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A. N. Sissakian, A. V. Tarasov, H. T. Torosyan*,
O. O. Voskresenskaya

e^+e^- PAIR PRODUCTION IN RELATIVISTIC ION COLLISIONS AND ITS CORRESPONDENCE TO ELECTRON–ION SCATTERING

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Сисакян А. Н. и др. E2-2004-192

Образование e^+e^--пар в столкновениях
релятивистских ионов и его связь с электрон-ионным рассеянием

Показано, что амплитуды процессов рассеяния электронов (позитронов) и образования e^+e^--пар в кулоновских полях двух релятивистских ионов могут быть выражены в терминах амплитуд рассеяния лептонов в кулоновском поле ионов с помощью разложения Ватсона. Получены компактные выражения для этих амплитуд в пределе высоких энергий. Обсуждаются также соотношения перекрестной симметрии.

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e^+e^- Pair Production in Relativistic Ion Collisions
and Its Correspondence to Electron–Ion Scattering

It is shown that the amplitudes of the processes of electron (positron) scattering and e^+e^- pair production in the Coulomb field of two relativistic ions can be expressed through the amplitudes of lepton scattering off the ion Coulomb field via the Watson expansion. We obtained compact expressions for these amplitudes valid in the high-energy limit and discuss the crossing symmetry relations among the considered processes.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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INTRODUCTION

A lot of work has been done in past years [1–12] on the investigation of lepton pair production in the Coulomb field of two colliding relativistic ions with charge numbers Z_1, Z_2:

$$Z_1 + Z_2 \to e^+e^- + Z_1 + Z_2. \tag{1}$$

The main goal of this investigation is to obtain the compact expression for the amplitude of the process (1) accounting for final state interaction of the produced pair with ions in all orders of fine structure constant $\alpha = e^2/4\pi$.

Solving of this issue can help to understand a very important and unsolved problem of accounting for the final state interaction of quarks and gluons in QCD. Unfortunately even in QED up to now this problem is not solved due to its complexity and so any progress in this direction is very useful.

The investigation of the process (1) becomes much simpler in the ultrarelativistic limit because of strong Lorentz contraction of electromagnetic fields of ions moving with the velocity close to the speed of light. An example of such a simple solution, which leads to an amplitude different from the Born one only by phase factor, was obtained in [1–3]. By virtue of procedure of the crossing symmetry, the amplitude of the process (1) has been obtained from the so-called «exact» result for the amplitude of electron scattering in the Coulomb field of two colliding ions, which has been extracted from «exact» solution of Dirac equation for electron in this field.

The further analysis of this problem in the framework of a more familiar Feynman diagram technique led to the conclusion that the result of [1–3] was incorrect. This led some authors [5, 12] to a surprising conclusion that crossing symmetry property was violated beyond the Born approximation.

Despite the essential progress achieved in the summing of some class of main diagrams [8, 9], the general structure of amplitudes of electron scattering in the Coulomb field of two relativistic nuclei and e^+e^- production in this field has not been established yet even in the limit of ultrarelativistic energies. Taking into account the importance of this problem and growing interest in this issue from the scientific community, we would like to make some remarks, which, as we hope, will be useful for understanding of this problem.

1. ELECTRON SCATTERING AND PAIR PRODUCTION IN THE COULOMB FIELD OF TWO COLLIDING NUCLEI

The amplitudes of electron scattering and lepton pair production in the arbitrary external electromagnetic field $A_\mu(x)$ can be cast in the following form:

$$\begin{aligned} A^{(\mathrm{scat})} &= \bar{u}(p_f)\int d^4x_1 d^4x_2 e^{ip_i x_1 - ip_f x_2} T(x_2,x_1) u(p_i), \\ A^{(\mathrm{prod})} &= \bar{u}(p_2)\int d^4x_1 d^4x_2 e^{-ip_1 x_1 - ip_2 x_2} T(x_2,x_1) v(p_1); \end{aligned} \tag{2}$$

where the function $T(x_2, x_1)$ is the same in both cases and obeys the following equation:

$$T(x_2,x_1) = V(x_2,x_1) - \int d^4x d^4x' V(x_2,x) G(x,x') T(x',x_1) \tag{3}$$

or in the short notation

$$\begin{aligned} T &= V - V\otimes G\otimes T, \\ V\left(x_2,x_1\right) &= e\gamma_\mu \mathrm{A}_\mu\left(x_1\right)\delta^{(4)}\left(x_2-x_1\right), \\ G\left(x,x'\right) &= \frac{1}{\left(2\pi\right)^4}\int d^4k \frac{\hat{k}+m}{k^2-m^2+i0} e^{-ik\left(x-x'\right)}, \end{aligned} \tag{4}$$

where γ_μ and $u(p), v(p)$ are Dirac matrices and spinors.

From these relations, it follows that for the two-center problem ($\mathrm{A}_\mu\left(x\right) = \mathrm{A}^{(1)}_\mu\left(x\right) + \mathrm{A}^{(2)}_\mu\left(x\right)$) the amplitude $T(x_2, x_1)$ can be represented in the form of infinitive Watson series [13]:

$$\begin{aligned} T = T^{(1)} + T^{(2)} - T^{(1)}\otimes G\otimes T^{(2)} - T^{(2)}\otimes G\otimes T^{(1)} + \\ + T^{(1)}\otimes G\otimes T^{(2)}\otimes G\otimes T^{(1)} + T^{(2)}\otimes G\otimes T^{(1)}\otimes G\otimes T^{(2)}\ldots, \end{aligned} \tag{5}$$

where $T^{(1,2)}$ obey the equations

$$\begin{aligned} T^{(1)} &= V^{(1)} - V^{(1)}\otimes G\otimes T^{(1)}, \\ T^{(2)} &= V^{(2)} - V^{(2)}\otimes G\otimes T^{(2)}. \end{aligned} \tag{6}$$

In high-energy limit, when Lorentz factor of colliding ions $\gamma = \dfrac{E}{M} \to \infty$, these equations can be solved with the result

$$T^{(1)}(x_2,x_1)=\gamma_+\Bigg[U_1(x_1)\delta^4(x_2-x_1)+ \\ +\frac{i}{2\pi}\delta^2(\mathbf{x}_2-\mathbf{x}_1)U_1(x_2)U_1(x_1)\exp\left(i\int\limits_{x_{1+}}^{x_{2+}}U_1(x)dx_+\right)\times \\ \times\int\limits_{-\infty}^{\infty}dk_+\big(\theta(k_+)\theta(x_{2+}-x_{1+})- \\ -\theta(-k_+)\theta(x_{1+}-x_{2+})\big)\exp\left(-\frac{ik_+(x_{2-}-x_{1-})}{2}\right)\Bigg], \quad (7)$$

$$T^{(2)}(x_2,x_1)=\gamma_-\Bigg[U_2(x_1)\delta^4(x_2-x_1)+ \\ +\frac{i}{2\pi}\delta^2(\mathbf{x}_2-\mathbf{x}_1)U_2(x_2)U_2(x_1)\exp\left(i\int\limits_{x_{1-}}^{x_{2-}}U_2(x)dx_-\right)\times \\ \times\int\limits_{-\infty}^{\infty}dk_-\big(\theta(k_-)\theta(x_{2-}-x_{1-})- \\ -\theta(-k_-)\theta(x_{1-}-x_{2-})\big)\exp\left(-\frac{ik_-(x_{2+}-x_{1+})}{2}\right)\Bigg], \quad (8)$$

where

$$U_1(x)=e\gamma\Phi_1\left(\sqrt{(\mathbf{b}_1-\mathbf{x})^2+\gamma^2x_+^2}\right), \\ U_2(x)=e\gamma\Phi_2\left(\sqrt{(\mathbf{b}_2-\mathbf{x})^2+\gamma^2x_-^2}\right). \quad (9)$$

Here $\Phi_{1,2}(r)$ are the Coulomb potentials of ions $Z_{1,2}$ in their rest frames; $\gamma_\pm=\gamma_0\pm\gamma_z$ is the Dirac matrices and we use the light cone definition of momenta and coordinates $k_\pm=k_0\pm k_z$, $x_{i\pm}=x_{i0}\pm x_{iz}$; $\mathbf{b}_1,\mathbf{b}_2$ are the impact parameters of ions and $\mathbf{x}_1,\mathbf{x}_2$ are the transverse coordinates of relevant four-vectors. There are no other simplifications in high-energy limit. In particular, there are no truncations of infinite Watson series in contrast to the statement done in [5].

2. THE CROSSING SYMMETRY RELATIONS

Let us discuss the property of crossing symmetry using as example the simplest crossed reactions — electron and positron scattering in the Coulomb field of ion. The amplitudes of these reactions read

$$\begin{aligned} A(e^-Z\to e^-Z) &= \bar{u}(p_f^-)T^{(1)}(p_f^-,p_i^-)u(p_i^-), \\ A(e^+Z\to e^+Z) &= -\bar{v}(p_i^+)T^{(1)}(-p_i^+,-p_f^+)v(p_f^+), \end{aligned} \tag{10}$$

where

$$\begin{aligned} T^{(1)}(p_2,p_1) = \int d^4x_1 d^4x_2 e^{ip_1x_1-ip_2x_2}T(x_2,x_1) = (2\pi)^2\delta(p_{2+}-p_{1+})\times \\ \times\gamma_+[\theta(p_{1+})f^+(\mathbf{p}_2-\mathbf{p}_1)-\theta(-p_{1+})f^-(\mathbf{p}_2-\mathbf{p}_1)], \end{aligned} \tag{11}$$

$$\begin{aligned} f^{\pm}(\mathbf{q}) &= \tfrac{i}{2\pi}\int d^2x e^{i\mathbf{qx}}[1-e^{\pm i\chi(\mathbf{b}-\mathbf{x})}], \\ \chi(\mathbf{b}-\mathbf{x}) &= e\int\limits_{-\infty}^{\infty}\Phi\left(\sqrt{(\mathbf{b}-\mathbf{x})^2+z^2}\right)dz. \end{aligned} \tag{12}$$

The crossing symmetry for reactions (10) means that the amplitudes for electron and positron scattering in the Coulomb field are expressed through universal function $T(p_2,p_1)$ calculated at different values of its arguments. If this function was an analytical function of its arguments, then it would be possible to express one amplitude through the other one using a simple substitution $p_i^-\to -p_f^+$; $p_f^-\to -p_i^+$ and a trivial change of the spinors $u\to v$. But discontinuity of $T(p_2,p_1)$ in the variable p_+ does not allow us to do this in general case. Such a substitution can be done only on the level of Born approximation as is easily seen from (11).

For the same reason, the amplitude of pair production cannot be derived from the amplitude of electron scattering by procedure of crossing symmetry beyond the Born approximation.

CONCLUSIONS

There are two wrong points in the derivation of expressions for e^+e^- pair production amplitude in [1–3]. The first one is the oversimplified expression for electron scattering amplitude derived in these papers. Really the authors have omitted all higher terms of Watson expression (5) except the four first ones. The other mistake, which has led the authors to the wrong statement about absence of Coulomb corrections to the Born approximation, is the wrong application of crossing symmetry property as we explained above.

Thus the problem of final state interaction to all orders in the fine structure constant even in Abelian theory is a complex issue and demands the further and deeper investigation which will be done elsewhere.

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141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@pds.jinr.ru
www.jinr.ru/publish/