Pis'ma v ZhETF, vol. 82, iss. 2, pp. 57 – 62 © 2005 July 25

An approach to NLO QCD analysis of the semi-inclusive DIS data with the modified Jacobi polynomial expansion method

A. N. Sissakian, O. Yu. Shevchenko[1)]*, O. N. Ivanov*

Joint Institute for Nuclear Research, 141980 Dubna, Russia

Submitted 17 May 2005
Resubmitted 6 June 2005

It is proposed the modification of the Jacobi polynomial expansion method (MJEM) which is based on the application of the truncated moments instead of the full ones. This allows to reconstruct with a high precision the local quark helicity distributions even for the narrow accessible for measurement Bjorken x region using as an input only four first moments extracted from the data in next to leading order QCD. It is also proposed the variational (extrapolation) procedure allowing to reconstruct the distributions outside the accessible Bjorken x region using the distributions obtained with MJEM in the accessible region. The numerical calculations encourage one that the proposed variational (extrapolation) procedure could be applied to estimate the full first (especially important) quark moments.

PACS: 13.60.Hb, 13.85.Ni, 13.88.+e

The extraction of the quark helicity distributions is one of the main tasks of the semi-inclusive deep inelastic scattering (SIDIS) experiments (HERMES [1], COMPASS [2]) with the polarized beam and target. At the same time it was argued [3] that to obtain the reliable distributions at relatively low average Q^2 available to the modern SIDIS experiments[2)], the leading order (LO) analysis is not sufficient and next to leading order (NLO) analysis is necessary. In ref. [4] the procedure allowing the direct extraction from the SIDIS data of the first moments of the quark helicity distributions in NLO QCD was proposed. However, in spite of the special importance of the first moments, it is certainly very desirable to have the procedure of reconstruction in NLO QCD of the polarized densities themselves. However, it is extremely difficult to extract the local in x_B distributions directly, because of the double convolution product entering the NLO QCD expressions for semi-inclusive asymmetries (see [4] and references therein). On the other hand, operating just as in ref. [4], one can directly extract not only the first moments, but the Mellin moments of any required order. The simple extension of the procedure proposed in ref. [4] gives for the n-th moments $\Delta_n q \equiv \int_0^1 dx\, x^{n-1} q(x)$ of the valence distributions the equations

$$\Delta_n u_V = \frac{1}{5}\frac{\mathcal{A}_p^{(n)} + \mathcal{A}_d^{(n)}}{L_{(n)1} - L_{(n)2}};\ \ \Delta_n d_V = \frac{1}{5}\frac{4\mathcal{A}_d^{(n)} - \mathcal{A}_p^{(n)}}{L_{(n)1} - L_{(n)2}}, \quad (1)$$

where all quantities in the right-hand side are the same as in ref. [4] (see Eqs. (18)–(23)) with the replacement of

$$\int_0^1 dx \ \text{ by } \ \int_0^1 dx\, x^{n-1}.$$

It should be noticed that in reality one can measure the asymmetries only in the restricted x_B region $0 < a < x < b < 1$, so that the approximate equations for the truncated moments

$$\Delta'_n q \equiv \int_a^b dx\, x^{n-1} q(x) \quad (2)$$

of the valence distributions have a form (1) with the replacement of the full integrals by the sums over bins covering accessible x_B region $a < x < b$, so that

$$\mathcal{A}_p^{(n)} \simeq$$
$$\simeq \sum_{i=1}^{N_{\rm bins}} x^{n-1} \Delta x_i\, A_p^{\pi^+-\pi^-}(x_i)\Big|_Z (4u_V - d_V)(x_i) \times$$
$$\times \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi}\tilde{C}_{qq}\otimes](D_1 - D_2), \quad (3)$$

and analogously for $\mathcal{A}_d^{(n)}$.

Thus, one can directly extract from the data the n-th Mellin moments of valence distributions. The question arises: is it sufficient to reconstruct the local in x_B distributions?

There exist several methods allowing to reconstruct the local in x_B quantities (like structure functions, polarized and unpolarized quark distributions, etc) knowing their n-th Mellin moments. All of them use the expansion of the local quantity in the series over the orthogonal polynomials (Bernstein, Legendre, Jacobi, etc). The

[1)] e-mail: shev@mail.cern.ch

[2)] For example, HERMES data [1] on semi-inclusive asymmetries is obtained at $Q^2_{\rm average} = 2.5\ {\rm GeV}^2$.

most successful in applications (reconstruction of the local distributions from the evolved with GLAP moments and investigation of Λ_{QCD}) occurred the Jacobi polynomial expansion method (JEM) proposed in the pioneer work by Parisi and Sourlas [5] and elaborated[3)] in refs. [6, 7]. Within JEM the local in x_B functions (structure functions or quark distributions) are expanded in the double series over the Jacobi polynomials and Mellin moments – see Eq. (A.1) in the Appendix. For what follows it is of importance that the moments entering Eq. (A.1) are the *full* moments, i.e., the integrals over the entire x_B region $0 < x < 1$: $M[j] = \int_0^1 dx x^{j-1} F(x)$. Until now nobody investigated the question of applicability of JEM to the rather narrow x_B region available to the modern polarized SIDIS experiments. So, let us try to apply JEM to the reconstruction of $\Delta u_V(x)$ and $\Delta d_V(x)$ in the narrow x_B region[4)] $a = 0.023 < x < b = 0.6$ available to HERMES, and to investigate is it possible to safely replace the full moments by the truncated ones. To this end we perform the simple test. We choose[5)] GRSV2000**NLO** (symmetric sea) parametrization [9] at $Q^2 = 2.5\,\text{GeV}^2$. Integrating the parametrization over the HERMES x_B region we then calculate twelve truncated moments of the u and d valence distributions given by Eq. (2) with $a = 0.023, b = 0.6$. Substituting these moments in the expansion (A.1) with $N_{\max} = 12$, we look for optimal values of parameters α and β corresponding to the minimal deviation of reconstructed curves for $\Delta u_V(x)$ and $\Delta d_V(x)$ from the input (reference) curves corresponding to input parametrization. To find these optimal values α_{opt} and β_{opt} we use the program MINUIT [10]. To control the quality of reconstruction we introduce the parameter

$$\nu = \frac{\int_a^b dx |F_{\text{reconstructed}}(x) - F_{\text{reference}}(x)|}{|\int_a^b dx F_{\text{reference}}|} \cdot 100\%, \quad (4)$$

where $F_{\text{reference}}(x)$ corresponds to the input parametrization and $F_{\text{reconstructed}}(x) \equiv F_{N_{\max}}(x)$ in Eq. (A.1) from the Appendix. The comparison of reconstructed and input (reference) curves shows that even for a such high number of used moments $N_{\max} = 12$ they strongly differ from each other: $\nu\big|_{JEM} = 6.24\%$ for Δu_V and $\nu\big|_{JEM} = 5.52\%$ for Δd_V. Thus, the substitution of truncated moments instead of exact ones in the expansion (A.1) is a rather crude approximation at least for HERMES x_B region. Fortunately it is possible to modify the standard JEM in a such way that new series contains the truncated moments instead of the full ones. The new expansion looks as (see the Appendix)

$$F(x) \simeq F_{N_{\max}}(x) = \left(\frac{x-a}{b-a}\right)^{\beta} \left(1 - \frac{x-a}{b-a}\right)^{\alpha} \times$$
$$\times \sum_{n=0}^{N_{\max}} \Theta_n^{(\alpha,\beta)} \left(\frac{x-a}{b-a}\right) \sum_{k=0}^{n} c_{nk}^{(\alpha,\beta)} \frac{1}{(b-a)^{k+1}} \times$$
$$\times \sum_{l=0}^{k} \frac{k!}{l!(k-l)!} M'[l+1](-a)^{k-l}, \quad (5)$$

where we introduce the notation (c.f. Eq. (2))

$$M'[j] \equiv M'_{[a,b]}[j] \equiv \int_a^b dx x^{j-1} F(x) \quad (6)$$

for the moments truncated to accessible for measurement x_B region. It is of great importance that now in the expansion enter not the full (unavailable) but the truncated (accessible) moments. Thus, having at our disposal first few truncated moments extracted in NLO QCD (see Eqs. (1)), and using MJEM (Eq. (5)), one can reconstruct the local distributions in the accessible for measurement x_B region.

Let us check how well MJEM works. To this end let us repeat the simple exercises with reconstruction of the known GRSV2000**NLO** (symmetric sea) parametrization and compare the results of $\Delta u_V(x)$ and $\Delta d_V(x)$ reconstruction with the usual JEM and with the proposed MJEM. To control the quality of reconstruction we again use[6)] the parameter ν given by Eq. (4), where now $F_{\text{reconstructed}}(x) \equiv F_{N_{\max}}(x)$ in Eq. (5). We perform the reconstruction with both very high number of used moments $N_{\max} = 12$ and small number $N_{\max} = 4$. Notice that the last choice $N_{\max} = 4$ is especially important because of peculiarities of the data on asymmetries provided by the SIDIS experiments. Indeed, the number of used moments should be as small as possible because first, the relative error $|\delta(M'[j])/M'[j]|$ on $M'[j]$ becomes higher with increase of j and second, the high moments becomes very sensitive to the

[3)] JEM with respect to polarized quark densities was first applied in ref. [8].

[4)] We choose here the most narrow HERMES x_B region where the difference between JEM and its modification MJEM (see below) application becomes especially impressive. However, even with the more wide x_B region (for example, COMPASS [2] region $0.003 < x < 0.7$) it is of importance to avoid the additional systematical errors caused by the replacement of the full (unaccessible) moments in JEM (A.1) by the accessible truncated moments.

[5)] Certainly, one can choose for testing any other parametrization.

[6)] Calculating ν we just cut off the boundary distortions which hold for MJEM in the small vicinities of the boundary points (see the Appendix), and decrease the integration region, respectively. To be more precise, one can apply after cutting some extrapolation to the boundary points. However, the practice shows that the results on ν calculation are practically insensitive to the way of extrapolation since the widthes of the boundary distortion regions are very small (less than 10^{-3}).

replacement of integration by the sum over the bins. The results of $\Delta u_V(x)$ and $\Delta d_V(x)$ reconstruction with MJEM for $N_{\max} = 12$ demonstrate that, on the contrary to the usual JEM, MJEM gives excellent agreement between the reference and reconstructed curves: $\nu|_{MJEM} = 0.06\%$ for $\Delta u_V(x)$ and $\nu|_{MJEM} = 0.08\%$ for $\Delta d_V(x)$.

In the case $N_{\max} = 4$ the difference in quality of reconstruction between JEM and MJEM (see Fig.1) becomes especially impressive[7]. While for standard JEM the reconstructed and reference curves strongly differ from each other, the respective curves for MJEM are in a good agreement. Thus, one can conclude that dealing with the truncated, available to measurement, x_B region one should apply instead of the usual JEM the proposed modified JEM to obtain the reliable results on the local distributions.

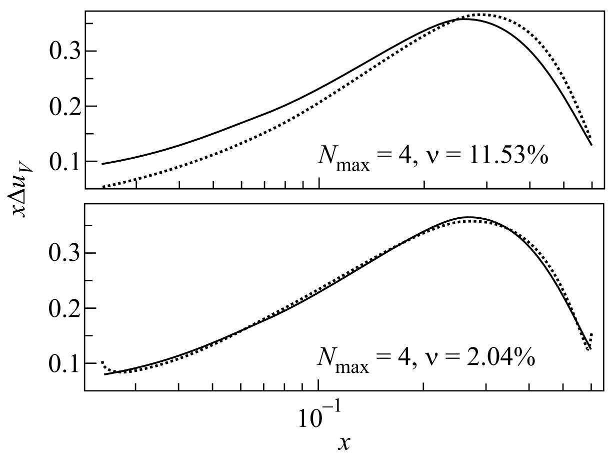

Fig.1. Comparison of the quality of $\Delta u_V(x)$ reconstruction with the usual JEM (top) and with MJEM (bottom). Solid lines correspond to input (reference) parametrization. Dotted lines correspond to the distributions reconstructed with JEM (top) and with MJEM (bottom)

Until now we looked for the optimal values of parameters α and β entering MJEM using explicit form of the reference curve (input parametrization). Certainly, in reality we have no any reference curve to be used for optimization. However, one can extract from the data in NLO QCD first few moments (see Eqs. (1)). Thus, we need some criterium of MJEM optimization which would use for optimization of α and β only the known (extracted) moments entering MJEM.

On the first sight it seems to be natural to find the optimal values of α and β minimizing the difference of *reconstructed* with MJEM (5) and input[8] (*entering* MJEM expansion (5)) moments. However, it is easy to prove[9] that this difference is equal to zero identically:

$$M'_{[a,b]}[n]\Big|_{\text{reconstructed}} \equiv M'_{[a,b]}[n]\Big|_{\text{input}}, \quad n \le N_{\max}, \quad (7)$$

i.e. all reconstructed moments with $n \le N_{\max}$ are identically equal to the respective input moments *for any* α and β. Fortunately, we can use for comparison the reference "twice-truncated" moments

$$M''[n] \equiv M''_{[a+a',b-b']}[n] \equiv \int_{a+a'}^{b-b'} dx\, x^{n-1} F(x)$$
$$(a < a + a' < b - b' < b), \qquad (8)$$

i.e. the integrals over the region less than the integration region $[a,b]$ for the "once-truncated" moments $M'_{[a,b]}$ entering MJEM (5). The respective optimization criterium can be written in the form

$$\sum\nolimits_{j=0}^{N_{\max}} \left| M''_{(\text{reconstructed})}[j] - M''_{(\text{reference})}[j] \right| = \min. \quad (9)$$

The "twice truncated" reference moments should be extracted in NLO QCD from the data in the same way as the input (entering MJEM (5)) "once truncated" moments. In reality one can reconstruct from the data "twice-truncated" moments using Eq. (1) and removing, for example, first and/or last bin from the sum in Eq. (3).

Let us now check how well the optimization criterium (9) works. To this end we again perform the simple numerical test. We choose GRSV2000**NLO** parametrization at $Q^2 = 2.5\,\text{GeV}^2$ with both broken and symmetric sea scenarios. We then calculate four first "once-truncated" and four first "twice-truncated" moments defined by Eqs. (6) and (8), and substitute them in the optimization criterium (9). To find the optimal values of α and β we use the MINUIT [10] program. The results are presented[10] by Fig.2. It is seen that the optimization criterium works well for both symmetric and broken sea scenarios. The deviations of the reconstructed curves from the reference curves (input parametrization) near the boundary points are unavoidable since MJEM is correctly defined in the entire region (a,b) except for the small vicinities of boundary points (see the Appendix).

[7] For Δd_V we obtained even more impressive difference between JEM and MJEM application with $N_{\max} = 4$: $\nu|_{JEM} = 13.33\%$ while $\nu|_{MJEM} = 1.2\%$.

[8] In practice one should reconstruct these input moments from the data using Eqs. (1). The reference "twice-truncated" moments (8) should be reconstructed from the data in the same way.

[9] It can be proved by analogy with the case of the usual JEM, where Eq. (7) with $[a,b] = [0,1]$ holds (see, for example, [7]).

[10] For Δd_V we also get very good agreement between input and reconstructed curves: $\nu = 0.3\%$ and $\nu = 0.07\%$ for symmetric and broken sea scenarios, respectively.

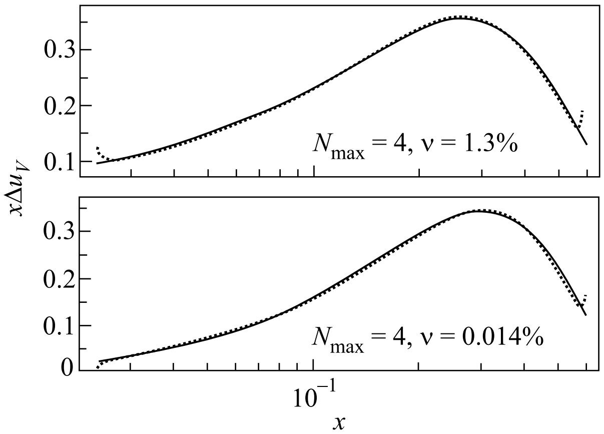

Fig.2. The results of Δu_V reconstruction for GRSV2000**NLO** parametrization for both symmetric (top) and broken sea (bottom) scenarios. Solid line corresponds to the reference curve (input parametrization). Dotted line is reconstructed with MJEM and criterium (9) inside the accessible for measurement region ([0.023,0.6] here). Optimal values of α and β are $\alpha_{\rm opt} = -0.15555$, $\beta_{\rm opt} = -0.097951$ and $\alpha_{\rm opt} = -0.209346$, $\beta_{\rm opt} = 0.153417$ for symmetric and broken sea scenarios, respectively

Fortunately, these distortions occur in very small vicinities of the boundary points, and the curves are in very good agreement in the practically entire accessible x_B region. Notice that for the procedure of extrapolation outside the accessible x_B region one just should cut off these unphysical boundary distortions (see below).

Thus, one can conclude that MJEM can be successfully applied for reconstruction of the local distributions knowing only first few truncated Mellin moments. Notice, however, that by construction MJEM reproduces the local distributions only in the accessible for measurement x_B region. The question arises: could one attempt to reconstruct the local distributions outside the accessible region (i.e. to perform extrapolation) using the obtained with MJEM distributions as an input? To this end we propose to solve the following variational task. We apply MJEM, Eq. (5), to the maximally[11)] extended x_B region $[a_{\rm min}, b_{\rm max}]$ replacing the moments $M'_{[a_{\rm min},b_{\rm max}]}[j]$ by $M'_{[a,b]}[j] + \epsilon_j$, where $\epsilon_j (j = 1 \ldots 4)$ are the free variational parameters (ϵ_j should be considered as unknown "tails" of the full moments). Then, using MINUIT program [10], one finds parameters ϵ_j requiring the minimal deviation of the reconstructed with ϵ_j curve from the input (reconstructed with criterium (9)) curve inside the accessible for measurement region $[a, b]$.

[11)]For a moment, we restrict ourselves by the x_B region $[a_{\rm min} = 10^{-4}, b_{\rm max} = 1]$ which is typical for the most known parametrizations on the quark helicity distributions.

The reconstructed in this way quantities $M'_{[a,b]}[j] + \epsilon_j$ should be compared with the reference (obtained by direct integration of the input parametrization) moments $M'_{[a_{min},b_{\rm max}]}[j]\Big|_{\rm reference}$. In ideal case (ideal reconstruction of "tails" ϵ_j) these quantities would coincide.

Let us test this variational (extrapolation) procedure by the simple numerical exercise. We choose GRSV2000**NLO** parametrization (for both broken and symmetric sea scenarios) at $Q^2 = 2.5\,{\rm GeV}^2$ as the reference one. Since the allowed [9] x_B region for this parametrization is $[10^{-4}, 1]$ we choose $[a_{\rm min}, b_{\rm max}] = [10^{-4}, 1]$, and for the truncated region $[a, b]$ we again choose the accessible for HERMES x_B region $[a, b] = [0.023, 0.6]$. Notice that performing the variational (extrapolation) procedure we cut off the boundary distortions of the curve (which enters the variational procedure as an input) obtained with MJEM and criterium (9) inside the accessible x_B region.

Results of first four moments of Δu_V reconstruction in the region $[a_{\rm min} = 10^{-4}, b_{\rm max} = 1]$ for the GRSV2000NLO parametrization for both symmetric (top) and broken sea (bottom) scenarios

n	$M'^{\rm input}_{[0.023,0.6]}$	$M'^{\rm output}_{[10^{-4},1]}$	$M'^{\rm reference}_{[10^{-4},1]}$
1	0.749	0.904	0.917
2	0.153	0.164	0.167
3	0.047	0.053	0.055
4	0.017	0.021	0.023
1	0.570	0.609	0.605
2	0.137	0.150	0.149
3	0.044	0.052	0.052
4	0.017	0.023	0.022

The results of the variational (extrapolation) procedure application are presented by Fig.3 and Table. Comparing the reconstructed curve with the input parametrization for $\Delta u_V(x)$ (see Fig.3) one can see that they are in a good agreement. First four reconstructed moments are also in a good agreement with the respective reference (obtained by direct integration of the input parametrization) moments – see Table. For Δd_V the quality of reconstruction is also very good for symmetric sea scenario and a little bit worse[12)] in the case of broken sea scenario. In any case, the reconstructed first moments (the most important for understanding of the proton spin structure) are in a good agreement with the

[12)]The point is that in the case of broken sea scenario the moments of Δd_V are very small quantities and, besides, $\Delta d_V(x)$ changes the sign at small x_B [9]. Thus, the application of proposed variational (extrapolation) procedure in this case becomes more complicated.

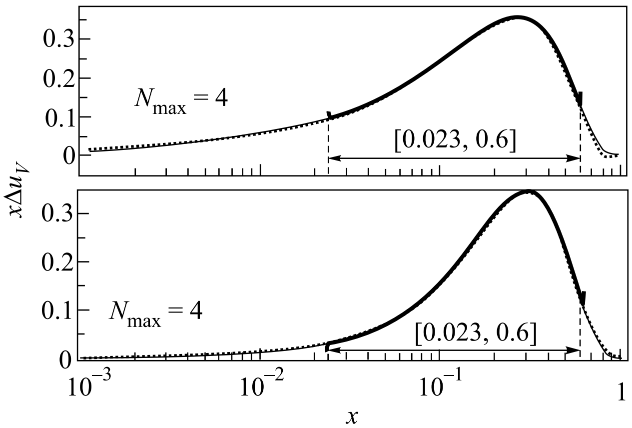

Fig.3. The results of Δu_V reconstruction in the region $[a_{\min} = 10^{-4}, b_{\max} = 1]$ for GRSV2000**NLO** parametrization for both symmetric (up) and broken sea (down) scenarios. Solid line corresponds to the reference curve (input parametrization). Dotted line corresponds to the curve reconstructed in the entire $[a_{\min} = 10^{-4}, b_{\max} = 1]$ region with the requirement of minimal deviation from the curve (bold solid line) reconstructed with MJEM and criterium (9) inside the accessible for measurement region ([0.023,0.6] here)

respective reference moments of both Δu_V and Δd_V distributions.

Thus, all numerical tests confirm that the proposed modification of the Jacobi polynomial expansion method, MJEM, allows to reconstruct with a high precision the quark helicity distributions in the accessible for measurement x_B region. We consider this as the main result of the paper. Besides, the numerical calculations encourage one that the proposed variational (extrapolation) procedure based on MJEM could become the reliable extrapolation procedure. Certainly, here the careful additional investigations are necessary.

First of all, we plan to apply the proposed method to HERMES data on the pion production with both proton and deutron targets. As it was shown above, MJEM (rather than usual JEM) should be applied for reconstruction of the local in x_B distributions from NLO QCD extracted moments in all modern semi-inclusive DIS experiments (such as COMPASS experiment) with the restricted accessible x_B region, and it becomes absolutely necessary for the rather narrow HERMES x_B region. To extract the valence quark helicity distributions in NLO QCD with the proposed method we will use so-called "difference asymmetries" (on essential advantages of these asymmetries see [4] and references therein) which now are constructed by HERMES. At present, the extended paper with the simulations corresponding to HERMES kinematics is in preparation.

The authors are grateful to R. Bertini, O. Denisov, A. Korzenev, V. Krivokhizhin, E. Kuraev, A. Maggiora, A. Nagaytsev, A. Olshevsky, G. Piragino, G. Pontecorvo, I. Savin, A. Sidorov and O. Teryaev, for fruitful discussions. Two of us (O.S., O.I.) thanks the RFBR grant 05-02-17748.

Appendix. The JEM is the expansion of the x-dependent function (structure function or quark density) in the series over Jacobi polynomials $\Theta_n^{(\alpha,\beta)}(x)$ orthogonal with weight $\omega^{(\alpha,\beta)}(x) = x^\beta(1-x)^\alpha$ (see [5–7] for details):

$$F(x) \simeq F_{N_{\max}}(x) = \omega^{(\alpha,\beta)} \sum_{k=0}^{N_{\max}} \Theta_k^{(\alpha,\beta)}(x) \times$$

$$\times \sum_{j=0}^{k} c_{kj}^{(\alpha,\beta)} M(j+1), \qquad \text{(A.1)}$$

where $M[j] = \int_0^1 dx\, x^{j-1} F(x)$ and

$$\int_0^1 dx \omega^{(\alpha,\beta)}(x) \Theta_n^{(\alpha,\beta)}(x) \Theta_m^{(\alpha,\beta)}(x) = \delta_{nm}. \qquad \text{(A.2)}$$

The details on the Jacobi polynomials

$$\Theta_k^{(\alpha,\beta)}(x) = \sum_{j=0}^{k} c_{kj}^{(\alpha,\beta)} x^j \qquad \text{(A.3)}$$

can be found in refs. [5] and [6]. Expansion (A.1) becomes exact when $N_{\max} \to \infty$. However, in practice one truncates the series (A.1) living in the expansion only finite number of moments $N_{\max}$. The experience shows [7] that JEM produces good results (for entire x_B region) even with the small number $N_{\max}$.

The idea of modified expansion is to reexpand $F(x)$ in the series over the truncated moments $M'_{[ab]}[j]$ given by Eq. (6), performing the rescaling $x \to a + (b-a)x$ which compress the entire region $[0,1]$ to the truncated region $[a,b]$. To this end let us apply the following ansatz[13)]

$$F(x) = \left(\frac{x-a}{b-a}\right)^\beta \left(1 - \frac{x-a}{b-a}\right)^\alpha \times$$

$$\times \sum_{n=0}^{\infty} \tilde{f}_n \Theta_n^{(\alpha,\beta)} \left(\frac{x-a}{b-a}\right) \qquad \text{(A.4)}$$

and try to find the coefficients $\tilde{f}_n$. Multiplying both parts of Eq. (A.4) by $\Theta_k^{(\alpha,\beta)}((x-a)/(b-a))$, integrating

13)Notice that ansatz (A.4) (as well as the expansion Eq. (5) itself) is correctly defined inside the entire region (a,b) except for the small vicinities of boundary points (absolutely the same situation holds for the usual JEM, Eq. (A.1), applied to the quark distributions in the region $(0,1)$). In practice, the respective boundary distortions are just cut off when one performs the extrapolation procedure.

over x in the limits $[a, b]$ and performing the replacement $t = (x - a)/(b - a)$, one gets

$$\int_a^b dx\, F(x)\Theta_k^{(\alpha,\beta)}\left(\frac{x-a}{b-a}\right) =$$

$$= (b-a)\sum_{n=0}^{\infty}\tilde{f}_n\int_0^1 dt\, t^\beta(1-t)^\alpha\Theta_n^{(\alpha,\beta)}(t)\Theta_k^{(\alpha,\beta)}(t),$$

so that with the orthogonality condition Eq. (A.2) one obtains

$$\tilde{f}_n = (b-a)^{-1}\int_a^b dx\, F(x)\Theta_n^{(\alpha,\beta)}\left(\frac{x-a}{b-a}\right). \quad \text{(A.5)}$$

Substituting Eq. (A.5) in the expansion (A.4), and using Eq. (A.3) one eventually arrives at Eq. (5) (with $N_{\max} \to \infty$) of the main text.

1. A. Airapetyan et al. (HERMES collab.), Phys. Rev. D **71**, 012003 (2005)
2. G. Baum et al. (COMPASS collab.), *COMPASS: A proposal for a common muon and proton apparatus for structure and spectroscopy*, CERN-SPSLC-96-14, 1996.
3. A. N. Sissakian, O. Yu. Shevchenko, and O. N. Ivanov, Phys. Rev. D **68**, 031502 (2003).
4. A. N. Sissakian, O. Yu. Shevchenko, and O. N. Ivanov, Phys. Rev. D **70**, 074032 (2004).
5. G. Parisi and N. Sourlas, Nucl. Phys. **B151**, 421 (1979).
6. I. S. Barker, C. S. Langensiepen, and G. Shaw, Nucl. Phys. B **186**, 61 (1981); CERN-TH-2988.
7. V. G. Krivokhizhin et al., Z. Phys. C **36**, 51 (1987); JINR-E2-86-564.
8. E. Leader, A. V. Sidorov, and D. B. Stamenov, Int. J. Mod. Phys. A**13**, 5573 (1998).
9. M. Gluck, E. Reya, M. Stratmann, and W. Vogelsang, Phys. Rev. D **63**, 094005 (2001); hep-ph/0011215.
10. F. James and M. Roos, Comput. Phys. Commun. **10**, 343 (1975).