NLO QCD procedure of the semi-inclusive deep inelastic scattering data analysis with respect to the light quark polarized sea

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The semi-inclusive deep inelastic scattering (SIDIS) process is considered. A theoretical procedure is proposed allowing the direct extraction from the SIDIS data of the first moments of the polarized valence distributions and of the first moment difference of the light sea quark polarized distributions in the next to leading QCD order. The validity of the procedure is confirmed by the respective simulations.

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I. INTRODUCTION

The extraction of the polarized quark and gluon densities is one of the main tasks of the semi-inclusive deep inelastic scattering (SIDIS) experiments with the polarized beam and target. Of special importance for the modern SIDIS experiments are the questions of strange quark and gluon contributions to the nucleon spin, and also the sea quark share as well as the possibility of broken sea scenario. Indeed, it is known [1] that the unpolarized sea of light quarks is asymmetric, so that the first moments of the unpolarized $\bar{u}$ and $\bar{d}$ quark densities do not equal each other:

$$\int_0^1 dx[\bar{d}(x) - \bar{u}(x)] = 0.147 \pm 0.039 \neq 0.$$

The question arises: Does the analogous situation occur in the polarized case, i.e., are the polarized density $\Delta\bar{u}$ and its first moment[1] $\Delta_1\bar{u} \equiv \int_0^1 dx\,\Delta\bar{u}$, respectively, equal to $\Delta\bar{d}$ and $\Delta_1\bar{d}$ or not?

In Ref. [2], the possibility of a broken sea scenario was analyzed, considering the results of SIDIS experiments on Δq with respect to their consistence with the Bjorken sum rule (BSR) predictions. It was shown [2] that, using the results of [3] on the valence quark distributions $\Delta_1 q_V$ obtained in the leading (LO) QCD order, one can immediately estimate the first moment difference of the u and d sea quark polarized distributions:

$$\Delta_1\bar{u} - \Delta_1\bar{d} = 0.235 \pm 0.097. \quad (1)$$

At the same time, it was stressed in [2] that this is just a speculation, and, to get the reliable results on Δq from the data obtained at the relatively small average $Q^2 = 2.5\ \mathrm{GeV}^2$ [3], one should apply next to leading (NLO) QCD analysis. The main goal of this paper is to present such a NLO QCD procedure allowing the *direct* extraction of the quantity $\Delta_1\bar{u} - \Delta_1\bar{d}$ from the SIDIS data.

It is known that the description of semi-inclusive DIS processes turns out to be much more complicated in comparison to the inclusive polarized DIS. First, the fragmentation functions are involved, for which the information is limited.[2] Second, the extraction of the quark densities in NLO QCD order turns out to be rather difficult, since the double convolution products are involved. So, to achieve a reliable description of the SIDIS data it is very desirable, on the one hand, to exclude from consideration the fragmentation functions, whenever possible, and, on the other hand, to try to simplify the NLO considerations as much as possible.

It is well known (see, for example, [4] and references therein) that within LO QCD approximation one can completely exclude the fragmentation functions from the expressions for the valence quark polarized distributions Δq_V through experimentally measured asymmetries. To this end, instead of the usual virtual photon asymmetry $A^h_{\gamma N} \equiv A^h_{1N}$ [which is expressed in terms of the directly measured asymmetry $A^h_{\mathrm{exp}} = (n^h_{\downarrow\uparrow} - n^h_{\uparrow\uparrow})/(n^h_{\downarrow\uparrow} + n^h_{\uparrow\uparrow})$ as $A^h_{1N} = (P_B P_T f D)^{-1} A^h_{\mathrm{exp}}$], one has to measure so-called "difference asymmetry" $A^{h-\bar{h}}_N$ which is expressed in terms of the respective counting rates[3] as

$$A^{h-\bar{h}}_N(x, Q^2; z) = \frac{1}{P_B P_T f D}\,\frac{(n^h_{\uparrow\downarrow} - n^{\bar{h}}_{\uparrow\downarrow}) - (n^h_{\uparrow\uparrow} - n^{\bar{h}}_{\uparrow\uparrow})}{(n^h_{\uparrow\downarrow} - n^{\bar{h}}_{\uparrow\downarrow}) + (n^h_{\uparrow\uparrow} - n^{\bar{h}}_{\uparrow\uparrow})}, \quad (2)$$

where the event densities $n^h_{\uparrow\downarrow(\uparrow\uparrow)} = dN^h_{\uparrow\downarrow(\uparrow\uparrow)}/dz$, i.e., $n^h_{\uparrow\downarrow(\uparrow\uparrow)}dz$, are the numbers of events for antiparallel (parallel) orientations of incoming lepton and target nucleon spins for the hadrons of type h registered in the interval dz. Quantities P_B and P_T, f and D are the beam and target polarizations, dilution and depolarization factors, respectively (for details on these quantities see, for example, [5,6] and references therein). Then, the LO theoretical

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[1]From now on, the notation $\Delta_1 q \equiv \int_0^1 dx\,\Delta q$ will be used to distinguish the local in Bjorken x polarized quark densities $\Delta q(x)$ and their first moments.

[2]For discussion of this subject see, for example, [4] and references therein.

[3]As usual, one should realize the quantities $n^h_{\uparrow\uparrow}(\uparrow\downarrow)$ entering Eq. (2) not as the pure event densities but as the event densities multiplied by the respective luminosities which, in general, do not cancel out (see the appendix).

expressions for the difference asymmetries look like (see, for example, COMPASS proposal [7], Appendix A)

$$A_p^{\pi^+-\pi^-} = \frac{4\Delta u_V - \Delta d_V}{4u_V - d_V}, \qquad A_d^{\pi^+-\pi^-} = \frac{\Delta u_V + \Delta d_V}{u_V + d_V},$$
$$A_n^{\pi^+-\pi^-} = \frac{4\Delta d_V - \Delta u_V}{4d_V - u_V}, \qquad A_p^{K^+-K^-} = \frac{\Delta u_V}{u_V}, \tag{3}$$
$$A_d^{K^+-K^-} = A_d^{\pi^+-\pi^-},$$

i.e., on the one hand, they contain only valence quark polarized densities, and, on the other hand, have the remarkable property to be free of any fragmentation functions.

II. THEORETICAL BASIS OF THE PROCEDURE

Let us start NLO consideration with the known [4,8–10] theoretical expressions for the difference asymmetries

$$A_N^{h-\bar{h}}(x, Q^2; z) = \frac{g_1^{N/h} - g_1^{N/\bar{h}}}{\tilde{F}_1^{N/h} - \tilde{F}_1^{N/\bar{h}}} \qquad (N = p, n, d), \tag{4}$$

where the semi-inclusive analogs of the structure functions g_1^N and F_1^N, functions $g_1^{N/h}$ and $\tilde{F}_1^{N/h}$, are related to the respective polarized and unpolarized semi-inclusive differential cross sections as follows [9]:

$$\frac{d^3\sigma^h_{N\downarrow\downarrow}}{dxdydz} - \frac{d^3\sigma^h_{N\uparrow\uparrow}}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}(2-y)g_1^{N/h}(x, z, Q^2), \tag{5}$$

$$\frac{d^3\sigma^h_N}{dxdydz} = \frac{2\pi\alpha^2}{Q^2}\,\frac{1+(1-y)^2}{y}\,2\tilde{F}_1^{N/h}(x, z, Q^2). \tag{6}$$

The semi-inclusive structure functions $g_1^{p(n)/h}$ are given in NLO by

$$g_1^{p/h} = \sum_{q,\bar{q}} e_q^2 \Delta q \Big[1 + \otimes \frac{\alpha_s}{2\pi}\delta C_{qq} \otimes\Big] D_q^h + \Big(\sum_{q,\bar{q}} e_q^2 \Delta q\Big) \otimes \frac{\alpha_s}{2\pi}\delta C_{gq} \otimes D_g^h + \Delta g \otimes \frac{\alpha_s}{2\pi}\delta C_{qg} \otimes \Big(\sum_{q,\bar{q}} e_q^2 D_q^h\Big), \tag{7}$$

$$g_1^{n/h} = g_1^{p/h}|_{u\leftrightarrow d}, \tag{8}$$

where the double convolution product is defined as

$$[\Delta q \otimes \delta C \otimes D](x, z) \equiv \int_{\mathcal{D}}\int \frac{dx'}{x'} \times \frac{dz'}{z'}\,\Delta q\Big(\frac{x}{x'}\Big)\delta C(x', z')D\Big(\frac{z}{z'}\Big). \tag{9}$$

The respective expressions for $2\tilde{F}_1^{p(n)/h}$ have the form analogous to Eq. (7) with the substitution $\Delta q \to q$, $\delta C \to \tilde{C}$. The expressions for the Wilson coefficients $\delta C_{qq(qg,gq)}$ and $\tilde{C}_{qq(qg,gq)} \equiv C^1_{qq(qg,gq)} + 2(1-y)/[1+(1-y)^2]C^L_{qq(qg,gq)}$ can be found, for example, in [9], Appendix C.

It is remarkable that, due to the properties of the fragmentation functions,

$$D_1 \equiv D_u^{\pi^+} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+} = D_d^{\pi^-},$$
$$D_2 \equiv D_{\bar{d}}^{\pi^+} = D_d^{\pi^-} = D_u^{\pi^-} = D_{\bar{u}}^{\pi^+}, \tag{10}$$

in the differences $g_1^{p/\pi^+} - g_1^{p/\pi^-}$ and $\tilde{F}_1^{p/\pi^+} - \tilde{F}_1^{p/\pi^-}$ (and, therefore, in the asymmetries $A_p^{\pi^+-\pi^-}$ and $A_d^{\pi^+-\pi^-}$), only the contributions containing the Wilson coefficients δC_{qq} and $\tilde{C}_{qq}$ survive. However, even then the system of double integral equations

$$A_p^{\pi^+-\pi^-}(x, Q^2; z) = \frac{(4\Delta u_V - \Delta d_V)[1 + \otimes\alpha_s/(2\pi)\delta C_{qq}\otimes](D_1 - D_2)}{(4u_V - d_V)[1 + \otimes\alpha_s/(2\pi)C_{qq}\otimes](D_1 - D_2)}, \qquad A_n^{\pi^+-\pi^-}(x, Q^2; z) = A_p^{\pi^+-\pi^-}(x, Q^2; z)|_{u_V\leftrightarrow d_V},$$

proposed by Christova and Leader [4], is rather difficult to solve directly[4] with respect to the local quantities $\Delta u_V(x, Q^2)$ and $\Delta d_V(x, Q^2)$. Besides, the range of integration $\mathcal{D}$ used in [4] has a very complicated form, namely,

$$\frac{x}{x + (1-x)z} \le x' \le 1 \quad \text{with } z \le z' \le 1,$$

if $x + (1-x)z \ge 1$, and, additionally, range

$$x \le x' \le x/[x + (1-x)]z,$$

with $x(1-x')/[x'(1-x)] \le z' \le 1$ if $x + (1-x)z \le 1$. Such an enormous complication of the convolution integral range occurs if one introduces (to take into account the target fragmentation contributions[5] and to exclude the cross-section singularity problem at $z_h = 0$) a new hadron kinematic variable $z = E_h/E_N(1-x)$ (γp c.m. frame) instead of the usual semi-inclusive variable $z_h = (Ph)/(Pq) = (\text{lab system})E_h/E_\gamma$. However, both problems compelling us to introduce z, instead of z_h, can be

[4]It seems that the only real possibility to extract polarized distributions from the data in NLO QCD order is to use some proper fit. At the same time, it is known that such a procedure is rather ambiguous since the sea distributions are very sensitive to the choice of initial functions for Δq parametrization (and, especially, for $\Delta\bar{q}$ parametrization).

[5]Then, one should also add the target fragmentation contributions to the right-hand side of Eq. (7).

avoided (see, for example [9,10]) if one, just to neglect the target fragmentation, applies a proper kinematical cut $Z < z_h \leq 1$, i.e., properly restricts the kinematical region covered by the final state hadrons.[6] Then, one can safely use, instead of z, the usual variable z_h, which at once makes the integration range $\mathcal{D}$ in the double convolution product (9) very simple: $x \leq x' \leq 1, z_h \leq z' \leq 1$. Note that in applying the kinematical cut it is much more convenient to deal with the total numbers of events (multiplied by the respective luminosities—see footnote 3 and the appendix)

$$N^h_{\uparrow\downarrow(\uparrow\uparrow)}(x, Q^2)|_Z = \int_Z^1 dz_h n^h_{\uparrow\downarrow(\uparrow\uparrow)}(x, Q^2; z_h), \tag{11}$$

within the entire interval $Z \leq z_h \leq 1$ and the respective integral difference asymmetries[7]

$$A_N^{h-\bar{h}}(x, Q^2)|_Z = \frac{1}{P_B P_T f D} \frac{(N^h_{\uparrow\downarrow} - N^{\bar{h}}_{\uparrow\downarrow}) - (N^h_{\uparrow\uparrow} - N^{\bar{h}}_{\uparrow\uparrow})}{(N^h_{\uparrow\downarrow} - N^{\bar{h}}_{\uparrow\downarrow}) + (N^h_{\uparrow\uparrow} - N^{\bar{h}}_{\uparrow\uparrow})}\Bigg|_Z \tag{12}$$

$$= \frac{\int_Z^1 dz_h (g_1^{N/h} - g_1^{N/\bar{h}})}{\int_Z^1 dz_h (\tilde{F}_1^{N/h} - \tilde{F}_1^{N/\bar{h}})} \qquad (N = p, n, d), \tag{13}$$

than with the local in z_h quantities $n_{\uparrow\downarrow(\uparrow\uparrow)}(x, Q^2; z_h)$ and $A_N^{h-\bar{h}}(x, Q^2; z_h)$. So, the expressions for the proton and deutron integral difference asymmetries assume the form[8]

$$A_p^{\pi^+-\pi^-}(x, Q^2)|_Z = \frac{(4\Delta u_V - \Delta d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \delta C_{qq} \otimes](D_1 - D_2)}{(4u_V - d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2)}, \tag{14}$$

$$A_d^{\pi^+-\pi^-}(x, Q^2)|_Z = \frac{(\Delta u_V + \Delta d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \delta C_{qq} \otimes](D_1 - D_2)}{(u_V + d_V) \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2)}, \tag{15}$$

where the double convolution product reads

$$[\Delta q \otimes \delta C \otimes D](x, z_h) = \int_x^1 \frac{dx'}{x'} \times \int_{z_h}^1 \frac{dz'}{z'} \Delta q\left(\frac{x}{x'}\right) C(x', z') D\left(\frac{z_h}{z'}\right). \tag{16}$$

With such a simple convolution region, one can apply the well-known property of the nth Melin moments $M^n(f) \equiv \int_0^1 dx x^{n-1} f(x)$ to split the convolution product into a simple product of the Melin moments of the respective functions:

$$M^n[A \otimes B] \equiv \int_0^1 dx x^{n-1} \int_x^1 \frac{dy}{y} A\left(\frac{x}{y}\right) B(y) = M^n(A) M^n(B). \tag{17}$$

So, applying the first moment to the difference asymmetries $A_p^{\pi^+-\pi^-}(x, Q^2)|_Z$ and $A_d^{\pi^+-\pi^-}(x, Q^2)|_Z$, given by (14) and (15), one gets a system of two equations for $\Delta_1 u_V \equiv \int_0^1 dx \Delta u_V$ and $\Delta_1 d_V \equiv \int_0^1 dx \Delta d_V$:

$$(4\Delta_1 u_V - \Delta_1 d_V)(L_1 - L_2) = \mathcal{A}_p^{\exp}, \tag{18}$$

$$(\Delta_1 u_V + \Delta_1 d_V)(L_1 - L_2) = \mathcal{A}_d^{\exp}, \tag{19}$$

with the solution

$$\Delta_1 u_V = \frac{1}{5} \frac{\mathcal{A}_p^{\exp} + \mathcal{A}_d^{\exp}}{L_1 - L_2}; \qquad \Delta_1 d_V = \frac{1}{5} \frac{4\mathcal{A}_d^{\exp} - \mathcal{A}_p^{\exp}}{L_1 - L_2}. \tag{20}$$

Here we introduce the notation

$$\mathcal{A}_p^{\exp} \equiv \int_0^1 dx A_p^{\pi^+-\pi^-}|_Z (4u_V - d_V) \int_Z^1 dz_h \times \left[1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes\right](D_1 - D_2),$$
$$\mathcal{A}_d^{\exp} \equiv \int_0^1 dx A_d^{\pi^+-\pi^-}|_Z (u_V + d_V) \int_Z^1 dz_h \times \left[1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes\right](D_1 - D_2), \tag{21}$$

$$L_1 \equiv L_u^{\pi^+} = L_{\bar{u}}^{\pi^-} = L_{\bar{d}}^{\pi^+} = L_d^{\pi^-}, \qquad L_2 \equiv L_d^{\pi^+} = L_{\bar{d}}^{\pi^-} = L_u^{\pi^-} = L_{\bar{u}}^{\pi^+}, \tag{22}$$

where

[6] This is just what was done in the HERMES and SMC experiments, where the applied kinematical cut was $z_h > Z = 0.2$.

[7] Namely, the integral spin symmetries $A_{1N}^h = \int_Z^1 dz_h g_{1N}^h / \int_Z^1 dz_h \tilde{F}_{1N}^h$ were measured by SMC and HERMES experiments (see [3,5,6] and also [10]).

[8] Here one uses the equality $g_1^{d/h} \simeq g_1^{p/h} + g_1^{n/h}$ which is valid up to corrections of order $O(\omega_D)$, where $\omega_D = 0.05 \pm 0.01$ is the probability to find a deutron in the D state.

$$L_q^h \equiv \int_Z^1 dz_h \left[D_q^h(z_h) + \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz'}{z'} \Delta_1 C(z') D_q^h\left(\frac{z_h}{z'}\right) \right], \quad (23)$$

with the coefficient $\Delta_1 C(z) \equiv \int_0^1 dx \delta C_{qq}(x, z)$.

Now one may do the last step to get the NLO QCD equation for the extraction of the quantity $\Delta_1\bar{u} - \Delta_1\bar{d}$ in which we are interested. Namely, one can use the equivalent of BSR (see [2] and references therein for details) rewritten in terms of the valence and sea distributions:

$$\Delta_1\bar{u} - \Delta_1\bar{d} = \frac{1}{2} \left| \frac{g_A}{g_V} \right| - \frac{1}{2}(\Delta_1 u_V - \Delta_1 d_V). \quad (24)$$

Using Eqs. (20)–(24), one gets a simple expression for the quantity $\Delta_1\bar{u} - \Delta_1\bar{d} \equiv \int_0^1 dx[\Delta\bar{u}(x, Q^2) - \Delta\bar{d}(x, Q^2)]$ in terms of experimentally measured quantities, which is valid in NLO QCD :

$$\Delta_1\bar{u} - \Delta_1\bar{d} = \frac{1}{2} \left| \frac{g_A}{g_V} \right| - \frac{2\mathcal{A}_p^{\exp} - 3\mathcal{A}_d^{\exp}}{10(L_1 - L_2)}. \quad (25)$$

It is easy to see that all the quantities present in the right-hand side of (25), with the exception of the two difference asymmetries $A_p^{\pi^+-\pi^-}|_Z$ and $A_d^{\pi^+-\pi^-}|_Z$ (entering $\mathcal{A}_p^{\exp}$ and $\mathcal{A}_d^{\exp}$, respectively) can be extracted from unpolarized[9] semi-inclusive data and can, thus, be considered here as a known input. So, the only quantities that have to be measured in polarized semi-inclusive DIS are the difference asymmetries $A_p^{\pi^+-\pi^-}|_Z$ and $A_d^{\pi^+-\pi^-}|_Z$ which, in turn, are just simple combinations of the directly measured counting rates.

III. ERRORS ON THE DIFFERENCE ASYMMETRIES

To check the validity of the proposed procedure, let us perform the respective simulations. To this end, one can use the polarized event generator PEPSI [13]. First of all, let us clarify the very important issue of the errors on the difference asymmetries. At first sight it could seem that the difference asymmetries suffer from the much larger errors in comparison to the usual asymmetries. Indeed, the approximate formula for the estimation of the statistical error on the difference asymmetries reads [see Eqs. (A15) and (A16) in the appendix]

$$\delta(A_{p(n,d)}^{\pi^+-\pi^-}) \sim \frac{\sqrt{N^{\pi^+} + N^{\pi^-}}}{N^{\pi^+} - N^{\pi^-}}. \quad (26)$$

So one can see that, contrary to the usual asymmetries, the difference of the total (for both parallel and antiparallel beam and target polarizations) counting rates for π^+ and π^- production, $N^{\pi^+} \equiv N_{\uparrow\downarrow}^{\pi^+} + N_{\uparrow\uparrow}^{\pi^+}$ and $N^{\pi^-} \equiv N_{\uparrow\downarrow}^{\pi^-} + N_{\uparrow\uparrow}^{\pi^-}$, occurs in the denominator of the expression for $\delta A_{p(n,d)}^{\pi^+-\pi^-}$, and it could lead to the large statistical errors on this asymmetry. Such a situation indeed occurs for the neutron target. However, fortunately, for the proton and deutron targets there is an important circumstance which rescues the situation.

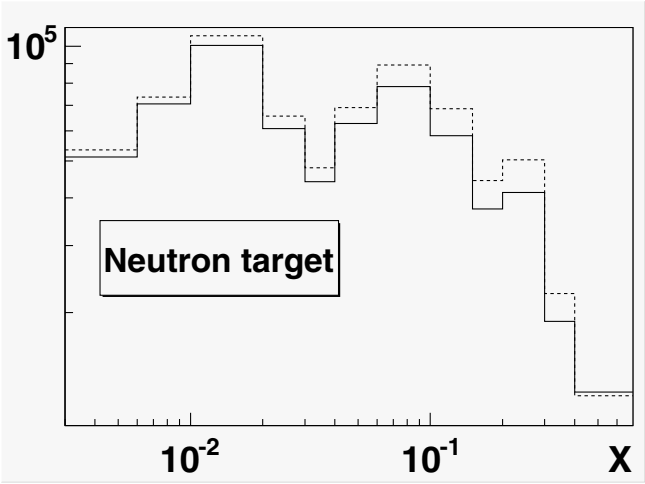

FIG. 1. N^{π^+} and N^{π^-} obtained with PEPSI for neutron target. The dashed and solid lines correspond to π^+, and π^- productions, respectively.

The point is that, unlike the neutron target case (see Fig. 1), the production of positive pions on the proton target (see Fig. 2), in the widest region in Bjorken x,[10] essentially exceeds the production of negative pions, whereas for the deuteron target the difference in π^+ and π^- production is not so drastic but is still essential (see Fig. 3).

It is of importance that, though the histograms in Figs. 1–3 are obtained using the PEPSI event generator (just summing the events with parallel and antiparallel beam and target polarizations), they represent the general, well-established experimental [14] picture (see Sec. 4.3 and Fig. 10 i n Ref. [14]), and peculiar to all known SIDIS event generators.[11] To be sure that concerning the strong asymmetry between $N_{p,d}^{\pi^+}$ and $N_{p,d}^{\pi^-}$ the PEPSI event generator

[9]The spin-independent fragmentation functions D can be taken either from independent measurements of e^+e^- annihilation into hadrons [11] or from the hadron production in unpolarized DIS [12].

[10]This occurs everywhere except for the vicinity of $x_B = 0$, where the ratio N^{π^+}/N^{π^-} approaches unity owing to the dominant contribution of the sea quarks [14]. However, let us stress that for the statistical error only the difference $N^{\pi^+} - N^{\pi^-}$ is of importance, and, at the statistics available to HERMES and COMPASS, $N^{\pi^+} - N^{\pi^-}$ is not a small quantity even in the vicinity of the minimal value $x_B = 0.003$ accessible to measurement; see below.

[11]Absolutely the same histograms for $N_{n,p,d}^{\pi^+}$ and $N_{n,p,d}^{\pi^-}$ are reproduced using, for example, the unpolarized event generator LEPTO [15].

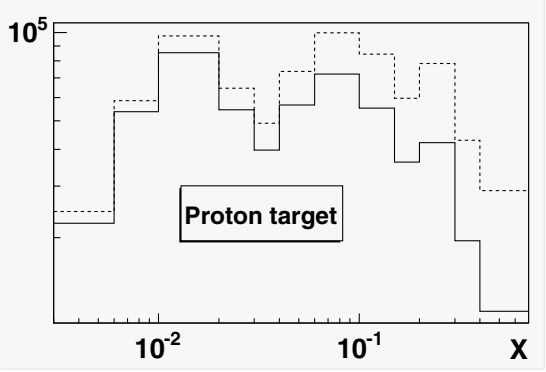

FIG. 2. N^{π^+} and N^{π^-} obtained with PEPSI for proton target. The dashed and solid lines correspond to π^+, and π^- productions, respectively.

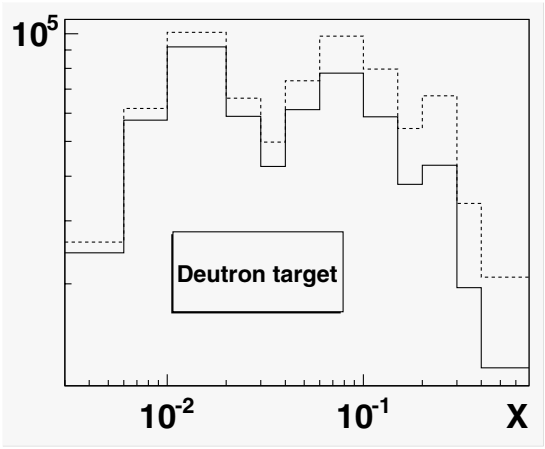

FIG. 3. N^{π^+} and N^{π^-} obtained with PEPSI for deutron target. The dashed and solid lines correspond to π^+, and π^- productions, respectively.

we deal with strictly follows the real [14] physical picture, one can also compare what we have obtained with PEPSI, Fig. 4, for the ratio of $N^{\pi^+}_{p,d}$ and $N^{\pi^-}_{p,d}$ with Fig. 10 in Ref. [14].

Thus, the differences between the total counting rates $N^{\pi^+} \equiv N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow}$ and $N^{\pi^-} \equiv N^{\pi^-}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\uparrow}$ are not small quantities[12] for both proton and deutron targets, and, besides, increase with the statistics. As a result, the respective statistical errors turn out to be quite acceptable.

Let us illustrate this statement by a simple LO example. Using the GRSV2000**LO** (broken sea) [16] parametrization entering the PEPSI event generator as the input, we generate 3×10^6 DIS events with $E_\mu = 160\ \mathrm{GeV}^2$ (COMPASS kinematics). We then construct the "experimental" asymmetries together with their statistical errors using Eqs. (A1) and (A7) from the appendix, respectively. These simulated asymmetries are compared with the theoretical ones given by Eq. (3)—see Fig. 5. One can see that the errors on the simulated asymmetries are quite acceptable and that the simulated and theoretical asymmetries are in good agreement within the errors. Furthermore, it is seen from Fig. 6 that the extracted valence[13] distributions are also in good accordance with the respective input parametrizations.

[12]Though in the vicinity $x_B = 0$ the ratio N^{π^+}/N^{π^-} approaches unity (see Fig. 4), with the applied statistics 3×10^6 DIS events (absolutely real for HERMES and COMPASS, see below), the difference $N^{\pi^+} - N^{\pi^-}$ entering the error significantly differs from zero even near the minimal value $x_B = 0.003$ accessible to measurement. Indeed, in the first bin $0.003 < x < 0.006$, the PEPSI event generator gives for the proton target $N^{\pi^+}/N^{\pi^-} = 1.085$ while $N^{\pi^+} - N^{\pi^-} = 2985$, and for deutron target $N^{\pi^+}/N^{\pi^-} = 1.053$ while $N^{\pi^+} - N^{\pi^-} = 2020$.

[13]Namely, the valence distributions are essential for what follows; see below.

IV. TESTING OF THE NLO QCD EXTRACTION PROCEDURE

A. Broken sea scenario

To perform the NLO QCD analysis, we first choose[14] the GRSV2000**NLO** (broken sea) [16] parametrization as an input. The conditions of simulations are presented in Table I and correspond to HERMES and[15] SMC (COMPASS) kinematics. Let us stress that all the cuts in Table I are the standard cuts applied[16] by SMC, HERMES, and COMPASS. The statistics 3×10^6 in Table I is the total number of DIS events for both proton and deutron target and for both longitudinal polarizations. Since the statistical error on the pion difference asymmetry depends on N^{π^+} and N^{π^-}, one needs to know the respective semi-inclusive statistics—the total (for all available x_B) number of pions $N^{\pi^+}_{\mathrm{tot}}$ and $N^{\pi^-}_{\mathrm{tot}}$ corresponding to 1.5×10^6 DIS events for each target. With all the cuts indicated in Table I, the PEPSI event generator gives

$$N^{\pi^+}_{\mathrm{tot}}|_{\mathrm{proton}} = 551281, \qquad N^{\pi^-}_{\mathrm{tot}}|_{\mathrm{proton}} = 358654;$$
$$N^{\pi^+}_{\mathrm{tot}}|_{\mathrm{deutron}} = 526\,747, \qquad N^{\pi^-}_{\mathrm{tot}}|_{\mathrm{deutron}} = 383\,826,$$

for $E_l = 27$ GeV, while

[14]Note that at present the broken sea scenario is argued as the most probable one [16] [see also the discussion on this subject at the beginning of this paper and Eq. (1)].

[15]Since the COMPASS muon beam energy 160 GeV is almost the same as that of SMC, 190 GeV, the COMPASS low x_B boundary achieved at the asymmetry measurements also should be about the same as the SMC 0.003.

[16]For example, the important cut on invariant mass $W^2 > 10\ \mathrm{GeV}^2$ is applied by these collaborations to exclude the events coming from the resonance region.

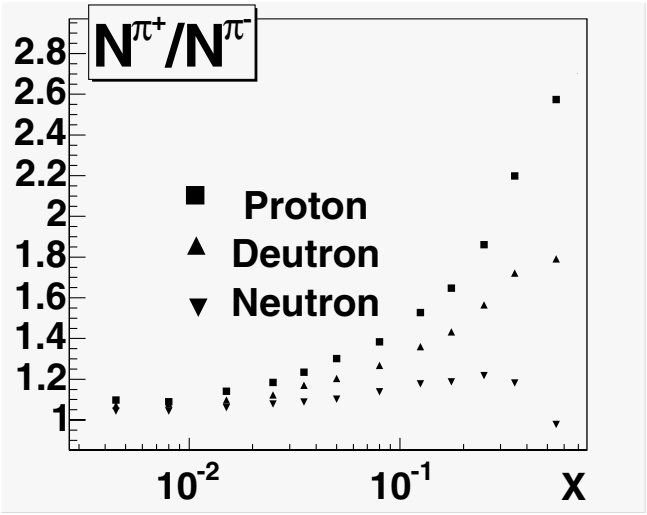

FIG. 4. Ratios of $N^{\pi^+} \equiv N^{\pi^+}_{\downarrow\uparrow} + N^{\pi^+}_{\uparrow\uparrow}$ and $N^{\pi^-} \equiv N^{\pi^-}_{\downarrow\uparrow} + N^{\pi^-}_{\uparrow\uparrow}$ obtained with the polarized event generator PEPSI for the different targets. The picture is in good agreement with the respective EMC result (Fig. 10 in Ref. [14].)

$$N^{\pi^+}_{\rm tot}|_{\rm proton} = 582\,913, \qquad N^{\pi^-}_{\rm tot}|_{\rm proton} = 420\,709;$$
$$N^{\pi^+}_{\rm tot}|_{\rm deutron} = 559\,494, \qquad N^{\pi^-}_{\rm tot}|_{\rm deutron} = 447\,599,$$

for $E_l = 160$ GeV. It is of importance that these numbers are absolutely realistic for HERMES (in 2000 HERMES already achieved for deutron target $N^{\pi^+}_{\rm tot} = 493\,492$, $N^{\pi^-}_{\rm tot} = 402\,479$—see Table 5.4 in Ref. [17]) and even much less than expected by COMPASS (see COMPASS proposal [7], p. 90).

To extract the quantities $\Delta_1 u_V$, $\Delta_1 d_V$, and, eventually, $\Delta_1\bar{u} - \Delta_1\bar{d}$ from the simulated asymmetries, one should first construct the difference asymmetries together with

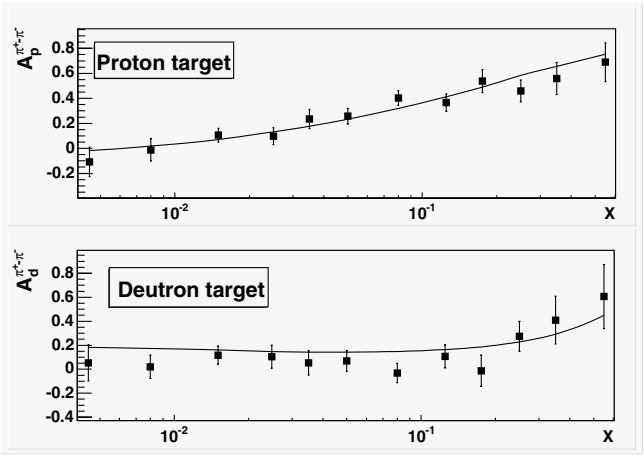

FIG. 5. Simulated (squares) and theoretical pion difference asymmetries for proton and deutron targets. The solid lines correspond to the theoretical asymmetries obtained from Eq. (3) with GRSV2000**LO** (broken sea) parametrizations for the valence distributions.

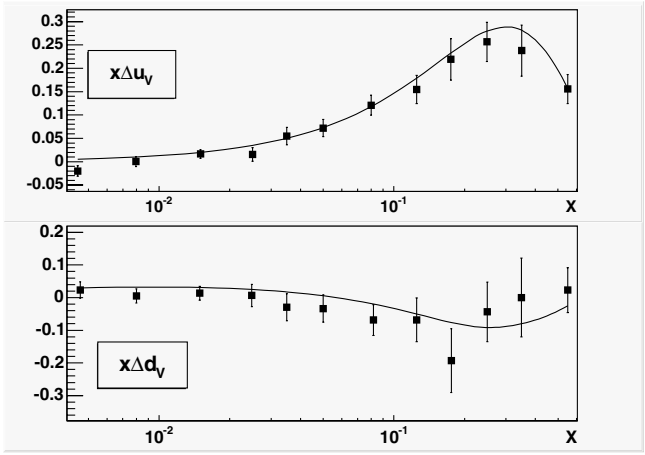

FIG. 6. The reconstructed polarized valence distributions (squares). The solid lines correspond to the respective GRSV2000**LO** (broken sea) parametrizations.

their statistical errors using Eqs. (A1) and (A7) from the appendix, and then calculate the quantities $\mathcal{A}^{\rm exp}_{p(d)}$ and $L_1 - L_2$ entering Eqs. (20) and (25). For $\mathcal{A}^{\rm exp}_p$ one should use, instead of integral formula (21), the equation (and analogously for $\mathcal{A}^{\rm exp}_d$)

$$\mathcal{A}^{\rm exp}_p = \sum_{i=1}^{N_{\rm bins}} \Delta x_i A^{\pi^+-\pi^-}_p(x_i) \Big|_Z (4u_V - d_V)(x_i) \times \int_Z^1 dz_h [1 + \otimes \frac{\alpha_s}{2\pi} \tilde{C}_{qq} \otimes](D_1 - D_2), \qquad (27)$$

where Δx_i is the ith bin width. The parametrizations [18] for the fragmentation functions and [19] for unpolarized quark distributions are used. Note that here one should not use the usual "+" prescription in the Wilson coefficients C_{qq}, but its generalization, the so-called "A" prescription [20]. The calculation of L_1,L_2 is rather simple and can be done using any numerical method.

Let us introduce the additional notation $\Delta^*_1 q_V = \int_{x_{\rm min}}^{x_{\rm max}} dx \Delta q_V$ and rewrite[17] BSR in the form (24) as

$$\Delta_1\bar{u} - \Delta_1\bar{d} = [\Delta^*_1\bar{u} - \Delta^*_1\bar{d}]_{\rm BSR} - \frac{1}{2}\int_0^{x_{\rm min}} dx(\Delta u_V - \Delta d_V), \qquad (28)$$

$$[\Delta^*_1\bar{u} - \Delta^*_1\bar{d}]_{\rm BSR} = \frac{1}{2}\left|\frac{g_A}{g_V}\right| - \frac{1}{2}(\Delta^*_1 u_V - \Delta^*_1 d_V), \qquad (29)$$

where $x_{\rm min}$ and $x_{\rm max}$ are the boundary points of the available Bjorken x region. It is obvious that, dealing with the restricted available Bjorken x regions, one can

[17] As usual, we neglect contributions to $\Delta_1 q$ from the unmeasured large x_B region $0.6(0.7) < x_B < 1$ because their upper limits given by the unpolarized distributions are very small there; see [3,6].

TABLE I. Simulation conditions. A and B correspond to HERMES and SMC (COMPASS) kinematics, respectively. Here x_B and x_F are the Bjorken and Feynman x variables, respectively, z_h = is the standard hadronic variable and W is the invariant mass of the final hadronic state.

Kinematics	E_{lepton}	x_B	x_F	z_h	W^2	Events
A	27.5 GeV	$0.023 < x_B < 0.6$	$x_F > 0.1$	$z_h > Z = 0.2$	$W^2 > 10\ \text{GeV}^2$	3×10^6
B	160 GeV	$0.003 < x_B < 0.7$	$x_F > 0.1$	$z_h > Z = 0.2$	$W^2 > 10\ \text{GeV}^2$	3×10^6

TABLE II. GRSV2000**NLO** (broken sea) parametrization. Results on $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\text{BSR}}$ extracted from the simulated difference asymmetries applying the proposed NLO procedure.

Kinematics	Q^2_{mean}	$\Delta_1^* u_V$	$\Delta_1^* d_V$	$[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\text{BSR}}$
A	$2.4\ \text{GeV}^2$	0.585 ± 0.017	-0.147 ± 0.037	0.268 ± 0.020
B	$7.0\ \text{GeV}^2$	0.602 ± 0.032	-0.110 ± 0.080	0.278 ± 0.040

directly extract from the measured difference asymmetries, namely, the quantities $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\text{BSR}}$, while the "tail" contributions $\int_0^{x_{\min}} dx \Delta u_V$, $\int_0^{x_{\min}} dx \Delta d_V$, and $\frac{1}{2} \int_0^{x_{\min}} dx (\Delta u_V - \Delta d_V)$ should be studied separately, applying the proper extrapolation procedure (see below).

The results on $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\text{BSR}}$ extracted from the simulated difference asymmetries using the presented NLO procedure, are given in Table II.

It is obvious that, to be valid, the extraction procedure, being applied to the simulated asymmetries, should yield results maximally close to the ones obtained directly from the parametrization entering the generator as an input. The results for the respective parametrization functions integrated over the total $0 < x < 1$ region in Bjorken x and over the regions $0.023 < x < 0.6$ (HERMES [3] kinematics) and $0.003 < x < 0.7$ (COMPASS kinematics) are presented by Table III.

Let us now compare the results from Tables II and III.

First of all notice that, contrary to the actual experiment conditions, the simulations give the possibility to check the validity of the extraction method comparing the results of the extraction from the simulated asymmetries with an *exact* answer. Namely, in our case this is the integral over the total region of the difference of the parametrizations for $\Delta\bar{u}$ and $\Delta\bar{d}$ entering the generator as an input:

$$[\Delta_1 \bar{u} - \Delta_1 \bar{d}]_{\text{exact}} \simeq [\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{25} = \int_{0.0001}^{0.99} dx [\Delta\bar{u} - \Delta\bar{d}]_{\text{parametrization}} = 0.310, \tag{30}$$

where the symbol $[\cdots]_{nm}$ denotes the nth line and mth column of Table III.

The second point is that the integral taken *directly* (without using BSR) from the $\Delta\bar{u}$ and $\Delta\bar{d}$ parametrization difference over the available to HERMES region is almost 2 times less than the exact answer (30):

$$[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{35} = \int_{0.023}^{0.6} dx [\Delta\bar{u} - \Delta\bar{d}]_{\text{parametrization}} = 0.170. \tag{31}$$

This is a direct indication that the HERMES interval in Bjorken x is too narrow[18] to extract the quantity $\Delta_1\bar{u} - \Delta_1\bar{d}$ in which we are directly interested.

However, there is a possibility to avoid this trouble and essentially improve the analysis on $\Delta_1\bar{u} - \Delta_1\bar{d}$ even with the narrow HERMES x_B region, applying BSR for $\Delta_1\bar{u} - \Delta_1\bar{d}$ extraction. Indeed, applying Eq. (29) to the HERMES x_B region, considering that $|g_A/g_V| = 1.2670 \pm 0.0035$, and calculating the integrals of the valence quark parametrizations over the region $0.023 < x < 0.6$, one gets

$$[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{36}^{\text{BSR}} = 0.292, \tag{32}$$

and this result [contrary to Eq. (31)] is in good agreement with the exact one, Eq. (30).

The reason for this good agreement of Eq. (32) with the exact answer Eq. (30) is that, contrary to the sea distributions, the valence distributions gather far from the low boundary $x_B = 0$ (see, for example, [14] and references therein).

Thus, this exercise with the integrals of the parametrization functions shows that, at least within the broken sea scenario, the application of Eq. (24) for $\Delta_1\bar{u} - \Delta_1\bar{d}$ extraction could give a reliable result on this quantity even with the narrow HERMES x_B region. Namely, one should

[18]Note that the proposed NLO extraction procedure has nothing to do with that problem—we just compare the integrals of the parametrization over the different Bjorken x regions.

TABLE III. Results on $\Delta_1^* u_V$, $\Delta_1^* d_V$, $\Delta_1^*\bar{u} - \Delta_1^*\bar{d}$, and $[\Delta_1^*\bar{u} - \Delta_1^*\bar{d}]_{\rm BSR}$ obtained from integration of the GRSV2000**NLO**(broken sea) parametrization of the quark distributions over the total and experimentally available Bjorken x regions. The fifth column is obtained by direct integration of the respective parametrizations. The sixth column is obtained using BSR and the parametrizations for the valence distributions.

x_B	Q^2	$\Delta_1^* u_V$	$\Delta_1^* d_V$	$\Delta_1^*\bar{u} - \Delta_1^*\bar{d}$	$[\Delta_1^*\bar{u} - \Delta_1^*\bar{d}]_{\rm BSR}$
$0.0001 < x_{Bj} < 0.99$	2.4 GeV^2	0.605	−0.031	0.310	0.315
$0.023 < x_{Bj} < 0.6$	2.4 GeV^2	0.569	−0.114	0.170	0.292
$0.0001 < x_{Bj} < 0.99$	7.0 GeV^2	0.604	−0.032	0.309	0.315
$0.003 < x_{Bj} < 0.7$	7.0 GeV^2	0.598	−0.065	0.262	0.302

first extract in the accessible x_B region the truncated moments of the valence distributions, and only then get the quantity $[\Delta_1^*\bar{u} - \Delta_1^*\bar{d}]_{\rm BSR}$ applying Eq. (29).

One can also compare elements 55 and 56 from Table III corresponding to the SMC (COMPASS) Bjorken x region with the exact answer, element 45 from the Table III. It is seen that even though the integral over the experimentally available region taken directly of the sea parametrization difference is now much closer to the exact answer, the application of BSR instead of direct extraction significantly improves the situation even for this much wider x_B region.

Returning now to the proposed NLO extraction procedure, let us recall that the application of BSR in the form (24) [see derivation of Eq. (25)] is one of the essential elements of the procedure. Comparing the result of Table II on $[\Delta_1^*\bar{u} - \Delta_1^*\bar{d}]_{\rm BSR}$ obtained from the simulated asymmetries with the HERMES kinematics (element 25 from Table II) with both Eqs. (30) and (31), one can see that they are in good agreement with each other. Besides,

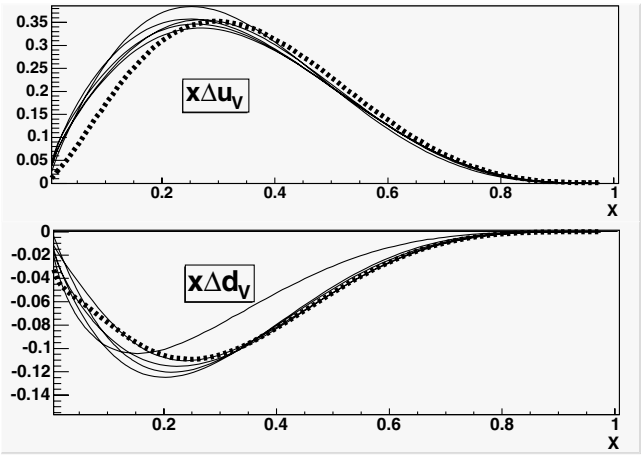

FIG. 7. Polarized valence distributions given by the different parametrizations with the symmetric and weakly broken sea. Solid lines correspond to the purely symmetric parametrizations from Refs. [16,22], while the dashed line corresponds to parametrization FS2000 [set (i −) in Ref. [10]] with a weakly broken sea. All of these parametrizations demonstrate quite similar behavior.

comparing the results of Tables II and III for the COMPASS kinematics, one can see that the results on reconstructed $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $\Delta_1^*\bar{u} - \Delta_1^*\bar{d}$ are still in good agreement with the respective quantities obtained by direct integration of the input parametrization over both the total $0 < x_B < 1$ and experimentally available $0.003 < x_B < 0.7$ regions.

It is also of importance that, even without BSR application, the moments of the valence distributions (interesting in themselves) extracted in NLO in the accessible x_B regions are in good agreement with the input parametrization for both HERMES and COMPASS kinematics.

B. Symmetric sea scenario

Until now, we dealt with the broken sea scenario (which seems to be the most probable one; see footnote 12). However, one certainly should also investigate an alternative opportunity—the symmetric sea scenario. Notice that all the known parametrizations with the symmetric or weakly broken[19] polarized sea essentially differ from the only presently known parametrization with the strongly broken sea, GRSV2000 (broken sea), we dealt with previously. However, they differ rather little from each other; see Fig. 7. So, for self-consistence, we again choose GRSV2000NLO (but with the symmetric sea) parametrization as an alternative input. The respective analysis is presented in Table IV.

Let us analyze the results from Table IV. First, one can see that, for both A and B kinematics, the results of reconstruction in the accessible x_B region of all the quantities presented in this table are again in good agreement with the input parametrization. Thus, the analysis performed within the symmetric sea scenario again confirms

[19]The only such parametrization which we know is FS2000 parametrization [10]. $\Delta_1\bar{u} - \Delta_1\bar{d} < 0.1$ within this parametrization, and with respect to the valence distribution it behaves quite analogously to the purely symmetric parametrizations; see Fig. 7. At the same time, the behavior of the parametrization with the strongly broken sea, GRSV2000 (broken sea), is absolutely different; contrary to the parametrizations with the symmetric or weakly broken sea, Δd_V changes the sign at small x_B, where the contribution of the sea quarks become dominant.

TABLE IV. The upper part presents the results on $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\mathrm{BSR}}$ obtained from integration of the GRSV2000**NLO** (symmetric sea) [16] parametrization. The lower part presents the results on $\Delta_1^* u_V$, $\Delta_1^* d_V$, and $[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\mathrm{BSR}}$ extracted from the simulated difference asymmetries applying the proposed NLO procedure with parametrization GRSV2000**NLO** (symmetric sea) entering the generator as the input.

x_B	Q^2	$\Delta_1^* u_V$	$\Delta_1^* d_V$	$[\Delta_1^* \bar{u} - \Delta_1^* \bar{d}]_{\mathrm{BSR}}$
$0.023 < x_{Bj} < 0.6$	2.4 GeV^2	0.749	−0.276	0.121
$0.003 < x_{Bj} < 0.7$	7.0 GeV^2	0.866	−0.320	0.041
$0.0001 < x_{Bj} < 0.99$	2.4 GeV^2	0.916	−0.339	0.006
$0.0001 < x_{Bj} < 0.99$	7.0 GeV^2	0.914	−0.339	0.007
Kinematics	Q^2_{mean}	$\Delta_1^* u_V$	$\Delta_1^* d_V$	$\Delta_1^* \bar{u} - \Delta_1^* \bar{d}$
A	2.4	0.736 ± 0.017	-0.310 ± 0.037	0.111 ± 0.020
B	7.0 GeV^2	0.842 ± 0.032	-0.300 ± 0.069	0.063 ± 0.038

that the proposed NLO extraction procedure satisfies the main criterion of validity—to reconstruct the quark moments in the experimentally available x_B region.

On the other hand, performing the reconstruction of the entire quantity $\Delta_1 \bar{u} - \Delta_1 \bar{d}$, one certainly should not roughly put it to $\Delta_1^* \bar{u} - \Delta_1^* \bar{d}$. It is necessary to carefully estimate the unmeasured tail $\frac{1}{2} \int_0^{x_{\min}} dx(\Delta u_V - \Delta d_V)$ entering Eq. (28), especially dealing with such a narrow Bjorken x region as the HERMES. It is clearly seen from the Table IV where the result on $\Delta_1^* \bar{u} - \Delta_1^* \bar{d}$ is quite close to the exact answer, zero, for the COMPASS x_B region, but in the case of HERMES kinematics it indicates rather essential deviation from the zero value.

C. Low x_B uncertainties

Let us stress that the problem of the unmeasured tail estimation is the common and long-standing problem which, however, in any case should be somehow solved if we wish to definitely answer the question of whether the sea is symmetric or not. Nowadays, the state of the art is such that the polarized SIDIS experiments use the only method of the low x_B contribution estimations (see, for example, [3,6]): The proper fit to the obtained data on Δq is performed with the subsequent extrapolation of the fitting function to unavailable low x_B. On the other hand, the low x_B tails of all the existing parametrizations on Δq are obtained using a quite analogous procedure. Namely, the parametrization on Δq is extracted in the accessible x_B region from the fit to the measured inclusive asymmetries and/or structure functions and then is extrapolated to low x_B. It is also of importance that the degree of the reliability of the low x_B estimations applied in the existing parametrizations increases because all the parametrizations are constructed in strict accordance with the sum rules on a_3 and a_8 nonsinglet combinations. Besides, the constructed parametrizations meet the requirement of agreement with the existing DIS data on Γ_1^p and Γ_1^n [21].

So, we propose to perform the respective estimation of the quantity $\int_0^{x_{\min}} dx(\Delta u_V - \Delta d_V)$ using the maximal number of the latest available NLO parametrizations. The results are presented in Table V, where the parametrizations from Refs. [10,16,22] are used.

Looking at Table V, one can conclude that for the HERMES x_B region

$$\frac{1}{2} \int_0^{0.023} dx|(\Delta u_V - \Delta d_V)| \lesssim 0.13, \tag{33}$$

while for the COMPASS x_B region the upper boundary is approximately twice as less:

$$\frac{1}{2} \int_0^{0.003} dx|(\Delta u_V - \Delta d_V)| \lesssim 0.06. \tag{34}$$

TABLE V. Low-x contributions to $\frac{1}{2}(\Delta_1 u_V - \Delta_1 d_V)$ for the different NLO parametrizations.

NLO parametrization	$\int_0^{x_{\min}} dx(\Delta u_V - \Delta d_V)/2$ $x_{\min} = 0.023$, $Q^2 = 2.4$ GeV^2	$x_{\min} = 0.003$, $Q^2 = 7.0$ GeV^2
GRSV2000 (broken sea)	−0.035	−0.016
GRSV2000 (symmetric sea)	0.110	0.033
FS2000 (i +)	0.104	0.036
FS2000 (i −)	0.080	0.031
LSS2001	0.098	0.032
AAC2000	0.116	0.046
AAC2003	0.127	0.055

Notice that one can estimate only the absolute value of $\int_0^{x_{\min}} dx(\Delta u_V - \Delta d_V)$, because we do not know which scenario (symmetric or not) is realized in nature. For example, the well-known broken sea parametrization GRSV2000 gives the negative sign for this quantity while all the symmetric sea parametrizations give the positive sign. It is also seen that the restrictions (33) and (34) based on the results of Table V are rather strong. For safety, we deliberately overestimate the upper boundaries choosing the largest numbers from the Table V instead of performing the averaging procedure over all used parametrizations (just as was done by SMC [6]). Notice also that the restriction (34) is consistent with the respective estimation made by SMC (see Table 5 in Ref. [6]), whereas the upper boundary given by Eq. (33) is 4 times larger than the HERMES estimation.[20]

Thus, to extract the entire quantity $\Delta_1\bar{u} - \Delta_1\bar{d}$ we propose to include the upper boundaries on $\frac{1}{2} \times \int_0^{x_{\min}} dx(\Delta u_V - \Delta d_V)$ given by the inequalities (33) and (34) into the respective systematical errors, so that the additional low x_B contributions into the systematical errors of HERMES and COMPASS appear as

$$\delta_{\text{low } x}|_{\text{HERMES}} = \pm 0.13, \tag{35}$$

$$\delta_{\text{low } x}|_{\text{COMPASS}} = \pm 0.06. \tag{36}$$

Certainly, Eqs. (35) and (36) should not be considered as strict estimations. This is just an attempt roughly, but with all possible precautions, to estimate if HERMES and (or) COMPASS could, under their real conditions, answer the question if the sea is broken or not.

V. DISCUSSION AND CONCLUSION

With the (rather overestimated) uncertainties given by Eqs. (35) and (36), it is quite possible that HERMES would not see within the total error that $\Delta_1\bar{u} - \Delta_1\bar{d} \neq 0$, if this quantity happens to be too small in reality (for example, about 0.2). However, if this quantity will be about 0.3 (as it assumed by GRSV2000 broken sea parametrization) and higher, it could still be possible to see this quantity even with the HERMES x_B region. On the other hand, it is seen from Tables II, III, and IV and Eq. (36) that the COMPASS x_B region could allow one to catch even a small difference (if any) between $\Delta_1\bar{u}$ and $\Delta_1\bar{d}$.

In any case, analyzing such a tiny quantity as $\Delta_1\bar{u} - \Delta_1\bar{d}$, it is very desirable to perform a combined analysis with both HERMES and COMPASS data. For example, having at one's disposal data on the difference asymmetries in the accessible to HERMES x_B region, one could involve in the analysis on $\Delta_1\bar{u} - \Delta_1\bar{d}$ the respective COMPASS data from the region $0.003 < x_B < 0.023$. The point is that a high statistics, especially at low x_B is claimed as one of the COMPASS advantages [7] as compared with SMC and HERMES experiments.

Thus, we have tested the proposed NLO QCD extraction procedure performing the simulations corresponding to both the broken and symmetric sea scenarios. This analysis confirms that the procedure meets the main requirement: to reconstruct the quark moments in the accessible to measurement x_B region. On the other hand, even with the overestimated low x_B uncertainty (35), one can conclude that the question if $\Delta_1\bar{u} - \Delta_1\bar{d}$ is equal to zero or not could be answered even with the HERMES kinematics in the case of a strongly asymmetric polarized sea. In any case, the situation is much better with the available to COMPASS x_B region.

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APPENDIX

Calculating the asymmetries given by Eq. (12) together with their statistical errors, one should have in mind that, contrary to SMC and COMPASS experiments, in the HERMES conditions the quantities $N_{\uparrow\uparrow(\uparrow\downarrow)}^{\pi^+(\pi^-)}$ entering Eq. (12) are not the pure counting rates, but the counting rates multiplied[21] by the respective luminosities [3]. Thus, in the general case, one should use instead of Eq. (12) the equation[22]

$$A_{p(n,d)}^{\pi^+-\pi^-} = \frac{1}{D}\left[\frac{(N_{\uparrow\downarrow}^{\pi^+} - N_{\uparrow\downarrow}^{\pi^-})L_{\uparrow\uparrow} - (N_{\uparrow\uparrow}^{\pi^+} - N_{\uparrow\uparrow}^{\pi^-})L_{\uparrow\downarrow}}{(N_{\uparrow\downarrow}^{\pi^+} - N_{\uparrow\downarrow}^{\pi^-})L_{\uparrow\uparrow} + (N_{\uparrow\uparrow}^{\pi^+} - N_{\uparrow\uparrow}^{\pi^-})L_{\uparrow\downarrow}}\right], \tag{A1}$$

where luminosities $L_{\uparrow\uparrow(\uparrow\downarrow)}$ are defined as

$$L_{\uparrow\uparrow(\uparrow\downarrow)} = (n\Phi)_{\uparrow\uparrow(\uparrow\downarrow)}, \tag{A2}$$

where n is the area density of the nucleons in the target

[20] Notice that it is dangerous to use the simple Regge parametrization for extrapolation in the rather large x_B region unavailable to HERMES. The estimations obtained in this way [3] $\int_0^{0.023} dx\Delta u_V \simeq 0.03$, $\int_0^{0.023} dx\Delta d_V \simeq -0.03$ seem to be rather underestimated (see discussion on this subject in Ref. [2]).

[21] The cancellation of the luminosities is possible only with the special target setup, like the SMC and COMPASS ones [6,7].

[22] Here the beam and target are assumed to be ideal which means that $P_B = P_T = f = 1$. Namely, this assumption is adopted in the PEPSI event generator [13] we use for simulations.

and Φ is the beam flux. Within the paper we do not study the specific peculiarities of the different experimental setups and deal only with the event generator where the acceptance a is equal to unity, so that

$$N_{\uparrow\downarrow(\uparrow\uparrow)} = (n\Phi)_{\uparrow\downarrow(\uparrow\uparrow)} a\sigma_{\uparrow\downarrow(\uparrow\uparrow)} \to (n\Phi)_{\uparrow\downarrow(\uparrow\uparrow)}\sigma_{\uparrow\downarrow(\uparrow\uparrow)}. \tag{A3}$$

Thus, Eq. (A2) for luminosities is rewritten in the following, suitable for simulations, form:

$$L_{\uparrow\uparrow(\uparrow\downarrow)} = N_{\uparrow\uparrow(\uparrow\downarrow)}/(\sigma_{\uparrow\uparrow(\uparrow\downarrow)}), \tag{A4}$$

where $N_{\uparrow\uparrow(\uparrow\downarrow)}$ are the numbers of inclusive events and $\sigma_{\uparrow\uparrow(\uparrow\downarrow)}$ are the inclusive cross sections automatically calculated by PEPSI (see Ref. [13]) for given sets of the kinematic conditions.

Choosing as the variables in Eq. (A1) the set $N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow}, N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow}$ and using the general formula (see, for example, [23]) for the statistical error on the function F of variables $x_1, x_2, \ldots,$

$$\delta^2[F(x_1, x_2, \ldots)] = \left(\frac{\partial F}{x_1}\right)^2\delta^2(x_1) + \left(\frac{\partial F}{x_2}\right)^2\delta^2(x_2) + 2\left(\frac{\partial F}{x_1}\right)\left(\frac{\partial F}{x_2}\right)\mathrm{cov}(x_1, x_2) + \cdots, \tag{A5}$$

one gets

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{1}{D^2Y^4}\{(Y-X)^2L^2_{\uparrow\uparrow}[\delta^2(N^{\pi^+}_{\uparrow\downarrow}) + \delta^2(N^{\pi^-}_{\uparrow\downarrow}) - 2\mathrm{cov}(N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow})] + (Y+X)^2L^2_{\uparrow\downarrow}[(\delta^2N^{\pi^+}_{\uparrow\uparrow}) + \delta^2(N^{\pi^-}_{\uparrow\uparrow})] - 2\mathrm{cov}(N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow})\}, \tag{A6}$$

where X and Y are the numerator and denominator in the square brackets in Eq. (A1). If the distributions of hadrons $N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow}, N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow}$ are Poissonian (low multiplicities n^+, n^-; see [23]), $\delta(N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}) = \sqrt{N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}}$, then one can neglect in Eq. (A6) the covariations $\mathrm{cov}(N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow})$ and $\mathrm{cov}(N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow})$ with a result

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{4L^2_{\uparrow\uparrow}L^2_{\uparrow\downarrow}}{D^2}\frac{(N^{\pi^+}_{\uparrow\uparrow} - N^{\pi^-}_{\uparrow\uparrow})^2[N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\downarrow}] + (N^{\pi^+}_{\uparrow\downarrow} - N^{\pi^-}_{\uparrow\downarrow})^2[N^{\pi^+}_{\uparrow\uparrow} + N^{\pi^-}_{\uparrow\uparrow}]}{[(N^{\pi^+}_{\uparrow\downarrow}L_{\uparrow\uparrow} + N^{\pi^+}_{\uparrow\uparrow}L_{\uparrow\downarrow}) - (N^{\pi^-}_{\uparrow\downarrow}L_{\uparrow\uparrow} + N^{\pi^-}_{\uparrow\uparrow}L_{\uparrow\downarrow})]^4}. \tag{A7}$$

Operating absolutely analogously, one gets for the error on the usual spin asymmetry the equation

$$\delta^2(A^{\pi^+}_{p(n,d)}) = \frac{1}{D^2}\frac{4L^2_{\uparrow\downarrow}L^2_{\uparrow\uparrow}(N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow})N^{\pi^+}_{\uparrow\downarrow}N^{\pi^+}_{\uparrow\uparrow}}{(N^{\pi^+}_{\uparrow\downarrow}L_{\uparrow\uparrow} + N^{\pi^+}_{\uparrow\uparrow}L_{\uparrow\downarrow})^4}. \tag{A8}$$

Notice that, namely, the equation (A8) for the statistical error on the usual semi-inclusive asymmetry was used by HERMES [17].

For the COMPASS experiment $L_{\uparrow\uparrow} = L_{\uparrow\downarrow}$ and Eq. (A7) reduces to

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{4}{D^2}\frac{(N^{\pi^+}_{\uparrow\uparrow} - N^{\pi^-}_{\uparrow\uparrow})^2[N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\downarrow}] + (N^{\pi^+}_{\uparrow\downarrow} - N^{\pi^-}_{\uparrow\downarrow})^2[N^{\pi^+}_{\uparrow\uparrow} + N^{\pi^-}_{\uparrow\uparrow}]}{[(N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow}) - (N^{\pi^-}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\uparrow})]^4}. \tag{A9}$$

Let us now choose as the variables the set $X_1 = N^{\pi^+}_{\uparrow\downarrow} - N^{\pi^-}_{\uparrow\downarrow}$, $X_2 = N^{\pi^+}_{\uparrow\uparrow} - N^{\pi^-}_{\uparrow\uparrow}$, so that the equation (A1) for asymmetry is rewritten as

$$A^{\pi^+-\pi^-}_{p(n,d)} = \frac{1}{D}\frac{X_1L_{\uparrow\uparrow} - X_2L_{\uparrow\downarrow}}{X_1L_{\uparrow\uparrow} + X_2L_{\uparrow\downarrow}}. \tag{A10}$$

Then Eq. (A5) gives

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{4L^2_{\uparrow\uparrow}L^2_{\uparrow\downarrow}}{D^2}\frac{X_2^2\delta^2(X_1) + X_1^2\delta^2(X_2)}{(X_1L_{\uparrow\uparrow} + X_2L_{\uparrow\downarrow})^4}, \tag{A11}$$

with

$$\delta^2(X_1) = \delta^2(N^{\pi^+}_{\uparrow\downarrow}) + \delta^2(N^{\pi^-}_{\uparrow\downarrow}) - 2\mathrm{cov}(N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow}), \tag{A12}$$

$$\delta^2(X_2) = \delta^2(N^{\pi^+}_{\uparrow\uparrow}) + \delta^2(N^{\pi^-}_{\uparrow\uparrow}) - 2\mathrm{cov}(N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow}). \tag{A13}$$

Again, with a standard assumption that the distributions of hadrons $N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}$ are Poissonian: $\delta(N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}) = \sqrt{N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}}$, one can neglect in Eqs. (A12) and (A13) the covariations $\mathrm{cov}(N^{\pi^+}_{\uparrow\uparrow}, N^{\pi^-}_{\uparrow\uparrow})$ and $\mathrm{cov}(N^{\pi^+}_{\uparrow\downarrow}, N^{\pi^-}_{\uparrow\downarrow})$. Then Eq. (A11) exactly transforms to Eq. (A7).

Let us involve an additional approximation (see [23], p. 7):

$$X_1 \simeq X_2 \simeq \tilde{Y}/2, \tag{A14}$$

where the quantity[23] $\tilde{Y}$ is defined as $\tilde{Y} \equiv (N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow}) - (N^{\pi^-}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\uparrow}) \equiv N^{\pi^+} - N^{\pi^-}$. Then Eq. (A11) reads

[23]It is easy to see that $\tilde{Y}$ up to luminosities coincides with denominator Y in the square brackets in Eq. (A1).

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{1}{D^2}\frac{16L^2_{\uparrow\downarrow}L^2_{\uparrow\uparrow}}{\tilde{Y}^2(L_{\uparrow\downarrow}+L_{\uparrow\uparrow})^4}\delta^2(\tilde{Y}) \equiv \frac{16L^2_{\uparrow\downarrow}L^2_{\uparrow\uparrow}}{D^2(L_{\uparrow\downarrow}+L_{\uparrow\uparrow})^4}\frac{1}{(N^{\pi+}-N^{\pi-})^2} \times \delta^2(N^{\pi+}-N^{\pi-}) = \frac{N^{\pi^+}+N^{\pi^-}}{(N^{\pi^+}-N^{\pi^-})^2}\frac{16L^2_{\uparrow\downarrow}L^2_{\uparrow\uparrow}}{D^2(L_{\uparrow\downarrow}+L_{\uparrow\uparrow})^4}. \quad \text{(A15)}$$

With the SMC (COMPASS) target setup $L_{\uparrow\downarrow} = L_{\uparrow\uparrow}$ and

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{1}{D^2}\frac{N^{\pi^+}+N^{\pi^-}}{(N^{\pi^+}-N^{\pi^-})^2}. \quad \text{(A16)}$$

It is instructive to reproduce Eq. (A16) choosing other variables in Eq. (A1): $\Delta N^{\pi^+} = N^{\pi^+}_{\uparrow\downarrow} - N^{\pi^+}_{\uparrow\uparrow}$, $\Delta N^{\pi^-} = N^{\pi^-}_{\uparrow\downarrow} - N^{\pi^-}_{\uparrow\uparrow}$, $N^{\pi^+} = N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow}$, $N^{\pi^-} = N^{\pi^-}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\uparrow}$, so that with $L_{\uparrow\downarrow} = L_{\uparrow\uparrow}$ (COMPASS target setup) Eq. (A1) for asymmetry reads

$$A^{\pi^+-\pi^-}_{p(n,d)} = \frac{1}{D}\frac{\Delta N^{\pi^+}-\Delta N^{\pi^-}}{N^{\pi^+}-N^{\pi^-}} \equiv \frac{1}{D}\frac{\tilde{X}}{\tilde{Y}}. \quad \text{(A17)}$$

Then the respective statistical error appears as

$$\delta^2(A^{\pi^+-\pi^-}_{p(n,d)}) = \frac{1}{D^2\tilde{Y}^2}\left\{\delta^2(\Delta N^{\pi^+}) + \delta^2(\Delta N^{\pi^-}) + \frac{\tilde{X}^2}{\tilde{Y}^2}[\delta^2(N^{\pi^+}) + \delta^2(N^{\pi^-})]\right\} \quad \text{(A18)}$$

$$\simeq \frac{1}{D^2\tilde{Y}^2}\left(1+\frac{\tilde{X}^2}{\tilde{Y}^2}\right)(N^{\pi^+}+N^{\pi^-}), \quad \text{(A19)}$$

where it is again adopted that distributions of hadrons $N^{\pi^+(\pi^-)}_{\uparrow\uparrow(\uparrow\downarrow)}$ are Poissonian, so that $\delta(\Delta N^{\pi^+}) = \delta(N^{\pi^+}) = \sqrt{N^{\pi^+}_{\uparrow\downarrow} + N^{\pi^+}_{\uparrow\uparrow}}$, $\delta(\Delta N^{\pi^-}) = \delta(N^{\pi^-}) = \sqrt{N^{\pi^-}_{\uparrow\downarrow} + N^{\pi^-}_{\uparrow\uparrow}}$, and one can neglect $\mathrm{cov}(\Delta N^{\pi^+}, N^{\pi^-})$ and $\mathrm{cov}(\Delta N^{\pi^-}, N^{\pi^+})$. By virtue of Eq. (A14),

$$\left|\frac{\tilde{X}}{\tilde{Y}}\right| = \left|\frac{X_1 - X_2}{X_1 + X_2}\right| \ll 1, \quad \text{(A20)}$$

so that one can neglect[24] $(\tilde{X}/\tilde{Y})^2$ in Eq. (A19). Thus, one again arrives at the approximate formula (A16) for the error on the difference asymmetries.

[24]Let us recall (see footnotes 10 and 12) that with the applied statistics 3×10^6 DIS events, the quantity $\tilde{Y} \equiv N^{\pi^+} - N^{\pi^-}$ essentially differs from zero even in the vicinity of the minimal value $x_B = 0.003$ accessible for measurement.

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