Discussion on "Dynamical Chern-Simons term generation at finite density" and "Chern-Simons term at finite density"

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Abstract

We discuss the comment by V.Zeitlin on our recent papers concerning Chern-Simons term generation at finite density.

The point of the V.Zeitlin's comment [1] is contradiction of our result [2] with the results obtained in papers [3] for QED_{2+1}. However, it isn't quite correct statement. As in papers [3] so as in our articles [2] mathematics is consistent. However, the investigated objects are essentially different. In [3] it was calculated not parity violating covariant form in action – Chern-Simons (CS) topological term, but the charge density which includes as parity odd as well parity even parts [3]. The parity odd part (we are studding) leads to CS term and mass generation of the gauge field in effective action. The parity even part reads

$$J^0_{\rm even} = \frac{|eB|}{2\pi}\left(\mathrm{Int}\left[\frac{\mu^2 - m^2}{2|eB|}\right] + \frac{1}{2}\right)\theta(\mu - |m|) \qquad (1)$$

(which is parity invariant because under parity $B \to -B$). It is clear that this parity even term does contribute neither to the parity anomaly nor to the mass of the gauge field. In our articles we are interested in parity odd covariant topological CS term in action, changing on winding number under gauge transformation and leading to the mass generation of a gauge field.

Moreover, in [3] calculations are done in the pure magnetic background and scalar magnetic field occurs in the argument's denominator of the cumbersome function. So, the parity even term seems to be "noncovariantizable", i.e. it can't be converted in covariant form in effective action.

In our papers [2] CS term was calculated at finite density as in abelian so as in nonabelian gauge theory. Besides, in [2] the calculations were performed in apparently covariant form for arbitrary background gauge field. Calculations were done as in 3–dimensional theory so as in 5–dimensional one. The result was generalized to any odd

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[3]When we, for abbreviation, speak about parity invariance properties of local objects we, certainly, have in mind symmetries of the corresponding action parts.

dimensional gauge theory. It was shown that μ–dependent CS term coefficient reveals the amazing property of universality. Namely, it does depend on neither dimension of the theory nor abelian or nonabelian gauge theory is studied.

In our papers [2] it was shown that in any odd dimension CS term coefficient has the same form

$$S_{eff}^{\mathrm{C.S}} = \frac{m}{|m|}\theta(m^2 - \mu^2)\pi W[A], \qquad (2)$$

where $W[A]$ is the CS term in any odd dimension. Since only the lowest orders of perturbative series contribute to CS term at finite density (the same situation is well-known at zero density), the result obtained by using formally perturbative technique appears to be nonperturbative. Thus, the μ–dependent CS term coefficient reveals the amazing property of universality. Namely, it does depend on neither dimension of the theory nor abelian or nonabelian gauge theory is studied.

The arbitrariness of μ gives us the possibility to see Chern–Simons coefficient behavior at any masses. It is very interesting that $\mu^2 = m^2$ is the crucial point for Chern–Simons. Indeed, it is clearly seen from (2) that when $\mu^2 < m^2$ μ–influence disappears and we get the usual Chern-Simons term $I_{eff}^{\mathrm{C.S}} = \pi W[A]$. On the other hand when $\mu^2 > m^2$ the situation is absolutely different. One can see that here the Chern-Simons term disappears because of non–zero density of background fermions. We'd like to emphasize the important massless case $m = 0$ considered in [4]. Then even negligible density, which always take place in any physical processes, leads to vanishing of the parity anomaly.

In conclusion, let us stress again that we nowhere have used any restrictions on μ. Thus we not only confirm result in [4] for Chern–Simons in QED_3 at small density, but also expand it on arbitrary μ, nonabelian case and arbitrary odd dimension.

References

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