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Multiplicity distribution tails at high energies

E.A.Kuraev, J.Manjavidze[1], *A.Sissakian*
JINR, Dubna, Russia

Abstract

The idea that the hard channels may dominate in the very high multiplicity processes is investigated. Quantitative realization of the 'hard Pomeron', deep inelastic scattering and large-angle annihilation mechanism combinations are considered in the pQCD frame for this purpose.

1 Introduction

Investigation of the multiplicity distributions was popular since seventies [1]. The modern hadron theory based on the local QCD Lagrangians [2] and the experimental consequences was given in the review papers [3]. The very high multiplicity (VHM) processes, as the attempt to get beyond this standard multiperipheral hadron physics, was offered in [4]. It considered as a possible physical program for LHC experiments.

It was shown in seventieth that the multiperipheral kinematics dominates inclusive cross sections. Moreover, the created particles spectra do not depend on s at high energies in the multiperipheral region:

$$f(s,p_c) = 2E_c\frac{d\sigma}{d^3p_c} = \int\frac{dt_1dt_2s_1s_2\phi_1(t_1)\phi_2(t_2)}{(2\pi)^2s(t_1-m^2)^2(t_2-m^2)^2},\ s_1s_2 = sE^2_{c\perp}, E^2_{c\perp} = m^2_c + \vec{p}^{\,2}_{c\perp}.$$

Here $s_1 = (p_a+p_c)^2, s_2 = (p_b+p_c)^2$, and $\phi_i(t_i)$ are the impact factors of hadrons. So the particle c forgot the details of its creation. It was found experimentally that the ratio

$$\frac{f(\pi^+p\to\pi_-+...)}{\sigma(\pi^+p)} = \frac{f(K^+p\to\pi_-+...)}{\sigma(K_+p)} = \frac{f(pp\to\pi_-+...)}{\sigma(pp)} \tag{1.1}$$

is universal [5]. This take place due to the two Pomeron multiperipheral exchange providing the nonvanishing contribution in the s asymptotics to the cross section. It was implied that the Pomeron intercept is exactly equal to one. Just this kinematics leads to $c_m = \gamma_m(c_1)^m$ (the correlators c_m are introduced in (1.8)), i.e. to the KNO-scaling [6].

We begin with general analysis to formulate an aim of this paper. Let $\sigma_n(s)$ be the cross section of n particles (hadrons) creation at the total CM energy $\sqrt{s}$. We introduce the generating function:

$$T(s,z) = \sum_{n=1}^{n_{max}} z^n\sigma_n(s),\ s = (p_1+p_2)^2 >> m^2,\ n_{max} = \sqrt{s}/m. \tag{1.2}$$

[1] Permanent address: Inst.of Phys., Tbilisi, Georgia

So, the total cross section and the averaged multiplicity will be:

$$\sigma_{tot}=T(s,1)=\sum\sigma_n,\ \ \sigma_{tot}\bar{n}=\sum n\sigma_n=\left.\frac{d}{dz}T(s,z)\right|_{z=1}. \tag{1.3}$$

At the same time

$$\sigma_n=\frac{1}{2\pi i}\oint\frac{dz}{z^{n+1}}T(s,z)=\frac{1}{2\pi i}\oint\frac{dz}{z}e^{(-n\ln z+\ln T(s,z))}. \tag{1.4}$$

The essential values of z in this integral are defined by the equation (of state):

$$n=z\frac{\partial}{\partial z}\ln T(z,s). \tag{1.5}$$

Considering the tail, i.e. $n>>\bar{n}$, let us assume that one can find such values of $n<<n_s$ at high energies $\sqrt{s}>>m$ that we can neglect in (1.2) dependence on the upper boundary n_{max}. This formal trick allows to consider $T(z,s)$ as the nontrivial function of z. Then the asymptotics over n ($n<<n_s$ is assumed) is governed by the mostleft situated singularity z_s of $T(z,s)$:

$$\sigma_n(s)\propto e^{-n\ln z_c(n,s)}, \tag{1.6}$$

where $z_c(n,s)$ is the smallest solution of eq.(1.5). It is important that

$$z_s(n,s)\rightarrow z_c \text{ at } n\rightarrow\infty. \tag{1.7}$$

One can put this method of asymptotic estimation in the basis of VHM processes phenomenology.

We may distinguish following possibilities at $n\rightarrow\infty$:
1)$z_s=1$: $\sigma_n>O(e^{-n})$;
2)$z_s=\infty$: $\sigma_n<O(e^{-n})$;
3)$i<z_s<\infty$: $\sigma_n=O(e^{-n})$.

The asymptotics 1) assumes the condensation phenomena [9]. The asymptotics 2) belong to the multiperipheral processes kinematics: created particles form jets moving in the CM frame with different velocities along the incoming particles directions, i.e. with restricted transverse momentum. The third type asymptotics is predicted by stationary Markovian processes with the QCD jets kinematics of the high transverse momentum particles creation. It is evident that the n asymptotics should be governed by largest among 1) - 3). Just under this idea we hope that we get beyond the multiperipheral kinematics in the VHM region.

Additional information is coded in the expansion over binomial moments $c_m(s)$:

$$\ln T(s,z)=\sum\frac{(z-1)^m}{m!}c_m(s). \tag{1.8}$$

So, for instance, if all $c_m=0,\ m>1$, then we have the Poisson distribution:

$$\sigma_n=\sigma_{tot}\frac{(\bar{n})^n}{n!}e^{(-\bar{n})}.$$

But if $c_m = \gamma_m (c_1)^m$, where γ_m is the some constant, then the so called KNO scaling take place:

$$\bar{n}\sigma_n = \sigma_{tot}\Psi(n/\bar{n}).$$

The multiple production processes are typical inelastic reactions of the initial kinetic energy dissipation into the particles mass. Consequently, the mean multiplicity $\bar{n}(s)$ is the measure of entropy $\mathcal{S}$ production at given energy. Experimentally $\bar{n}(s) \leq \ln^2 s << n_{max}$. This testify to the incomplete energy dissipation in the mostly probable channels of hadrons production.

This phenomena is explained naturally by presence of the space-time local non-Abelian symmetry constraints. So, it is known [8] that there is not thermalization phenomena in the completely integrable systems. But, at all evidence, the quantum Yang-Mills theory is not completely integrable, i.e. it admits the dissipation, but nevertheless the symmetry constraints play essential role.

It is natural to assume that $\mathcal{S}$ exceed its maximum if $n >> \bar{n}(s)$. So, our essentially inelastic process is happened so rapidly that the non-Abelian symmetry constraints becomes frozen. On other hand, maximum of entropy $\mathcal{S}$ means that the final state of the dissipation process is equilibrium.

Last one means relaxation of energy correlations, i.e. absence of the macroscopical energy flows in the system, and the Gauss energy spectrum of created particles. We would like to say that in such a state one get to the VHM.

The aim of this article is to build the suitable mechanism of maximal initial energy dissipation, where the correlations are relaxed and the energy spectra are Gaussian.. The classification 1)-3) is mostly general and we will put it in the basis of consideration. So, our main purpose is to show as the multiperipheral kinematics transform in the VHM domain.

2 Pomeron,DIS and Double-Logarithmic kinematics

Let us consider process of type $2 \to 2 + n$ in different kinematics

$$A(P_1) + B(P_2) \to A'(p_1') + B'(p_2') + h_1(k_1) + h_2(k_2) + \cdots + h_n(k_n), s = (p_1 + p_2)^2 >> m^2.$$

We will distinguish the peripheral,deep inelastic and large-angles kinematical regions with different physical content. First we build two light-like 4-momenta from the momenta of initial particles $p_{1,2} = P_{1,2} - P_{2,1}m^2_{2,1}/s$ and present the 4-momenta of final particles in form:

$$p_1' = \alpha_1' p_2 + \beta_1' p_1 + p_{1\perp}'; a_\perp p_{1,2} = 0; p_2' = \alpha_2' p_2 + \beta_2' p_1 + p_{2\perp}'; k_i = \alpha_i p_2 + \beta_i p_1 + k_{i\perp}'. \quad (2.1)$$

Sudakov's parameters α, β are not independent.The mass shell conditions and the conservation low give the relations:

$$s\alpha_1'\beta_1' = m_1^2 + \vec{p}_1^{\prime 2} = E_{1\perp}^2, \quad (2.2)$$

$$s\alpha_2'\beta_2' = E_{2\perp}^2, s\alpha_i\beta_i = E_{i\perp}^2,$$
$$\vec{a}^2 = -a_\perp^2 > 0; \alpha_1' + \alpha_2' + \sum \alpha_i = 1;$$
$$\beta_1' + \beta_2' + \sum \beta_i = 1.$$

Consider first the peripheral kinematics. For it is characteristic the weak dependence of differential cross sections on the center of mass (CM) total energy $2E = \sqrt{s}$,the strict ordering of parameters α, β

$$1 \approx \beta_1' >> \beta_1 >> ... >> \beta_n >> \beta_2' \sim \frac{m^2}{s}; \frac{m^2}{s} << \alpha_1' << \alpha_1 << ... << \alpha_n << \alpha_2' \approx 1$$

and the restrictiveness on the transvers momenta $|\vec{k}_i| \sim m$. It corresponds to small emission angles moving along 3-momentum $\vec{P}_1$

$$\theta_i = \frac{|\vec{k}_i|}{E\beta_i} << 1, |\beta_i| >> |\alpha_i|,$$

and the similar expression for particles moving in opposite direction $|\beta_i| << |\alpha_i|$. The central region $|\alpha_i| \sim |\beta_i| \sim E_{i\perp}/E << 1$ corresponds to particles of low energies moving at large angles(in cm frame). The differential cross section have the form:

$$\begin{aligned} d\sigma_{2\to 2+n} &= \frac{(2\alpha_s)^{2+n}}{16\pi^{2n}} C_V^n \frac{d^2q_1}{q_1^2+m^2} \frac{d^2q_2}{(q_1-q_2)^2+\lambda^2} ... \\ &\times \frac{d^2 q_{n+1}}{(q_n - q_{n+1})^2 + \lambda^2} \frac{1}{q_{n+1}^2+\lambda^2} \frac{d\alpha_1}{\alpha_1}\theta(\alpha_2-\alpha_1)... \\ &\times \frac{d\alpha_n}{\alpha_n} \prod_{i=1}^{n+1} (\frac{s_i}{m^2})^{2\alpha(q_i^2)} = \frac{1}{q_{n+1}^2+\lambda^2} dZ_n, \end{aligned} \tag{2.3}$$

where $C_V = 3$ and we imply q_i the two-dimensional euclidean vectors,the 4-momentum of the i-th emitted particle (gluon)is

$$k_i = (\alpha_i - \alpha_{i+1})p_2 + (\beta_i - \beta_{i+1})p_1 + (q_i - q_{i+1})_\perp = -\alpha_{i+1}p_2 + \beta_i p_1 + (q_i - q_{i+1})_\perp; \tag{2.4}$$

s_i are the partial squares of invariant mass of nearest emitted particles:

$$s_1 = (p_1' + k_1)^2 = s|\alpha_2|, s_{n+1} = (k_n + p_2')^2 = \frac{E_{n\perp}^2}{\alpha_n}, s_i = E_{\perp,i-1}^2 \frac{\alpha_{i+1}}{\alpha_{i-1}}, s_1 s_2 ... s_n = s E_{1\perp}^2 ... E_{n\perp}^2, \tag{2.5}$$

and the trajectory of reggeized gluon is

$$\alpha(q^2) = \frac{q^2\alpha_s}{2\pi^2} \int \frac{d^2k}{(k^2+\lambda^2)((q-k)^2+\lambda^2}.$$

Here λ is gluon mass.It was shown that the infrared singularities absent in the limit $\lambda \to 0$.In this point we will suggest the gluon to be massive: $\lambda \to M$ and will decay to the jet of hadrons (pions) with the probability to create n particles $dW_n(M) = \frac{c}{\bar{n}} exp(-n/\bar{n})dn, \bar{n} = \ln(M^2/m_\pi^2)$.

The Monte Carlo simulation shows a tendency to minimization of the number of rungs at large n.

For the pure deep inelastic case, when one of the initial hadrons is scattered at the angle θ have the energy E' in the cms of beams whereas the another is scattered at small angle and the large transfer momentum $Q = 4EE' \sin^2(\theta/2) >> m^2$, is distributed to the some number of the emitted particles due to evolution mechanism we have [13](θ is small):

$$d\sigma_n^{DIS} = \frac{4\alpha^2 E'^2}{Q^4 M} dD_n dE' d\cos\theta,$$

$$dD_n = (\frac{\alpha_s}{4\pi})^n \int_{m^2}^{Q^2} \frac{dk_n^2}{k_n^2} \int_{m^2}^{k_n^2} \frac{dk_{n-1}^2}{k_{n-1}^2} \cdots \int_{m^2}^{k_2} \frac{dk_1^2}{k_1^2} \int_x^1 d\beta_n \Theta_n^{(1)} \int_{\beta_n}^1 d\beta_{n-1} \Theta_{n-1}^{(1)} \ldots$$
$$\times \int_{\beta_2}^1 d\beta_1 \Theta_1^{(1)} P(\frac{\beta_n}{\beta_{n-1}}) \ldots P(\beta_1), \quad P(z) = 2\frac{1+z^2}{1-z}, \tag{2.6}$$

where the limits of integrals show the intervals of variation and the integrand is the differential cross section. Again the rapidities β_i are arranged and the transverse momenta square are rigorously arranged. Here $\Theta^{(i)} = \theta(\theta_{i+1} - \theta_i)$ the condition which forbids the destructive interference of jets and jets emission angles are $\theta_i = |\vec{k}_i|/(E \max|\alpha_i|, |\beta_i|)$ regarding the beam axe direction.

Compared with peripheral production regime DIS one gives the contribution which fall down with increasing Q^2. Using the analogy to statistical nonequilibrium processes we may consider DIS regime as an nonequilibrium process of diminishing of large virtualities to small ones by means of evolution. The peripheral regime may be associated with equilibrium process when the random fluctuations on the transvers momenta values become essential.

Let consider now the regime of hard particles production at large angles. It is known as a double-logarithm regime. Any exclusive process is suppressed in this regime by Sudakov form factor. For inclusive set-up of experiments when arbitrary number of photons (gluons) may be emitted the Sudakov's form factor suppression disappears and we obtain the cross section of the form [14]:

$$d\sigma(s) = \frac{1}{s} F(\alpha \ln^2 \frac{s}{m^2}). \tag{2.7}$$

For instance the cross section of annihilation of electron-positron to muon pair accompanied by emission of arbitrary number of photons is

$$\sigma(s)_{e\bar{e}\to\mu\bar{\mu}+\ldots} = \frac{4\pi\alpha^2}{3s} chx, x^2 = \frac{2\alpha}{\pi} \ln^2(\frac{s}{m_\mu m_e}).$$

The characteristic squares of transvers momenta of created particles in this regime are big and of order of s.

The typical process -annihilation of electron -positron pair to n hard photons accompanied by emission of any number of soft and virtual photons have a form [14]:

$$d\sigma_n = \frac{2\pi\alpha^2}{s} dF_n^{DL}, \qquad (2.8)$$

$$dF_n^{DL} = (\frac{\alpha}{2\pi})^n \prod_{i=1}^{i=n} dx_i dy_i \theta(y_i - y_{i-1})\theta(x_i - y_i)\theta(x_i)\theta(y_i)\theta(\rho - x_n)\theta(\rho - x_n),$$

$$x_i = \ln\frac{\vec{q}_i^2}{m^2}, y_i = \ln\frac{1}{\beta_i}, \rho = \ln\frac{s}{m^2}.$$

3 Various scenario

Keeping in mind the kinematical restriction $\Pi s_j \sim s$ we may build the combinations of regimes considered above. Le construct the relevant cross sections. It is convenient to separate them to the classes

a) Pomeron regime (P);

b) Evolution regime (DIS);

c) Double logarithmic regime (DL);

d) DIS+P regime;

e) P+DL+P regime.

The description of every regime may be performed in terms of effective ladder-type Feynman diagrams (The set of relevant FD depends on the gauge chosen and include much more number of them).

The Pomeron is treated as a (infinite) set of particles emitted close to the CM beams direction (within the small angles of order $\theta_i \sim 2m_h/\sqrt{s} << 1$). We expect that these type of particles will not be detected by the detectors since they are move into the beams pipe. The collider experiment detectors locate at finite angles $\theta_D \sim 1$ and will measure the products only of particle c decay.

What will happened when instead of one particle (see (1.1)) a set of particles with invariant mass square s_t is created at large angles? Then the cross section will have a form:

$$d\sigma_n = \frac{\alpha_s^2}{s_t} N dF_n^{DL}(\alpha_s \ln^2(\frac{s_t}{s_0})), \; N = (\frac{s}{s_t})^{\Delta},$$
$$\Delta = \alpha_P - 1 = \frac{12\ln 2}{\pi}\alpha_s \approx 0.55, \; \alpha_s = 0.2 \qquad (3.1)$$

Radiative corrections to the intercept was calculated [11] in recent time. The resulting value is $\Delta \approx 0.2$.

The way to obtain detected large multiplicity is to organize DIS-like experiments, expecting the large-angle scattered hadrons in the detectors. Large transfers momenta will be decreasing by ordinary evolution mechanism to the value of order m_π and then the Pomeron mechanism of peripheral scattering of the created hadrons from the pionization region will start.

This phenomena is quite close to flea-dog model of Euhrenfest (see [9]): in the nonequilibrium process (DIS regime) the fluctuations are suppressed and they take place (Pomeron regime) when the equilibrium take place.

What the characteristic multiplicities expected from Pomeron mechanism with the intercept exceeding unity, $\Delta \sim 0.2$? It is the quantity of order $(s/m_\pi^2)^\Delta \approx 200$ for $\sqrt{s} = 14TeV$. This rather rough estimation is in agreement with the phenomenological analysis of A.Kaidalov [1], based on multi-pomeron exchange in the scattering channel.

Construct now the cross sections of combined processes. When the one of the initial particles h_1 is scattered on small but sufficient enough angle to fit the detectors and other is scattered almost forward the combination of DIS and Pomeron regimes take place:

$$d\sigma_{n,m} = d\sigma_n^{DIS} dZ_m, |q_n|^2 \sim m^2 \tag{3.2}$$

provided that the virtuality of the last step of evolution regime of order of hadron mass. For the kinematical case of almost forward scattering of both initial hadrons the situation may be realized with large angles hadron production from the central region (see (3.1)):

$$d\sigma_{n,m,k} = dZ_n d\sigma_m^{DL} dZ_k. \tag{3.3}$$

4 Discussion

Every regime $a) - e)$ considered above provide creation of $(n+2)$ gluons and theirs subsequent decay in the universal way on jets.

Let us discuss more carefully the large-angle moving jets creation mechanism due to annihilation of initial partons into the hadrons system. The intermediate state of single heavy photons decay into jet was analyzed by A.Polyakov [15]. It was shown that the scaling regime works, providing the behavior of the n jets creation cross section and the mean multiplicity as:

$$\sigma_n(s) \sim \frac{1}{s}(\frac{m^2}{s})^\delta F(n(\frac{m^2}{s})^\delta), \bar{n} \sim (\frac{s}{m^2})^\delta, 0 < \delta < \frac{1}{2}. \tag{4.4}$$

Another mechanism of multiple production [16] takes into account the channels of incident partons annihilation onto the arbitrary large number of real and virtual gluons (photons). On this way some intermediate regime between the single logarithm (the renormalization group approach) and the double logarithmical, $\alpha \ln^\rho(s/m^2) \sim 1, 1 < \rho < 2$, is realized.

All the considered regimes of many particles state production describes the hard stage of process. The hadronization stage will impose its features which can not be expressed in terms of pQCD as well as it concern the confinement region. Here the identity of produced particles must be taken into account [17]. The effective Lagrangian approach also may be applied here [10]. We hope to consider last questions in another work.

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