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Remarks on polarized quark distributions extracted from SIDIS experiments

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The results of SIDIS experiments concerning the first moments of the polarized quark distributions are considered. The possible reasons for the deviation from the fundamental restrictions such as the Bjorken sum rule and the ways to properly improve the analysis of measured SIDIS asymmetries are discussed. The possibility of a broken polarized sea scenario is analyzed.

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The extraction of the polarized quark and gluon densities is the main task of the SIDIS experiments with a polarized beam and target. Of special importance for the modern SIDIS experiments are the questions of the strange quark and gluon contributions to the nucleon spin, and also the sea quark share as well as the possibility of a broken sea scenario. Indeed, it is known [1] that the unpolarized sea of light quarks is essentially asymmetric, and, thus, the question arises: does the analogous situation occurs in the polarized case, i.e., whether the polarized density $\Delta\bar{u}$ is equal to $\Delta\bar{d}$ or not.

The crucial tests for the polarized quark distributions extracted from the SIDIS data are the sum rules dictated by $\mathrm{SU}_f(2)$ and $\mathrm{SU}_f(3)$ symmetries. While $\mathrm{SU}_f(3)$ symmetry (and, as a consequence, the respective sum rule) is rather approximate (see, for example [2], and references therein), $\mathrm{SU}_f(2)$ symmetry may be regarded as almost exact as well as the respective sum rule—Bjorken sum rule (BSR).

Let us recall that the Bjorken sum rule written in terms of the first moments of the structure functions $\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x,Q^2)$ and $\Gamma_1^n(Q^2) \equiv \int_0^1 dx g_1^n(x,Q^2)$ contains Q^2 dependent quantity C_1^{NS} on the right-hand side:[1]

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6}\left|\frac{g_A}{g_V}\right| C_1^{NS}(Q^2), \tag{1}$$

$$C_1^{NS} = 1 - \left(\frac{\alpha_s(Q^2)}{\pi}\right) - 3.5833\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.2153\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 - 130\left(\frac{\alpha_s(Q^2)}{\pi}\right)^4 + O(\alpha_s^5). \tag{2}$$

However (this is of great importance for what follows), the first moments of polarized quark distributions satisfy the respective form of the BSR without C_1^{NS} on the right-hand side irrespectively in which QCD order they are extracted. Namely, the equivalent of BSR written in terms of polarized quark distributions reads

$$\Delta q_3 \equiv a_3 = [\Delta_1 u(Q^2) + \Delta_1 \bar{u}(Q^2)] - [\Delta_1 d(Q^2) + \Delta_1 \bar{d}(Q^2)] = \left|\frac{g_A}{g_V}\right| = F + D = 1.2670 \pm 0.0035 \quad \text{in all QCD orders}, \tag{3}$$

where the notation $\Delta_1 q \equiv \int_0^1 dx \Delta q$ is used to distinguish the local in Bjorken x polarized quark densities $\Delta q(x)$ and their first moments.

Notice that well known fact of nonrenormalizability (i.e., Q^2 independence) of the quantity Δq_3 directly follows from its definition:

$$\frac{s_\mu}{2}\Delta q_3 = \langle ps | A_\mu^3 | ps \rangle \tag{4}$$

due to conservation[2] of the flavor nonsinglet axial-vector current A_μ^3. This fact is also confirmed by the explicit next to leading order (NLO) calculations of the respective nonsinglet anomalous dimension which is just zero [6], so that[3]

$$\frac{d\Delta q_3}{d\ln(Q^2/\Lambda^2)} = \frac{\alpha_s}{2\pi}\,\delta\gamma_{NS,\eta}^{(n)}\Big|_{n=1,\eta=-1}\Delta q_3 = 0.$$

Let us analyze to what extent the results of the modern polarized SIDIS experiments are in agreement with the sum rule predictions. Such detailed analysis with respect to the sum rule based on $\mathrm{SU}_f(3)$ symmetry,

$$\Delta q_8 \equiv a_8 = 3F - D,$$

was performed in [2], so that we will concentrate here on the equivalent of BSR (3) which, using that $\Delta q = \Delta q_V + \Delta\bar{q}$, may be rewritten in the form convenient for analysis:

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[1] See, for example, excellent theoretical overview in [3], and references therein. The $O(\alpha_s^3)$ correction for C_1^{NS} was calculated in [4] and $O(\alpha_s^4)$ correction was estimated in [5].

[2] It is important to remind that while the first moments of the *nonsinglet* densities Δq_3 [$\mathrm{SU_f}(2)$ symmetry] and Δq_8 [$\mathrm{SU_f}(3)$ symmetry] *must be* conserved, i.e., are independent of Q^2 (corresponding to the conservation of the nonsinglet axial-vector Cabibbo currents), the *singlet* axial charge $a_0(Q^2)$ depends on Q^2 because of the axial anomaly.

[3] Here the notation of Ref. [6] for the anomalous dimension is used.

TABLE I. The SMC results on $\Delta_1 q$ within the unbroken sea assumption (the partially reproduced Table 5 of Ref. [7]).

$\Delta\bar{u}(x)=\Delta\bar{d}(x)$	x	0–0.003	0.003–0.7	0–1
	$\Delta_1 u_V$	0.04 ± 0.04	$0.73\pm0.10\pm0.07$	$0.77\pm0.10\pm0.08$
	$\Delta_1 d_V$	-0.05 ± 0.05	$-0.47\pm0.14\pm0.08$	$-0.52\pm0.14\pm0.09$
	x	$0.0-0.003$	$0.003-0.3$	$0-1$
	$\Delta_1\bar{q}$	0.0 ± 0.02	$0.01\pm0.04\pm0.03$	$0.01\pm0.04\pm0.03$

$$\Delta_1\bar{u}-\Delta_1\bar{d}=\frac{1}{2}\left|\frac{g_A}{g_V}\right|-\frac{1}{2}(\Delta_1 u_V-\Delta_1 d_V)\ \text{in all QCD orders.} \tag{5}$$

Let us first consider the SMC results [7]. SMC has performed two types of analyses on Δq, with broken and unbroken sea scenarios, respectively. Unfortunately, the SMC analysis within the broken sea scenario suffers from too big errors[4] because the full number of measured asymmetries and achieved statistics were not quite sufficient to negate the restriction $\Delta\bar{u}=\Delta\bar{d}$. So, let us look at the SMC results for the first moments of polarized quark distributions obtained within the unbroken sea scenario, where the respective table of first moments looks as (see Table 5 of Ref. [7]) in Table I.

Taking the first moments of valence distributions directly from the table, one gets

$$\Delta_1 u_V-\Delta_1 d_V=1.3\pm0.17\pm0.12, \tag{6}$$

and this result is in a good agreement with the equivalent of BSR (5) which within the unbroken sea approximation is rewritten as

$$\Delta_1 u_V-\Delta_1 d_V=\Delta_1 u-\Delta_1 d=\left|\frac{g_A}{g_V}\right|=1.2670\pm0.0035.$$

Let us now perform the similar analysis of HERMES results for the first moments of the polarized quark distributions published in Table 1 of Ref. [8] which we, for convenience, partially reproduce here (Table. II).

Directly from the table one gets

$$\Delta q_3\equiv(\Delta_1 u+\Delta_1\bar{u})-(\Delta_1 d+\Delta_1\bar{d})=0.82\pm0.06\pm0.06, \tag{7}$$

whereas the right-hand side ought to be equal to $|g_A/g_V|=1.2670\pm0.0035$ in accordance with the equivalent of BSR (3).

Thus, the HERMES distributions do not satisfy the real equivalent of BSR (3) (without any Q^2 dependence on the right-hand side). Instead these distributions are rather claimed to be in agreement with the sum rule [see Eq. (13) of Ref. [8]]

$$\Delta q_3=\int_0^1\Delta q^{NS}dx=|g_A/g_V|\times C_{QCD},$$

where $\Delta q_{NS}(x,Q^2)\equiv\Delta u(x,Q^2)+\Delta\bar{u}(x,Q^2)-[\Delta d(x,Q^2)+\Delta\bar{d}(x,Q^2)]$, and $C_{QCD}\equiv C_1^{NS}(Q^2)$ is the nonsinglet coefficient function[5] given by Eq. (2) which is incorrect.[6]

To understand what happens let us briefly recall the HERMES procedure of the polarized density extraction from the measured SIDIS asymmetries. To this end the method of purities is used at HERMES average $Q^2=2.5\ \text{GeV}^2$. Within this method the leading order (LO) expression for SIDIS asymmetry

$$A_1^h(x,Q^2)=\frac{\sum_f e_f^2\Delta q_f(x,Q^2)\int_{0.2}^1 dzD_f^h(z,Q^2)}{\sum_f e_f^2 q_f(x,Q^2)\int_{0.2}^1 dzD_f^h(z,Q^2)}$$

is rewritten via purities $P_f^h(x,Q^2)$ as

$$A_1^h(x,Q^2)=\sum_f\frac{\Delta q_f}{q_f}P_f^h,$$

$$P_f^h(x,Q^2)\equiv\frac{e_f^2 q_f(x,Q^2)\int_{0.2}^1 dzD_f^h(z,Q^2)}{\sum_f e_f^2 q_f(x,Q^2)\int_{0.2}^1 dzD_f^h(z,Q^2)},$$

so that one can see that the application of the purity method is equivalent to the LO QCD analysis.

Thus both SMC and HERMES Collaborations use LO QCD analysis to extract polarized distributions from the measured SIDIS asymmetries. However, there is an important distinction between SMC and HERMES analysis conditions. Namely, whereas the SMC analysis is performed at

[4]Indeed, for $\Delta_1\bar{d}$ the Table 5 of Ref. [7] gives the value 0.01 with ±0.14 and ±0.12 for the statistical and systematical errors, respectively.

[5]The quantity $C_{\rm QCD}$ in Eq. (13) of Ref. [8] is, namely, the nonsinglet coefficient function C_1^{NS} given by Eq. (2) in fourth order of QCD expansion, so that at $\alpha_s(2.5\ \text{GeV}^2)=0.35\pm0.04$ the right-hand side of Eq. (13) in Ref. [8] reads $C_1^{NS}|g_A/g_V|=1.01\pm0.05$ (just as in [8]). For details see [9], Sec. 5.5.4, Eq. (5.22), Appendix A.7, Eq. (A.44), and also [10], Sec. 2.5

[6]Notice that the HERMES result (7) differs by about 2 standard deviations even from this incorrect sum rule whose right-hand side reads $|g_A/g_V|\times C_1^{NS}(2.5\ \text{GeV}^2)=1.01\pm0.05$ (just as in [8]).

TABLE II. The HERMES results on $\Delta_1 q$ (the partially reproduced Table 1 of Ref. [8]).

	Measured region	Low x	Total integral
$\Delta_1 u+\Delta_1\bar{u}$	$0.51\pm0.02\pm0.03$	0.04	$0.57\pm0.02\pm0.03$
$\Delta_1 d+\Delta_1\bar{d}$	$-0.22\pm0.06\pm0.05$	-0.03	$-0.25\pm0.06\pm0.05$
$\Delta_1 s+\Delta_1\bar{s}$	$-0.01\pm0.03\pm0.04$	0.00	$-0.01\pm0.03\pm0.04$
$\Delta_1\bar{u}$	$-0.01\pm0.02\pm0.03$	0.00	$-0.01\pm0.02\pm0.03$
$\Delta_1\bar{d}$	$-0.02\pm0.03\pm0.04$	0.00	$-0.02\pm0.03\pm0.04$
Δq_3	$0.74\pm0.07\pm0.06$	0.07	$0.84\pm0.07\pm0.06$
Δq_8	$0.32\pm0.09\pm0.10$	0.01	$0.32\pm0.09\pm0.10$
$\Delta_1 u_V$	$0.52\pm0.05\pm0.08$	0.03	$0.57\pm0.05\pm0.08$
$\Delta_1 d_V$	$-0.19\pm0.11\pm0.13$	-0.03	$-0.22\pm0.11\pm0.13$

average $Q^2=10$ GeV^2, i.e., when LO QCD is a quite good approximation, the HERMES uses LO analysis to extract the polarized distributions from the respective asymmetries measured at relatively low average $Q^2=2.5$ GeV^2 value. So the inconsistency of HERMES result on Δq_3 with BSR can serve as a direct indication that LO analysis is not sufficient and NLO analysis is necessary at such conditions.

It is illustrative to show how one can arrive at the incorrect sum rule using the purity method at low average Q^2 value.

Since the application of this method with respect to SIDIS asymmetries is just LO QCD analysis, the first moments of the DIS structure functions $\Gamma_1^{p,n}$ have LO QCD expressions via HERMES distributions:

$$\Gamma_1^p(2.5\ \ \mathrm{GeV}^2)=\frac{1}{2}\sum_{q,\bar{q}} e_q^2\Delta_1 q(2.5\ \ \mathrm{GeV}^2), \quad \Gamma_1^n=\Gamma_1^p|_{u\leftrightarrow d}. \tag{8}$$

On the other hand, the exact expression for the physical (independently measurable) quantity $\Gamma_1^p-\Gamma_1^n$ has a form (1), where C_1^{NS} differs essentially from the LO value 1 at so low Q^2.

The extraction of the quark distributions from the SIDIS asymmetries in NLO order means that the respective DIS structure functions are expressed via these distributions as

$$g_1^p(x,Q^2)=\frac{1}{2}\sum_{q,\bar{q}} e_q^2\bigg(\Delta q+\frac{\alpha_s(Q^2)}{2\pi} \times[\delta C_q\otimes\Delta q+\delta C_g\otimes\Delta g]\bigg)(x,Q^2).$$

Then, using the explicit values of the first moments of the respective $\overline{MS}$ Wilson coefficients [6] $M^1(\delta C_q)=-2$, $M^1(\delta C_g)=0$, one gets in NLO QCD

$$M^1[g_1^p]\equiv\Gamma_1^p=\frac{1}{2}\sum_{q,\bar{q}} e_q^2\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\Delta_1 q, \quad \Gamma_1^n=\Gamma_1^p|_{u\leftrightarrow d}. \tag{9}$$

Substituting this on the left-hand side of Eq. (1) with C_1^{NS} given by Eq. (2) reduced to NLO QCD: $C_1^{NS}=1-\alpha_s/\pi$, one can see that α_s dependent multipliers $[1-\alpha_s(Q^2)/\pi]$ cancel out precisely in the left- and right-hand sides, so that one arrives at Eq. (3) without any logarithmic corrections in the right-hand side (see the Appendix).

Let us now analyze the results of Table 1 of Ref. [8] on $\Delta_1\bar{q}$. First of all notice that HERMES uses the assumption that the relative polarization of sea quarks is independent of flavor

$$\frac{\Delta\bar{u}}{\bar{u}}=\frac{\Delta\bar{d}}{\bar{d}}=\frac{\Delta\bar{s}}{\bar{s}}=\frac{\Delta s}{s}, \tag{10}$$

and this assumption is used to extract almost all first moments of the Table 1 of Ref. [8].[7] It is of importance that this assumption already implies the asymmetry of the polarized light sea quark distributions. Indeed, the equality $\Delta\bar{u}/\bar{u}=\Delta\bar{d}/\bar{d}$, together with the well known result[8] [1] $\bar{u}(x)\neq\bar{d}(x)$ immediately give rise to $\Delta\bar{u}\neq\Delta\bar{d}$. So, the results of Table 1 of Ref. [8] for light sea quarks should be asymmetric. However, taking the first moments of the polarized light sea quark distributions directly from Table 1, one gets

$$\Delta_1\bar{u}-\Delta_1\bar{d}=(-0.01+0.02)\pm0.061=0.01\pm0.061, \tag{11}$$

which is just zero within the errors.

This disagreement now seems to be not too surprising because the results of Table 1 of Ref. [8] do not satisfy the equivalents of BSR (3) and (5) [see discussion on Eq. (7)].

Let us now do some speculation assuming, for a moment, that at least the first moments of the valence quark distributions from Table 1 of Ref. [8] are close to the real ones [satisfying the real equivalents of BSR (3) and (5)]. Then, substituting values $\Delta_1 u_V$ and $\Delta_1 d_V$ taken from Table 1 into the BSR written in the form (5), one arrives at rather amazing result

$$\Delta_1\bar{u}-\Delta_1\bar{d}=0.235\pm0.097, \tag{12}$$

i.e., the quantity $\Delta_1\bar{u}-\Delta_1\bar{d}$ we are interested in, is not zero as compared with the total error (2.42 standard deviations), and the polarized sea of light quarks is asymmetric with respect to u and d quark polarized distributions.

Certainly, this is just a speculation based on the abovementioned assumption. We rather believe that all this is a direct indication that the HERMES data for asymmetries should be properly reanalyzed. First, the low-x region should

[7]Except for the quantity Δq_8^* (see comment for Table 1 of Ref. [8]) where the symmetric sea assumption $\Delta\bar{u}=\Delta\bar{d}=\Delta s=\Delta\bar{s}$ is used.

[8]Notice that the equation $\bar{u}\neq\bar{d}$ is implicitly used in [8] since it is included in the parametrization CTEQ low Q^2 applied for the data analysis.

be treated more carefully[9] and second, the NLO QCD procedure is necessary at so low Q^2 to properly extract so tiny quantities as $\Delta_1 s$ and $\Delta_1\bar{u}-\Delta_1\bar{d}$.

Besides, there is a good lesson here for another polarized SIDIS experiments, in particular, for the COMPASS experiment [11]. On the one hand, the low x_B boundary should be as small as possible to achieve the maximal accuracy for the first moments. On the other hand, it is extremely desirable to maximally increase the average Q^2 value in order to the simple LO analysis would become applicable. Otherwise, while the SIDIS asymmetries are measured at average Q^2 which is still about 2 GeV^2, the LO analysis is not sufficient and NLO analysis is necessary to get reliable polarized distributions consistent with the fundamental restrictions such as the Bjorken sum rule.

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APPENDIX: CANCELLATION OF NLO QCD CORRECTIONS IN EQUIVALENT OF BSR (3)

The NLO DIS proton structure function reads

$$g_1^p(x,Q^2)=\frac{1}{2}\sum_{q,\bar{q}} e_q^2\bigg(\Delta q^{(\mathrm{NLO})}+\frac{\alpha_s(Q^2)}{2\pi}[\delta C_q\otimes\Delta q^{(\mathrm{NLO})} + \delta C_g\otimes\Delta g^{(\mathrm{NLO})}]\bigg)(x,Q^2), \tag{A1}$$

where

$$(A\otimes B)(x)\equiv\int_x^1\frac{dy}{y}A\left(\frac{x}{y}\right)B(y)$$

is the convolution product. The $\overline{MS}$ nth Melin moments $M^n(f)\equiv\int_0^1 dx x^{n-1}f(x)$ of the Wilson coefficients $\delta C_{q,g}$ appear as [6]

$$\begin{aligned}\delta C_q^n &\equiv M^n[\delta C_q]\\ &= C_F\Bigg[-S_2(n)+[S_1(n)]^2+\left(\frac{3}{2}-\frac{1}{n(n+1)}\right)\\ &\quad\times S_1(n)+\frac{1}{n^2}+\frac{1}{2n}+\frac{1}{n+1}-\frac{9}{2}\Bigg],\end{aligned}$$

$$\delta C_g^n = 2T_f\left[-\frac{n-1}{n(n+1)}[S_1(n)+1]-\frac{1}{n^2}+\frac{2}{n(n+1)}\right],$$

where $C_F=4/3$, $T_f=3/2$ for the number of active flavors $f=3$, and $S_k(n)=\sum_{j=1}^n 1/j^k$. Then, for the first moments ($n=1$) of the Wilson coefficients, one has

$$\delta C_g^1\equiv M^1[\delta C_g]=\int_0^1 dx\,\delta C_g=0, \quad \delta C_q^1=-2. \tag{A2}$$

Taking the first moment of Eq. (A1), using the property of the Melin nth moments to split the convolution product into a simple product of the Melin moments of the respective functions: $M^n[C\otimes f]=M^n(C)M^n(f)$, and also Eq. (A2), one gets in NLO QCD

$$M^1[g_1^p]=\frac{1}{2}\sum_{q,\bar{q}} e_q^2\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\int_0^1 dx\Delta q^{(\mathrm{NLO})},$$

and the same for g_1^n with the replacement $u\leftrightarrow d$. Thus

$$\begin{aligned}\Gamma_1^p-\Gamma_1^n=\frac{1}{6}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)&[\Delta_1 u^{(\mathrm{NLO})}(Q^2)+\Delta_1\bar{u}^{(\mathrm{NLO})}(Q^2)\\ &-\{\Delta_1 d^{(\mathrm{NLO})}(Q^2)+\Delta_1\bar{d}^{(\mathrm{NLO})}(Q^2)\}].\end{aligned} \tag{A3}$$

Substituting Eq. (A3) into the left-hand side of BSR (1) with C_1^{NS} given by Eq. (2) reduced to NLO QCD: $C_1^{NS}=1-\alpha_s/\pi$, one can see that the α_s dependent multipliers $[1-\alpha_s(Q^2)/\pi]$ cancel out precisely in the left- and right-hand sides. So, one arrives at Eq. (3) without any Q^2 dependence:

$$\begin{aligned}\Delta q_3^{(\mathrm{NLO})}&=[\Delta_1 u^{(\mathrm{NLO})}(Q^2)+\Delta_1\bar{u}^{(\mathrm{NLO})}(Q^2)]\\ &\quad-[\Delta_1 d^{(\mathrm{NLO})}(Q^2)+\Delta_1\bar{d}^{(\mathrm{NLO})}(Q^2)]\\ &=\left|\frac{g_A}{g_V}\right|.\end{aligned}$$

[9]Indeed, the unmeasured low-x region of HERMES is $0<x_B<0.023$, and in this rather large region HERMES uses the simple Regge fit without the estimation of systematical errors.

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