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On the Calculation of Phase Space Integral

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We investigate the phase space integral. Such type integrals arrive when the topological cross-section is calculated.

$$Z_n = \int \left\{ \prod_{i=1}^{n} \frac{d^3 k_i}{2\sqrt{k_i^2 + m^2}} \right\} \delta^4 \left(P - \sum k_i \right) f_n(k_1, \ldots, k_n)$$

We will consider the simplest case when the module square of amplitude f_n has the form:

$$f_n(k_1, \ldots, k_n) = \prod_{i=1}^{n} \exp\left(-r_0^2 \, k_{t,i}^2\right)$$

$P \equiv (E, 0, 0, 0)$; $k_{t,i} = \sqrt{k_{i,x}^2 + k_{i,y}^2}$ is the transverse momentum; r_0 is the cutting parameter - phenomenological constant.

Therefore, the aim of my talk is to formulate the effective method of calculation of the longitudinal phase space integral for the case when the number n is high enough.

The theory of calculation of integrals of this type have a long history. Note, that all these methods can't work for big (≥ 100) values of n. Considering the very high multiplicity, it is important to have an universal method of calculation of the phase space integrals. It can be useful in a wide range of multiplicity.

We neglect the momenta conservation low and leave only energy conservation low:

$$\int \ldots \delta^4 \left(P - \sum_{i=1}^{n} k_i \right) \to \int \ldots \delta \left(E - \sum_{i=1}^{n} \sqrt{k_i^2 + m^2} \right)$$

After introducing spherical coordinates

$$k_{i,x} = k_{t,i} \cos(\phi_i)$$
$$k_{i,y} = k_{t,i} \sin(\phi_i)$$

and integration on ϕ_i we receive:

$$Z_n(E) = (\pi/2)^n \int \left\{ \prod_{i=1}^{n} \frac{d(k_{t,i}^2) dk_z}{\sqrt{k_{t,i}^2 + k_{z,i}^2 + m^2}} e^{-r_0^2 k_{t,i}^2} \right\} \times$$
$$\times \delta \left(E - \sum_{i=1}^{n} \sqrt{k_{t,i}^2 + k_{z,i}^2 + m^2} \right)$$

We may include to the previous the integral equality:

$$\int d\mathcal{E}_i \delta \left(\mathcal{E}_i - \sum_{i=1}^{n} \sqrt{k_{t,i}^2 + k_{z,i}^2 + m^2} \right) \equiv 1$$

and after some transformations receive:

$$Z_n(E) = (\pi/r_0)^n [m(n_{max} - n)]^{n-1} \times$$
$$\times \left\{ \prod_{i=1}^{n} \int_0^1 dy_i F(r_0 m \sqrt{(n_{max} - n) y_i ((n_{max} - n) y_i + 2)} \right\} \delta \left(1 - \sum_{i=1}^{n} y_i \right)$$

where $n_{max} = E/m$, F(x) - is a Dawson's integral:

$$F(x) = e^{-x^2} \int_0^x e^{t^2} dt$$

Using Fourier representation of Dirac delta-function we receive final equation:

$$Z_n(E) = (\pi/r_0)^n [m(n_{max} - n)]^{n-1} \times \\ \times \int_0^\infty d\alpha \cos[-\alpha + n \arctan(\phi_s/\phi_c)][\phi_s^2 + \phi_c^2]^{n/2}$$

where

$$\phi_s = \frac{1}{\pi} \int_0^1 dy \sin(\alpha y) F\left(r_0 m \sqrt{(n_{max} - n)y((n_{max} - n)y + 2)}\right)$$

$$\phi_c = \frac{1}{\pi} \int_0^1 dy \cos(\alpha y) F\left(r_0 m \sqrt{(n_{max} - n)y((n_{max} - n)y + 2)}\right)$$

On the Figures 1,2 behavior of $\phi_c(\alpha, E)$ and $\phi_s(\alpha, E)$ are demonstrated.

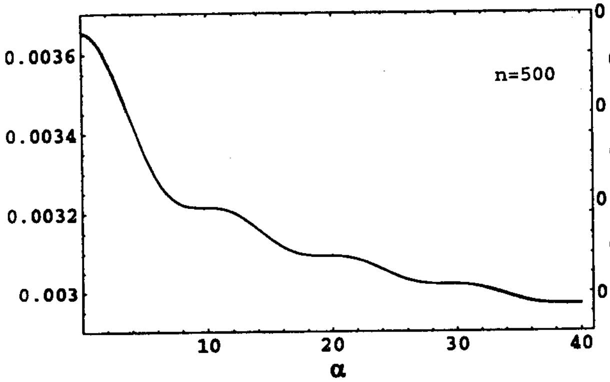

Figure 1:

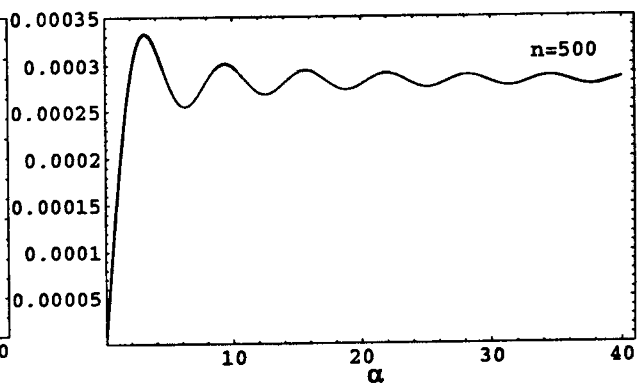

Figure 2:

Calculations are realized by the package "Mathematica" 4.1 with range of precision ~ 40. We meet with the serious difficulty when we execute numerical calculations. Integrand in (11) is a fast oscillatory function, see Figure 3 for normalized function.

For its precision computing we find roots of integrand and represent integral as a sum of small integrals between neighboring roots. We received unexpected result on this way - the sum of positive small integrals are equal sum of the negative small integrals with 34 number of mantissa digits. On the numerical analysis such result have the name "Roundoff Error".

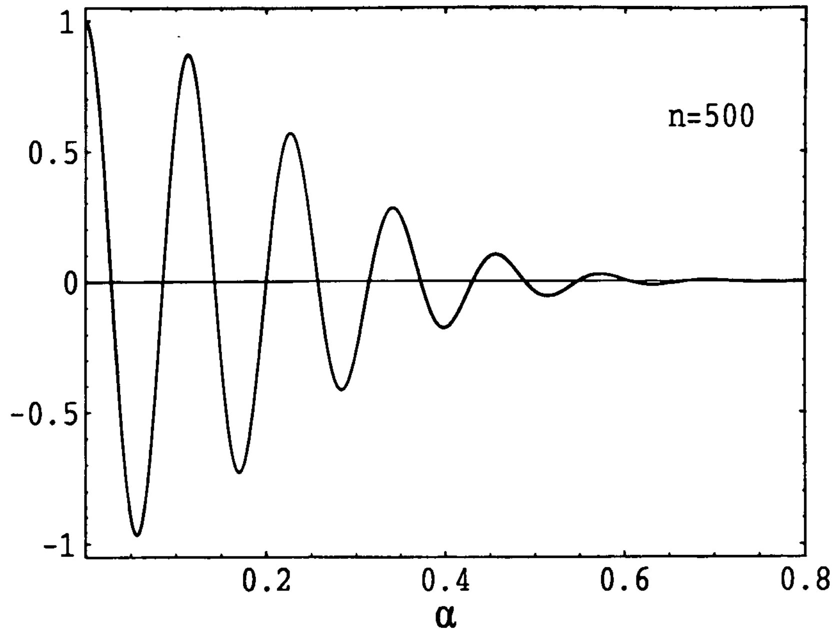

Figure 3: .

To find a way out of the impasse we use another method. After substitutions to the primordial integral Fourier representation of Dirac delta-function and replacement of variable we receive:

$$Z_n = -\frac{i}{2\pi}\int_{-i\infty}^{+i\infty} d\beta e^{\beta E + n\ln[K(\beta)]} = -\frac{i}{2\pi}\int_{-i\infty}^{+i\infty} d\beta e^{L(\beta)}$$

where

$$K(\beta) = \int \frac{d^3k}{2\sqrt{k^2+m^2}} e^{-\beta\sqrt{k^2+m^2} - r_0^2 k_t^2}$$

For last integral we use foregoing formalizm and receive:

$$K(\beta) = \frac{\pi}{r_0^3}\int_0^{u_{max}} e^{-\beta\sqrt{(u/r_0)^2+m^2}} \frac{uF(u)}{\sqrt{(u/r_0)^2+m^2}} du$$

where

$$u_{max} = r_0 m\sqrt{(n_{max}-n)(n_{max}-n+2)}$$

We estimate value of our integral by the method of pass:

M.V.Fedoryuk, Metod perevala."Nauka",1977,Moscow

Let β_0 is a root of equation:

$$\frac{\partial}{\partial\beta} L(\beta) = 0$$

Then we expand the range of exponent near point β_0 and write Z_n as:

$$Z_n = \frac{i}{2\pi}\int_{-i\infty}^{+i\infty} d\beta e^{L(\beta_0)+L''(\beta_0)/2(\beta-\beta_0)^2+L'''(\beta_0)/6(\beta-\beta_0)^3+\dots}$$

If restrict oneself by the third part in exponent we receive:

$$Z_n = \frac{e^{L(\beta_0)}}{\sqrt{2\pi L''(\beta_0)}}\sum_{k=0}^{\infty}\frac{(6k-1)!!}{12^k(2k)!}R^{3k}$$

where

$$R = [L'''(\beta_0)]^{2/3}/L''(\beta_0)$$

Since series (19) is a Borel type, we may write condition of its semiconvergence:

$$R \ll 1$$

On the Figures 4,5 the behavior of R for $r_0 = 0.2$ and $r_0 = 1.0$ for various n are demonstrated.

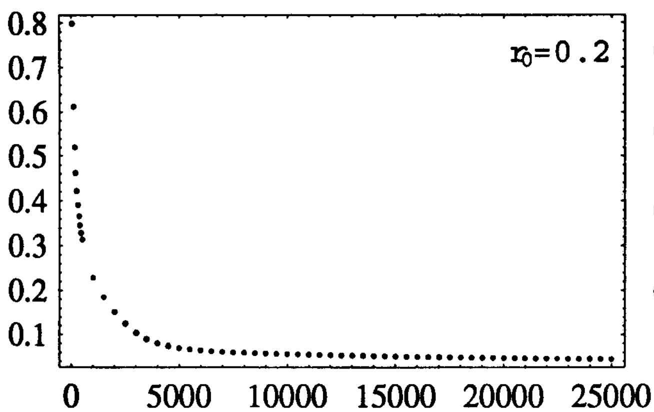

Figure 4:

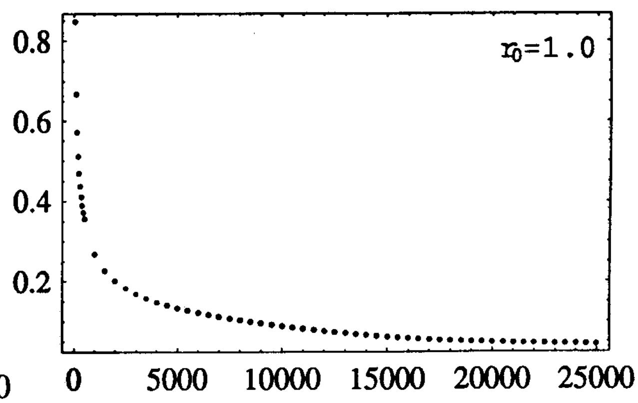

Figure 5:

On the figure 6 behavior of Z_n as function of n are demonstrated:

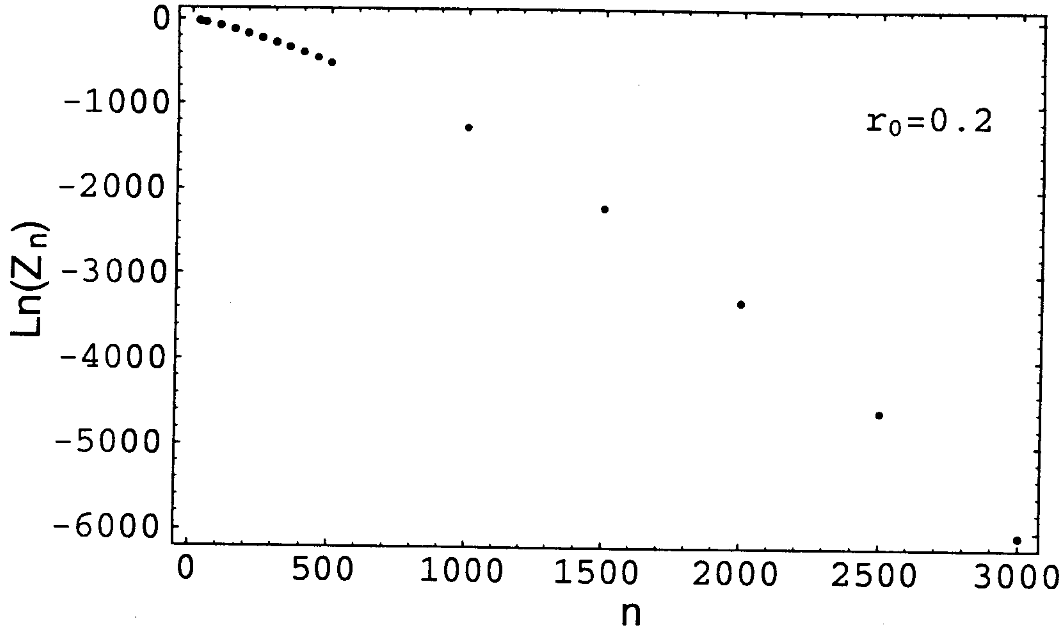

Figure 6: .

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