Properties of lightest mesons at finite temperature and quark/baryon chemical potential in instanton model of QCD vacuum

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The thermal and quark/baryon chemical potential dependences of quark condensate and masses of π- and σ-mesons are studied in the instanton model of the QCD vacuum in precritical region. The impact of phonon-like excitations of instanton liquid on the characteristics of σ-meson in such an environment is also examined.

Nowadays with the common belief that the hadron world and its symmetries are driven by QCD our understanding of the lightest mesons origin, at least from the hadron side, is tightly related to the spontaneous breakdown of chiral symmetry inherent to the corresponding Lagrangian. Then the pions appear as the massless Goldstone bosons while the scalars acquire a non-zero vacuum expectation values reflecting the sophisticated structure of QCD vacuum. At the level of QCD fundamental fields we have phenomenologically learnt that this symmetry breaking is dominated by the non-zero values of quark condensates [1] and the scalar meson attributes result from the dynamical generation of light quark masses. However, at rather high temperature T, as the lattice QCD studies have taught [2], the phase transition restoring the chiral symmetry occurs (and a quark chiral condensate is just its order parameter) together with the transition deconfining the quarks and gluons and another transition of colour superconductivity (characterized by a diquark condensate) at high quark/baryonic chemical potential μ. Clearly, the last phase transition is scarcely feasible in the terrestrial conditions and is relevant rather in the astrophysics observations of compact stars. As to two others they are intensively explored in ultrarelativistic heavy ion collisions [3].

The properties of various phases of hot and dense matter are dependent on the interplay between the competing processes of quark-quark and quark-antiquark pairing (influenced by the QCD vacuum) and, as expected, could be clarified by studying the QCD phase diagram on (μ–T)-plane in the lattice simulations (albeit a very idea is relevant for the infinite matter and in thermodynamic equilibrium). The first results of lattice phase diagram studies were quite schematic but,at the same time, indicative enough because they generated very important question about the structure of phases around the deconfinement critical temperature T_c, i.e. at $|T-T_c| \ll T_c$. Today as become especially clear when it was found that J/ψ charmonium survives at rather high temperatures beyond T_c [4]. In fact, this result has been confirmed by the RHIC experimetal observations and has led to the concept of strongly coupled quark-gluon plasma [5] revealing the existence of enormous number of bound states at $T > T_c$ in deconfined phase.

Phenomenological analysis of strongly interacting matter under the extreme conditions is complicated by the comparatively low credibility of dynamical hadron models (linear and nonlinear sigma-models, the Nambu–Jona-Lasinio model and its modificatios etc.) on the ($\mu - T$)-plane. Their predictions are not likely to be reliable at approaching the critical line of phase transition restorating the chiral symmetry. At small values of μ and T the parameters of those models are tuned in such a way to reproduce some experimental data and to obey some general constraints rooted in

QCD. However, extending the models to the phase transition regions leads sometimes to conflicting conclusions (see, for example, the recent discussion in [6]) At the same time the analysis of precritical regions could be entirely informative which was demonstrated in [7] where the properties of lightest mesons in hot and dense environment making use the virial expansion were estimated in chiral perturbation theory. In the present paper similar questions are discussed within the instanton liquid model of the QCD vacuum [8].

The quark generating functional for stochastic ensemble of gluon configurations being defined as (the sign of double averaging means the corresponding average over the gluon fields)

$$\mathcal{Z}_\psi \simeq \int D\psi^\dagger D\psi \;\; \langle\langle\, e^{S(\psi,\psi^\dagger,A)}\, \rangle\rangle_A \;,$$

at nonzero temperature and finite chemical potential is estimated by the saddle point method and could be presented (up to the details inessential for our consideration here) with an auxiliary integration over saddle point parameter λ (we are dealing here with $SU(3)$ gauge group and quarks of two flavours $N_f = 2$) [9] in the following form

$$\begin{aligned}
&\mathcal{Z}_\psi \simeq \int d\lambda \int D\psi^\dagger D\psi \;\exp\left\{n\; V_4 \left(\ln\frac{n\bar\rho^4}{\lambda} - 1\right)\right\} \times \\
&\times \exp\left\{T \sum_{l=-\infty}^{\infty} \int \frac{d\boldsymbol{k}}{(2\pi)^3}\; \sum_{f=1}^{2} \psi_f^\dagger(k)\; (-\hat k - i\hat\mu)\; \psi_f(k) + \mathcal{L}_{int}\right\} \;, \qquad (1)\\
&\mathcal{L}_{int} = i\lambda\; (\psi_1^{\dagger L}\; L_1\; \psi_1^L)(\psi_2^{\dagger L}\; L_2\; \psi_2^L) + i\lambda\; (\psi_1^{\dagger R}\; R_1\; \psi_1^R)(\psi_2^{\dagger R}\; R_2\; \psi_2^R) \;.
\end{aligned}$$

Here $\psi_f^T = (\psi_f^R, \psi_f^L)$, $f = 1, 2$ are the quark fields composed by the spinors of fixed chirality $\psi_f^{L,R} = P_\pm\; \psi_f$, $P_\pm = \frac{1 \pm \gamma_5}{2}$, n is the instanton liquid density, $\bar\rho$ is the mean size of instanton liquid pseudoparticles, $V_4 = TL^3$ denotes the 4-volume of system and $\mu_\nu = (\mathbf{0}, \mu)$. Working in the Euclidean space we are summing up over l by introducing the Matsubara frequencies $k_4(l) = 2\pi T\left(l + \frac{1}{2}\right)$ with the convention $\Delta k_4 = T$. Summing up and integrating over spatial momentum components in Eq.(1) will further be marked as $\sum\!\!\!\!\!\!\int$. Then the Lagrangian $\mathcal{L}_{int}$ of 4-fermion quark interaction is given with the chiral components as

$$(\psi_f^{\dagger L}\; L_f\; \psi_f^L) = \int \frac{dp_f dq_f}{(2\pi)^8}\; \psi^{\dagger L}_{f\alpha_f i_f}(p_f)\; L^{\beta_f j_f}_{\alpha_f i_f}(p_f, q_f; \mu)\; \psi_f^{L\beta_f j_f}(q_f) \;,$$

and for the right hand fields one should change $L \to R$.

Currently, the corresponding effective Lagrangians for the L и R components are set in terms of the quark zero modes. At zero values of temperature and chemical potential the zero modes approach [10], being combined with the mean field approximation [11], is providing us with very reasonable phenomenological results. But at finite temperature the superposition of (anti-)calorons which are the periodic solutions of the Yang-Mills equations [12] looks much more relevant to be used as the saturating configuration of suitable generating functional. At high temperature, meanwhile, the chromoelectrical part of the gluon field is expected to develop the mass term $\frac{m_{el}^2}{2}\; A_4^2$ in the effective Lagrangian (the corresponding term in one-loop approximation has been already found out in [13]). It implies the saturating configurations should be mainly rooted in the chromomagnetic sector. (In the framework of instanton QCD vacuum model the alternative scenario has also been developed with assuming the formation of instanton-anti-instanton molecules [14].) The recent lattice studies of correlators of the topological configurations corroborated [15] the chromomagnetic character of gauge fields beyond the critical temperature.

Unfortunately, even the simplest caloron zero mode (in the momentum space) at zero value of quark chemical potential μ may not be presented by the special functions and, hence, is impractical for the analytical exploration. It is the major reason for us to develop here the approximate procedure in which we substitute the explicit expression of zero mode $\psi[A(T,\mu);T,\mu]$ for its simplified form $\psi[A(0,\mu);0,\mu]$ and imply the corresponding corrections could be calculated perturbatively. Such an oversimplification results in the $L^{\beta_f j_f}_{\alpha_f i_f}$ kernels which are defined by the zero modes (the solutions of the Dirac equations with a chemical potential μ) in the form of h_i-functions as regards

$$\begin{aligned} h_4(k_4,k;\mu) &= \frac{\pi}{4k}\{(k-\mu-ik_4)[(2k_4+i\mu)f_1^- + i(k-\mu-ik_4)f_2^-] + \\ &+ (k+\mu+ik_4)[(2k_4+i\mu)f_1^+ - i(k+\mu+ik_4)f_2^+]\}\ , \end{aligned}$$

$$\begin{aligned} h_i(k_4,k;\mu) &= \frac{\pi\ k_i}{4k^2}\left\{(2k-\mu)(k-\mu-ik_4)f_1^- + (2k+\mu)(k+\mu+ik_4)f_1^+ + \right. \\ &+ \left[2(k-\mu)(k-\mu-ik_4) - \frac{1}{k}(\mu+ik_4)[k_4^2+(k-\mu)^2]\right] f_2^- + \\ &+ \left.\left[2(k+\mu)(k+\mu+ik_4) + \frac{1}{k}(\mu+ik_4)[k_4^2+(k+\mu)^2]\right] f_2^+\right\}\ , \end{aligned}$$

with $i=1,\dots,3$ and $k=|\boldsymbol{k}|$, if one considers the spatial components of 4-vector k_ν, and

$$f_1^\pm = \frac{I_1(z^\pm)K_0(z^\pm)-I_0(z^\pm)K_1(z^\pm)}{z^\pm}\ , f_2^\pm = \frac{I_1(z^\pm)K_1(z^\pm)}{z^2_\pm}\ ,\quad z^\pm = \frac{\bar\rho}{2}\sqrt{k_4^2+(k\pm\mu)^2}\ ,$$

with $I_i,\ K_i$ $(i=0,1)$ as the modified Bessel functions. Introducing in the same time the scalar function $h(k_4,k;\mu)$ which is related to the three-dimensional components as $h_i(k_4,k;\mu)=h(k_4,k;\mu)\ \frac{k_i}{k}\ , i= 1,2,3$ (when it does not mislead we omit the arguments of h_i functions at all) we have then

$$L^{\beta j}_{\alpha i}(p,q;\mu) = S^{ik}(p;\mu)\epsilon^{kl}\ U^\alpha_l\ U^{\dagger\sigma}_\beta\,\epsilon^{\sigma n}S^+_{nj}(q;-\mu)\ ,$$

with $S(p;\mu)=(p+i\mu)^-\ h^+(p;\mu)\ ,\quad S^+(p;-\mu)=\overset{*}{h}{}^-(p;-\mu)(p+i\mu)^+$, moreover, for the conjugated function it is valid $\overset{*}{h}_\mu\ (p;-\mu)=h_\mu(p;\mu)$, and ϵ is entirely antisymmetric matrix $\epsilon_{12}=-\epsilon_{21}=1$. Here $p^\pm$ and similar symbols are used for the 4-vectors spanned on the matrices $\tau^\pm_\nu$ where $\tau^\pm_\nu=(\pm i\boldsymbol{\tau},1)$ and $\boldsymbol{\tau}$ is the 3-vector of Pauli matrices, $p^\pm=p^\nu\tau^\pm_\nu$ and U is a matrix of rotations in the colour space. Surely, we have the similar relations for the right hand components

$$(\psi^{\dagger R}_f\ R_f\ \psi^R_f) = \int \frac{dp_f dq_f}{(2\pi)^8}\ \psi^{\dagger R}_{f\alpha_f i_f}(p_f)\ R^{\beta_f j_f}_{\alpha_f i_f}(p_f,q_f;\mu)\ \psi^{R\beta_f j_f}_f(q_f)$$

with the kernel as

$$R^{\beta j}_{\alpha i}(p,q;\mu) = T^{ik}(p;\mu)\epsilon^{kl}\ U^\alpha_l\ U^{\dagger\sigma}_\beta\,\epsilon^{\sigma n}T^+_{nj}(q;-\mu)\ ,$$

where $T(p;\mu)=(p+i\mu)^+\ h^-(p;\mu)\ ,\quad T^+(p;-\mu)=\overset{*}{h}{}^+(p;-\mu)(p+i\mu)^-$. As far as the vector-function $\boldsymbol{h}(p)$ is spanned on the vector $\boldsymbol{p}$ only the components of matrices $(p+i\mu)^\pm$ and $h^\mp(p;\mu)$ are permutable. As a result it is easy to understand the validity of the following identities

$$T(p;\mu)=S^+(p;-\mu)\ ,\quad T^+(p;-\mu)=S(p;\mu)\ .$$

In what follows we omit the μ-dependence of matrices $S,T,\ S^+,T^+$ because the chemical potential enters the matrices S^+,T always with the positive sign only and the matrices S,T^+ with the negative one.

After averaging over the colour orientation the leading term of the N_c^{-1} expansion of four-fermion interaction contribution looks like the following

$$\begin{aligned} \left\langle\left(\psi^{\dagger L}_1\ L_1\ \psi^L_1\right)\ \left(\psi^{\dagger L}_2\ L_2\ \psi^L_2\right)\right\rangle_U &= \left(\psi^{\dagger L}_1(p_1)\ S(p_1)\ S^+(q_1)\ \psi^L_1(q_1)\right)\left(\psi^{\dagger L}_2(p_2)\ S(p_2)\ S^+(q_2)\ \psi^L_2(q_2)\right) - \\ &- \left(\psi^{\dagger L}_1(p_1)\ S(p_1)\ S^+(q_2)\ \psi^L_1(q_2)\right)\left(\psi^{\dagger L}_2(p_2)\ S(p_2)\ S^+(q_1)\ \psi^L_1(q_1)\right)\ , \end{aligned}$$

(and, of course, a similar expression for the right hand chiral component). Eventually, the interaction term may be rewritten as

$$
\begin{aligned}
\mathcal{L}_{int} &= \frac{i\lambda}{4}\left(\psi_f^{\dagger L}(p_1)\ S(p_1)\ S^+(q_1)\ (\tau_a^+)_{ff'}\ \psi_{f'}^L(q_1)\right)\left(\psi_g^{\dagger L}(p_2)\ S(p_2)\ S^+(q_2)\ (\tau_a^+)_{gg'}\ \psi_{g'}^L(q_2)\right) - \\
&- \frac{i\lambda}{4}\left(\psi_f^{\dagger L}(p_1)\ T(p_1)\ T^+(q_1)\ (\tau_a^+)_{ff'}\ \psi_{f'}^L(q_1)\right)\left(\psi_g^{\dagger L}(p_2)\ T(p_2)\ T^+(q_2)\ (\tau_a^+)_{gg'}\ \psi_{g'}^L(q_2)\right)\ .
\end{aligned}
\tag{2}
$$

Performing the auxiliary integration over the bosonic fields L_a и R_a the four-fermion interaction can be transformed to the Gauss-type integral (analogously for right hand chiral fields and scalar field R_a)

$$
\frac{\lambda}{4}\left(\psi^{\dagger L}\ SS^+\ \tau_a^+\ \psi^L\right)\left(\psi^{\dagger L}\ SS^+\ \tau_a^+\ \psi^L\right) \to -\lambda\left(\psi^{\dagger L}\ SS^+\ \tau_a^+\ \psi^L\right)\ L_a - \lambda L_a^2\ ,
$$

which is quite convenient to obtain the effective Lagrangian in the terms of hadronic degrees of freedom

$$
\begin{aligned}
\hat{L} &= (1+\sigma+\eta)\ U\ V\ , \quad & \hat{R} &= (1+\sigma-\eta)\ V\ U^+\ , \\
U &= e^{i\ \pi^a\tau^a}\ , & V &= e^{i\ \sigma^a\tau^a}\ .
\end{aligned}
$$

Then the corresponding Lagrangian density incorporating the interaction of quarks and hadrons is

$$
\begin{aligned}
\mathcal{L}'_{int} = &- i\lambda\ \psi^\dagger(p)\ \Big[S(p)\ S^+(q)\ e^{i(p-q)x}\ \ (1+\sigma(x)+\eta(x))\ U(x)\ V(x)\ P_+ - \\
&- T(p)\ T^+(q)\ e^{i(p-q)x}\ \ (1+\sigma(x)-\eta(x))\ V(x)\ U^+(x)\ P_-\Big]\ \psi(q)\ ,
\end{aligned}
\tag{3}
$$

and calculating two well-known diagrams with two external hadron lines one may extract the necessary hadronic correlators according to the corresponding effective Lagrangian [10]

$$
\begin{aligned}
R_{\pi_a}(p) &= 4N_c \sum\!\!\!\!\!\!\int \frac{dk}{(2\pi)^4}\ \frac{M^2(k)}{(k+i\mu)^2+M^2(k)} - \\
&- 4N_c \sum\!\!\!\!\!\!\int \frac{(dk_1dk_2)}{(2\pi)^4}\ \frac{[(k_1+i\mu)(k_2+i\mu)+M_1M_2]\ M(k_1,k_2)M(k_2,k_1)}{[(k_1+i\mu)^2+M_1^2]\,[(k_2+i\mu)^2+M_2^2]}\ , \\
R_\sigma(p) &= n\bar\rho^4 - 4N_c \sum\!\!\!\!\!\!\int \frac{(dk_1dk_2)}{(2\pi)^4}\ \frac{[(k_1+i\mu)(k_2+i\mu)-M_1M_2]\ M(k_1,k_2)M(k_2,k_1)}{[(k_1+i\mu)^2+M_1^2]\,[(k_2+i\mu)^2+M_2^2]}\ , \\
R_\eta(p) &= n\bar\rho^4 + 4N_c \sum\!\!\!\!\!\!\int \frac{(dk_1dk_2)}{(2\pi)^4}\ \frac{[(k_1+i\mu)(k_2+i\mu)+M_1M_2]\ M(k_1,k_2)M(k_2,k_1)}{[(k_1+i\mu)^2+M_1^2]\,[(k_2+i\mu)^2+M_2^2]}\ , \\
R_{\sigma_a}(p) &= 4N_c \sum\!\!\!\!\!\!\int \frac{dk}{(2\pi)^4}\ \frac{M^2(k)}{(k+i\mu)^2+M^2(k)} + \\
&+ 4N_c \sum\!\!\!\!\!\!\int \frac{(dk_1dk_2)}{(2\pi)^4}\ \frac{[(k_1+i\mu)(k_2+i\mu)-M_1M_2]\ M(k_1,k_2)M(k_2,k_1)}{[(k_1+i\mu)^2+M_1^2]\,[(k_2+i\mu)^2+M_2^2]}\ .
\end{aligned}
\tag{4}
$$

The momentum integration here includes the δ-function $(dk_1dk_2) = dk_1dk_2\ \delta(k_1-k_2-p)$ carrying the external momentum p. The vertex function $M(k_1,k_2) = \lambda S(k_1)S^+(k_2) = \lambda v(k_1,k_2)$ has the following structure

$$
v(k_1,k_2) = A + iB\ \frac{\boldsymbol{k}_1\ \boldsymbol{\tau}}{|\boldsymbol{k}_1|} + iC\ \frac{\boldsymbol{k}_2\ \boldsymbol{\tau}}{|\boldsymbol{k}_2|} + iD\ \frac{\boldsymbol{k}_1\times\boldsymbol{k}_2\ \boldsymbol{\tau}}{|\boldsymbol{k}_1|\ |\boldsymbol{k}_2|}\ ,
\tag{5}
$$

with the functions

$$
A = [k_1h+(k_1+i\mu)h_4]\,[k_2g+(k_2+i\mu)g_4] + [(k_1+i\mu)h-k_1h_4]\,[(k_2+i\mu)g-k_2g_4]\ \frac{(\boldsymbol{k}_1\boldsymbol{k}_2)}{|\boldsymbol{k}_1|\ |\boldsymbol{k}_2|}\ ,
$$

$$
B = [(k_1+i\mu)h-k_1h_4][k_2g+(k_2+i\mu)g_4]\ ,
$$

$$C = -[k_1 h + (k_1 + i\mu) h_4][(k_2 + i\mu) g - k_2 g_4] \ ,$$
$$D = \ [(k_1 + i\mu) h - k_1 h_4][(k_2 + i\mu) g - k_2 g_4],$$

where $h = h(k_1; \mu)$, $h_4 = h_4(k_1; \mu)$, $g = h(k_2; \mu)$, $g_4 = h_4(k_2; \mu)$, and in the shortened form one should imply $M_1 = M(k_1, k_1)$, $M_2 = M(k_2, k_2)$. The product of matrices $v(k_1, k_2)v(k_2, k_1)$ is spanned on the unit matrix because

$$v(k_2, k_1) = A - iB \ \frac{\boldsymbol{k}_1 \ \boldsymbol{\tau}}{|\boldsymbol{k}_1|} - iC \ \frac{\boldsymbol{k}_2 \ \boldsymbol{\tau}}{|\boldsymbol{k}_2|} - iD \ \frac{\boldsymbol{k}_1 \times \boldsymbol{k}_2 \ \boldsymbol{\tau}}{|\boldsymbol{k}_1| \ |\boldsymbol{k}_2|} \ ,$$

and denotion $M(k_1, k_2)M(k_2, k_1)$ means the corresponding scalar to be singled out. Undoubtedly, all expressions of Eq.(4) are approximate and going to improve them one remembers the matrices $S(k_1)S^{+}(k_2)$ (and $T(k_1)T^{+}(k_2)$ as well) should be linked with the corresponding quark propagators in between (the vertices might be interchanged only when the momentum equality $k_1 = k_2$ is valid) which finally leads to the additional terms of the following form

$$Tr(A + i\boldsymbol{N}\boldsymbol{\Sigma}) \left\{ \begin{matrix} 1 \\ \gamma_5 \end{matrix} \right\} (\hat{k}_1 + i\hat{\mu} + iM_1)(A - i\boldsymbol{N}\boldsymbol{\Sigma}) \left\{ \begin{matrix} 1 \\ \gamma_5 \end{matrix} \right\} (\hat{k}_2 + i\hat{\mu} + iM_2) \ ,$$

where $\boldsymbol{N} = B \ \frac{\boldsymbol{k}_1}{|\boldsymbol{k}_1|} + C \ \frac{\boldsymbol{k}_2}{|\boldsymbol{k}_2|} + D \ \frac{\boldsymbol{k}_1 \times \boldsymbol{k}_2}{|\boldsymbol{k}_1| \ |\boldsymbol{k}_2|}$. However, numerical analysis performed shows the additional terms proportional to the corresponding traces of matrices of the spin type $\Sigma_i = \frac{i}{2} \varepsilon_{ijk} \ \gamma_j \gamma_k$ are negligible.

Now going to analyse the thermal and finte μ evolution of quark mass and quark condensate we would like to emphasize that we do not expect our approach to be valid in the chiral limit but reasonably reliable in the precritical region. As usual the saddle point is determined by the solution of the following equation

$$4N_c \sum\!\!\!\!\!\!\int \frac{dk}{(2\pi)^4} \ \frac{M^2(k)}{(k + i\mu)^2 + M^2(k)} = n \ . \tag{6}$$

At zero temperature and particular value of the chemical potential ($\mu_c \simeq 350$ MeV) there appears the singularity in the denominator of Eq.(6) which is rooted in the solution of equation

$$\boldsymbol{k}^2 - \mu^2 + M(|\boldsymbol{k}|) = 0$$

and quarks are going to fill the Fermi sphere in. If we do not calculate the imaginary part of the correlators (4) the corresponding integrals might be calculated as the principal values only. Obviously the accurate calculation foresees an analysis of all poles and cuts in the imaginary p_4-plane. However, for the verteces generated by instantons this task, unfortunately, is not feasible today because of the fairly complicated analytical structure of the zero modes in the complex plane (see, for example, [16] where unrealistic (low) thresholds of chiral symmetry restoration have been received for the zero mode forms considered).

The masses of σ- and π-mesons are extracted from hadron correlator expansions at small values of momenta (4) $R_\pi(p) = \beta_\pi p^2 + \ldots$, $R_\sigma(p) = \alpha_\sigma + \beta_\sigma p^2 + \ldots$ and $M_\sigma^2 = \frac{\alpha_\sigma}{\beta_\sigma}$ but the π-meson mass is given by the Gell-Mann-Oakes-Renner relation. In our paper the coefficients α and β are numerically calculated and the precision which we are able to provide for the meson characteristics is at the level of 10% only (it is exracted from the result dependence on the spacing of numerical differentiation). It looks quite satisfactory for the qualitative appraisals which here we intend to. The typical thermal behaviours of M_σ and M_π are depicted in Fig.1 for the several values of chemical potentials $\mu = 100, 160, 240, 260$ MeV (a polarization was taken trivial $|\boldsymbol{p}| = 0$ while calculating) and the upper curve there corresponds to $\mu = 100$ MeV. The mass of σ-meson diminishes because of α decreasing and β increasing at the same time. The distinctive feature of our approximate calculation is an absence of characteristic structure having usually observed at the crossing of the σ- and π-meson curves while chiral symmetry is restored [17]. Two curves of the dynamical mass behaviour

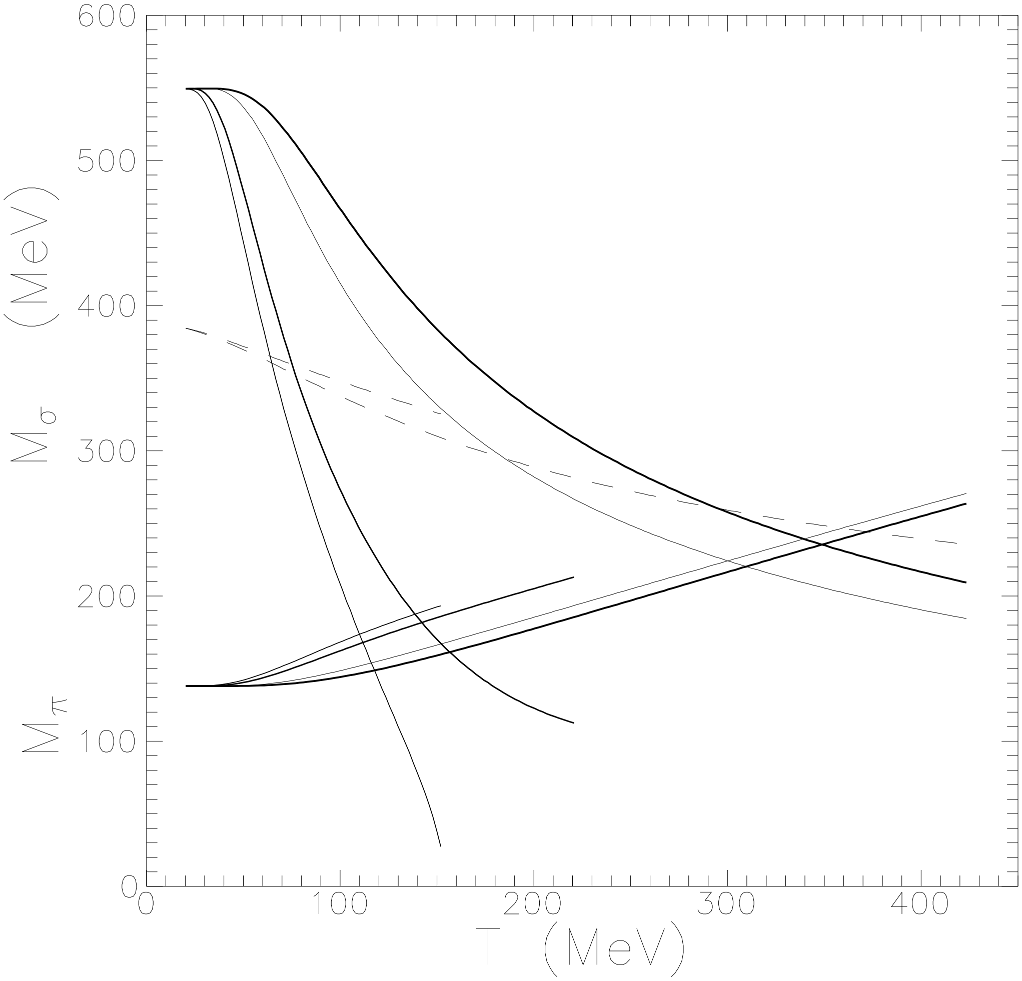

Рис. 1: The masses of σ- (upper solid lines) and π- mesons (lower solid lines) as the temperature functions for four values of the chemical potential $\mu = 100, 160, 240, 260$ MeV, the upper line corresponds to $\mu = 100$ MeV. Two dashed lines show the behaviour of dynamical quark mass corresponding to two extreme values of chemical potential (upper line corresponds to larger value.

$M(k_4(0);\mu)$ for $\mu = 100$ MeV (lower dashed curve) and $\mu = 260$ MeV (upper dashed curve) are shown on the same plot demonstrating a slow decrease with the temperature increasing. It is interesting to notice the σ-meson mass is in the domain of $\frac{M_\sigma}{M}$ expansion validity with μ and T increasing which allows us to hope that the results of analysis with an effective Lagrangian proposed might be taken not only as qualitative ones. It is well seen that with μ increasing the curves describing the σ-meson mass become steeper and steeper and around $\mu_c \simeq 350$ MeV where the process of filling in the Fermi sphere starts σ-meson is simply degenerated. Moreover the hadronic correlator $R_\sigma(p) = \alpha_\sigma + \beta_\sigma p^2 + \ldots$ develops almost constant behaviour at $\beta_\sigma \to 0$. With μ further increasing the correlation function of π_a-meson becomes degenerate but its chiral partner σ_a-meson appears to be the 'physical' one. Fig.2 shows the correlation functions of $R_{\sigma_a}(p)$ and $R_\pi(p)$ mesons as the functions of of momentum $|\boldsymbol{p}|$ (MэВ) for three chemical potential values $\mu = 0, 510, 680$ MeV.

The curve where $M_\sigma = 2M_\pi$ is depicted (dashed line) in Fig.3 together with the line $M_\sigma = M_\pi$ (solid curve). Unfortunately, more or less reliable theoretical predictions for their locations on the (μ – T)-plane do not exist nowadays. For example, in the Nambu–Jona-Lasinio model these curves have the intersection point with the $\mu = 0$-axis around rather 'high' critical temperature and their changes while the perturbation expansion in the saturating configuration is applied is of special interest. In the model developed, the curves should move down on the plot and if σ-meson appears beyond the $M_\sigma = 2M_\pi$ curve the decay channel $\sigma \to 2\pi$ becomes impossible which favours its experimental identification. In general we could conclude that our results as to behaviour of σ- and π- mesons

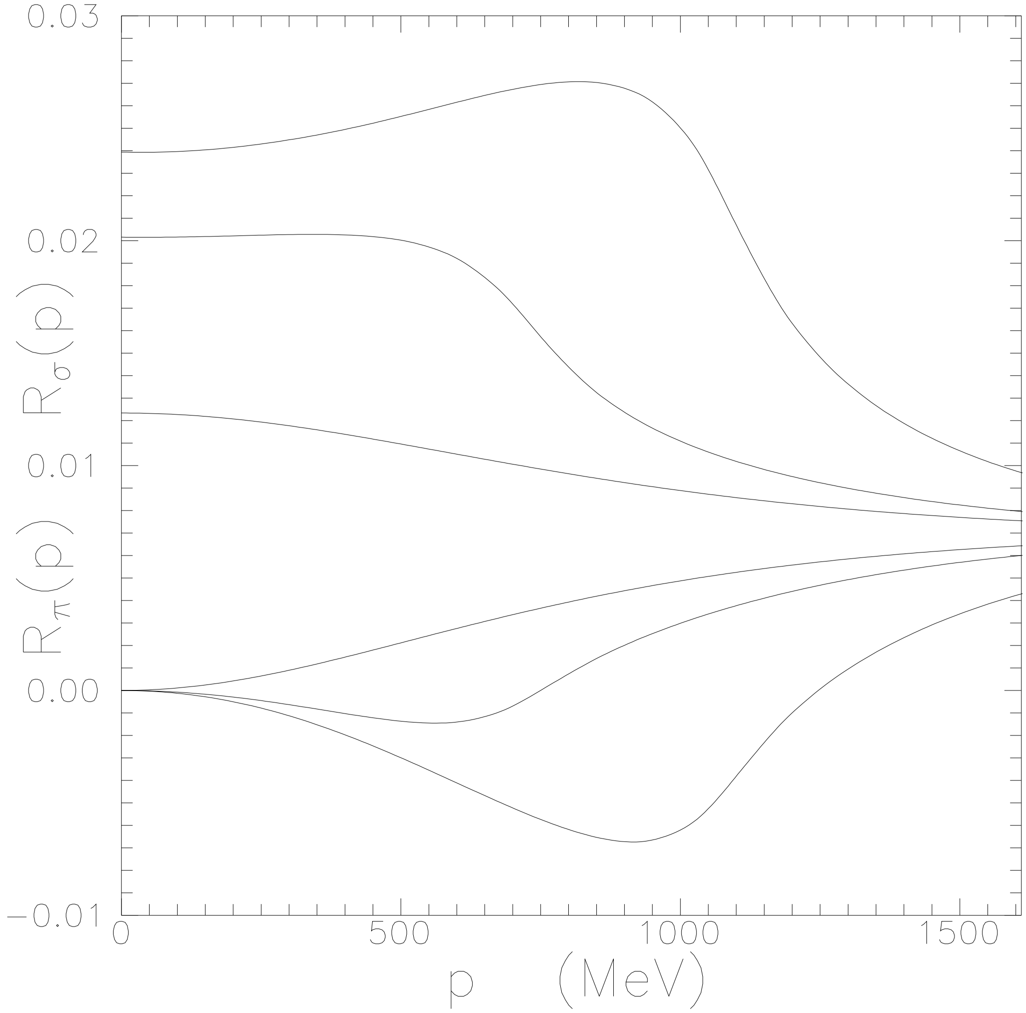

Рис. 2: Correlation functions $R_{\sigma_a}(p)$ (upper solid lines) and $R_\pi(p)$-mesons (lower solid lines)as the momentum $|\boldsymbol{p}|$ (MeV) functions for three magnitudes of chemical potentials $\mu = 0, 510, 680$ MeV.

on the $\mu - -T$-plane are in reasonable agreement with the results obtained in other papers (see, for example, [18]) although one distinction is obvious. It concerns the quark mass behaviour (see, the respective dashed lines in Fig.1). Aiming to clarify this point we have studied the generating functional at zero temperature in the mean field approximation as it was formulated in [11]. Its considerable advantage comes from the possibility to calculate the quark condensate (dynamical quark mass) without appealing the saddle point equation. However, in this exercise the zero mode approximation leads to the unphysical increase of quark condensate when the chemical potential value approaches dynamical quark mass magnitude. It seems this result dictates a necessity to take into account the terms neglected even though partly (remember, we have made use the approximation for zero modes) and to investigate the role of non-zero modes. But we believe studying the mechanism of filling the Fermi sphere in gives more illumination to the origin of chiral symmetry restoration in this case.

In our previous paper [19] we were analysing the possible mechanism of mixing σ-meson and the phonon-like excitations of instanton liquid and demonstrated that notwithstanding the seeming smallness of coupling constant (small change of dynamical quark mass) the effects of mixing could be powerful enough. The specific feature of these effects while depending on μ and T is the alternation of the heavy and light component roles. Initially σ-meson is the heavier component but gn heating (or compressing) this field loses its mass quickly, as was demonstrated, and at reaching a certain threshold magnitude the phonon-like excitations play the role of heavier components.

We are not preoccupied with the claim for the accurate quantative estimates of particular

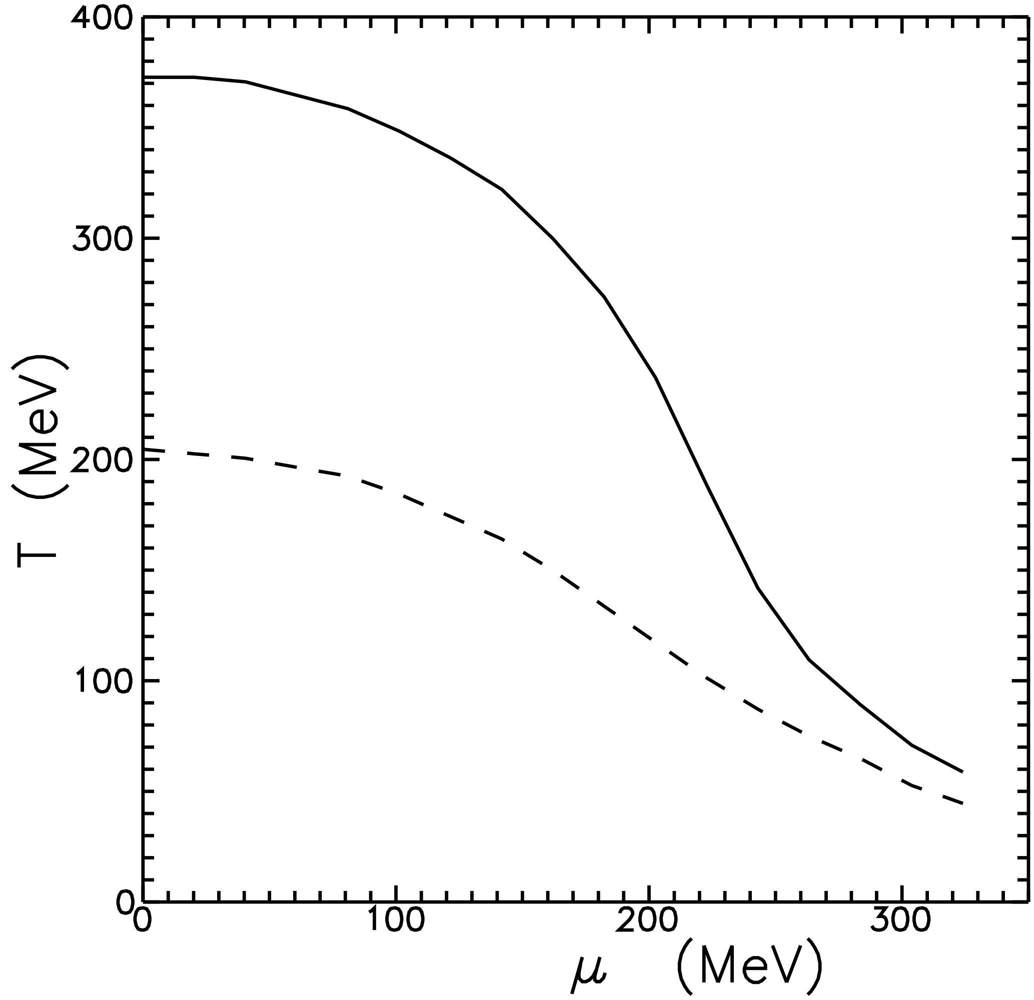

Рис. 3: The curve of $M_\sigma = 2M_\pi$ (dashed line) and the curve of $M_\sigma = M_\pi$ (solid line).

masses or widths because we realize the approximate character of perturbative scheme proposed. Nevertheless, we believe this approach could be useful to clarify the possibility of continuing the estimates obtained to the region of large chemical potentials (larger than dynamical quark mass) and the limits of applicability of zero mode approximation.

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