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e^+e^- ANNIHILATION INTO HADRONS AT LOW ENERGIES AND τ DECAY IN THE NONPERTURBATIVE APPROACH TO QUANTUM CHROMODYNAMICS

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Abstract

We formulate a systematic, nonperturbative expansion in quantum chromodynamics using a new small parameter. We consider the process of e^+e^- annihilation into hadrons at low energies applying the "smearing" method to compare the obtained theoretical prediction with experimental data. The method is applied to the inclusive semileptonic decay of the τ–lepton.

The solution of many physical problems is based on approximation of a quantity under consideration by a finite number of terms of a certain series. In quantum field theory this is conventionally an expansion into a perturbative series. This approach combined with the renormalization procedure is now a basic method for computations. As is well-known, perturbative series for many interesting models including realistic models are not convergent. Nevertheless, at small values of the coupling constant these series may be considered as asymptotic series and could provide a useful information. However, even in the theories with a small coupling constant, for instance, in quantum electrodynamics there exist problems which cannot be solved

by perturbative methods. Also, a lot of problems of quantum chromodynamics require nonperturbative approaches.

Many approaches have been devoted to the development of nonperturbative methods. Among them is the summation of a perturbative series (see reviews [1] and monograph [2]). The difficulty is that the procedure of summation of asymptotic series is not unique as it contains a functional arbitrariness. A correct formulation of the problem of summation is ensured by further information on the sum of a series [3]. At present information of that kind is known only for the simplest field-theoretical models [4].

There have been approaches that are not directly based on the perturbative series. Many of nonperturbative approaches make use of a variational procedure for finding the leading contribution. However, in this case there is no always an algorithm of calculating corrections to the value found by a variational procedure, and this makes difficult to answer the question how adequate is the so-called main contribution to the object under investigation and what is the range of applicability of the obtained estimations. Therefore, useful approaches to the study of the nonperturbative structure of quantum field theory are the methods that combine an expansion of a given quantity in a series that defines the algorithm of calculating the correction with an optimizing procedure.

More accurate results obtained in Ref. [5] for the $R(s)$-ratio for the process of e^+e^- annihilation into hadrons stimulated further theoretical study of that ratio [6-9]. Higher orders of perturbation theory for starting from the three-loop order become dependent on the renormalization scheme. Therefore, for the correct application of perturbative results it is necessary to use a certain procedure of optimization, for instance, the principle of minimal sensitivity [10]. On the one hand, the dependence of a physical quantity on the renormalization scheme thus arising can be considered as a regrettable fact; on the other hand, there appears an extra degree of freedom that can be used for constructing an optimal expansion that has a wider range of applicability in a particular scheme as compared to perturbation theory. Along this line, as shown in Ref. [7], it has been possible to advance towards the low-energy region.

In this paper, to consider the processes of e^+e^- annihilation and the inclusive τ decay we will apply a non-perturbative method proposed in Ref. [11]. This approach based on a new small expansion parameter a connected with the initial coupling constant g by the following equation [11]

$$\lambda = \frac{g^2}{(4\pi)^2} = \frac{1}{C}\frac{a^2}{(1-a)^3}\,, \tag{1}$$

where C is a positive constant. The Green function can be written as follows

$$\begin{aligned} G(\cdots) &= \sum_n \sum_{k=0}^{n} \frac{1}{(n-k)!}\Big(-\frac{\partial}{\partial\kappa}\Big)^{n-k} \frac{i^k}{k!}\,\frac{1}{[1+\kappa(\zeta^{-1}-1)]^{\nu/2}} \\ &\times \int D_{\rm QCD}(\cdots)\,[g_3\,S_3(A,q,\varphi)]^k \exp\{\,i\,[\,S_0(A,q,\varphi) + g_4^2\,S_4(A)\,]\,\}\,, \end{aligned} \tag{2}$$

where

$$g_3 = \frac{g}{[1 + \kappa((1-a)^{-1} - 1)]^{3/2}}, \qquad g_4 = \frac{g}{[1 + \kappa((1-a)^{-3} - 1)]^{1/2}}, \tag{3}$$

and we will set $\kappa = 1$ after all calculations. The term S_0 is the free action functional, S_3 and S_4 generate, respectively, three-line and four-gluon vertices. As follows from (1), at any values of the coupling constant g, the new expansion parameter a obeys the inequality $0 \leq a < 1$. For small values of the coupling constant g, the Nth–partial sum of the new series reproduces the Nth order of standard perturbation theory to within $O(g^{N+1})$. In the nonperturbative region, when the running coupling constant of QCD becomes large and perturbation theory does not work, the new expansion parameter a remains small and the method remains still valid.

We apply these results to describe the process of e^+e^- annihilation into hadrons using the renormalization scheme in which the pole quark masses are fixed. The corresponding consideration in the $\overline{MS}$–scheme has been performed in Ref. [12]. We will consider the range of $Q = \sqrt{s}$ from 0 to 6 GeV (like in Ref. [7]) and compare with experiment by using the smearing method [13]. The renormalization scale dependence of the running expansion parameter $a = a(\mu^2)$ is defined by the following equation [14]

$$C\,[U(a) - U(a_0)] = 11 \ln\frac{\mu^2}{\mu_0^2} - \frac{2}{3}\sum_f \left[I\left(\frac{\mu^2}{m_f^2}\right) - I\left(\frac{{\mu_0}^2}{m_f^2}\right)\right], \tag{4}$$

where μ_0 is some normalization point, $a_0 = a(\mu_0^2)$, $I(\mu^2/m^2)$ is well known one–loop integral, and the function $U(a)$ has the following form

$$U(a) = \frac{1}{a^2} - \frac{3}{a} - 12\ln a + \frac{3}{4}\ln(1-a) + \frac{45}{4}\ln(1+3a). \tag{5}$$

According to Ref. [13], we consider the following smeared quantity

$$R_\Delta(Q) = \frac{\Delta}{\pi}\int_0^\infty ds \frac{R(s)}{(s-Q^2)^2 + \Delta^2}. \tag{6}$$

For comparison with experimental data, we will also use the function

$$W_\Delta(q^2) = \frac{dR_\Delta(q^2)}{dq^2} = -\frac{1}{2\mathrm{i}}\left[\frac{D(q^2 + \mathrm{i}\Delta)}{q^2 + \mathrm{i}\Delta} - \frac{D(q^2 - \mathrm{i}\Delta)}{q^2 - \mathrm{i}\Delta}\right], \tag{7}$$

where $D(q^2)$ is the Adler function.

In Fig. 1, the smeared quantity (6) obtained in the first nontrivial order of our approximation is shown for $\Delta = 3\,\mathrm{GeV}^2$ (solid line). For the quark masses we use the following values: $m_u = m_d = 0.330$ GeV, $m_s = 0.510$ GeV, $m_c = 1.35$ GeV, $m_b = 4.5$ GeV and $m_t = 174$ GeV, and the parameter $C = 39$ that comes from meson spectroscopy [14]. The experimental curve is taken from Ref. [7] (short–dashed line);

we also report the theoretical results from this paper (long–dashed line). In Fig. 2 we show the analogous results for the quantity (7) with $\Delta = 4\,\mathrm{GeV}^2$. As can be seen, our result obtained in the first order quite well reproduces the experimental curve. It is of interest that the ratio we have derived almost coincides with the relevant result of [7] obtained on the basis of optimization of the third order of standard perturbation theory.

The τ decay process with hadronic final states represents an important test of quantum chromodynamics. Due to the inclusive character of the process, the ratio R_τ is a very convenient quantity both for a theoretical investigation and for the definition of the QCD coupling constant $\alpha_s(M_\tau^2)$. A detailed theoretical analysis of this problem has been given in Ref. [15] (see also Refs. [16-19], in which different aspects of the problem are discussed).

The starting point of the theoretical analysis is the expression

$$R_\tau = 2\int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\tilde{R}(s)\,, \tag{8}$$

where

$$\begin{aligned}\tilde{R}(s) &= \frac{\mathrm{N}}{2\pi\mathrm{i}}\left[\Pi(s+\mathrm{i}\epsilon)-\Pi(s-\mathrm{i}\epsilon)\right],\\ \Pi(s) &= \sum_{q=d,s}|V_{uq}|^2\left(\Pi_{uq,V}(s)+\Pi_{uq,A}(s)\right).\end{aligned} \tag{9}$$

The normalization factor N is defined so that in zeroth order perturbation theory $\tilde{R}^{(0)}_{\mathrm{pert}} = 3$. In the framework of standard perturbation theory the integral (8) cannot be evaluated directly since the integration region in (8) includes small values of momentum for which perturbation theory is invalid[1]. Instead of Eq. (8), the expression for R_τ may be rewritten, using Cauchy's theorem, as a contour integral in the complex s-plane with the contour running clockwise around the circle $|s| = M_\tau^2$. It seems that this trick allows one to avoid the problem of calculating the nonperturbative contribution, which is needed if one uses Eq. (8). However, the application of Cauchy's theorem is based on specific analytic properties of Π(s) or the Adler D function

$$D(q^2) = q^2\,(-\frac{d}{dq^2})\,\mathrm{N}\,\Pi(q^2)\,. \tag{10}$$

The function $D(q^2)$ is an analytic function in the complex q^2–plane with a cut along the positive real axis. It is clear that the approximation of the D–function by perturbation theory breaks these analytic properties. For example, the one-loop approximation for the QCD running coupling constant has a singularity at $Q^2 = \Lambda^2_{QCD}$, the existence of which prevents the application of Cauchy's theorem. Moreover, to define the running coupling constant in the timelike domain, one usually uses the

[1]In Ref.[20], the integral (8) has been calculated within the method of optimized perturbative series [21].

dispersion relation for the D function derived on the basis of the above-mentioned analytic properties. In the framework of perturbation theory, this method gives the so-called π^2–term contribution which plays an important role in the analysis of various processes [22-26]. However, the same problem arises: the perturbative approximation breaks the analytic properties of $\lambda^{\rm eff}(q^2)$ which are required to write the dispersion relation. In addition, there is the problem of taking into account of threshold effects. As follows from Eq. (8), the initial expression for R_τ "knows" about the threshold. But all the threshold information is lost if one rewrites Eq. (8) as a contour integral and uses a fixed number of flavours for the calculation of $\tilde{R}(s)$ on this contour.

Here we will concentrate on both aspects of the problem. In the framework of our approach there exists a well-defined procedure for defining the running coupling in the timelike domain which does not conflict with the dispersion relation [27]. We will use the following definitions: $\lambda^{\rm eff} = \alpha_{\rm QCD}/(4\pi)$ is the initial effective coupling constant in the t–channel (spacelike region) and $\lambda_s^{\rm eff}$ is the effective coupling constant in the s–channel (timelike region). ¿From the dispersion relation for the D–function we obtain

$$\lambda^{\rm eff}(q^2) = -q^2 \int_0^\infty \frac{ds}{(s-q^2)^2}\, \lambda_s^{\rm eff}(s)\ . \tag{11}$$

Thus, the initial running coupling constant $\lambda^{\rm eff}(q^2)$ is an analytic function in the complex q^2–plane with a cut along the positive real axis. This function does not exist for real positive q^2, so the definition of the running coupling constant in the timelike domain is a crucial question. Here we use the standard definition of $\lambda_s^{\rm eff}(s)$ in the s–channel based on the dispersion relation for the Adler D–function. In this case, parametrization of timelike quantities, for example $R_{e^+e^-}(s)$ or $\tilde{R}(s)$, by the function $\lambda_s^{\rm eff}(s)$ is similar to parametrization of spacelike processes by the function $\lambda^{\rm eff}(q^2)$.

The inverse relation of Eq. (11), given the analytic properties of $\lambda^{\rm eff}(q^2)$, is of the form

$$\lambda_s^{\rm eff}(s) = -\frac{1}{2\pi {\rm i}} \int_{s-{\rm i}\epsilon}^{s+{\rm i}\epsilon} \frac{dq^2}{q^2}\, \lambda^{\rm eff}(q^2)\ , \tag{12}$$

where the contour goes from the point $q^2 = s - {\rm i}\epsilon$ to the point $q^2 = s + {\rm i}\epsilon$ and lies in the region where $\lambda^{\rm eff}(q^2)$ is an analytic function of q^2. Equation (12) defines the running coupling constant in the timelike region which we must use to calculate $\tilde{R}(s)$ in Eq. (8).

To write Eq. (12), it was important that the function $\lambda^{\rm eff}(q^2)$ had the above-mentioned analytic properties. For example, to use the one-loop approximation, one needs to modify its infrared behavior at $Q^2 = \Lambda^2$ in an *ad hoc* manner so that the singularity at $Q^2 = \Lambda^2$ is absent in the new expression for $\lambda(Q^2)$. A self-consistent formulation of the analytic continuation problem is, however, possible within the scope of a systematic non-perturbative approach. Within this approach we can

maintain the mentioned above analytic properties [27,28]. Taking the experimental value $R_\tau = 3.552$ [29] as an input, we obtain $\alpha_s(M_\tau^2) = 0.37$ and $\alpha(M_\tau^2) = 0.40$. The values of the coupling constant in the s- and t–channels are clearly different from each other; the ratio is $\alpha_s(M_\tau^2)/\alpha(M_\tau^2) = 0.92$.

The experimentally measurable quantity R_τ can be parametrized both by the function $\alpha_s(s)$ defined in the time–like region and entering into the initial expression for R_τ and by the running coupling constant $\alpha(q^2)$ used in the contour integral. The perturbative expansion does not allow one to perform the integration in Eq.(8) directly because it involves a non-perturbative region. Instead, one usually uses the perturbative formula to evaluate the contour integral. However, we believe this to be inconsistent because the analytic properties which are required to write down the Cauchy integral are not respected by the perturbative formula. The method proposed allows one to evaluate both the initial integral for R_τ and the expression obtained by the use of Cauchy's theorem. Of course, as it should be, they are equal. We have also demonstrated that the distinction between the functions $\alpha_s(s)$ and $\alpha(q^2)$ is not simply a matter of the standard π^2 terms, which may be important for understanding certain discrepancies [30] arising in the determination of the QCD coupling constant from various experiments.

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SOME METHODS OF MINIMIZATION OF CALCULATIONS IN HIGH ENERGY PHYSICS

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Abstract

Two approaches to calculations' minimization in High Energy Physics are considered. The first one is the method of covariant calculations for the amplitudes of processes with polarized Dirac particles. The second one connects with the possibility to reduce the expressions for the traces of products of ten and more Dirac γ-matrices.

1 Introduction

It is well known that the high order calculation of the observables within the perturbative theory turns to the serious difficulties (especially, if we take into account the polarization effects). The reason is in necessity to evaluate the traces of products of great number of Dirac γ-matrices. So the problems arise in the both cases of analytical and numerical calculations of the different physical quantities (for cumbersomeness of their expressions). It is clear that direct calculation of the processes' amplitudes (see Section II) is one of the chanses to turn over such a problem.

In Section III the possibility to reduce the expressions for the traces of products of ten and more Dirac γ-matrices is disscussed.

Some details can be found in the papers [1]-[3].

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