# Transversity and its accompanying T-odd distribution from Drell-Yan processes with pion-proton collisions 

A. Sissakian, O. Shevchenko ${ }^{\text {a }}$, A. Nagaytsev, O. Denisov, O. Ivanov<br>Joint Institute for Nuclear Research, 141980 Dubna, Russia<br>Received: 12 December 2005 / Revised version: 11 January 2006 /<br>Published online: 9 March 2006 - © Springer-Verlag / Società Italiana di Fisica 2006


#### Abstract

This paper studies the possibility of direct extraction of the transversity and its accompanying T-odd parton distribution function (PDF) from Drell-Yan (DY) processes with unpolarized pion beam and with both unpolarized and transversely polarized proton targets. At present, such an extraction can be performed with the COMPASS experiment at CERN. The preliminary estimations performed for the COMPASS kinematic region demonstrate that it is quite realistic to extract both transversity and its accompanying T-odd PDF under COMPASS conditions.


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The advantage of DY process for the extraction of PDFs is that there is no need for any fragmentation functions. It is well-known that the double transversely polarized DY process $H_{1}^{\uparrow} H_{2}^{\uparrow} \rightarrow l^{+} l^{-} X$ allows us to directly extract the transversity distributions (see [1] for review). In particular, at PAX [2] it is planned to study the double polarized DY process with an antiproton beam. However, it is a rather difficult task to produce an antiproton beam with a sufficiently high degree of polarization. So, it is certainly desirable to have an alternative (complementary) possibility allowing us to extract the transversity PDF from unpolarized and single-polarized DY processes. This could be a matter of special interest for the COMPASS experiment [3], where the possibility of studying DY processes with an unpolarized pion beam and with both unpolarized and transversely polarized proton targets $\pi^{-} p \rightarrow \mu^{+} \mu^{-} X$, $\pi^{-} p^{\uparrow} \rightarrow \mu^{+} \mu^{-} X$ is currently under discussion.

The original expressions for unpolarized and singlepolarized DY cross-sections [4] are very inconvenient in application, since all $k_{T}$-dependent PDFs enter there in the complex convolution. To avoid this problem, in [5] the $q_{T}$ integration approach $[6-8]$ was applied. As a result, the procedure proposed in [5] allows us to extract the transversity $h_{1}$ and the first moment

$$
\begin{equation*}
h_{1 q}^{\perp(1)}(x) \equiv \int \mathrm{d}^{2} \mathbf{k}_{T}\left(\frac{\mathbf{k}_{T}^{2}}{2 M_{\pi}^{2}}\right) h_{1 q}^{\perp}\left(x_{\pi}, \mathbf{k}_{T}^{2}\right) \tag{1}
\end{equation*}
$$

of T-odd distribution $h_{1}^{\perp}$ directly, without any model assumptions about $k_{T}$-dependence of $h_{1}^{\perp}\left(x, k_{T}^{2}\right)$.

[^0]The general procedure proposed in [5] applied to unpolarized DY process $\pi^{-} p \rightarrow \mu^{+} \mu^{-} X$ gives ${ }^{1}$

$$
\begin{align*}
& \left.\hat{k}\right|_{\pi^{-} p \rightarrow \mu^{+} \mu^{-} X}= \\
& 8 \frac{\sum_{q} e_{q}^{2}\left[\left.\left.\bar{h}_{1 q}^{\perp(1)}\left(x_{\pi}\right)\right|_{\pi^{-}} h_{1 q}^{\perp(1)}\left(x_{p}\right)\right|_{p}+\left(x_{\pi} \leftrightarrow x_{p}\right)\right]}{\sum_{q} e_{q}^{2}\left[\left.\left.\bar{f}_{1 q}\left(x_{\pi}\right)\right|_{\pi^{-}} f_{1 q}\left(x_{p}\right)\right|_{p}+\left(x_{\pi} \leftrightarrow x_{p}\right)\right]} \tag{2}
\end{align*}
$$

where $\hat{k}$ is the coefficient at the $\cos 2 \phi$ dependent part of the properly integrated over $q_{T}$ ratio of unpolarized crosssections:

$$
\begin{align*}
& \hat{R}=\frac{\int \mathrm{d}^{2} \mathbf{q}_{T}\left[\left|\mathbf{q}_{T}\right|^{2} / M_{\pi} M_{p}\right]\left[\mathrm{d} \sigma^{(0)} / \mathrm{d} \Omega\right]}{\int \mathrm{d}^{2} \mathbf{q}_{T} \sigma^{(0)}}  \tag{3}\\
& \hat{R}=\frac{3}{16 \pi}\left(\gamma\left(1+\cos ^{2} \theta\right)+\hat{k} \cos 2 \phi \sin ^{2} \theta\right) \tag{4}
\end{align*}
$$

At the same time, in the case of a single polarized DY process $\pi^{-} p^{\uparrow} \rightarrow \mu^{+} \mu^{-} X$, operating just as in [5], one obtains

$$
\begin{equation*}
\hat{A}_{h}=-\frac{1}{2} \frac{\sum_{q} e_{q}^{2}\left[\bar{h}_{1 q}^{\perp(1)}\left(x_{\pi}\right) h_{1 q}\left(x_{p}\right)+\left(x_{\pi} \leftrightarrow x_{p}\right)\right]}{\sum_{q} e_{q}^{2}\left[\bar{f}_{1 q}\left(x_{\pi}\right) f_{1 q}\left(x_{p}\right)+\left(x_{\pi} \leftrightarrow x_{p}\right)\right]} \tag{5}
\end{equation*}
$$

[^1]where the single spin asymmetry (SSA) $\hat{A}_{h}$ is defined as ${ }^{2}$
\[

$$
\begin{align*}
\hat{A}_{h}= & \left\{\int \mathrm{d} \Omega \mathrm{~d} \phi_{S_{2}} \int \mathrm{~d}^{2} \mathbf{q}_{T}\left(\left|\mathbf{q}_{T}\right| / M_{\pi}\right)\right. \\
& \left.\times \sin \left(\phi+\phi_{S_{2}}\right)\left[\mathrm{d} \sigma\left(\mathbf{S}_{2 T}\right)-\mathrm{d} \sigma\left(-\mathbf{S}_{2 T}\right)\right]\right\} / \\
& \left\{\int \mathrm{d} \Omega \mathrm{~d} \phi_{S_{2}} \int \mathrm{~d}^{2} \mathbf{q}_{T}\left[\mathrm{~d} \sigma\left(\mathbf{S}_{2 T}\right)+\mathrm{d} \sigma\left(-\mathbf{S}_{2 T}\right)\right]\right\} \tag{6}
\end{align*}
$$
\]

In (2)-(6), the quantity $h_{1 q}^{\perp(1)}\left(x_{\pi}\right)$ is defined by (1). All other notations are the same as in [5] (see [1] for details on kinematics in the Collins-Soper frame that we are dealing with).

Neglecting strange quark PDF contributions, squared sea contributions of $u$-quark PDF $\left.\left.h_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{\pi^{-}} \bar{h}_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}$, $\left.\left.f_{1 u}\left(x_{\pi}\right)\right|_{\pi^{-}} \bar{f}_{1 u}\left(x_{p}\right)\right|_{p}$, and cross terms containing the products of sea and valence $d$-quark PDFs (additionally suppressed by the charge factor $1 / 4$ ), one arrives at the simplified equations

$$
\begin{align*}
& \left.\hat{k}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-}} \simeq 8 \frac{\left.\left.\bar{h}_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{\pi^{-}} h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}}{\left.\left.\bar{f}_{1 u}\left(x_{\pi}\right)\right|_{\pi^{-}} f_{1 u}\left(x_{p}\right)\right|_{p}}  \tag{7}\\
& \left.\hat{A}_{h}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-} p^{\uparrow}} \simeq-\frac{1}{2} \frac{\left.\left.\bar{h}_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{\pi^{-}} h_{1 u}\left(x_{p}\right)\right|_{p}}{\left.\left.\bar{f}_{1 u}\left(x_{\pi}\right)\right|_{\pi^{-}} f_{1 u}\left(x_{p}\right)\right|_{p}} \tag{8}
\end{align*}
$$

Notice that while two equations corresponding to unpolarized and single-polarized antiproton-proton DY processes completely determine the transversity and its accompanying T-odd PDF in proton [5], the two equations (7) and (8) contain three unknown quantities $\left.\bar{h}_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{\pi^{-}}$, $\left.h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}$ and $h_{1 u}\left(x_{p}\right)$. Nevertheless, from (7) and (8) it immediately follows that

$$
\begin{equation*}
\frac{\left.h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}}{\left.h_{1 u}\left(x_{p}\right)\right|_{p}}=-\frac{1}{16} \frac{\left.\hat{k}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-} p}}{\left.\hat{A}_{h}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-} p^{\uparrow}}} \tag{9}
\end{equation*}
$$

Thus, using only unpolarized pion beam colliding with unpolarized and transversely polarized protons, it is possible to extract the ratio $\left.h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p} /\left.h_{1 u}\left(x_{p}\right)\right|_{p}$. However, it is certainly desirable to extract $\left.h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}$ and $\left.h_{1 u}\left(x_{p}\right)\right|_{p}$ in separation

The simplest way to solve this problem is to use in (7) and (8) the quantity $\left.h_{1 u}^{\perp(1)}\right|_{p}$ extracted from $\hat{k}$ measured in the unpolarized DY process, $\bar{p} p \rightarrow l^{+} l^{-} X$ (in the

[^2]way proposed in [5]). However, if one wishes to extract all quantities within the experiment with the pion beam (here COMPASS), the additional assumptions connecting pion and proton PDFs are necessary. Taking into account the probability interpretations of $h_{1 q}^{\perp}$ and $f_{1 q}$ PDFs, we assume the relation
\[

$$
\begin{equation*}
\frac{\left.\bar{h}_{1 u}^{\perp(1)}(x)\right|_{\pi^{-}}}{\left.h_{1 u}^{\perp(1)}(x)\right|_{p}}=C_{u} \frac{\left.\bar{f}_{1 u}(x)\right|_{\pi^{-}}}{\left.f_{1 u}(x)\right|_{p}} \tag{10}
\end{equation*}
$$

\]

Notice that the assumption given by (10) is in accordance (but it is much more weak restriction) with Boer's model (see (50) in [4]), where $C_{u}=M_{p} c_{\pi}^{u} / M_{\pi} c_{p}^{u}$.

As we will see below, one should put $C_{u}$ to be about unity

$$
\begin{equation*}
C_{u} \simeq 1 \tag{11}
\end{equation*}
$$

to reconcile the results on $h_{1 u}^{\perp(1)}$ in the proton obtained from the simulated $\left.\hat{k}\right|_{\pi^{-} p}$ with the respective results [5] obtained from the simulated $\left.\hat{k}\right|_{\bar{p} p}$, as well as with the upper bound [5] on this quantity.

Thus, (7) and (8) are rewritten as (cf. (19), (20) in [5])

$$
\begin{align*}
& \left.\hat{k}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-} p} \simeq 8 \frac{\left.\left.h_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{p} h_{1 u}^{\perp(1)}\left(x_{p}\right)\right|_{p}}{\left.\left.f_{1 u}\left(x_{\pi}\right)\right|_{p} f_{1 u}\left(x_{p}\right)\right|_{p}},  \tag{12}\\
& \left.\hat{A}_{h}\left(x_{\pi}, x_{p}\right)\right|_{\pi^{-} p^{\uparrow}} \simeq-\frac{1}{2} \frac{\left.\left.h_{1 u}^{\perp(1)}\left(x_{\pi}\right)\right|_{p} h_{1 u}\left(x_{p}\right)\right|_{p}}{\left.\left.f_{1 u}\left(x_{\pi}\right)\right|_{p} f_{1 u}\left(x_{p}\right)\right|_{p}} \tag{13}
\end{align*}
$$

Looking at (12) and (13), one can see that now the number of equations is equal to the number of variables to be found. Measuring the quantity $\hat{k}$ in unpolarized DY ((3) and (4)), and using (12), one can obtain the quantity $\left.h_{1 u}^{\perp(1)}\right|_{p}$. Then, measuring SSA, (6), and using in (13) the quantity $\left.h_{1 u}^{\perp(1)}\right|_{p}$ obtained from unpolarized DY, one can eventually extract the transversity distribution $\left.h_{1 u}\right|_{p}$.

To deal with (12) and (13) in practice, one should consider them at the points ${ }^{3} x_{\pi}=x_{p} \equiv x$ (i.e., $x_{F} \equiv x_{\pi}-$ $x_{p}=0$ ), so that

$$
\begin{equation*}
h_{1 u}^{\perp(1)}(x)=f_{1 u}(x) \sqrt{\frac{\left.\hat{k}(x, x)\right|_{\pi^{-} p}}{8}} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
h_{1 u}(x)=-4 \sqrt{2} \frac{\left.\hat{A}_{h}(x, x)\right|_{\pi^{-} p^{\uparrow}}}{\sqrt{\left.\hat{k}(x, x)\right|_{\pi^{-} p}}} f_{1 u}(x) \tag{15}
\end{equation*}
$$

where now all PDFs refer to protons.
${ }^{3}$ The different points $x_{F}=0$ can be reached by changing the $Q^{2}$ value at fixed $s \equiv Q^{2} / x_{1} x_{2} \equiv Q^{2} / \tau$.

To estimate the possibility of the $h_{1 u}^{\perp(1)}$ measurement, the special simulation of unpolarized DY events with the COMPASS kinematics is performed. The pion-proton collisions are simulated with the PYTHIA generator [10]. Two samples are prepared corresponding to 60 GeV and 100 GeV pion beams. Each sample contains about 100 K pure Drell-Yan events. The events are weighted (see [5] for details) with the ratio of DY cross-sections given by (see $[4,11]$ )

$$
\begin{align*}
R \equiv & \frac{d \sigma^{(0)} / d \Omega}{\sigma^{(0)}}  \tag{16}\\
R= & \frac{3}{16 \pi}\left[1+\cos ^{2} \theta\right. \\
& \left.+(\nu / 2) \cos 2 \phi \sin ^{2} \theta\right] \quad(\nu \equiv 2 \kappa) \tag{17}
\end{align*}
$$

where $\nu$ dependencies of $q_{T}$ and $x_{\pi}$ are taken from $[11,12]$.
The angular distributions of $\hat{R}((3)$ and (4)) for both samples are studied just as was done in [11] with respect to $R((16),(17))$. The results are shown in Fig. 1. The values of $\hat{k}$ at averaged $Q^{2}$ for both energies are found to be $0.7 \pm$ 0.1 for 60 GeV and $0.9 \pm 0.1$ for 100 GeV pion beams.


Fig. 1. $\hat{k}$ versus $x_{\pi}$ for $x_{F} \simeq 0$. Data is obtained with MC simulations for 60 GeV (closed circles) and 100 GeV (open circles) pion beams


Fig. 2. $h_{1 u}^{\perp(1)}$ versus $x_{\pi}$ for $x_{F} \simeq 0$. Data is obtained with MC simulations for 60 GeV (closed circles) and 100 GeV (open circles) pion beams

The quantity $h_{1 u}^{\perp(1)}$ is reconstructed from the obtained values of $\hat{k}$ using (14) with $x_{F}=0 \pm 0.04$. The results are shown in Fig. 2. Let us recall that to obtain $h_{1 u}^{\perp(1)}$ from $\left.\hat{k}\right|_{\pi^{-} p}$, we have chosen $C_{u} \simeq 1$. Notice that this choice of $C_{u}$ is consistent with the results on $h_{1 u}^{\perp(1)}$ obtained in [5] from simulated $\left.\hat{k}\right|_{\bar{p} p}$ (compare Fig. 2 with Fig. 4 of [5]), and also with the upper bound on $h_{1 u}^{\perp(1)}$ estimated in that paper. Otherwise, if $C_{u}$ essentially differs from unity, one should multiply the results on $h_{1 u}^{\perp(1)}$ by the factor $1 / \sqrt{C_{u}}$, which would lead to disagreement of the results on $h_{1 u}^{\perp(1)}$ obtained from the simulated quantities $\left.\hat{k}\right|_{\pi^{-} p}$ and $\left.\hat{k}\right|_{\bar{p} p}$.

Certainly, all conclusions made on the basis of simulations are very preliminary. A reliable conclusion about $C_{u}$ can be made only from the future measurements of $\hat{k}$ for both DY processes with $\bar{p}$ and $\pi^{-}$participation.

Using the obtained magnitudes of $h_{1 u}^{\perp(1)}$ we estimate the expected SSA given by (13). The results are shown in Figs. 3 and 4. For estimation of $h_{1 u}$ entering SSA together with $h_{1 u}^{\perp(1)}$ (see (13)), we follow the procedure of [13] and use the (rather crude) "evolution model" [1, 13], where


Fig. 3. SSA given by (13) versus $x_{F}$ for the 100 GeV pion beam $\left(Q_{\text {average }}^{2}=6.2 \mathrm{GeV}^{2}\right)$


Fig. 4. SSA given by (13) versus $x_{F}$ for the 60 GeV pion beam $\left(Q_{\text {average }}^{2}=5.5 \mathrm{GeV}^{2}\right)$
there are no estimations of uncertainties. That is why in (purely qualitative) Figs. 3 and 4 we present the solid curves instead of points with error bars. To obtain these curves we reproduce the $x$-dependence of $h_{1 u}^{\perp(1)}$ in the considered region, using Boer's model ((50) in [4]), properly numerically corrected in accordance with the simulation results.

It should be noticed that the estimations of the $\hat{k}$ and $\hat{A}_{h}$ magnitudes obtained in this paper are very preliminary and just show the order of values of these quantities. For more precise estimations one needs the Monte-Carlo generator, which is more suitable for studies of DY processes (see, for example [14]) than the PYTHIA generator that we used (with the proper weighting of events) here.

In summary, it is shown that the procedure proposed in [5] can be applied to DY processes: $\pi^{-} p \rightarrow \mu^{+} \mu^{-} X$ and $\pi^{-} p^{\uparrow} \rightarrow \mu^{+} \mu^{-} X$, which could be studied in the COMPASS experiment at CERN. The preliminary estimations for the COMPASS kinematical region show the possibility of measuring both $\hat{k}$ and SSA $\hat{A}_{h}$ and then extracting the quantities $h_{1}^{\perp(1)}$ and $h_{1}$ that we are interested in.

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[^0]:    ${ }^{\text {a }}$ e-mail: shev@mail.cern.ch

[^1]:    ${ }^{1}(2)$ is obtained within the quark parton model. It is of importance that the large values of $\nu$ cannot be explained by leading and next-to-leading order perturbative QCD corrections, or by the high twists effects (see [4] and references therein).

[^2]:    ${ }^{2}$ Notice that $\mathrm{SSA} \hat{A}_{h}$ is analogous to asymmetry $A_{U T}^{\sin \left(\phi-\phi_{S}\right) \frac{q_{T}}{M_{N}}}$ (weighted with $\sin \left(\phi-\phi_{S}\right)$ and the same weight $\left.q_{T} / M_{N}\right)$ applied in [9] with respect to Sivers function extraction from the single-polarized DY processes.

