_a W^{\beta A} = W^{\gamma C} J_{\gamma C\ \alpha}{}^A .$$

- and bosonic equations plus Binachi identities

$$\mathcal{D}^b F_{bc}\gamma^c_{\alpha\beta}\delta_B{}^A + \frac{1}{2}\mathcal{D}_{[a}F_{bc]}\gamma^{abc}_{\alpha\beta}\delta_B{}^A - i\epsilon_{\alpha\beta\gamma\delta}\{W^\gamma_C\, ,\, W^{\delta C}\}\delta_B{}^A - \\ -\frac{1}{2}F_{ab}J^{ab\ A}_{\beta\alpha B} + iW^{\gamma C}J_{\beta\gamma\alpha\ BC}{}^A ,$$

- with contributions of the 'geometric' degrees of freedom and fluxes of background SUGRA + SO(32) SYM enclosed inside $J_{\beta\gamma\alpha\ BC}{}^A$, $J^{ab\ A}_{\beta\alpha B}$ and $J_{\gamma C\ \alpha}{}^A$.

SO(32) H5-brane equations of motion: hypermultiplet in (2,32)

SO(32) H5-brane equations of motion: hypermultiplet in (2,32)

- The hypermultiplet equations are also on-shell: $\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_C{}^A \psi_\gamma^{\tilde{B}J} \Rightarrow$

SO(32) H5-brane equations of motion: hypermultiplet in (2,32)

- The hypermultiplet equations are also on-shell: $\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_C{}^A \psi_\gamma^{\tilde{B}J} \Rightarrow$

$$\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J} W^{\alpha}_{A}{}_{\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}^{\alpha}_{A}{}^{IJ}\right) - \\ -\frac{i}{12} H^{A\tilde{B}J}\tilde{\gamma}^{b\alpha\beta}\left(4\mathcal{D}^B_\beta f_{bAB} - \mathcal{F}^B_{\beta}{}_{b\,BA}\right) - \\ +\frac{1}{24}\tilde{\gamma}^{b\alpha\beta}\left(8t_{\beta A\,b}{}^{\gamma A} - r_{b\,cd}\gamma^{cd}{}_\beta{}^\gamma\right)\psi_\gamma^{\tilde{B}J}.$$

and also bosonic equation.

SO(32) H5-brane equations of motion: hypermultiplet in (2,32)

- The hypermultiplet equations are also on-shell: $\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_C{}^A \psi_\gamma^{\tilde{B}J} \Rightarrow$

$$\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J}W^{\alpha}_{A\,\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}^{\alpha\,IJ}_{A}\right) - \\ -\frac{i}{12}H^{A\tilde{B}J}\tilde{\gamma}^{b\alpha\beta}\left(4\mathcal{D}^B_\beta f_{bAB} - \mathcal{F}^B_{\beta\;b\,BA}\right) - \\ +\frac{1}{24}\tilde{\gamma}^{b\alpha\beta}\left(8t_{\beta A\,b}{}^{\gamma A} - r_{b\,cd}\gamma^{cd}{}_\beta{}^\gamma\right)\psi_\gamma^{\tilde{B}J}.$$

  and also bosonic equation.

- These describe interaction with SUGRA, SO(32) gauge fields ($\hat{\mathcal{W}}^{\alpha\,IJ}_{A}$), geometric d-o-f.s (also given by hypermultiplet) **and** *SU*(2) SYM.

SO(32) H5-brane equations of motion: hypermultiplet in (2,32)

- The hypermultiplet equations are also on-shell: $\mathcal{D}_{\gamma C} H^{A\tilde{B}J} = 4i\delta_C{}^A \psi_\gamma^{\tilde{B}J} \Rightarrow$

$$\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J} W_{A\ \tilde{A}}^{\alpha\ \ \tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}_A^{\alpha\ IJ}\right) - $$
$$- \frac{i}{12} H^{A\tilde{B}J}\tilde{\gamma}^{b\alpha\beta}\left(4\mathcal{D}_\beta^B f_{bAB} - \mathcal{F}_{\beta\ b\,BA}^B\right) - $$
$$+ \frac{1}{24}\tilde{\gamma}^{b\alpha\beta}\left(8t_{\beta A\, b}{}^{\gamma A} - r_{b\, cd}\gamma^{cd}{}_\beta{}^\gamma\right)\psi_\gamma^{\tilde{B}J}.$$

  and also bosonic equation.
- These describe interaction with SUGRA, SO(32) gauge fields ($\hat{\mathcal{W}}_A^{\alpha\ IJ}$), geometric d-o-f.s (also given by hypermultiplet) **and** *SU*(2) SYM.
- Here we have a problem indicating that our present description of H5-brane is approximate.

Our $SO(32)$ H5-brane equations of motion are approximate

- Here we have a problem indicating that our present description of H5-brane is approximate.

Our $SO(32)$ H5-brane equations of motion are approximate

- Here we have a problem indicating that our present description of H5-brane is approximate.
- As hypermultiplet is minimally coupled to SU(2) SYM ('charged'), $(\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J}W^\alpha_{A}{}_{\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}^{\alpha}_{A}{}^{IJ}\right) - ....$

Our $SO(32)$ H5-brane equations of motion are approximate

- Here we have a problem indicating that our present description of H5-brane is approximate.
- As hypermultiplet is minimally coupled to SU(2) SYM ('charged'), $(\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J}W^{\alpha}_{A}{}_{\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}^{\alpha}_{A}{}^{IJ}\right) - ....$
- But: the SYM constraints are on-shell. And they produce eqs.

$$\mathcal{D}^b F_{bc}\gamma^c_{\alpha\beta}\delta_B{}^A + \frac{1}{2}\mathcal{D}_{[a}F_{bc]}\gamma^{abc}_{\alpha\beta}\delta_B{}^A - i\epsilon_{\alpha\beta\gamma\delta}\{W^\gamma_C\,,\, W^{\delta C}\}\delta_B{}^A - \\ -\frac{1}{2}F_{ab}J^{ab}_{\beta\alpha B}{}^A + iW^{\gamma C}J_{\beta\gamma\alpha\; BC}{}^A\,,$$

with no hypermultiplet contributions.

Our $SO(32)$ H5-brane equations of motion are approximate

- Here we have a problem indicating that our present description of H5-brane is approximate.
- As hypermultiplet is minimally coupled to SU(2) SYM ('charged'), $(\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J}W_A^\alpha{}_{\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}_A^{\alpha\, IJ}\right) - ....$
- But: the SYM constraints are on-shell. And they produce eqs.

$$\mathcal{D}^b F_{bc}\gamma^c_{\alpha\beta}\delta_B{}^A + \frac{1}{2}\mathcal{D}_{[a}F_{bc]}\gamma^{abc}_{\alpha\beta}\delta_B{}^A - i\epsilon_{\alpha\beta\gamma\delta}\{W^\gamma_C\,,\, W^{\delta C}\}\delta_B{}^A - \\ -\frac{1}{2}F_{ab}J^{ab}_{\beta\alpha B}{}^A + iW^{\gamma C}J_{\beta\gamma\alpha\; BC}{}^A\,,$$

  with no hypermultiplet contributions.
- Such an approximate description may be useful as it is (it is certainly approximate in the SU(2) SYM sector)

Our *SO*(32) H5-brane equations of motion are approximate

- Here we have a problem indicating that our present description of H5-brane is approximate.
- As hypermultiplet is minimally coupled to SU(2) SYM ('charged'), $(\tilde{\gamma}^{a\alpha\beta}\mathcal{D}_a\psi_\beta^{\tilde{B}J} = \frac{1}{2}\left(H^{A\tilde{A}J}W^\alpha_{A\;\tilde{A}}{}^{\tilde{B}} + H^{A\tilde{B}I}\hat{\mathcal{W}}^{\alpha\, IJ}_A\right) - \ldots.$
- But: the SYM constraints are on-shell. And they produce eqs.

$$\mathcal{D}^b F_{bc}\gamma^c_{\alpha\beta}\delta_B{}^A + \frac{1}{2}\mathcal{D}_{[a}F_{bc]}\gamma^{abc}_{\alpha\beta}\delta_B{}^A - i\epsilon_{\alpha\beta\gamma\delta}\{W^\gamma_C\,,\, W^{\delta C}\}\delta_B{}^A - \\ -\frac{1}{2}F_{ab}J^{ab\;A}_{\beta\alpha B} + iW^{\gamma C}J_{\beta\gamma\alpha\;BC}{}^A\,,$$

  with no hypermultiplet contributions.
- Such an approximate description may be useful as it is (it is certainly approximate in the SU(2) SYM sector)
- but it is tempting to speculate that the use of the GIKOS harmonic superfield formalism might help to make the SYM constraints 'off-shell' - or, at least, 'on-any-shell' - allowing for incorporation of the terms describing the hypermultiplet contributions.

Outline

1. Introduction
   - SUSY extended objects
   - 'Simple' D=10, N=1 5-brane and heterotic 5–branes
2. Superembedding approach for 'simple' N=1, D=10 5-brane
   - Worldvolume superspace and superembedding equation
   - Moving, and spinor moving frame and geometry induced by superembedding
3. 'Simple' 5-brane equations of motion from superembedding approach
4. Superembedding description of the $SO(32)$ heterotic 5-brane
   - Basic superfield equations of the $SO(32)$ heterotic 5-brane
   - From basic superfield equations for $SO(32)$ heterotic 5-brane to equations of motion.
5. Conclusions and outlook

Conclusions and outlook

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.
- These are the constraints of SU(2) SYM and superfield eqs. for hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$ on curved superspace $\mathcal{W}^{(6|8)}$ identical or similar to the w/v SSP of simple 5-brane

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.
- These are the constraints of SU(2) SYM and superfield eqs. for hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$ on curved superspace $\mathcal{W}^{(6|8)}$ identical or similar to the w/v SSP of simple 5-brane
- (at least as the first stage) the embedding $\mathcal{W}^{(6|8)} \subset \Sigma^{(8|16)}$ is defined by superembedding equation (the same as for 'simple' 5-brane)

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.
- These are the constraints of SU(2) SYM and superfield eqs. for hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$ on curved superspace $\mathcal{W}^{(6|8)}$ identical or similar to the w/v SSP of simple 5-brane
- (at least as the first stage) the embedding $\mathcal{W}^{(6|8)} \subset \Sigma^{(8|16)}$ is defined by superembedding equation (the same as for 'simple' 5-brane)
- and $\Sigma^{(8|16)}$ is characterized by the standard N=1 10D SUGRA constraints (+10D SYM).

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.
- These are the constraints of SU(2) SYM and superfield eqs. for hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$ on curved superspace $\mathcal{W}^{(6|8)}$ identical or similar to the w/v SSP of simple 5-brane
- (at least as the first stage) the embedding $\mathcal{W}^{(6|8)} \subset \Sigma^{(8|16)}$ is defined by superembedding equation (the same as for 'simple' 5-brane)
- and $\Sigma^{(8|16)}$ is characterized by the standard N=1 10D SUGRA constraints (+10D SYM).
- Then, after studying the simplest possibility, the modification of both superembedding equations and supergravity constraints.

Conclusions and outlook

- We are studying the possibility to search for equations of motion of the SO(32) heterotic 5-brane (H5-brane) in the framework of superembedding approach
- We have proposed the basic superfield equations of SO(32) H5-brane.
- These are the constraints of SU(2) SYM and superfield eqs. for hypermultiplet in **(2,32)** of $SU(2) \times SO(32)$ on curved superspace $\mathcal{W}^{(6|8)}$ identical or similar to the w/v SSP of simple 5-brane
- (at least as the first stage) the embedding $\mathcal{W}^{(6|8)} \subset \Sigma^{(8|16)}$ is defined by superembedding equation (the same as for 'simple' 5-brane)
- and $\Sigma^{(8|16)}$ is characterized by the standard N=1 10D SUGRA constraints (+10D SYM).
- Then, after studying the simplest possibility, the modification of both superembedding equations and supergravity constraints.
- Our approach is able to describe the interaction of heterotic 5-brane with background D=10 SUGRA and SO(32) SYM fluxes.

Outlook

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:
- The hypermultiplet equations describe its coupling to $SU(2)$ SYM but the SYM equations remain 'free'.

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:
- The hypermultiplet equations describe its coupling to $SU(2)$ SYM but the SYM equations remain 'free'.
- This is because the SYM constraints are on-shell.

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:
- The hypermultiplet equations describe its coupling to $SU(2)$ SYM but the SYM equations remain 'free'.
- This is because the SYM constraints are on-shell.
- The possible way out might lay through reformulating our approach with the use of GIKOS harmonic superspace formalism

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:
- The hypermultiplet equations describe its coupling to $SU(2)$ SYM but the SYM equations remain 'free'.
- This is because the SYM constraints are on-shell.
- The possible way out might lay through reformulating our approach with the use of GIKOS harmonic superspace formalism
- Some kind of superembedding of harmonic superspaces?

Outlook

- On the other hand, as it is formulated now, our approach provides an approximate description of H5-brane:
- The hypermultiplet equations describe its coupling to $SU(2)$ SYM but the SYM equations remain 'free'.
- This is because the SYM constraints are on-shell.
- The possible way out might lay through reformulating our approach with the use of GIKOS harmonic superspace formalism
- Some kind of superembedding of harmonic superspaces?
- The properties of the SO(32) H5-brane equations as they follow from the present superembedding approach as well as search for their possible generalizations are under study now.

THANK YOU FOR YOUR ATTENTION!