

# Phase transitions signal in inelastic hadron collisions

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## Abstract

The consequence of vacuum instability against particles creation is described using lattice gas model. It is shown that the tail of multiplicity distribution should decrease slower than any power of  $\exp\{-n\}$ , but faster than any power of  $1/n$  if the vacuum is unstable against particles creation.

## 1 Introduction

The aim of this paper is to find the experimentally observable consequences of collective phenomena in the high energy hadrons inelastic collision. We will pay the main attention on the phase transitions, leaving out other possible interesting collective phenomena.

First of all, the statistics experience dictate that we should prepare the system to the phase transition. The temperature in a critical domain and the equilibrium media are just this conditions. It is evident, they are not trivial requirement considering the hadrons inelastic collision at high energies.

The collective phenomena by definition suppose that the kinetic energy of particles of media are comparable, or even smaller, than the potential energy of their interaction. It is a quite natural condition noting that, for instance, the kinetic motion may destroy, even completely at a given temperature  $T$ , necessary for the phase transition long-range order. This gives, more or less definitely, the critical domain.

The same idea as in statistics seems natural in the multiple production physics. We will assume (**A**) that the collective phenomena should be seen just in the very high multiplicity (VHM) events, where, because of the energy-momentum conservation laws, the kinetic energy of created particles can not be high.

We will lean at this point on the  $S$ -matrix interpretation of statistics [1]. It based on the  $S$ -matrix generalization of the Wigner function formalism of Carruzers and Zachari-zen [2] and the real-time finite temperature field theory of Schwinger and Keldysh [3, 4].

First of all, the  $n$ -particle partition function in this approach coincide with the  $n$  particles production cross section  $\sigma_n(s)$  (in the appropriate normalization condition). Secondly,  $\sigma_n(s)$  can be calculated from the  $n$ -point Wigner function  $W_n(X_1, X_2, \dots, X_n)$ . In the relativistic case  $X_k = (u, q)_k$  are the 4-vectors. So, the external particles are considered as the 'probes' to measure the interacting fields state, i.e. the low mean energy of probes means that the system is 'cold'.

The multiple production phenomena may be considered also as the thermalization process of incident particles kinetic energy dissipation into the created particles mass. From this point of view the VHM processes are highly nonequilibrium since the final state of this case is very far from initial one. It is known in statistics [5] that such process aspire to be the stationary Markovian with high level of entropy production. In the case of complete thermalization the final state is equilibrium.

The equilibrium we will classify as the condition in frame of which the fluctuations of corresponding parameter are Gaussian. So, in the case of complete thermalization the probes should have the Gauss energy spectra. In other terms, the necessary and sufficient condition of the equilibrium is smallness of mean value of energy correlators [6, 1]. From physical point of view, absence of this correlators means depression of the macroscopic energy flows in the system.

The multiple production experiment shows that created particles energy spectrum is far from Gauss law, i.e. the final states are far from equilibrium. The natural explanation of this phenomena consist in presence of (hidden) conservation laws in the interacting Yang-Mills fields system: it is known that presence of sufficient number of first integrals in involution prevents thermalization completely.

But nevertheless the VHM final state may be equilibrium (**B**) in the above formulated sense. This means that the forces created by the non-Abelian symmetry conservation laws may be frozen during thermalization process (remembering its stationary Markovian character in the VHM domain). We would like to take into account that the entropy  $\mathcal{S}$  of a system is proportional to number of created particles and, therefore,  $\mathcal{S}$  should tend to its maximum in the VHM region [7].

One may consider following small parameter  $(\bar{n}(s)/n) \ll 1$ , where  $\bar{n}(s)$  is the mean value of multiplicity  $n$  at given CM energy  $\sqrt{s}$ . Another small parameter is the energy of fastest hadron  $\varepsilon_{max}$ . One should assume that in the VHM region  $(\varepsilon_{max}/\sqrt{s}) \rightarrow 0$ . So, the conditions:

$$\frac{\bar{n}(s)}{n} \ll 1, \quad \frac{\varepsilon_{max}}{\sqrt{s}} \rightarrow 0 \quad (1.1)$$

would be considered as the mark of the processes under consideration. We can hope to organize the perturbation theory over them having this small parameters. In this sense VHM processes may be ‘simple’, i.e. one can use for theirs description semiclassical methods.

So, considering VHM events one may assume that the conditions (**A**) and (**B**) are hold and one may expect the phase transition phenomena. The paper is organized as follows. In Sec.2 we will offer the qualitative picture of expected phase transition. In sec.3 we will describe quantitatively this phenomena to find the predictions for experiment. In Sec.4 we will give formal derivation of the formalism used in Sec.3.

## 2 Condensation and singularity at $z = 1$

The  $S$ -matrix interpretation of statistics is based on following definitions. First of all, let us introduce the generating function [8]:

$$T(z, s) = \sum_n z^n \sigma_n(s). \quad (2.1)$$

Summation is performed over all  $n$  up to  $n_{max} = \sqrt{s}/m$  and, at finite CM energy  $\sqrt{s}$ ,  $T(z, s)$  is a polynomial function of  $z$ . Let us assume now that  $z$  is sufficiently small and by this reason  $T(z, s)$  depends on upper boundary  $n_{max}$  weakly. In this case one may formally extend summation up to infinity and in this case  $T(z, s)$  may be considered as a whole function. This possibility is important being equivalent of thermodynamical limit and it allows to classify the asymptotics over  $n$  in accordance with position of singularities over  $z$ .

Let us consider  $T(z)$  (prove will be given below) as the big partition function, where  $z$  is ‘activity’. It is known [9] that  $T(z)$  should be regular in the circle of unite radii. The leftist singularity lie at  $z = 1$ . This singularity is manifestation of the first order phase transition [9, 5, 10].

The origin of this singularity was investigated carefully in the paper [5]. It was shown that position of singularities over  $z$  depends on the number of particles  $n$  in the system: the two complex conjugated singularity moves to the real  $z$  axis with rising  $n$  and in the thermodynamical limit  $n = \infty$  they pinch point  $z = 1$ . More careful analysis [10] shows that if the system is equilibrium then  $T(z)$  may be singular at  $z = 1$  and  $z = \infty$  only.

The position of singularity over  $z$  and the asymptotic behavior of  $\sigma_n$  are related closely. Indeed, for instance, inserting into (2.1)  $\sigma_n \propto \exp\{-cn^\gamma\}$  we find that  $T(z)$  is singular at  $z = 1$  if  $\gamma < 1$ . Generally, using Mellin transformation,

$$\sigma_n = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} T(z) \quad (2.2)$$

This integral can be calculated expanding it in vicinity of  $z_c$ , where  $z_c$  is smallest real positive solution of equation:

$$n = z \frac{\partial}{\partial z} \ln T(z). \quad (2.3)$$

Then integral (2.2) have following estimation:

$$\sigma_n \propto e^{-n \ln z_c(n)}, \quad z_c > 1. \quad (2.4)$$

Therefore, to have the singularity at  $z = 1$  we should consider  $z_c(n)$  as a decreasing function of  $n$ . On other hand, at constant temperature,  $\ln z_c(n) \sim \mu_c(n)$  is the chemical potential, i.e. is a work necessary for one particle creation. So, the singularity at  $z = 1$  means that the system is unstable: the less work is necessary for creation of one more particle if  $\mu(n)$  is the decreasing function of  $n$ .

The physical explanation of this phenomena is following, see also [11]. Generating function  $T(z)$  have following expansion:

$$T(z) = \exp\left\{\sum_l z^l b_l\right\}, \quad (2.5)$$

where  $b_l$  are known as the Mayer's group coefficients. They can be expressed through the inclusive correlation functions and may be used to describe formation of droplets of correlated particles. So, if droplet consist from  $l$  particles, then

$$b_l \sim e^{-\beta \xi l^{(d-1)/d}} \quad (2.6)$$

is the mean number of such droplets. Here  $\xi l^{(d-1)/d}$  is the surface energy of  $d$ -dimensional droplet.

Inserting this estimation into (2.5),

$$\ln T(z) \sim \sum_l e^{\beta(l\mu - \xi l^{(d-1)/d})}, \quad \beta\mu = \ln z. \quad (2.7)$$

First term in the exponent is the volume energy of droplet and being positive it try to enlarge the droplet. The second surface term try to shrink it. Therefore, singularity at  $z = 1$  is the consequence of instability: at  $z > 1$  the volume energy abundance leads to unlimited grow of the droplet.

### 3 Condensation and type of asymptotics over multiplicity

It is important for VHM experiment to have upper restriction on the asymptotics. We wish to show that  $\sigma_n$  decrease faster than any power of  $1/n$ :

$$\sigma_n < O(1/n). \quad (3.1)$$

To prove this estimation one should know the type of singularity at  $z = 1$ . The detailed derivation of the model used for this purpose we will give in subsequent section.

One can imagine that the points, where the external particles are created, form the gas system. Here we assume that this system is equilibrium, i.e. there is not in this system macroscopical flows of energy, particles, charges and so on.

The lattice gas approximation is used to describe such system. This description is quite general and did not depend on details. Motion of the gas particles leads to necessity sum over all distributions of the particles on cells. For simplicity we will assume that only one particle can occupy the cell.

So, we will introduce the occupation number  $\sigma_i = \pm 1$  in the  $i$ -th cell:  $\sigma_i = +1$  we have not particle in the cell and  $\sigma_i = -1$  means that the particle exist. Assuming that the system is equilibrium we may use the ergodic hypothesis and sum over all 'spin' configurations of  $\sigma_i$ , with restriction:  $\sigma_i^2 = 1$ . It is well known [12] this restriction introduce the interactions.

Corresponding partition function in temperature representation [10]

$$\rho(\beta, H) = \int D\sigma e^{-S\lambda(\sigma)} \quad (3.2)$$

where integration is performed over  $|\sigma(x)| \leq \infty$  and, considering the continuum limit,  $D\sigma = \prod_x d\sigma(x)$ . The action

$$S_\lambda(\sigma) = \int dx \left\{ \frac{1}{2}(\nabla\sigma)^2 - \omega\sigma^2 + g\sigma^4 - \lambda\sigma \right\} \quad (3.3)$$

where

$$\omega \sim \left(1 - \frac{\beta_{cr}}{\beta}\right), \quad g \sim \frac{\beta_{cr}}{\beta}, \quad \lambda \sim \left(\frac{\beta_{cr}}{\beta}\right)^{1/2} \beta H. \quad (3.4)$$

and  $1/\beta_{cr}$  is the critical temperature.

### 3.1 Unstable vacuum

We start consideration from the case  $\omega > 0$ , i.e. assuming that  $\beta > \beta_{cr}$ . In this case the ground state is degenerate if  $H = 0$ . The extra term  $\sim \sigma H$  in (3.3) can be interpreted as the interaction with external magnetic field  $H$ . This term regulate number of ‘down’ spins with  $\sigma = -1$  and is related to the activity:

$$z^{1/2} = e^{\beta H}, \quad (3.5)$$

i.e.  $H$  coincide with chemical potential.

The potential

$$v(\sigma) = -\omega\sigma^2 + g\sigma^4, \quad \omega > 0, \quad (3.6)$$

has two minimums at

$$\sigma_\pm = \pm \sqrt{\omega/2g}.$$

If the dimension  $d > 1$  no tunnelling phenomena exist. But choosing  $H < 0$  the system in the right minimum (it correspond to the state without particles) becomes unstable. The system tunneling into the state with absolute minimum of energy.

The partition function  $\rho(\beta, z)$  becomes singular at  $H = 0$  because of this instability. The square root branch point gives

$$\text{Im}\rho(b, z) = \frac{a_1(\beta)}{H^4} e^{-a_2(\beta)/H^2}, \quad a_i > 0. \quad (3.7)$$

Note,  $\text{Im}\rho(b, z) = 0$  at  $H = 0$ . Deforming contour of integration in (2.2) on the branch line,

$$\rho_n(\beta) = \frac{1}{\pi} \int_1^\infty \frac{dz}{z^{n+1}} \frac{8a_1\beta^4}{\ln^4 z} e^{-4a_2\beta^2/\ln^2 z}. \quad (3.8)$$

In this integral

$$z_c \propto \exp \left\{ \frac{8a_2\beta^2}{n} \right\}^{1/3} \quad (3.9)$$

is essential. This leads to following estimation:

$$\rho_n \propto e^{-3(a_2\beta^2)^{1/3} 1/3n^{2/3}} < O(1/n). \quad (3.10)$$

- It is useful to note at the end of this section that
- (i) The value of  $\rho_n$  is defined by  $\text{Im}\rho(b, z)$  and the metastable states, decay of which gives contribution into  $\text{Re}\rho(b, z)$ , are not important.
  - (ii) It follows from (3.9) that in the VHM domain

$$H \sim H_c \sim \ln z_c \sim (1/n)^{1/3} \rightarrow 0. \quad (3.11)$$

So, the calculations are performed for the weak external field case, when the degeneracy is weekly broken. It is evident that the life time of the unstable (without particles) state is large in this case and by this reason the used semiclassical approximation is rightful. This is important consequence of (1.1).

### 3.2 Stable vacuum

Let us consider now  $\omega < 0$ , i.e.  $\beta\beta_{cr}$ . Potential (3.6) have only one minimum at  $\sigma = 0$  in this case. Inclusion of external field shifts the minimum to the point  $\sigma_c = \sigma_c(H)$ . In this case the expansion in vicinity of  $\sigma_c$  should be useful. In result,

$$\rho(\beta, z) = \exp\left\{\int dx \lambda \sigma_c - W(\sigma_c)\right\}, \quad (3.12)$$

where  $W(\sigma_c)$  can be expanded over  $\sigma_c$ :

$$W(\sigma_c) = \sum_l \frac{1}{l} \int \prod_k \{dx_k \sigma_c(x_k; H)\} \tilde{b}_l(x_1, \dots, x_l). \quad (3.13)$$

In this expression  $\tilde{b}_l(x_1, \dots, x_l)$  is the one-particle irreducible  $l$ -point Green function, i.e.  $\tilde{b}_l$  is the virial coefficient. Then  $\sigma_c$  can be considered as the effective activity of the correlated  $l$ -particle group.

The sum in (3.13) should be convergent and, therefore,  $|s_c| \rightarrow \infty$  if  $|H| \rightarrow \infty$ . But in this case the virial decomposition is equivalent of the expansion over inverse density of particles [13]. In the VHM region it is high and the mean field approximation becomes rightful. In result,

$$\sigma_c \simeq - \left(\frac{|\lambda|}{4g}\right)^{1/3} : |s_c| \rightarrow \infty \text{ if } |\lambda| \rightarrow \infty, \quad (3.14)$$

and

$$\rho(\beta, z) \propto e^{\frac{3|\lambda|^{4/3}}{(4g)^{1/3}}} \left\{ 12g \left(\frac{|\lambda|}{4g}\right)^{2/3} \right\}^{-1/2}. \quad (3.15)$$

We can use this expression to calculate  $\rho_n$ . In this case

$$z_c \propto e^{4gn^3} \rightarrow \infty \text{ at } n \rightarrow \infty, \quad (3.16)$$

is essential and in the VHM domain

$$\rho_n \propto e^{-gn^4} < O(e^{-n}). \quad (3.17)$$

This result is evident consequence of vacuum stability. It should be noted once more that the conditions (1.1) considerably simplify calculations.

## 4 Conclusion

In conclusion we wish to formulate once more the main assumptions.

(I). It was assumed first of all that the system under consideration is equilibrium. This condition may be naturally reached in the statistics, where one can wait the arbitrary time till the system becomes equilibrium. Note, in the critical domain the time of relaxation  $t_r \sim (T_c/(T - T_c))^\nu$ ,  $(T - T_c) \rightarrow +0$ ,  $\nu > 0$ ,  $T_c$  is the critical temperature.

We can not give the guarantee that in the high energy hadron collisions the final state system is equilibrium. The reason of this uncertainty is the finite time the inelastic processes and presence of hidden (confinement) constraints on the dynamics.

But we may formulate the quantitative conditions, when the equilibrium is hold [6]. One should have the Gauss energy spectra of created particles. If this condition is hardly investigated on the experiment, then one should consider the relaxation of 'long-range' correlations. This excludes usage of relaxation condition for the 'short-range' (i.e. resonance) correlation

(II). The second condition consist in requirement that the system should be in the critical domain, where the (equilibrium) fluctuations of system becomes high. Having no theory of hadron interaction at high energies we can not define where is lie the 'critical domain' and even exist it or not.

But anyway, having the VHM 'cold' final state we can hope that the critical domain is achieved. Moreover, noting that the entropy reach its maximum at given incident energy, we can hope that the VHM state is equilibrium.

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