# Two-dimensional analysis of the Bose-Einstein correlations in $e^{+} e^{-}$annihilation at the $Z^{0}$ peak 

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#### Abstract

The study of the directional dependence of two-particle correlations in the hadronic decays of the $Z^{0}$ boson is performed using the data collected by the DELPHI experiment in the 1992-1995 running periods. The comparison between the transverse, $R_{\perp}$, and longitudinal, $R_{\|}$, correlation radii confirms the string model prediction that the transverse correlation length is smaller than the longitudinal one, with the measured values of $R_{\perp}=0.53 \pm 0.08 \mathrm{fm}$ and $R_{\|}=0.85 \pm 0.08 \mathrm{fm}$, for selected $Z^{0} \rightarrow q \bar{q}$ events. © 2000 Published by Elsevier Science B.V. All rights reserved.


## 1. Introduction

Detailed studies of the two-particle Bose-Einstein correlations (BEC) in $Z^{0}$ hadronic decays in $e^{+} e^{-}$ annihilation allow the determination of the shape of the source of bosons, which gives the possibility to analyse the spatial and temporal characteristics of the hadronisation region. These studies are of considerable interest mainly due to the recent predictions of possible influence of BEC on the measured value of the $W$ boson mass in $e^{+} e^{-}$annihilation [1,2]. Estimates of the strength of this effect have been made using the Monte Carlo particle generator Jetset [3], involving a simple algorithm for the two-particle BEC simulation which uses a correlation function in terms of the invariant four-momentum difference of identical partons, $Q$. This algorithm is known to reproduce well basic features of BEC in experimental data, like the shape of the correlation function in terms of $Q$ [4] and the shift of the $\rho^{0}$ mass [5], but it does not describe other related effects, like the higher order correlations [6], neither it reproduces its own input parameters in a wide range [7]. More detailed tests are necessary in order to establish the extent of applicability of the mentioned algorithm and the reliability of its predictions.

In the two-jet hadronic decays $Z^{0} \rightarrow q \bar{q}$, the comparison between the transverse and longitudinal radii of the BEC (with respect to the initial parton direction of motion) can test the string model prediction [8] that the transverse correlation length is considerably smaller than the longitudinal one.

Until recently, studies of the identical-boson correlations in $e^{+} e^{-}$annihilation process at LEP energies have concentrated on the shape of the two-particle correlation function in terms of $Q$ [4]. At lower energies, several collaborations have studied BoseEinstein correlations using two-dimensional distributions of components of $Q$ [9]. Multidimensional analyses of the BEC are now being made by the LEP experiments as well. Studies performed by the L3 [10] experiment and preliminary results by DELPHI [11] and OPAL [12] at LEP1 energies indicate that the transverse size of the boson source in $e^{+} e^{-}$ annihilation is smaller than the longitudinal one.

Here, the two-dimensional analysis of BEC in $Z^{0}$ hadronic decays is presented, using DELPHI data collected in the 1992-1995 running periods. Twoparticle correlations are studied in terms of different components of the four-momentum difference. Results are compared to those obtained from the analysis of events generated by Jetset.

## 2. Correlation function definition

The correlation function, $C_{2}$, of two identical bosons is defined as [13]

$$
\begin{equation*}
C_{2}\left(p_{1}, p_{2}\right)=\frac{P\left(p_{1}, p_{2}\right)}{P\left(p_{1}\right) P\left(p_{2}\right)}, \tag{1}
\end{equation*}
$$

where $p_{1}$ and $p_{2}$ are the four-momenta of the two particles, $P\left(p_{1}, p_{2}\right)$ is the two-particle probability
density and $P\left(p_{1}\right)$ and $P\left(p_{2}\right)$ represent single-particle probability densities. The invariant fourmomentum difference $Q$ is defined as
$Q=\sqrt{\left(E_{1}-E_{2}\right)^{2}-\left(\boldsymbol{p}_{1}-\boldsymbol{p}_{2}\right)^{2}}$,
where $\boldsymbol{p}_{1}$ and $\boldsymbol{p}_{2}$ are the momenta of the two particles, and $E_{1}, E_{2}$ are their energies. As long as bosons, which are subject to BEC, have similar momenta, one would expect to observe an enhanced production of pairs with low values of $Q$ as compared to the non-correlated case.

The most commonly used [13] parametrization of $C_{2}$ is:
$C_{2}(Q)=n\left(1+\lambda e^{-R^{2} Q^{2}}\right)$,
where the parameter $\lambda$ is interpreted as the strength of the correlation, and $R$ as the size of the source of bosons, or the correlation radius. $n$ is the overall normalisation.

In expression (3), $R$ corresponds to an average over the spatial and temporal source dimensions. To probe the actual shape of a boson source, Bose-Einstein correlations must be studied in terms of the various components of the three-momentum difference $\boldsymbol{Q}=\boldsymbol{p}_{\boldsymbol{I}}-\boldsymbol{p}_{2}$ in a chosen coordinate system.

For this purpose, the Longitudinal Centre-of-Mass System [8,14] (LCMS) is often used. The LCMS is defined for each pair of particles as the system in which the sum of the two particles' momenta is perpendicular to a selected reference axis (see Fig. 1). The reference axis has to be a physical axis of the process: for example, in $e^{+} e^{-}$annihilation it can be the direction of a primary parton, or of the corresponding jet. In this analysis, the thrust axis was chosen as the reference (see Section 4). In such a system, $\boldsymbol{Q}$ is decomposed into the following components: $Q_{\text {long }}$, parallel to the thrust axis; $Q_{t, \text { out }}$, collinear with the sum of the two particles' momenta, and the complementary $Q_{t, \text { side }}$, perpendicular to both $Q_{\text {long }}$ and $Q_{t, \text { out }}$. This system is convenient for calculations and interpretations. The projection of the momentum sum of the two particles is non-zero only in the ' $t$,out' direction. The spatial dimensions of the source effect all components of $Q$. However the energy difference and hence the temporal dimension of the source, couples only to the $Q_{t, \text { out }}$ component. If the string model is considered, the longitudi-


Fig. 1. The Longitudinal Centre-of-Mass System is defined, for each pair of particles, as the system in which the sum of the two particles' momenta is perpendicular to a selected reference axis. The reference axis has to be a physical axis of the process.
nal direction of the LCMS has to be aligned with the direction of motion of the initial partons, so that the system itself will be the local rest frame of a string.

By analogy with Eq. (3), the three-dimensional correlation function in LCMS can be parametrized as:

$$
\begin{align*}
& C_{2}\left(Q_{t, \text { out }}, Q_{t, \text { side }}, Q_{\text {long }}\right) \\
& \quad=n\left(1+\lambda e^{-Q_{t, \text { out }}^{2} R_{t, \text { out }}^{2}-Q_{t, \text { side }}^{2} R_{t, \text { side }}^{2}-Q_{\text {long }}^{2} R_{\text {long }}^{2}}\right) \tag{4}
\end{align*}
$$

In this analysis, the two-dimensional projection of the LCMS is used, with longitudinal component $Q_{\|} \equiv Q_{\text {long }}$ and perpendicular component $Q_{\perp}$ $=\sqrt{Q_{t, \text { out }}^{2}+Q_{t, \text { side }}^{2}}$. The parametrization of $C_{2}$ in the two-dimensional case is chosen here as:
$C_{2}\left(Q_{\perp}, Q_{\|}\right)=n\left(1+\lambda e^{-Q_{\perp}^{2} R_{\perp}^{2}-Q_{\|}^{2} R_{\|}^{2}}\right)$.

## 3. Data selection

Data collected by the DELPHI detector [15] in 1992-1995 at centre-of-mass energies around $\sqrt{s}=$ 91.2 GeV were used.

Only charged particles in hadronic events were considered in the analysis [15]. The tracks were taken into account if their impact parameter was below 1 cm in the transverse plane and below 5 cm along the beam axis (to reduce contributions from
long-living resonance decays), the measured track length was above 50 cm , the momentum $p$ was in the range of $0.1 \mathrm{GeV} / c<p<50 \mathrm{GeV} / c$ and the polar angle between $25^{\circ}$ and $155^{\circ}$. All particles were assumed to be pions.

Hadronic events were selected by requiring that: (a) they contained at least 5 charged particles with momentum above $0.2 \mathrm{GeV} / c$; (b) the total energy of all charged particles exceeded 15 GeV ; (c) each hemisphere with respect to the sphericity axis contained a total energy of charged particles larger than 3 GeV ; (d) the polar angle of the sphericity axis was between $40^{\circ}$ and $140^{\circ}$, so that the events are well contained inside the TPC.

In this analysis two-jet events were selected in order to compare the result with the theoretical prediction [8]. These events are also convenient because the procedures of preparing the reference sample (see Section 4) and the definition of LCMS are easier to apply and to understand in this case. Since the thrust axis of the two-jet events is well aligned with the direction of motion of the initial partons, its direction can be selected as the physical axis of the hadronization process, and the possible influence of hard gluon radiation can be neglected. The two-jet event selection was done using the LUCLUS [3] clustering algorithm (with parameter $d_{\text {join }}=8$ $\mathrm{GeV} / c$ ), requiring that the thrust value be more than 0.95 , and, that the jet acollinearity shall not exceed $5^{\circ}$. A total of about 810000 events satisfied these criteria.

To purify the reference sample and to reduce the background, additional selection criteria were applied for each pair of particles. To stay away from the two-particle phase-space limits, where kinematic correlations are significant, a pair of tracks was selected for the analysis, if both particles had momenta below $5 \mathrm{GeV} / c$. To exclude the partially overlapping tracks which can be poorly reconstructed, the angle between tracks was required to exceed $2^{\circ}$. To reduce the correlations caused by the local transverse momentum compensation, pairs were rejected if the angle between tracks in a plane, transverse to the thrust axis, was more than $120^{\circ}$. In addition, to reduce the contribution from resonance decays and to eliminate the region where the Coulomb correction is substantial, pairs were rejected if their $Q$ was less than 0.06 GeV .

## 4. Correlation function measurement

The measurement of the correlation function (1) in the two-dimensional LCMS requires accumulation of the double-differential distributions $d^{2} N^{ \pm \pm /}$ $d Q_{\perp} d Q_{\|}$, where $N^{ \pm \pm}$is the number of like-sign pairs. All the data were corrected for detector effects. Events generated with the Jetset 7.3 PS model with DELPHI tuning [16] were used to estimate the acceptance corrections and to account for effects arising from the limited detector resolution. The selected events were passed through the DELSIM [17] detector simulation and the same selection criteria were used as for real data. Correction coefficients $c\left(Q_{\perp}\right.$ , $Q_{\|}$) were calculated as the ratios of distributions at the generation level (JeTSET only) to those at the reconstruction level (JETSET + DELSIM):
$c\left(Q_{\perp}, Q_{\|}\right)=\frac{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\mathrm{gen}}}{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\mathrm{rec}}}$,
where indices 'gen' and 'rec' refer to the generation and reconstruction level respectively.

The two-particle correlation function definition in Eq. (1) requires the knowledge of the product of the single-particle probability densities, $P\left(p_{1}\right) P\left(p_{2}\right)$. Due to the phase space limitations, it is difficult to construct this product. Therefore it is often replaced by $P_{0}\left(p_{1}, p_{2}\right)$, which is equal to $P\left(p_{1}\right) P\left(p_{2}\right)$ in a hypothetical case of no correlations. Technically this means that one has to construct an artificial reference sample of particles which are not subject to BoseEinstein correlations, but obey the same kinematics as a regular event. Several techniques for obtaining a reference sample can be considered, like using the unlike-sign particle combinations, Monte Carlo simulated events without the BEC effect, or the eventmixing technique. It has been established [9] that the latter is the most reliable method. To construct the mixed reference sample, all events are rotated to a new coordinate system, which has the $z$ axis along the thrust axis. The sample is then obtained by combining a particle from one event randomly with a like-charge particle from another.

The mixed reference sample is prepared using the same set of hadronic events as for the real data. Within the applied selection criteria, the mixed sample does not contain BEC and satisfies most of the basic requirements for the reference sample [9]. It has no additional dynamical correlations, like those coming from the $K^{0}$ and $\rho^{0}$ decays in the case of unlike-charge reference sample. Since only two-jet events are used, and the detector corrections are applied, the mixed sample contains the correlation due to the jet structure of events. Correlations due to energy-momentum conservation are also included, since the pairs close to the phase-space limits are removed (see Section 3). However, the mixing procedure does not conserve energy and momentum in general, affects the normalisation, and destroys not only the Bose-Einstein correlation but some other kinds of correlations, like those coming from the local transverse momentum compensation. Fig. 2 shows the effect of the mixing on the original $Q$ distributions in detector-corrected DELPHI data, which contain physical BEC, and Jetset generated events without BEC simulation. Enhancement with respect to the reference distribution in the region of $Q<0.25 \mathrm{GeV}$ is readily seen in data, manifesting the presence of BEC. In the case of the BEC-free Monte Carlo events (Fig. 2(b)), no such enhancement can be observed, with the original and the reference distributions being identical at small $Q$ values. This illustrates the reliability of the mixing technique.

From Fig. 2(b) one can see the unwanted feature of the mixing procedure at $Q>0.25 \mathrm{GeV}$ : the reference sample distribution deviates from the original one. This difference however is not essential for the analysis, since the region of genuine two-particle BEC lies below that value [4]. To correct for this effect, the measured two-particle correlation function $C_{2}(Q)$ is calculated as the double-ratio:

$$
\begin{equation*}
C_{2}\left(Q_{\perp}, Q_{\|}\right)=\frac{r_{\mathrm{data}}\left(Q_{\perp}, Q_{\|}\right)}{r_{\mathrm{noBE}}\left(Q_{\perp}, Q_{\|}\right)}, \tag{7}
\end{equation*}
$$

where

$$
r_{\mathrm{data}}\left(Q_{\perp}, Q_{\|}\right)=\frac{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\mathrm{data}}}{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\text {data }, \text { mix }}}
$$

and

$$
\begin{equation*}
r_{\mathrm{noBE}}\left(Q_{\perp}, Q_{\|}\right)=\frac{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\mathrm{noBE}}}{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\mathrm{noBE}, \text { mix }}} \tag{8}
\end{equation*}
$$

Here $\left(d^{2} N^{ \pm \pm} / d Q_{\perp} d Q_{\|}\right)_{\text {data }}$ is the $Q$-distribution of the pion pairs with the same charge in real data, while the subscript 'data, mix' denotes the same quantity but for pairs from the reference sample. The indices 'noBE' and 'noBE, mix' refer to analogous


Fig. 2. Comparison of the original $Q$ distributions of like-charge particle pairs, and the reference ones obtained by the mixing procedure: (a) in DELPHI data (data points are connected with lines for clarity) and (b) in JETSET generated events without BEC. All distributions are normalized by the total number of selected events, $N$.


Fig. 3. Two-dimensional correlation function, $C_{2}\left(Q_{\|}, Q_{\perp}\right)($ a $)$, as measured by DELPHI in hadronic decays of $Z^{0}$. Its transverse (b) and longitudinal (c) slices at the peak are shown together with the fit to the Eq. (5).
quantities in absence of BEC obtained from the Jetset sample without BEC.

The reference sample $\left(d^{2} N^{ \pm \pm} / d Q\right)_{\text {data,mix }}$ is corrected for the detector effects using a correction coefficient similar to (6):
$c_{\text {mix }}\left(Q_{\perp}, Q_{\|}\right)=\frac{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\text {gen, mix }}}{\left(\frac{d^{2} N^{ \pm \pm}}{d Q_{\perp} d Q_{\|}}\right)_{\text {rec,mix }}}$,
where 'mix' denotes the mixed samples. The final, corrected, correlation function is then evaluated from Eq. (7) as:
$C_{2}\left(Q_{\perp}, Q_{\|}\right)=\frac{r_{\text {data }}\left(Q_{\perp}, Q_{\|}\right)}{r_{\text {noBE }}\left(Q_{\perp}, Q_{\|}\right)} \frac{c\left(Q_{\perp}, Q_{\|}\right)}{c_{\text {mix }}\left(Q_{\perp}, Q_{\|}\right)}$.

## 5. Results and discussion

The correlation function (10) as measured from the DELPHI data is shown in Fig. 3. BEC manifest themselves as the enhancement of $C_{2}\left(Q_{\perp}, Q_{\|}\right)$at low values of the $Q$ components.

The quantitative evaluation of the two-dimensional correlation function parameters was made by fitting the parametrization (5) to the measured correlation function $C_{2}\left(Q_{\perp}, Q_{\|}\right)$. The fit has been performed in the enhancement region of $\left|Q_{\|}\right|<0.8$ GeV and $0 \mathrm{GeV}<Q_{\perp}<0.6 \mathrm{GeV}$. Variation of the fit parameters as a function of the selected fit region contributes to the systematic uncertainty of the analysis. To estimate this contribution, the maximal value of $\left|Q_{\|}\right|$was varied in the range from 0.6 GeV to 1.1 GeV , and the one of $Q_{\perp}$ - from 0.6 GeV to 1 GeV . Other sources of systematic uncertainties were evaluated varying the selection criteria. The biggest uncer-
tainty comes from varying the minimal thrust value requirement in the range between 0.93 and 0.97 . The total systematic error was evaluated by adding all the contributions in quadrature.

The following values for the correlation radius components were obtained:
$R_{\perp}=0.53 \pm 0.02 \pm 0.07 \mathrm{fm}$,
$R_{\|}=0.85 \pm 0.02 \pm 0.07 \mathrm{fm}$,
where the first error is statistical, and the second is the systematic uncertainty. The correlation strength is found to be $\lambda=0.261 \pm 0.007 \pm 0.010$, and it is slightly correlated (about $30 \%$ ) with the radii. The $\chi^{2}$ of the fit is 96 for 92 degrees of freedom. The ratio of the transverse and longitudinal radii from Eq. (11) is $R_{\perp} / R_{\|}=0.62 \pm 0.10$. This ratio can be obtained as the result of a direct fit, using $R_{\perp} / R_{\|}$ as a parameter, and $R_{\|}$as the complementary one. The correlation between the radii proves to be small (around $10 \%$ ), and the fit leads to the value of $R_{\|}$ identical with that of (11), and for the ratio
$R_{\perp} / R_{\|}=0.62 \pm 0.02 \pm 0.05$.
The values obtained are in qualitative agreement with the theoretical prediction of [8], according to which the longitudinal correlation length in $Z^{0} \rightarrow q \bar{q}$ hadronic decay has to be larger than the transverse one, if the string fragmentation model is used.

In the Jetset generator, BEC is simulated by changing the final state particle momenta so that the Gaussian distribution of Eq. (3) is reproduced [3]. The procedure is performed in terms of $Q$, not resolving it into components, hence $R_{\perp}$ and $R_{\|}$ ought to be similar in the Jetset generated events. Indeed, the two-dimensional fit to the correlation function evaluated from the Jetset generated decays of $Z^{0}$ gives a ratio of $R_{\perp} / R_{\|}$between 0.9 and 1.1, depending on the generator tuning. This is very different from the ratio of (12) and reflects the fact that the BEC implementation in Jetset is not appropriate for the multidimensional description of the correlation.

An elongation of the pion source was also observed by the L3 [10] and OPAL [12] collaborations at LEP1. L3 collaboration used all the hadronic events in the analysis, without applying additional selection criteria neither for two-jet events, nor for pairs of tracks. The contribution from the correla-
tions between particles produced in gluon jets and possibly between the two strings is expected to lead to a more spherical source shape and to a bigger value of the ratio of the radii, close to unity. The ratio measured by L3 $R_{t, \text { side }} / R_{\text {long }}=0.81 \pm 0.02_{-0.19}^{+0.03}$ is bigger then the $R_{\perp} / R_{\|}$reported in this work, which confirms these expectations. The OPAL Collaboration used the unlike-charge reference sample in their analysis of two-jet events, obtaining the ratio of radii $R_{\perp} / R_{\|}=0.77 \pm 0.02 \pm 0.07$.

The measurement of the shape of the BEC presented here makes use of the LCMS system to obtain a clear interpretation of the observed difference between transverse and longitudinal correlation radii. Together with analogous measurements done by other LEP experiments, it represents an improvement in BEC studies compared to previous studies at lower energies [9], which used the laboratory system. While, the TASSO and MARK-II collaborations, barely hinted at the possibility of the pion source in the process $e^{+} e^{-} \rightarrow$ hadrons being elliptical, this new result provides clear evidence for the elongation of the source. The results have implications for the modelling of hadronic final states performed by event generators.

## 6. Summary

Two-dimensional analysis of the Bose-Einstein effect using the 1992-1995 DELPHI data confirms the prediction that the longitudinal correlation length, $R_{\|}$, in $Z^{0} \rightarrow q \bar{q}$ decay is bigger than the transverse one, $R_{\perp}$, if the bosons produced in the string fragmentation are subject to Bose-Einstein correlations during the hadronization process. The measured values are:
$R_{\perp}=0.53 \pm 0.08 \mathrm{fm}, \quad R_{\|}=0.85 \pm 0.08 \mathrm{fm}$.
The measured ratio of the radii components is $R_{\perp} / R_{\|}=0.62 \pm 0.06$, which is consistent with qualitative predictions [8]. These results cannot be reproduced by the Jetset generator because this generator includes only a simplified algorithm for the BEC simulation, which does not distinguish be-
tween the directional components of the correlation radius.

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