QCD multi-jet processes and the multiplicity asymptotics

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Abstract

We will offer the formal arguments that the very high multiplicity (VHM) events are defined by the heavy QCD jets and, with exponential over multiplicity accuracy, the number of jets must decrease with the increasing multiplicity. This must rise the mean transverse momentum of secondaries with multiplicity. The effectiveness of the LLA in the VHM domain is also discussed.

1. The interest of experimentalists in the very high multiplicity (VHM) physics has grown for the last few years [1]. But in spite of all efforts there has not been any good quantitative theory of such events until now. The reason lies in the special kinematics of VHM events: the produced particles have approximately the same small momenta and for this reason the usage of LLA becomes problematic.

The purpose of the present paper is to give the formal arguments that (i) the hadron processes become hard in VHM domain and (ii) this tendency to the events hardness is conserved in the asymptotics over multiplicity.

2. It is useful to introduce the generating function

$$T(z,s;n_{max})=\sum_{n}^{\infty} z^n\sigma_n(s;n_{max}),\ \ \sigma_n(s;n_{max})=0 \text{ if } n>n_{max}, \tag{1}$$

where $n_{max}=\sqrt{s}/m$ is the maximal multiplicity of hadrons of mass m at given CM energy $\sqrt{s}$. It appears because of the energy-momentum conservation law. We will assume that the total energy $\sqrt{s}$ can be arbitrary large. The lepton and photon production is neglected.

It seems natural to omit the dependence from n_{max} if n is far from it: in the opposite case the dependence on the phase-space boundaries would be sizeable for all n. We will assume that z is so small that

$$n \ll n_{max} \tag{2}$$

are important in (1). Assuming (2), we can consider $T(z,s)$ as an *entire function* of z in the future.

One may turn (1) and define $\sigma_n(s)$ through $T(z,s)$ by the inverse Mellin transform:

$$\sigma_n(s)=\frac{1}{2\pi i}\oint\frac{dz}{z^{n+1}}T(z,s).$$

This integral can be calculated for large values of n by the stationary phase method. The corresponding saddle point is defined by the equation:

$$n=z\frac{\partial}{\partial z}\ln T(z,s). \tag{3}$$

Then the asymptotic estimation exists:

$$\sigma_n(s)\propto e^{-n\ln z_c(n,s)}, \tag{4}$$

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where $z_c(n,s)$ is the solution of (3). The estimation (4) exists if $z_c(n,s) > 1$, i.e. if n is larger than the mean multiplicity $\bar{n}(s)$; at the same time n can not be too large, see (2). The estimation (4) signifies that the quantity

$$\mu'(n,s) = -\frac{1}{n} \ln \sigma_n(s)/\sigma_{tot}(s) \tag{5}$$

is defined with the $O(1/n)$ accuracy only by the solution $z_c(n,s)$.

As follows from (3), with the increasing multiplicity $z_c(n,s)$ must tend to the singularities z_s of $T(z,s)$:

$$z_c(n,s) \to z_s \ . \tag{6}$$

We will assume that [2] $T(z,s)$ is regular in the complex z plane inside the unit circle (otherwise σ_n would be the increasing function of n inside the domain (2)). Keeping in mind (6), the estimation (4) means the following supposition:

(I). *The asymptotics over multiplicity is governed with the exponential accuracy by the leftmost singularity over* z.

It was shown [3] that the multiperipheral kinematics leads to $T(z,s)$ which is regular for the arbitrary finite z. For this reason the *soft* channel of hadrons production leads, as follows from (3), to $\mu'(n,s) \simeq \ln z_c(n,s)$ increasing with n. On the other hand the pQCD jets generating function gives the singularity at $z_s(s) - 1 \sim 1/\bar{n}_j(s) > 0$, where the mean multiplicity in the jet is $\bar{n}_j(s)$, $\ln \bar{n}_j(s) \propto \sqrt{\ln s}$ [4]. Therefore, for *hard* processes,

$$\mu'(n,s) = 1/\bar{n}_j(s) + O(1/n),$$

i.e. $\mu'(n,s)$ is the n independent quantity which *decreases* with the increasing energy.

We conclude that the VHM processes are mostly the hard processes and at first sight they can be described by the pQCD. Whether this is right or not is the question of our interest.

3. We will consider particles production in the "deep inelastic scattering" (DIS) kinematics. Let $\mathcal{F}_{ab}(x,q^2;\omega)$ be the generating functional:

$$\mathcal{F}_{ab}(x,q^2;\omega) = \sum_\nu \int d\Omega_\nu(k) \prod_{i=1}^{\nu} \omega^{r_i}(k_i^2) \left| a_{ab}^{r_1 r_2 \ldots r_\nu}(k_1,k_2,...,k_\nu) \right|^2 ,$$

where $a_{ab}^{r_1 r_2 \ldots r_\nu}$ is the production amplitude of ν partons ($r_i = (q,\bar{q},g)$) with momenta $(k_1,k_2,...,k_\nu)$ in the process of scattering of the parton a on the parton b; $d\Omega_\nu(k)$ is the phase space element; $\omega^r(k^2)$ is the "probe function", i.e. the correlation functions

$$N_{ab}^{r_1 r_2 \ldots r_\nu}(k_1^2,k_2^2,...,k_\nu^2;x,q^2) = \prod_{i=1}^{\nu} \frac{\delta}{\delta\omega^{r_i}(k_i^2)} \ln \mathcal{F}_{ab}(x,q^2;\omega) \bigg|_{\omega=1} .$$

The generating functional is normalized on the DIS structure function $\mathcal{D}_{ab}(x,q^2)$,

$$\mathcal{F}_{ab}(x,q^2;\omega=1) = \mathcal{D}_{ab}(x,q^2),$$

which is described in the leading logarithm approximation (LLA) by the ladder diagrams. We will consider the approximation when the cutting line passes only through the "steps" of the ladder diagram. In this case $\mathcal{D}_{ab}(x,q^2)$ has a meaning of the *probability* to find the parton a in the parton b.

Considering the VHM region, one must assume that the mass $|k_i|$ of the produced parton is large. For instance $k_i^2 \gg \lambda^2$, where λ is the virtuality of the parton of the pre-confinement phase. Then

$$\ln k_i^2 = \ln |q_{i+1}^2| \left(1 + O\left(\frac{\ln(1/x_i)}{\ln |q_{i+1}^2|}\right)\right). \tag{7}$$

The LLA means that

$$\lambda^2 \ll |q_{i+1}^2| \ll |q_i^2| \ll |q^2|. \tag{8}$$

Therefore, $|k_i|$ are also strongly ordered. The approximation (7) means that the produced partons longitudinal momenta are small:

$$\ln(1/x_i) < \ln |q_{i+1}^2|. \tag{9}$$

It is useful to consider the Laplace image over $\ln(1/x)$:

$$\mathcal{F}_{ab}(x, q^2; \omega) = \int \frac{dj}{2\pi i} \left(\frac{1}{x}\right)^j \mathbf{F}_{ab}(j, q^2; \omega). \tag{10}$$

Then, taking into account the above mentioned conditions, one may find the DGLAP evolution equation:

$$t\frac{\partial}{\partial t}\mathbf{F}_{ab}(j, t; \omega) = \sum_{c,r} \varphi_{ac}^r(j)\omega^r(t)\mathbf{F}_{cb}(j, t; \omega), \tag{11}$$

where $t = \ln(|q^2|/\Lambda^2)$,

$$\varphi_{ac}^r(j) \equiv \varphi_{ac}(j) = \int_0^1 dx\ x^{j-1}\ P_{ac}^r(x)$$

and $P_{ac}^r(x)$ are the regular kernels of the Bete-Solpiter equation for pQCD [5]. The equation (11) coincides at $\omega^r = 1$ with the habitual equation for Laplace transform of the structure function $\mathcal{D}_{ab}(x, q^2)$. While the equation (11) was being derived, only one additional assumption had been used for our problem $\omega^r = \omega^r(k^2)$.

The dominance of the gluon contributions for the case $x \ll 1$ will be used and for this reason all parton indexes will be omitted. One may find the solution of (11) in terms of the ν-gluon correlation functions $N^{(\nu)}$. Omitting the t dependence in the renormalized constant α_s, let us write:

$$\mathbf{F}(j, t; \omega) = \mathbf{D}(j, t) \exp\left\{\sum_\nu \frac{1}{\nu!} \int \prod_{i=1}^{\nu} dt_i(\omega(t_i) - 1)N^{(\nu)}(t_1, t_2, ..., t_\nu; x, t)\right\}.$$

In the VHM domain, where $x \ll 1$ is important, one must consider $(j-1) \ll 1$. Then

$$N^{(1)}(t_1; j, t) = \varphi(j) \sim \frac{1}{j-1} \gg 1.$$

The second correlator

$$N^{(2)}(t_1, t_2; j, t) = O\left(\max\left\{\left(\frac{t_1}{t}\right)^{\varphi(j)}, \left(\frac{t_2}{t}\right)^{\varphi(j)}, \left(\frac{t_1}{t_2}\right)^{\varphi(j)}\right\}\right)$$

is negligible at $(j-1) \ll 1$. Therefore, in the LLA,

$$\mathbf{F}(j, t; \omega) = \mathbf{D}(j, t) \exp\left\{\varphi(j) \int_{t_0}^{t} dt_1(\omega(t_1) - 1)\right\}.$$

Taking $\omega(t) = const$, one may find that $\mathbf{F}(j,t;\omega)$ has the Poisson distribution with the "mean multiplicity" $\sim \varphi(j)t$.

If the time necessary to confine a parton into the hadron is $\sim (1/\lambda)$, then the parton of the mass (virtuality) $|k| \gg \lambda$ must decay on the partons of lower mass since its life time is $\sim (1/|k|) << (1/\lambda)$. This is the reason of the pQCD jets formation.

If the quantity

$$\omega(t,z),\ \omega(t,1) = 1,\ t = \ln k^2/\lambda^2,$$

is the generating function of the pre-confinement partons multiplicity distribution

$$\omega_n(t) = \frac{\partial^n}{\partial z^n}\omega(t,z)\bigg|_{z=0},$$

then, as follows from derivation of $\mathbf{F}(j,t;\omega)$, the quantity

$$\mathbf{F}(j,t;\omega) = \mathbf{D}(j,t)\exp\left\{\frac{1}{j-1}\int_{t_0}^{t} dt(\omega(t,z)-1)\right\} \tag{12}$$

will describe the pre-confinement partons multiplicity distribution in the frame of LLA.

Inserting (12) into the integral (10), one can find that, if we will use the designation:

$$\bar{\omega}(t,z) \equiv \int_{t_0}^{t} dt\ \omega(t,z),$$

$j-1 = \{\bar{\omega}(t,z)/\ln(1/x)\}^{1/2}$ are essential in it. So, to justify the LLA approximation, one should assume that the "mobility"

$$\{\ln(1/x)/\bar{\omega}(t,z)\} \gg 1 \tag{13}$$

decreases with z or, it is the same, with the multiplicity n. This is the reason why the LLA for considered DIS kinematics has a restricted range of validity in the VHM region.

Nevertheless, in the frame of LLA conditions, as follows from (12), the generating functional $\mathcal{F}_{ab}(x,t;z)$ has the following estimation:$\ln\mathcal{F}_{ab}(x,t;z) \propto \{\ln(1/x)\bar{\omega}(t,z)\}^{1/2}$. Therefore, since the coupling is a constant, $\ln\mathcal{F}_{ab}(x,t;z=1) = \ln\mathcal{D}_{ab}(x,t) \propto (t\ln(1/x))^{1/2}$. This is a well known result.

4. The higher correlation functions $N_{ab}^{r_1 r_2 \ldots r_\nu}(t_1,t_2,...,t_\nu;x,q^2)$ must be taken into account if z is large: the smallness of this correlation functions can be compensated by the large value of $\omega^r(t,z)$. This leads to dominance of the multi-jet processes with jets of approximately equal masses. Such kinematics is outside of the LLA. Let us consider it in detail.

It is known [6] that $\omega(t,z)$ is singular at

$$z_s - 1 \sim 1/\bar{n}_j(t), \tag{14}$$

where $\bar{n}_j(t)$ is the mean multiplicity in the jet: $\ln\bar{n}_j(t) \sim \sqrt{t}$. As follows from (4), the singularity (14) gives the estimation: $\mu' = 1/\bar{n}_j(t) + O(1/n)$, i.e. the character of singularity is not important with $O(1/n)$ accuracy.

This means that, for instance, the multi-jet event with $k>1$ gives the same contribution to μ'. The difference may appear only in the pre-exponential factor. But actually the difference will appear since the singularity moves with t, $z_s = z_s(t)$. Indeed, if the energy conservation law is taken into account and if the produced k jets have the same energy, then $\bar{n}_j^{(k)}(t) < \bar{n}_j(t)$. Consequently, $z_s^{(k)})(t) > z_s(t)$ and $\mu_j'^{(k)} > \mu_j'$. This means that the heavy jet productions mechanism would dominate in the VHM region in accordance with the proposition **(I)**.

5. The above derived result means the increase of the mean transverse energy with the multiplicity n. The continuation of this result into the asymptotics over multiplicity leads to the deduction that only two heavy jets with masses $\sim \sqrt{s}/2$ are produced in the deep asymptotics over n. However, the value of such a process would be extremely small, $\propto (\alpha_s(s)/s)$.

Therefore, according to **(I)** it is most probable to expect the increase of the transverse energy with multiplicity, the experimental confirmation of this result was given in [7] , but apparently the limiting case of two-jet kinematics will never be exceeded.

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