Dual-Resonance Model and Multiple Production

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Abstract

The very high multiplicity (VHM) processes are considered to investigate consequences of the dual resonance models reach (exponential) mass spectrum. The virial decomposition is developed for description of the produced particles short-rang (resonance) correlations. It is shown that the reach mass spectrum is able to do more flat the VHM distribution: $\sigma_n = O(e^{-n})$ up to multiplicities $n \simeq \bar{n}^2$, $\bar{n}$ is the mean multiplicity. But for $n >> \bar{n}^2$ one should expect $\sigma_n < O(e^{-n})$.

1 Introduction

The last two decades of field theory development is marked by considerable efforts to avoid the problem of color charge confinement formulating a closed hadron field theory. The remarkable attempt based on the string model, in its various realizations [1]. But, in spite of remarkable success (in formalism especially) there is not an experimentally measurable predictions of this approach till now, e.g. [3].

The string model is a natural consequence of the old dual resonance model [2] and we hope that our toy approach includes main characteristic features of this model. We would like to describe in this paper production of 'stable' hadrons through decay of resonances. This channel was considered firstly in the papers [4].

Our consideration will use following assumptions.

A. The string interpretation of the dual-resonance model bring to the observation that the mass spectrum of resonances, i.e. the total number $\rho(m)$ of mass m resonance excitations, grows exponentially:

$$\rho(m) = (m/m_0)^{\gamma} e^{\beta_0 m}, \; \beta_0 = \text{const}, \; m > m_0. \qquad (1.1)$$

Note also that the same hadron mass spectrum (1.1) was predicted in the 'bootstrap' approach [5]. It predicts that

$$\gamma = -5/2. \qquad (1.2)$$

Another assumptions are based on the ordinary (resonance $\leftrightarrow$ Regge pole) duality.

B. The 'probability' of mass m resonance creation $\sigma^R(m)$ has the Regge pole asymptotics:

$$\sigma^R(m) = g^R \frac{m_0}{m}, \; m \geq m_0 \approx 0.2 \text{Gev}, \; g^R = \text{const}. \tag{1.3}$$

It was assumed here that the intercept of Regge pole trajectory $\alpha^R = 1/2$. So, the meson resonances only would be taken into account.

C. If $\sigma_n^R(m)$ describes decay of mass m resonance on the n hadrons, then the mean multiplicity of hadrons

$$\bar{n}^R(m) = \frac{\sum_n n \sigma_n^R(m)}{\sigma^R(m)}. \tag{1.4}$$

Following to the Regge model,

$$\bar{n}^R(m) = \bar{n}_0^R \ln \frac{m^2}{m_0^2}. \tag{1.5}$$

D. We will assume that there is a definite vicinity of $\bar{n}^R(m)$ where $\sigma_n^R(m)$ is defined by $\bar{n}^R(m)$ only. I.e. in this vicinity

$$\sigma_n^R(m) = \sigma^R(m) e^{-\bar{n}^R(m)} (\bar{n}^R(m))^n / n!. \tag{1.6}$$

This is the direct consequence of the Regge pole model, if m/m_0 is high enough.

The connection between S-matrix approach and the real-time statistics (finite temperature field theories) [8] will be used to formulate our model quantitatively. This interpretation will be useful since it allows to formulate the description in terms of a few parameters only. All this statistical parameters are expressed through created particles energies and momenta.

We will use also the virial decomposition technique. It was extremely effective for description of the phase transitions critical region, where the correlation radii tends to infinity. The Mayer' decomposition over 'connected groups' is well known in this connection [6].

Following to our idea, we will distinguish the short-range correlations among hadrons and the long-range correlations among resonances. The 'connected groups' would be described by resonances (strings) and the interactions among them should be described introducing for this purpose the correlation functions among strings. So, we will consider the 'two-level' model

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of hadrons creation: first level describes the short-range correlation among hadrons and the second level is connected to the correlations among strings.

2   Asymptotic estimations

Our purpose is to investigate the role of exponential spectrum (1.1) in the asymptotics over multiplicity n. In this case one can valid heavy resonances creation and such formulation of the problem have definite advantage.

(i) If creation of heavy resonances at $n \to \infty$ is expected, then one can neglect the dependence on resonances momentum $\mathbf{q_i}$. So, the 'low-temperature' expansion is valid in the VHM region.

(ii) Having the big parameter n one can construct the perturbations expanding over $1/n$. We will see that there is a wide domain for n, where one can neglect resonance correlations.

(iii) We will be able to show at the end the range of applicability of this assumptions. For this purpose following formal phenomena will be used. Let us introduce the 'grand partition function'

$$\Xi(z, s) = \sum_n z^n \sigma_n(s), \ \Xi(1, s) = \sigma_{tot}(s), \ n \le \sqrt{s}/m_0 \equiv n_{max}(s), \qquad (2.7)$$

and let us assume that just Ξ is known. Then, using the inverse Mellin transformation,

$$\sigma_n(s) = \frac{1}{2\pi i} \int \frac{dz}{z^{n+1}} \Xi(z, s). \qquad (2.8)$$

This integral will be computed expanding it in vicinity of solution $z_c > 0$ of equation:

$$n = z \frac{\partial}{\partial z} \ln \Xi(z, s). \qquad (2.9)$$

It is assumed, and this should be confirmed at the end, that the fluctuations in vicinity of z_c are Gaussian.

It is natural at first glance to consider $z_c = z_c(n, s)$ as the increasing function of n. Indeed, this immediately follows from positivity of $\sigma_n(s)$ and finiteness of $n_{max}(s)$ at finite s. But one can consider the limit $m_0 \to 0$. Theoretically this limit is rightful because of PCAC hypotheses and nothing should be happen if the pion mass $m_0 \to 0$. In this sense $\Xi(z, s)$ may be considered as the whole function of z. Then, $z_c = z_c(n, s)$ would be increasing function of n if and only if $\Xi(z, s)$ is regular function at $z = 1$.

The prove of this statement is as follows. We should conclude, as follows from eq.(2.9), that

$$z_c(n,s) \to z_s \text{ at } n \to \infty, \text{ and at } s = const, \qquad (2.10)$$

i.e. the singularity points z_s *attracts* z_c in asymptotics over n.

If $z_s = 1$, then $(z_c - z_s) \to +0$, when n tends to infinity [7]. But if $z_s > 1$, then $(z_c - z_s) \to -0$ in VHM region.

On may find the estimation:

$$-\frac{1}{n} \ln \frac{\sigma_n(s)}{\sigma_{tot}(s)} = \ln z_c(n,s) + O(1/n), \qquad (2.11)$$

where z_c is the *smallest* solution of (2.9). It should be underlined that this estimation is *independent* on the character of singularity, i.e. the position z_s only is important.

3   Partition function

Introducing the 'grand partition function' (2.7) the 'two-level' description means that

$$\ln \frac{\Xi(z,\beta)}{\sigma_{tot}(s)} = -\beta \mathcal{F}(z,s) =$$

$$= \sum_k \frac{1}{k!} \int \prod_{i=1}^{k} \left\{ \frac{d^3 q_i dm_i \xi(q_i, z) e^{-\beta \varepsilon_i}}{(2\pi)^3 2\varepsilon_i} \right\} N_k(q_1, q_2, ..., q_k; \beta), \qquad (3.12)$$

where $\varepsilon_i = \sqrt{q_i^2 + m_i^2}$. This is our virial decomposition. Indeed, by definition

$$\Xi(z,s)\big|_{\xi=1} = \sigma_{tot}(s). \qquad (3.13)$$

The quantity $\xi(q,z)$ may be considered as the local activity. So,

$$\frac{\delta \Xi}{\delta \xi(q,z)}\bigg|_{\xi=1} \sim \sigma_{tot} N_1(q) \qquad (3.14)$$

So, if decay of resonances form a group with 4-momentum q, then $N_1(q)$ is the mean number of such groups. The second derivative gives:

$$\frac{\delta^2 \Xi}{\delta \xi(q_1, z) \delta \xi(q_2, z)}\bigg|_{\xi=1} \sim \sigma_{tot}\{N_2(q_1, q_2) - N_1(q_1)N_1(q_2)\} \equiv \sigma_{tot} K_2(q_1, q_2)$$

$$(3.15)$$

where $K_2(q_1, q_2)$ is two groups correlation function, and so on.

The Lagrange multiplier β was introduced in (3.12) to each resonance: the Bolzmann exponent $\exp\{-\beta\varepsilon\}$ takes into account the energy conservation law $\sum_i \varepsilon_i = E$, where E is the total energy of colliding particles, $2E = \sqrt{s}$ in the CM frame. This conservation law means that β is defined by equation:

$$\sqrt{s} = \frac{\partial}{\partial\beta}\ln\Xi(z,\beta). \tag{3.16}$$

So, to define the state one should solve two equations of state (2.9) and (3.16).

The solution β_c of the eq.(3.16) have meaning of inverse temperature of gas of strings if and only if the fluctuations in vicinity of β_c are Gaussian.

On the second level we should describe the resonances decay onto hadrons. Using (1.6) we can write in some vicinity of $z = 1$:

$$\xi(q,z) = \sum_n z^n \sigma_N^R(q) = g^R(\frac{m_0}{m})e^{(z-1)\bar{n}(m)}, \quad m = |q|. \tag{3.17}$$

The assumptions **B** and **D** was used here.

So,

$$-\beta\mathcal{F}(z,s) = \sum_k \int \prod_{i=1}^{k}\{dm_i^2\xi(m_i,z)\}\tilde{N}_k(m_1,m_2,...,m_k;\beta), \tag{3.18}$$

where ξ was defined in (3.17) and

$$\tilde{N}_k(m_1,m_2,...,m_k;\beta) = \int \prod_{i=1}^{k}\left\{\frac{d^3q_i e^{-\beta\varepsilon_i(q_i)}}{2\varepsilon_i(q_i)}\right\} N_k(q_1,q_2,...,q_k;m_1,m_2,...,m_k). \tag{3.19}$$

Assuming now that $|q_i| << m$ are essential,

$$\tilde{N}_k(m_1,m_2,...,m_k;\beta) \simeq N_k'(m_1,m_2,...,m_k)\prod_{i=1}^{k}\left\{\sqrt{\frac{2m_i}{\beta^3}}e^{-\beta m_i}\right\} \tag{3.20}$$

Following to the duality assumption one may assume that

$$N_k'(m_1,m_2,...,m_k) = \bar{N}_k(m_1,m_2,...,m_k)\prod_{i=1}^{k}\left\{m_i^\gamma e^{\beta_0 m_i}\right\} \tag{3.21}$$

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and $\bar{N}_k(m_1, m_2, ..., m_k)$ is slowly varying function:

$$\bar{N}_k(m_1, m_2, ..., m_k) \simeq C_k$$

In result the low-temperature expansion looks as follows:

$$- \beta \mathcal{F}(z, s) = \sum_k \frac{2^{k/2} m_0^k (g^R)^k C_k}{\beta^{3k/2}} \left\{ \int_{m_0}^{\infty} dm\, m^{\gamma + 3/2} e^{(z-1)\bar{n}^R(m) - (\beta - \beta_0)m} \right\}^k .$$

(3.22)

We should assume that $(\beta - \beta_0) \geq 0$. In this sense one may consider $1/\beta_0$ as the limiting temperature and above mentioned constraint means that the string energies should be high enough.

4   Thermodynamical parameters

Remembering that the position of singularity over z is essential only, let us assume that the resonance interactions can not renormalize it. Then, living first term in the sum (3.22),

$$- \beta \mathcal{F}(z, s) = \frac{m_0 g^R C_1}{\beta^{3/2}} \int_{m_0}^{\infty} dm\, (m/m_0)^{\gamma + 3/2} e^{(z-1)\bar{n}^R(m) - (\beta - \beta_0)m} .$$

(4.23)

We expect that this assumption is hold if

$$n \to \infty, \; s \to \infty, \; \frac{n m_0}{\sqrt{s}} \equiv \frac{n}{n_{max}} << 1.$$

(4.24)

So, we would solve our equations of state with following 'free energy':

$$- \beta \mathcal{F}(z, s) = \frac{\alpha}{\beta^{3/2}} \int_{m_0}^{\infty} d(\frac{m}{m_0}) (\frac{m}{m_0})^{\gamma' - 1} e^{-\Delta(m/m_0)},$$

(4.25)

where, using (1.2),

$$\gamma' = \gamma + 2(z - 1)\bar{n}_0^R + 5/2 = 2(z - 1)\bar{n}_0^R, \; \Delta = m_0(\beta - \beta_0) \geq 0, \; \alpha = const.$$

(4.26)

We have in terms of this new variables following equation for z,

$$n = z \frac{2\alpha \bar{n}_0^R}{\beta^{3/2}} \frac{\partial}{\partial \gamma'} \frac{\Gamma(\gamma', \Delta)}{\Delta^{\gamma'}}.$$

(4.27)

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The equation for β takes the form:

$$n_{max} = \frac{\alpha m_0}{\beta^{3/2}} \frac{\Gamma(\gamma' + 1, \Delta)}{\Delta^{\gamma'+1}},$$ (4.28)

where $n_{max} = (\sqrt{s}/m_0)$ and $\Gamma(\Delta, \gamma')$ is the incomplete Γ-function:

$$\Gamma(\gamma', \Delta) = \int_{\Delta}^{\infty} dx \, x^{\gamma'-1} e^{-x}.$$

5 Asymptotic solutions

Following to physical intuition one should expect the cooling of the system when $n \to \infty$ (at fixed $\sqrt{s}$) and heating when $n_{max} \to \infty$ (at fixed n). But, as was mentioned above, since the solution of eq.(4.28) β_c is defined the value of total energy, one should expect that β_c decrease in both cases. So, the solution

$$\Delta_c \geq 0, \quad \frac{\partial \Delta_c}{\partial n} < 0 \text{ at } n \to \infty, \quad \frac{\partial \Delta_c}{\partial s} < 0 \text{ at } s \to \infty$$ (5.29)

is natural for our consideration.

The physical meaning of z is activity. It defines at $\beta = const$ the work needed for one particles creation. Then, if the system is stable and $\Xi(z, s)$ may be singular at $z > 1$ only,

$$\frac{\partial z_c}{\partial n} > 0 \text{ at } n \to \infty, \quad \frac{\partial z_c}{\partial s} < 0 \text{ at } s \to \infty.$$ (5.30)

One should assume solving equations (4.27) and (4.28) they

$$z_c \Delta^{\gamma'_c+1} \frac{\partial}{\partial \gamma'_c} \frac{\Gamma(\gamma'_c, \Delta_c)}{\Delta_c^{\gamma'_c}} << \Gamma(\gamma'_c + 1, \Delta_c).$$ (5.31)

This condition stands the physical requirement that $n << n_{max}$. In opposite case the finiteness of the phase space for $m_0 \neq 0$ should be taken into account.

As was mentioned above the singularity z_s attracts z_c at $n \to \infty$. By this reason one may consider following solutions.

A. $z_s = \infty$: $z_c >> \Delta$, $\Delta << 1$.

In this case

$$\Delta^{-\gamma'} \Gamma(\gamma', \Delta) \sim e^{\gamma' \ln(\gamma'/\Delta)}.$$ (5.32)

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This estimation gives following equations:

$$n = C_1 \gamma' \ln(\gamma'/\Delta) e^{\gamma' \ln(\gamma'/\Delta)},$$

$$\frac{n}{n_{max}} = C_2 \Delta \gamma' \ln(\frac{\gamma'}{\Delta}) << 1, \tag{5.33}$$

where $C_i = O(1)$ are the unimportant constants. The inequality is consequence of (5.31).

This equations have following solutions:

$$\Delta_c \simeq \frac{n}{n_{max} \ln n} << 1, \ \gamma'_c \sim \ln n >> 1. \tag{5.34}$$

Using this solution one can see from (2.11) that it gives

$$\sigma_n < O(e^{-n}). \tag{5.35}$$

B. $z_s = +1$: $z_c \to 1$, $\Delta_c << 1$.

One should estimate $\Gamma(\gamma', \Delta)$ near the singularity at $z = 1$ and in vicinity of $\Delta = 0$ to consider the consequence of this solution. Expanding $\Gamma(\gamma', \Delta)$ over Δ at $\gamma' \to 0$,

$$\Gamma(\gamma', \Delta) = \Gamma(\gamma') - \Delta^{\gamma'} e^{-\Delta} + O(\Delta^{\gamma'+1}) \simeq \frac{1}{\gamma'} + O(1). \tag{5.36}$$

This gives following equations for γ':

$$n = C'_1 \frac{\gamma' \ln(1/\Delta) - 1}{\gamma'} e^{\gamma' \ln(1/\Delta)}. \tag{5.37}$$

The equation for Δ has the form:

$$n_{max} = C'_2 e^{(\gamma'+1) \ln(1/\Delta)}. \tag{5.38}$$

Where $C'_i = O(1)$ are unimportant constants.

At

$$0 < \gamma' \ln(1/\Delta) - 1 << 1, \ \text{i.e. at} \ \ln(1/\Delta) << n << \ln^2(1/\Delta), \tag{5.39}$$

we find:

$$\gamma'_c \sim \frac{1}{\ln(1/\Delta_c)}. \tag{5.40}$$

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Inserting this solution into (5.38):

$$\Delta_c \sim \frac{1}{n_{max}}. \tag{5.41}$$

It is remarkable that Δ_c in the leading approximation is n independent. By this reason γ_c' becomes n independent also:

$$\gamma_c' \sim \frac{1}{\ln(n_{max})} \; : \; z_c = 1 + \frac{1}{\bar{n}_0^R \ln(n_{max})}. \tag{5.42}$$

This means that

$$\sigma_n = O(e^{-n}) \tag{5.43}$$

and obey the KNO scaling with mean multiplicity $\bar{n} = \bar{n}_0^R \ln(n_{max})$.

6 Conclusion

Comparing A and B solutions we can see the change of attraction points with rising n: at $n \simeq \bar{n}^2(s) = \bar{n}_0^R \ln(\sqrt{s}/m_0)$ the transition from (5.43) asymptotics to (5.35) should be seen. At the same time one should see the strong KNO scalings violation at the tail of multiplicity distribution.

The comparison of considered above model with experimental data at moderate and high energies will be given in subsequent papers.

We have neglect the strings interactions and the final state particles interactions deriving this results. This assumption seems natural since $z_c - 1 << 1$ is essential at $\bar{n} << n << \bar{n}^2$. By this reason one can neglect higher powers of $(z_c - 1)$ in expansion of $\ln \xi(z, m)$ over $(z_c - 1)$. Therefore, describing $\xi(x, m)$ we may restrict ourselves by Poisson distribution (1.6).

At the same time, at first glance, we can not neglect in (3.22) the contributions with $k > 1$ in the moderate region $\bar{n} << n << \bar{n}^2$. Indeed, k-th order in (3.22) is

$$\sim \Gamma^k(\gamma_c', \Delta_c) \sim (\frac{1}{\gamma_c'})^k \sim (\ln \Delta_c)^k \sim (\ln n_{max})^k \sim (\ln(s/m_0^2))^k >> 1$$

Nevertheless it can be shown that the higher terms with $k > 1$ can not change our semiqualitative conclusion. This question will be considered later.

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