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VERY HIGH MULTIPLICITY PHYSICS

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Status of very high multiplicity physics

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Abstract

It is argued that investigation of the very high multiplicity processes at high energies will allow

- (i) to check the pQCD in the infrared region,
- (ii) to investigate the soft gluons role in the final state properties formation and of the 'cold' colored plasma creation,
- (iii) to observe possibility of heavy pQCD jets creation,
- (iv) investigate the collective phenomena.

The possibility of experimental investigation of the very high multiplicity events is discussed.

1 Introduction

The infrared region of soft color partons interaction is a mostly important field of high energy hadron dynamics. Such fundamental problems as the infrared divergences of perturbative QCD (pQCD), collective phenomena in the colored particles system, symmetry breaking are the phenomena of infrared domain. Just this general idea forces us investigate the very high multiplicity (VHM) physics.

Another reason of our interest was comparable simplicity of the VHM physics. This connected with presence of small parameter $(\bar{n}(s)/n) \ll 1$, where $\bar{n}(s)$ is the mean multiplicity at given CM energy \sqrt{s} . Another small parameter is the energy of fastest hadron ε_{max} . In the VHM region $(\varepsilon_{max}/\sqrt{s}) \rightarrow 0$. Having this small parameters we can organize the perturbation theory over them.

We wish discuss a set of problems, mostly interesting from our point of view. In the second part of paper the brief discussion of experimental status of the VHM problem will be given.

A. Soft gluons problem

The standard (mostly popular) hadron theory considers perturbative QCD (pQCD) at small distances (in the scale of $\Lambda \simeq 0.2 \text{ Gev}$) as the exact theory. This statement is confirmed by a number of experiments, viz deep-inelastic scattering data, hard jets observation. The pQCD has finite range of validity since the non-perturbative effects should be taken into account at distances larger then $1/\Lambda$.

It is natural to assume, building the complete theory, that at large distances the non-perturbative effects *lay on*¹ the perturbative ones. In result pQCD lose its predictability, it becomes *shadowed* by the non-perturbative effects.

Exist another possibility. It should be noted here that the pQCD running coupling constant $\alpha_s(q^2) = 1/b \ln(q^2/\Lambda^2)$ becomes infinite at $q^2 = \Lambda^2$ and we do not know what happens with pQCD if $q^2 < \Lambda^2$. There is a few possibility. For instance, at $q^2 \sim \Lambda^2$ the properties are changed so drastically, i.e. the new vacuum appears, that even the notions of pQCD is *disappeared*. This means that pQCD should be truncated from below on the 'fundamental' scale Λ . Then it seems natural that this infrared cut-off would influence on the soft hadrons emission.

So, it is important to rise predictability of pQCD in the 'forbidden' area of large distances. For this purpose one should split experimentally the perturbative and non-perturbative effects at the large distances to check out quantitatively the role of soft color partons. One must realize for this purpose highly unusual condition that the non-perturbative effects must be negligible even if, formally, the distance among color charge is high.

It is a question of the leading non-perturbative effects in the multiple production process. Thus, the non-perturbative effect leads to the strong polarization of QCD vacuum and, in result, to the color charge confinement. This vacuum is unstable against creation of real quark-antiquark ($q\bar{q}$) pairs [2] if the distance among charges became large. This pairs are captured into the colorless hadrons and just emission of this 'vacuum' hadrons is the mostly important non-perturbative effect.

But, if the kinetic energy of colored partons is small, i.e. is comparable with hadron masses, creation of 'vacuum' hadrons should be negligible. Just this is the VHM process kinematics: because of the energy-momentum conservation law, produced (final-state) partons can not have high relative momenta and, if they was created at small distances, the production of 'vacuum' hadrons will be negligible (or did not play important role). Therefore, if the 'vacuum' channel is negligible, the pQCD contributions became "pure" [3, 4].

B. Dissipation problems

The multiple production process is the process of dissipation of the kinetic energy of incident particles into the created particles energies. From this point of view the VHM processes are highly nonequilibrium since the VHM final state is very far from initial one. It is known from statistics [5] that such process are close to the "stationary Markovian" ones.

In the pQCD terms last one means that the process of VHM formation should be enhanced, at least in asymptotics over multiplicity and energy, by jets. It is the general conclusion of nonequilibrium thermodynamics and it means that the very nonequilibrium initial state tends to equilibrium (thermalized) as fast as possible.

The entropy \mathcal{S} of a system is proportional to number of created particles and, therefore, \mathcal{S} should tend to its maximum in the VHM region [6]. But the maximum of entropy testify

¹The corresponding formalism was described e.g. in [1].

the equilibrium of a system and it is well known that only a few parameter is enough to describe such system. For this reason the VHM events may be 'simple'. But, introducing the infinite-range correlations, the soft massless gluons may destroy our hope [7].

C. Collective phenomena

The connection of the equilibrium and relaxation of correlations is well known [8]. Continuing this idea, if the VHM system is in equilibrium one may assume that the color charges in the pre-confinement phase of VHM event form the plasma. One should note here that expected plasma is 'cold' and by this reason no long-range confinement forces would act among color charges. Then, being 'cold', in such system various collective phenomena may be important. For instance, the phase transition (*colored partons* \rightarrow *colorless hadrons*) may be investigated.

We should underline that the collective phenomena may take place if and only if the particles interaction energy is comparable to the kinetic one. The considered VHM system is 'cold' and just for this reason the collective phenomena may be seen in it. The fundamental interest presents the problem of vacuum structure of Yang-Mills theory. For instance, if the process of cooling is 'fast', then one can consider a possibility of formation of domains with various chiral properties. As was mentioned above, the dissipation process of VHM final state formation should be as fast as possible [5]. Then decay of this domains may lead to large fluctuations, for instance, of the isotopic spin [10].

The paper is organized as follows. In the following Sec.2 we will give the list of problems with short comments. In the subsequent Sec.3 the the list of measurable predictions will be given. It should help read the Sec.4, where the experimental side of the VHM problem is described.

2 List of problems connected with VHM processes

It is not necessary to fix the multiplicity n exactly in the VHM domain: if the number n is too high, for example 10 000 then the physics should not changed drastically if, for instance, $n = 10000 \pm 100$. Therefore, it is natural to think that the theory prediction in the VHM domain should not exceed this accuracy.

Let us consider possible predictions for the topological cross sections σ_n asymptotics over multiplicity n from this point of view. We will expect that

$$\sigma_n(s) = \sigma_{tot}(s)P_n(s)e^{-n\mu'(n,s)}, \quad (2.1)$$

where $P_n = O(n)$ and $\mu'(n, s) = O(n)$ are some power functions of n . Then, the main quantity of investigations would be

$$-\frac{1}{n} \ln \left\{ \frac{\sigma_n(s)}{\sigma_{tot}} \right\} = \mu'(n, s) - \frac{1}{n} \ln P_n(s) = \mu'(n, s) + O(\ln n/n). \quad (2.2)$$

So, with $O(\ln n/n)$ accuracy we may investigate in the VHM domain the value of $\mu'(n, s)$.

Considering the VHM events we will use the S -matrix interpretation of thermodynamics. In this interpretation the created particles are probes, i.e. measuring created particles energy, momentum, charge we reproduce investigated system of interacting particles (fields). So, if we assume that the temperature of interacting fields coincide with mean energy of created particles, then exist one-to-one coincidence among cross-sections of S -matrix theory and corresponding partition functions? [3].

Taking this interpretation into account, μ' has evident physical meaning. Consider the multiple production state as the statistical system with variable number of particles, $\sigma_n(s)/\sigma_{tot}$ may be considered as the partition function [3] with fixed number of particles. This quantity should be $\sim e^{-n\mu'}$, where μ' is the dimensionless chemical potential. It has natural connection with so called activity z : $\mu' = \ln z$ see also [11] and the papers cited therein.

The ordinary thermodynamical definition of the chemical potential measures μ in the term of the particles mean kinetic energy units, $1/\beta$, the temperature, i.e. $\mu' = \beta\mu$. The question of introduction of the canonical chemical potential into the theory will be considered.

We would like show now that investigation of the VHM events may help to look inside a number of interesting problems. We divide them on three groups:

(A) *Hard pQCD problems for VHM processes*

(I) *The dissipation process of highly nonequilibrium state is the stationary Markovian;*

(We may use the general statement of nonequilibrium thermodynamics that if the initial state is very far from equilibrium then there is not fluctuations of flow to the equilibrium state [5]. One can say, that in this case the system tends to equilibrium as fast as possible. This means in the field-theoretical terms that the very high frequency phonons will only decay on the lower frequency modes and the inverse process is practically forbidden.)

In the considered hadrons collision process this statement is less evident since there is a two channel of incident energy dissipation. First one is dominate in the total cross sections and is connected to creation of 'vacuum' ($q\bar{q}$) pairs. It can be shown [3] that corresponding multiplicity n distribution σ_n^s have following asymptotics over n :

$$\sigma_n^s < O(e^{-n}) \text{ at } n \rightarrow \infty, \quad (2.3)$$

i.e. σ_n^s decrease faster than any power of e^{-n} . So, if

$$\sigma_n^s < O(e^{-n}) \text{ then : } \frac{\partial}{\partial n} \mu'(n, s) > 0. \quad (2.4)$$

Noting that a chemical potential is a work needed to create one particle, thus the estimation (2.3) means that the vacuum of this model is stable against particles creation.

At the same time, for all evidence [12], for the stationary Markovian processes we may find:

$$\sigma_n^j = O(e^{-n}) : \frac{\partial}{\partial n} \mu'(n, s) = 0, \quad (2.5)$$

i.e. no additional work (energy) is needed for particles creation in this case.

Comparing (2.3) and (2.5) we can conclude that one can find such high values of n that

$$\sigma_n^j \gg \sigma_n^s. \quad (2.6)$$

It should be noted that the influence of the finite phase space boundary can prevent (2.6) from observation and by this reason the energy \sqrt{s} should be high enough to have the maximal value of multiplicity at given energy

$$n_{max} = \frac{\sqrt{s}}{m} \gg \bar{n}(s) \sim \ln s, \quad (2.7)$$

where $m \simeq 0.2$ Gev is the typical hadron mass.

(II) *The heavy QCD jets.*

The prediction (2.5) is natural for QCD jets [12]. The explicit for one jet production looks as follows:

$$\sigma_n^{(1j)}(M) = a^{(1j)}(M, n) e^{-c_j n / \bar{n}_j(M)}, \quad n \geq \bar{n}_j(M), \quad (2.8)$$

where $a^{(1j)}(n, M)$ is the polynomial function of n , $\bar{n}_j(M)$ is the mean multiplicity in the mass M jet and c_j is a positive constant.

The linearity of the exponent in (2.8) over n have important consequences. So, let us assume that the total energy M is divided on two jets of masses M_1 and M_2 equally: $M_1 = M_2 = M/2$. If, for instance, $M_2 \ll M_1 \simeq M$ then the distribution will coincide with (2.8), but the second jet distribution would reorganize coefficient $a^{(1j)}$.

Then the multiplicity distribution in the two-jet event would be

$$\sigma_n^{(2j)}(M) = a^{(2j)}(M, n) e^{-c_j n / \bar{n}_j(M/2)}, \quad (2.9)$$

where $n_1 + n_2 = n$ is the total multiplicity. (If, for instance, $M_2 \ll M_1 \simeq M$ then the distribution will coincide with (2.8), but the second jet distribution reorganizes $a^{(1j)}$.)

Comparing (2.8) with (2.9) we can see that with exponential accuracy:

$$\sim \exp\left\{-c_j \frac{\bar{n}_j(M) - \bar{n}_j(M/2)}{\bar{n}_j(M)\bar{n}_j(M/2)} n\right\}$$

the (2.8) would dominate in the VHM domain since the mean multiplicity $\bar{n}_j(M)$ increase with M .

The experimental observation of this phenomena crucially depends on the value of a^{1j} , a^{2j} ,... But if (2.6) is hold then one can expect that the events in the VHM domain would be enhanced by QCD jets and the mass of jets would have a tendency to be high with growing multiplicity.

(B) *The pQCD problems connected with soft massless gluons.*

(III) *The pQCD physics: BFKL Pomeron predictions;*

Wishing to describe the VHM domain in the pQCD frame one should take into account that

- (a). Expected transverse momentum of created particle is, at least, larger than the Regge-pole model predicts;
- (b). The longitudinal momenta of created particles are comparatively smaller (in the given frame) than the Regge-pole model predicts.

Validity of such kinematics follows from (I). One can say that the VHM are 'beyond' the standard Regge (multiperipheral) hadron kinematics.

The ATLAS detectors are unable to cover all 4π geometry and some number of particles is escape being undetected. By this reason our description is 'semi-inclusive': we can guarantee the (a) and (b) conditions in the restricted (central) range of rapidities.

In terms of pQCD the following kinematical variables are used: the $\tau = \ln(1/|x|) \geq 0$, where $|x| \leq 1$ is the fraction of longitudinal momentum and $\xi = \ln(|q^2|/\Lambda^2) \geq 0$, where the transverse momentum is $\sim q^2$. The logarithmic variables are natural for renormalizable quantum field theories assuming that the main theoretical contributions are calculated with logarithmic accuracy.

The multiperipheral kinematics assumes that

$$\xi \ll 1, \tau \ll 1, \quad (2.10)$$

i.e. in the multiperipheral processes the transverse momenta are restricted (by exponential factor) and the longitudinal momenta are high. Just in this kinematics the BFKL Pomeron parameters was calculated [13]. It describes creation of particles close to CM beams direction (either small angles $\Theta_i \sim (2m/\sqrt{s}) \ll 1$) [4].

VHM are beyond this kinematics:

$$\xi \gg 1, \tau \gg 1. \quad (2.11)$$

and particles angles Θ_i are high. Moreover, with rising n the tendency to hardness should be conserved, see (II).

The natural attempt to generalize the BFKL Pomeron leader structure to include creation, at least, mini-jets means that the transverse momenta should be comparatively high and the longitudinal momentum low, i.e. the generalization of BFKL Pomeron kinematics on the case

$$\xi \sim \tau \sim 1 \quad (2.12)$$

is necessary. The naive attempts of such generalization shows the 'unstable' predictions.

We conclude, that we have not field theory formalism which be able to describe the soft hadron physics and can be continued in the VHM domain.

(IV) pQCD physics: DIS kinematics

To describe the hadron production in pQCD terms the parton-hadron duality is assumed. This means that the multiplicity, momentum etc. distributions of hadron and colored partons are the same. This reduce the problem practically on the level of QED.

Let us consider now n particles (gluons) creation the DIS. As usual, $D_{ab}(x, q^2)$ is the probability to find parton b with virtuality $q^2 < 0$ in the parton a . We always may chose q^2 and x so that the leading logarithm approximation (LLA) will be acceptable. Then $D_{ab}(x, q^2)$ described by ladder diagrams. From qualitative point of view this means

approximation of Markovian process of random walk over coordinate $\ln(1/x)$ and time is $\ln \ln |q^2|$. LLA means that the 'mobility' $\sim \ln(1/x)/\ln \ln |q^2|$ should be large

$$\ln(1/x) \gg \ln \ln |q^2|. \quad (2.13)$$

But, on other hand,

$$\ln(1/x) \ll \ln |q^2|. \quad (2.14)$$

The leading contributions give integration over wide range $\lambda^2 \ll k_i^2 \ll -q^2$, where $k_i^2 > 0$ is the 'mass' of real gluon. If the time needed to capture the parton into the hadron is $\sim (1/\lambda)$ than, then the gluon should decay if $k_i^2 \gg \lambda^2$. This leads to creation of (mini)jets. The mean multiplicity \bar{n}_j in the QCD jets is high if the gluon 'mass' $|k|$ is high: $\ln \bar{n}_j \simeq \sqrt{\ln(k^2/\Lambda^2)}$. By this reason the jet creation should dominate in the VHM domain.

But rising multiplicity should rise mean value of gluon masses $\langle |k_i| \rangle$ and this decrease the range of integrability over k_i^2 . The quantitative analysis gives:

$$\ln D_{gg}(n; x, q^2) \propto \sqrt{\ln(1/x) \bar{t}(\mu'(n), q^2)}, \quad (2.15)$$

where

$$\bar{t}(\mu, q^2) = \int_1^\tau \frac{d\tau'}{\tau'} t(\mu, \tau'), \quad \tau = \ln(-q^2/\lambda) \quad (2.16)$$

and $t(\mu, \tau)$ is the probability of mass $-q^2 = \lambda^2 e^\tau$ jets formation if the number of particles in the jet is defined by the 'chemical' potential μ' :

$$t(\mu', \tau) = \sum_n e^{n\mu'} t_n(\tau), \quad t_n > 0 \text{ for all } n. \quad (2.17)$$

The estimation (2.15) assumes that

$$\ln(1/x) \gg \bar{t}(\mu', q^2) \quad (2.18)$$

i.e. \bar{t} play the role of time.

By definition $t(\mu' = 0, q^2) = 1$ and in this case (2.18) coincide with (2.13). But with rising μ' or, it is the same, with rising n the 'time' \bar{t} increase since t should increase with μ'/m , see (2.17). So, increasing n we go out the validity of the LLA if $\ln(1/x)$ is fixed.

But if x is dynamical (fluctuating) variable, one can remain the LLA taking $x \rightarrow 0$. We may conclude, noting that the LLA gives main contribution, that the rising multiplicity leads to the infrared domain, where the soft gluons creation becomes dominant. Otherwise, if there is an infrared cut-off ($x > x_0$) the cross section should decrease and the next to leading order corrections should be taken into account.

(V) The problem of long-range correlation.

The S -matrix interpretation of the thermodynamics hides the assumption that the system can be described by few 'thermodynamical' parameter. Main of them is temperature $1/\beta_c$ and chemical potential μ_c . In thermodynamics this quantities have the fundamental character: the probability to find given state is defined just by the values of β_c and μ_c .

In particles physics the energies of particles, their number, etc. are fundamental parameters and the 'temperature', 'chemical potential' are secondary quantities. So, wishing to describe the multiple production state thermodynamically, one should be sure that the thermodynamical parameters are 'good' ones, i.e. they are able to describe whole system unambiguously.

This assumes that the fluctuations near β_c, μ_c should be Gaussian. This strong assumption [8, 14] leads to the smallness of particles energy correlations. Indeed, if the system is not in the thermal equilibrium, then there should be in it the macroscopic energy flows. It is evident, presence of energy flows would lead to the particles energy correlations. We expect that the entropy $S \sim n$ and in the VHM domain the system *relax* exceeding the entropy maximum. The same one may expect considering the relaxation over other parameters.

Otherwise the system would be unstable. The type of instabilities may be few. First of all the instability may be connected with the masslessness of gluons (one may distinguish few hundred instabilities in QED plasma) and may be seen in the color charge correlations. Another source of instability may be connected with complicated structure of Yang-Mills vacuum. So, the mostly interesting for us question is following: tend or not the VHM system to the equilibrium. As was explained this should be seen in the relaxation of energy, momentum, charge, jet-jet (i.e. color charge), etc. correlations.

(C) *The problems connected with collective phenomena in the 'pre-confinement' final state of VHM hadron collision.*

(VI) *The problem of cold color plasma (CCP) formation;*

We have a large number of colored particle in the preconfinement phase of VHM process and one may consider this system as a colored plasma state.

Indeed, by definition, the plasma is a *state* of *semi-free* charged particles. The *semi-free* condition means that the collision frequency is much less than the system characteristic frequencies. This means that particles are involved mostly into the collective motions.

The condition that the system of charges form a *state* means that the system should be 'describable', i.e. the only restricted number of parameters should be used for its complete description. Only in this case the system has a physical meaning. It is natural to describe the system introducing the correlation functions among particles (anyway, one can transform this description into arbitrary one [14]). It is known from thermodynamics that it is necessary and sufficient of correlations relaxation if the system tends to equilibrium [8]. In other words, the equilibrium is defined by correlation relaxation and the equilibrium state is described by few parameters; by temperature, chemical potential, density, pressure.

So, if we want to observe the plasma state, one should investigate the corresponding correlators. So, if we want to describe the system by the temperature, one should observe the energy correlations relaxation. This has evident physical meaning: if energy correlators are not small (in definite scale) then there should be energy flows in the system. Such system can not be equilibrium in this sense (but, for instance, can be equilibrium in the charge distribution sense).

Noting that the entropy of system should rise with rising multiplicity one can expect the (cold) color plasma formation. The signal of this plasma would be relaxation of corresponding correlators.

(VII) The first order phase transition.

Let us assume that the VHM state is equilibrium. Being cold this state is mostly useful for investigation of collective phenomena, since in this case the particles kinetic energy becomes comparable or even less than the potential energy. There is the old standing problem of the transition of colored state into the colorless one.

We can offer in the VHM domain the direct investigation of this question. So, the first order phase transition assumes that some part of quark systems energy is hidden into the hadrons. This means that there should be discrepancy in energy spectrum of quarks in the preconfinement state and hadrons.

The experimental side of this problem would be considered later. Here we note, it is evident that the VHM kinematics is especially convenient to observe this discrepancy.

(VIII) The vacuum instability against particles creation.

This situation was described in [15, 16]. Let us consider the lattice gas model. In this models the down spin means absence of particle in this junction and creation of particle flips spin up. The energy of this system is $\sim \sigma_i \sigma_{i+1}$, where σ_i is the spin in the i -th junction, $\sigma_i \in \pm 1$. The external field \mathcal{H} adds the energy $\sim \sigma_i \mathcal{H}$ and leads to polarization of the system. By this reason \mathcal{H} has the meaning of the chemical potential μ .

Let us assume that at $\mathcal{H} = 0$ the ground state of this system degenerate. This means that the system (of dimension $d = 3$) is divided on the stable domains of up and down spins. The thermal fluctuations create subdomains (bubbles) of opposite for given domain spins, but this bubbles are unstable and dissipates with time. Let us switching on the external field in such a way that the domains with down spin becomes unstable.

This means that in this domains the bubbles are stable and grow with time. It is a typical first order phase transition. The stability of the bubble is defined by the its volume energy $E_v \sim R^3$ and surface energy $E_s \sim r^2$. So, if $E_v > E_s$ then the bubble grow with time and its boundary accelerate.

For our VHM physics acceleration of boundary means that less energy (work) is needed to add into bubble (i.e. to create) a particle. But searching VHM events we extract the bubbles with very high multiplicity of up spins, i.e. the very large dimension bubbles. So, we conclude that in considered model the increasing n should decrease our chemical potential μ' :

$$\frac{\partial}{\partial n} \mu'(n, s) < 0. \quad (2.19)$$

Then, taking into account (2.2) we conclude, that in this case

$$\sigma_n > O(e^{-n}). \quad (2.20)$$

3 What must be measured

Following quantities should be measured:

a. Multiplicity distributions

It was shown in Sec.2I that the cross sections asymptotics over n gives important information. On Fig. we depict three type of possible asymptotics. One can conclude that in terms of pQCD

$$\frac{\partial}{\partial n} \mu'(n, s) \leq 0 \text{ at } s \rightarrow \infty, n \rightarrow \infty, \frac{n}{\sqrt{s}} \ll 1. \quad (3.1)$$

Last condition assumes that the finiteness of phase space should not influence on the particles creation dynamics. The inequality (3.1) means that, at least, jets creating govern cross sections asymptotics over n .

b. One-particle distribution

The estimation (3.1) means that the created particles transverse energy should rise with multiplicity.

c. Correlation functions

It is important to investigate the multiple correlation functions $C_k(X_1, X_2, \dots, X_k; n, s)$, $k \geq 2$, where X_i is the dynamical variable in the i -th cell of measuring device; for example, if $X_i = \varepsilon_i$ this means that we investigate the energy correlations and i may be considered as the 3-(or 4-)coordinate of calorimeter cell, where the energy of particle (or group of particles) measured. So, if the mean value of correlation functions

$$\langle C_k(i; n, s) \rangle = \int \prod_{i=1}^k dX_i C_k(X_1, X_2, \dots, X_k; n, s) \quad (3.2)$$

tend to zero for given set i , in the VHM domain then one can expect tendency to equilibrium over corresponding parameter.

So, the smallness of energy correlations means that the energy ε fluctuation near some mean value $1/\beta_c(n, s)$ should be Gaussian. This spectra contradict to the Pomeron predictions. Therefore, with rising multiplicity we should see transition from Pomeron $d\varepsilon/\varepsilon$ energy spectra to the exponential $d\varepsilon e^{-\beta_c \varepsilon}$. If the created particles are nonrelativistic this distribution coincide with Maxwell distribution.

If $\langle C_k(i; n, s) \rangle \approx 0$ for arbitrary i then the system is in equilibrium and in this case one may consider the set of color charges as the plasma state. One can consider the collective phenomena in this state.

References

- [1] J.Manjavidze and A.Sissakian, Theor. Math. Phys., 130 (2002) 153
- [2] A.Casher, H.Neuberger, S.Nassinov, Phys.Rev., D20 (1979) 179; E.Gurvich, Phys.Lett., B87 (1979) 386

- [3] J.Manjavidze, El. Part. At. Nucl., 16 (1985) 101
- [4] E.Kuraev, J.Manjavidze and A.Sissakian, hep-ph/0003074
- [5] M.Kac in: Probability and Related Topics (Intersciens Publ., London, New York, 1957)
- [6] E.Fermi, Progr. Theor. Phys., 4 (1950) 570, Phys. Rev., 81 (1950) 115, *ibid* 92 (1953) 452;
L.D.Landau, Izv. AN SSSR, 17 (1953) 85
- [7] Selected papers of F.Dyson (AMS,1996)
- [8] N.N.Bogolyubov, Studies in Statistical Mechanics, (North-Holland, Amsterdam, 1962)
- [9] L.Van Hove, Phys. Lett. B245 (1990) 605
- [10] J.D.Bjorken, Acta Phys. Polon., B28 (1997) 2773, hep-ph/9712434; A.Anselm and M.G.Ryskin, Phys. Lett. B266 (1991) 482
- [11] J.Manjavidze and A.Sissakian, Phys. Rep. 346 (2001) 1
- [12] K.Konishi, A.Ukawa and G.Veneziano, Phys.Lett., B80 (1979) 259; A.Basseto, M.Ciafaloni and G.Marchesini, Nucl.Phys., B163 (1980) 477; see also J.Taylor, Phys. Lett., 83B (1979) 207
- [13] E.Kuraev, L.Lipatov and V.Fadin, Sov. Phys. JETP, 44 (1976) 443; Zh. Eksp. Teor. Fiz., 71 (1976) 840; L.Lipatov, Sov.J. Nucl. Phys., 20 (1975) 94; V.Gribov and L.Lipatov, Sov.J. Nucl. Phys., 15 (1972) 438, 675; G.Altarelli and G.Parisi, Nucl. Phys., B126 (1977) 298; I.V.Andreev, Chromodynamics and Hard Processes at High Energies (Nauka, Moscow, 1981)
- [14] J.Manjavidze and A.Sissakian, Proc. of Bogol. Conf. Physics of Elementary Particles and Atomic Nuclei, 31,(2000) 104.
- [15] C.G. Callan, Jr. and S.R. Coleman, Phys.Rev.D16 (1977) 1762
- [16] T.D.Lee and C.N.Yang, Phys.Rev., 87 (1952) 404, 410; J.S.Langer, Ann.Phys., 41 (1967) 108