A new perspective on nonrelativistic gravity

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Overview

- Motivation
- GR by gauging the Poincaré algebra
- Newton-Cartan
- Newton-Cartan by gauging the Bargmann algebra
- Extension to strings

Motivation

- A lot of physics in our universe is nonrelativistic
- Problems can become simpler
- Possible applications to AdS/CFT?

General Relativity from Poincaré

- Poincaré algebra:

[P , P] = 0

[J , P] = P

[J , J] = J

	Gauge fields	Curvatures
	$P_a : \quad e_\mu{}^a$	$R_{\mu\nu}{}^a(P)$
	$J_{ab} : \quad \omega_\mu{}^{ab}$	$R_{\mu\nu}{}^{ab}(J)$

- $R_{\mu\nu}{}^a(P) = 0 \quad \rightarrow \quad (1)\ \ \omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e, \partial e)$

  $(2)\ P_a \ \rightarrow \ gct + local\ \ Lorentz$

- Impose vielbein postulate → General Relativity

Newton-Cartan : geometrizing Newton

- Spatial metric h and temporal metric τ
- Space and time decouple: $h^{\mu\nu}\tau_{\nu\rho} = 0$
- Introduce 'inverse metrics' $\tilde{h}_{\mu\nu}, \quad \tilde{\tau}^{\mu\nu}$
- Connection via $\begin{array}{l} \nabla_\rho h^{\mu\nu} = 0 \\ \nabla_\rho \tau_{\mu\nu} = 0 \end{array} \quad \Rightarrow \quad \Gamma^{\rho}_{\mu\nu}(h, \tilde{h}, \tau, \tilde{\tau}, K)$
- 'Ehlers conditions' to constrain K $\longrightarrow K_{\mu\nu} = 2\partial_{[\mu}m_{\nu]}$

Newton-Cartan : EOM

- Poisson eqn. : $R_{\mu\nu}(\Gamma) = 4\pi G \rho \tau_{\mu\nu}$
- Geodesic eqn. : $\ddot{x}^\rho + \Gamma^\rho_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = 0$
- Only nonzero connection: $\Gamma^i_{00} = \partial^i \phi$
- All degrees of freedom in ϕ

Newton-Cartan from Bargmann I

- Bargmann algebra generators: { H, P, G, J, M }

		Gauge fields	Curvatures
Time transl.	$H:$	τ_μ	$R_{\mu\nu}(H)$
Space transl.	$P_i:$	$e_\mu{}^i$	$R_{\mu\nu}{}^i(P)$
Boosts	$G_i:$	$\omega_\mu{}^{i0}$	$R_{\mu\nu}{}^{i0}(G)$
Space rot.	$J_{ij}:$	$\omega_\mu{}^{ij}$	$R_{\mu\nu}{}^{ij}(J)$
Central ext.	$M:$	m_μ	$R_{\mu\nu}(M)$

- Central extension : [G , P] = 0 $\rightarrow$ [G , P] ~ M

Newton-Cartan from Bargmann II

- R(P) = R(M) = R(H) = 0 such that

  {H,P} → gct + remaining transformations

- -R(M) = R(P) = 0: solve for spin connections

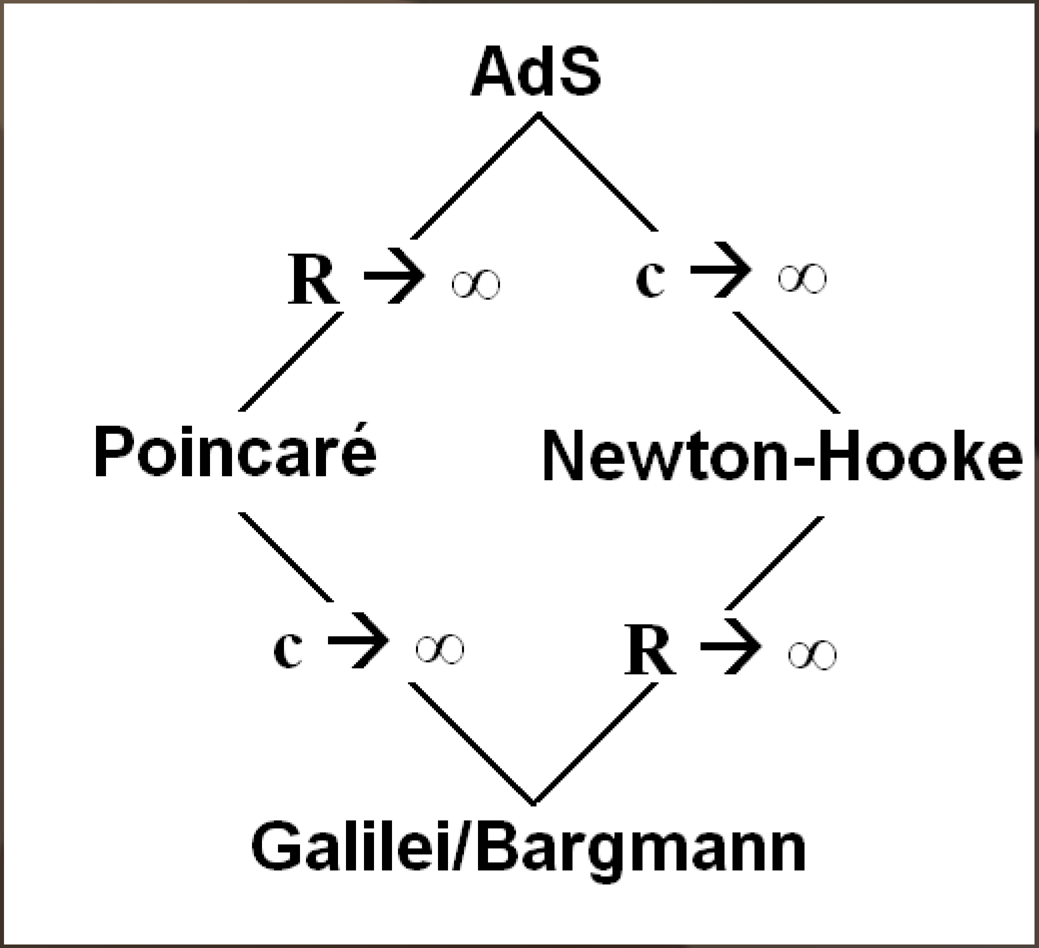
  -R(H) = 0: $\tau_\mu = \partial_\mu t$

- On-shell constraint: R(J)=0 → flat space
- Vielbein postulate
- $K_{\mu\nu} = 2\partial_{[\mu}m_{\nu]} \rightarrow$ Newton-Cartan!

Role of central extension M

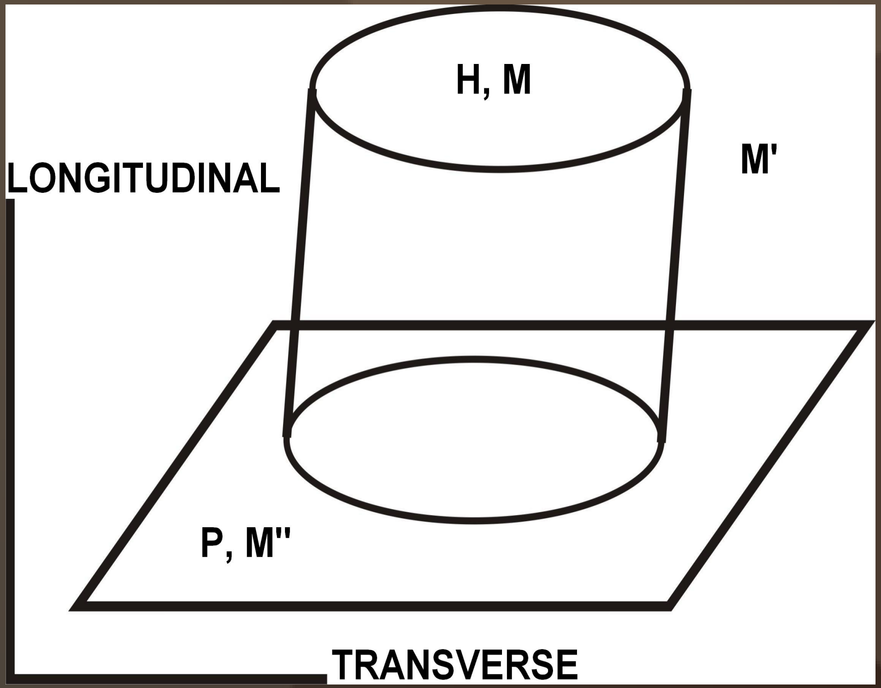
- 1) Without M we cannot solve for spin connections, we need both R(P) = 0 *and* R(M) = 0!
- 2) Remember that in Newton-Cartan the K in the connection is closed: $K_{\mu\nu} = 2\partial_{[\mu}m_{\nu]}$ $\rightarrow$ Curl of the gauge field of M!
- In short: M is crucial

Algebra contractions



- Newton-Hooke: nonrelativistic limit of AdS, cosmological constant in longitudinal direction!

The deformed string Galilei algebra



Generators:

H: long. translations

M: long. boost

P: transv. translation

M '': transv. rotation

M ': stringy 'boost'

Z: deformations

- Galilei: { H, M } form 2-dim. Poincaré algebra
- Newton-Hooke: { H, M } form 2-dim. AdS algebra

Stringy Newton-Cartan

- Idea: gauge deformed string Galilei algebra
- Add cosmological constant to equations of motion
- $\rightarrow$ Stringy Newton-Hooke-Cartan theory!

- Changes:

-Newton potential: $\phi \;\rightarrow\; \phi_{ab}$

-Deformation: $M \;\rightarrow\; \{Z_a, Z_{ab}\}$

-K in connection: $K_{\mu\nu} = 2\partial_{[\mu} m_{\nu]} \;\rightarrow\; K_{\mu\nu}{}^a = 2\partial_{[\mu} m_{\nu]}{}^a$

Stringy Newton-Cartan: EOM

- Poisson equation: $\partial^i \partial_i \phi_{ab} = (4\pi G\rho - \Lambda)\tau_{ab}$

- String geodesic equation

Role of deformations $\{Z_a, Z_{ab}\}$

- Without the Z's we can't solve for the three different spin connections!
- Ambiguity in connection now becomes
  $K_{\mu\nu}{}^a = 2\partial_{[\mu} m_{\nu]}{}^a$
- $m_\mu{}^{ab}$ appears as ambiguity of K, doesn't enter EOM
- In short: the deformation is crucial!

Conclusions

- Bargmann algebra can be gauged to obtain Newton-Cartan theory
- This method can be mimicked for strings, including a cosmological constant
- Algebra deformations play a crucial role
- Stringy Newton-Cartan theory with Newton-Hooke symmetry possibly interesting for AdS/CFT