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MODIFIED JACOBI POLYNOMIAL EXPANSION METHOD APPLIED TO SIDIS DATA ANALYSIS

A.N. Sissakian, O.Yu. Shevchenko and O.N. Ivanov

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Abstract

It is proposed the modification of the Jacobi polynomial expansion method (MJEM) which is based on the application of the truncated (available to experiment) moments instead of the full ones. The application of MJEM with respect to quark helicity distributions reconstruction from the SIDIS data is considered.

One of very important topics in the modern high energy physics is the investigation of the partonic spin structure of nucleon. In this connection, nowadays, there is a huge growth of interest to semi-inclusive DIS (SIDIS) experiments with longitudinally polarized beam and target such as SMC [1], HERMES [25], COMPASS [3].

It is argued (see, for example, Ref. [4]) that to obtain the reliable distributions at relatively low average Q^2 available to the modern SIDIS experiments[1], the leading order (LO) analysis is not sufficient and next to leading order analysis (NLO) is necessary. In ref. [5] the procedure allowing the direct extraction from the SIDIS data of the first moments (truncated to the accessible for measurement x_B region) of the quark helicity distributions in NLO QCD was proposed. However, in spite of the special importance of the first moments[2], it is certainly very desirable to have the procedure of reconstruction in NLO QCD of the polarized densities themselves. At the same time, it is extremely difficult to extract the local in Bjorken x distributions directly, because of the double convolution product entering the NLO QCD expressions for semi-inclusive asymmetries (see [5] and references therein). Fortunately, operating just as in ref. [5], one can directly extract not only the first moments, but the Mellin moments of any required order. Using the truncated moments of parton distribution functions (PDFs) and applying the modified Jacobi polynomial expansion method (MJEM) proposed in Ref.

[1]For example, HERMES data [25] on semi-inclusive asymmetries is obtained at $Q^2_{\text{average}} = 2.5 GeV^2$.

[2]Let us recall that namely these quantities, first moments, are of the most importance for understanding the proton spin puzzle because namely these quantities compose the nucleon spin.

[6] one can reconstruct PDFs themselves in the entire accessible for measurement Bjorken x region.

The standard JEM [7, 8, 9, 10] is the expansion of the studied function $F(x)$ (structure functions, PDFs) in the double series

$$F(x) \simeq F_{N_{max}}(x) = \omega^{(\alpha,\beta)} \sum\nolimits_{k=0}^{N_{max}} \Theta_k^{(\alpha,\beta)}(x) \sum\nolimits_{j=0}^{k} c_{kj}^{(\alpha,\beta)} M(j+1) \tag{1}$$

over the Jacobi polynomials $\Theta_k^{(\alpha,\beta)}(x)$ and the *full* Mellin moments

$$M[j] = \int_0^1 dx x^{j-1} F(x). \tag{2}$$

Here N_{max} is the number of moments left in the expansion.

However, the accessible for the modern SIDIS experiments Bjorken x region is rather narrow. For example, for the HERMES experiment the available region is $0.023 < x < 0.6$. In Ref. [6] it was shown that the application in the expansion (1) of the truncated to the such narrow region moments instead of the full ones leads to the strong disagreement between input and reconstructed distributions.

Fortunately it is possible to modify the standard JEM, Eq. (1), in a such way that new series contains the truncated moments instead of the full ones. The modified expansion [6] looks as

$$F(x) \simeq F_{N_{max}}(x) = \left(\frac{x-a}{b-a}\right)^{\beta} \left(1 - \frac{x-a}{b-a}\right)^{\alpha}$$

$$\times \sum_{n=0}^{N_{max}} \Theta_n^{(\alpha,\beta)} \left(\frac{x-a}{b-a}\right) \sum_{k=0}^{n} c_{nk}^{(\alpha,\beta)} \frac{1}{(b-a)^{k+1}} \sum_{l=0}^{k} \frac{k!}{l!(k-l)!} M'[l+1](-a)^{k-l}, \tag{3}$$

where the notation

$$M'[j] \equiv M'_{[a,b]}[j] \equiv \int_a^b dx x^{j-1} F(x) \tag{4}$$

is used for the moments truncated to accessible for measurement x_B region $[a, b]$. It is of great importance that now in the expansion enter not the full (unavailable) but the truncated (accessible) moments. Thus, having at our disposal few first truncated moments extracted in NLO QCD, and using MJEM Eq. (3), one can reconstruct the local distributions in the accessible for measurement x_B region.

The numerical tests (reconstruction of the parameterization GRSV2000NLO on Δu_V and Δd_V) presented by Fig. 1 demonstrate that for the truncated region

MJEM yields much better accuracy of the input parameterization reconstruction in comparison with JEM. To control the quality of reconstruction, the parameter

$$\nu = \frac{\int_a^b dx |F_{\text{reconstructed}}(x) - F_{\text{reference}}(x)|}{\int_a^b dx |F_{\text{reference}}|} \cdot 100\% \tag{5}$$

is used. Looking at Fig. 1 one can see that dealing with the truncated, available to measurement, x_B region one should apply the proposed MJEM to obtain the reliable results on the local distributions.

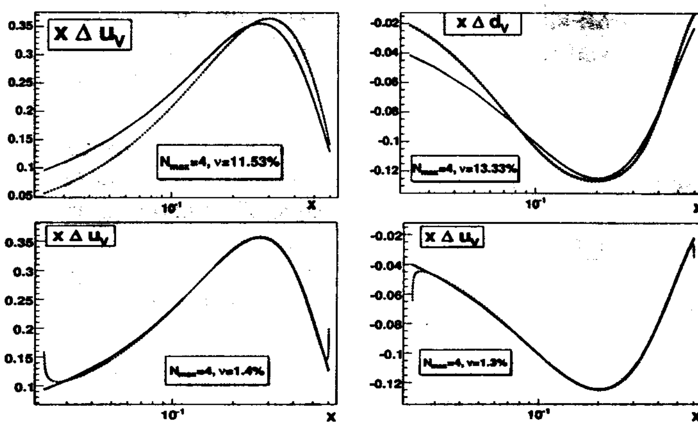

Figure 1: The top part corresponds to $\Delta u_V(x)$ and $\Delta d_V(x)$ reconstructed with the usual JEM. The bottom part corresponds to $\Delta u_V(x)$ and $\Delta d_V(x)$ reconstructed with MJEM. Solid lines correspond to input (reference) parameterization. Dotted lines correspond to the distributions reconstructed with JEM (top) and MJEM (bottom).

The important remark should be made here. The deviations of reconstructed with MJEM, Eq. (3), $F_{N_{max}}$ from F near the boundary points are unavoidable since MJEM is correctly defined in the entire region (a, b) except for the small vicinities of boundary points (see the Appendix). Fortunately, $F_{N_{max}}$ and F are in very good agreement in the practically entire accessible x_B region, while the boundary distortions are easily identified and controlled since they are very sharp and hold in very small vicinities of the boundary points (see Fig. 1). Thus, it is easy to cut off them and to extrapolate the function to the bounds of region. In particular, the such procedure was performed for the calculation of the parameter ν.

Until now we looked for the optimal values of parameters α and β entering MJEM using explicit form of the reference curve (input parameterization). Certainly, in reality we have no any reference curve to be used for optimization and, thus we need some special criteria to find α and β by using only experimental data. Such criteria was proposed in Ref. [6], and it allows to find the optimal values of α and β applying the condition[3]

$$\sum_{j=0}^{N_{max}} \left| M''_{(\text{reconstructed})}[j] - M''_{(\text{reference})}[j] \right| = min, \tag{6}$$

where

$$M''[n] \equiv M''_{[a+a',b-b']}[n] \equiv \int_{a+a'}^{b-b'} dx\, x^{n-1} F(x) \quad (a < a+a' < b-b' < b) \tag{7}$$

is so-called "twice-truncated" moments. The "twice truncated" reference moments should be extracted in NLO QCD from the data in the same way as the input (entering MJEM (3)) "once truncated" moments. In reality one can obtain "twice-truncated" moments removing, for example, first and/or last bin from the integration region. It was shown [6] that the application of criteria (7) allows to reconstruct the distributions with a good precision.

Let us now apply MJEM to the HERMES SIDIS data on the pion production. Here we would like just to test the applicability of MJEM to the experimental data, so that, for a moment, we do not like to deal with the such purely known objects as the fragmentation functions (even for the pion production). The most attractive object from this point of view is the difference asymmetries (see [5] and references therein), where the fragmentation functions are cancel out in the leading order, while in the next to leading order the difference asymmetry has only weak dependence of the difference of the favored and unfavored fragmentation functions (known with a good precision). At the same time the difference asymmetries are still not constructed[4]. So, let us apply a trick and to express the difference asymmetries

$$A_{p(d)}^{\pi^+-\pi^-} = \frac{1}{P_B P_T f D} \frac{(N_{\uparrow\downarrow}^{\pi^+} - N_{\uparrow\downarrow}^{\pi^-}) - (N_{\uparrow\uparrow}^{\pi^+} - N_{\uparrow\uparrow}^{\pi^-})}{(N_{\uparrow\downarrow}^{\pi^+} - N_{\uparrow\downarrow}^{\pi^-}) + (N_{\uparrow\uparrow}^{\pi^+} - N_{\uparrow\uparrow}^{\pi^-})} \tag{8}$$

via the standard virtual photon SIDIS asymmetries

$$A_{p(d)}^{\pi^\pm} = \frac{1}{P_B P_T f D} \frac{N_{\uparrow\downarrow}^{\pi^\pm} - N_{\uparrow\uparrow}^{\pi^\pm}}{N_{\uparrow\downarrow}^{\pi^\pm} + N_{\uparrow\uparrow}^{\pi^\pm}},$$

[3] On the first sight it seems to be natural to find the optimal values of α and β minimizing the difference of *reconstructed* with MJEM and input (extracted from the data) moments. However, it is easy to prove that these moments are equal – see Ref. [6].

[4] At present the such analysis is performed by HERMES collaboration.

which were measured by HERMES [25]. Namely, the difference asymmetries Eq. (8) can be rewritten as

$$A_{p(d)}^{\pi^+-\pi^-} = \frac{R}{R-1}A_{p(d)}^{\pi^+} - \frac{1}{R-1}A_{p(d)}^{\pi^-}, \tag{9}$$

where the ratio

$$R^{\pi^+/\pi^-} \equiv N^{\pi^+}/N^{\pi^-}, \quad N^{\pi^\pm} \equiv N_{\uparrow\downarrow}^{\pi^\pm} + N_{\uparrow\uparrow}^{\pi^\pm}$$

is taken from the unpolarized SIDIS data. This quantity is well known (much better than the fragmentation functions). We take its value from the LEPTO generator of unpolarized events [12], which reproduces the results on this quantity obtained by EMC collaboration [9]. First, for the sake of testing, we reconstruct the local valence distributions in the leading order. The results are presented in Fig. 2. One can see that reconstructed with MJEM curve is in a good agreement with both HERMES results and with the results of direct LO extraction from the difference asymmetries constructed with the application of Eq. (9).

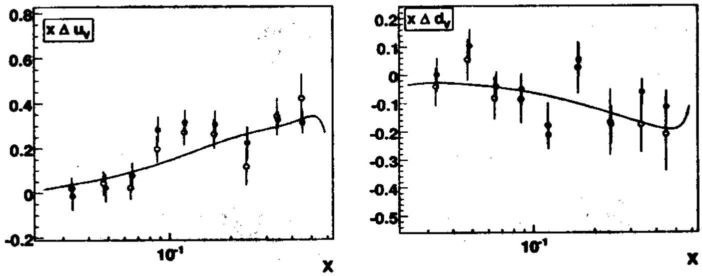

Figure 2: The results of LO reconstruction of $\Delta u_V(x)$ and $\Delta d_V(x)$. The closed circles correspond to the HERMES LO analysis [25]. The open circles correspond to direct LO extraction from the difference asymmetries constructed applying Eq. (9). The solid curves correspond to reconstruction with MJEM ($N_{max} = 4$).

The results of NLO reconstruction with MJEM of Δu_V and Δd_V from the difference asymmetries given by Eq. (9) are presented by Fig. 3 in comparison with the respective LO results. It is seen that the behavior of NLO and LO curves with respect to each other is in agreement with the predictions of existing parameterizations (see, for example, [11]).

Thus, all numerical tests confirm that the proposed modification of the Jacobi polynomial expansion method, MJEM, allows to reconstruct with a high precision the quark helicity distributions in the accessible for measurement x_B region. The performed preliminary analysis of the HERMES data on the pion production gives the reliable results on the valence quark helicity distributions, which are in

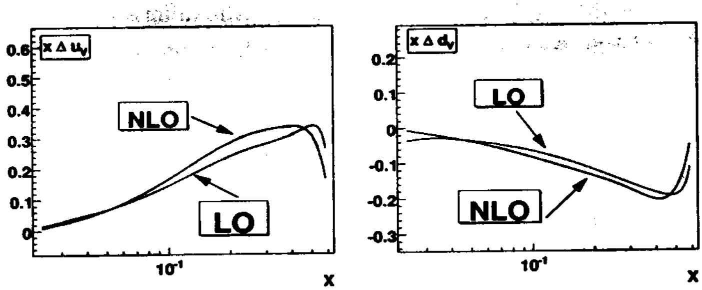

Figure 3: The results of Δu_V and Δd_V NLO reconstruction with MJEM from the HERMES data in comparison with the respective LO results.

accordance with the leading order HERMES results as well as with the existing NLO parametrizations on these quantities. Thus, MJEM occurs successful with respect to NLO extraction of the valence PDFs. In future we also plan to apply MJEM to NLO QCD extraction of the sea and strange quark helicity distributions from the published HERMES data and expected COMPASS data on the SIDIS asymmetries with the pion and kaon production.

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Discussion

Q. (O. Teryaev, JINR, Dubna, Russia): How your method is related to the earlier studies of the evolution of truncated method?

A. We did not consider the evolution due to limited Q^2 range of HERMES but we plan the apply MJEM to evolution of truncated moments later.