# Energy-energy correlations in hadronic final states from $\mathrm{Z}^{0}$ decays 

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We have studied the energy-energy angular correlations in hadronic final states from $Z^{0}$ decay using the DELPHI detector at LEP. From a comparison with Monte Carlo calculations based on the exact second order QCD matrix element and string frag-
 0.003 (stat.) $\pm 0.003$ (syst.) ${ }_{-0.0003}$ (theor.). The theoretical error stems from different choices for the renormalization scale of $\alpha_{s}$. In the Monte Carlo simulation the scale of $\alpha_{\mathrm{s}}$ as well as the fragmentation parameters have been optimized to described reasonably well all aspects of multihadron production.

## 1. Introduction

The asymmetry in the energy-energy correlation (AEEC) was introduced by Basham, Brown, Ellis and Love [1] as "good" observable to determine the
strong coupling constant $\alpha_{\mathrm{s}}$, since it is relatively insensitive to fragmentation effects, which mainly contribute symmetrically to the energy-energy correlations (EEC). Subsequently the second order corrections have been calculated [2-5], and found
to be reasonably small at the parton level. Many experiments have studied the EEC and determined $\alpha_{\text {s }}$ [6]. Most of the previous studies have been performed with Monte Carlo simulations based on the second order QCD matrix element using $\sqrt{s}$ as scale of $\alpha_{\mathrm{s}}$. However, it is known that this scale does not describe the four-jet contribution correctly [7,8]. The purpose of this paper is threefold: (a) to study the AEEC with an optimized scale, thus providing a correct contribution from the four-jet multiplicities; (b) to study the AEEC at higher energies, where fragmentation effects should be less important; (c) to determine the strong coupling constant from the AEEC at $\sqrt{s}=91 \mathrm{GeV}$.

## 2. The detector and data selection

The sample of events used in the analysis was collected by the DELPHI detector at the LEP $\mathrm{e}^{+} \mathrm{e}^{-}$collider. Features of the apparatus relevant for the analysis of multihadronic final states (with emphasis on the detection of charged particles ) are outlined in ref. [9]. The present analysis relies on the information provided by the charged particle detectors operating in a 1.2 T magnetic field.
Only charged particles fulfilling the following criteria were used:
(a) Impact parameter below 5 cm in radius $r$ from the beam axis and below 10 cm along the beam axis ( $z$ ).
(b) Momentum $p$ larger than $0.1 \mathrm{GeV} / c$.
(c) Measured track length above 50 cm .
(d) Polar angle $\theta$ between $25^{\circ}$ and $155^{\circ}$.

Hadronic events were then selected requiring that
$(\alpha)$ each of the two hemispheres $\cos \theta<0$ and $\cos \theta>0$ contained a total charged energy $E_{\mathrm{ch}}=\sum E_{i}$ larger than 3 GeV , where $E_{i}$ are the particle energies (assuming $\pi^{ \pm}$mass for the particles).
$(\beta)$ the total charged energy seen in both hemispheres exceeded 15 GeV .
$(\gamma)$ at least five charged particles were detected with momenta above $0.2 \mathrm{GeV} / c$.
( $\delta$ ) the polar angle $\theta$ of the sphericity axis was in the range $40^{\circ}<\theta<140^{\circ}$ (this cut ensures that the events are well contained inside the TPC).
$(\epsilon)$ the missing momentum $\left|\sum \boldsymbol{P}_{i}\right|$ did not exceed 20 GeV .

A total of 4158 events satisfied these cuts. Events due to beam-gas scattering and to $\gamma \gamma$ interactions have been estimated to be less than $0.1 \%$ of the sample; background from $\tau^{+} \tau^{-}$events was calculated to be less than $0.2 \%$.

## 3. Fragmentation models

The transformation of quarks and gluons into hadronic final states - called fragmentation - cannot be calculated perturbatively, because of the large value of the strong coupling constant. Therefore, the perturbative QCD calculations at the parton level have to be supplemented with phenomenological fragmentation models.

Several fragmentation models are available [5]. We have chosen the JETSET Monte Carlo [10]. This Monte Carlo has several options for the choice of the underlying QCD calculations and the subsequent fragmentation. For the parton generation we have used the exact second order QCD matrix element (ME \#1) option as well as the parton shower option (PS ${ }^{\# 2}$ ) based on a combination of the first order ME and the leading log approximation (LLA). This latter approximation is not valid for hard large angle gluon radiation; therefore the first gluon radiated in the Monte Carlo is generated with the first order matrix element. For the second order ME we have used the default value for the "optimized" scale to evaluate the coupling constant, i.e. $\alpha_{\mathrm{s}}\left(Q^{2}\right)$ is evaluated at $Q^{2}=0.002 s$ with $s$ as the centre of mass energy squared. Such a small scale ( $Q$ of the order of a few GeV ) and a correspondingly large value of $\alpha_{\mathrm{s}}$ are needed to get a proper description of the jet multiplicities [7,8], especially the four-jet rate. The scale of $\alpha_{\mathrm{s}}$ for the LLA option is roughly the transverse momentum of the branching, which is typically of the order of a few GeV too, and which describes the jet multiplicities equally well at lower energies.

For the transformation of the partons into hadrons we have used the string fragmentation model as implemented in the JETSET 7.2 Monte Carlo [10]. In this model the hadrons are formed along a string stretched between the outgoing partons. The string

[^0]tension represents the strength of the colour field (growing linearly with distance) and as soon as the tension becomes large enough, the energy is converted into mass by the formation of qa pairs at the breakpoints of the string.

## 4. Energy-energy correlations

Experimentally the energy-energy correlation (EEC) can be defined as a histogram of all angles between all pairs of particles, weighted with their energies:

$$
\begin{aligned}
& \operatorname{EEC}(\chi)=\frac{2}{N} \sum_{\mathrm{events}}^{N} \sum_{i}^{N_{\text {par }}-1} \sum_{j>i}^{N_{\text {var }}} \frac{E_{i} E_{j}}{E_{\mathrm{vis}}^{2}} \\
& \times\left(\frac{1}{\Delta \chi} \int_{x-\Delta x / 2}^{x+\Delta x / 2} \delta\left(\chi^{\prime}-\chi_{i j}\right) \mathrm{d} \chi^{\prime}\right),
\end{aligned}
$$

where $E_{i}$ is the energy of particle $i, \chi_{i j}$ is the angle between particles $i$ and $j, \chi$ is the opening angle for which one studies the correlation, $\Delta \chi$ is the bin width, $N$ is the number of events, $N_{\text {par }}$ is the number of particles in the event, and the weights are normalized to the visible energy $E_{\text {vis }}=\sum_{i}^{N_{\text {par }}} E_{i}$. The integral of the $\delta$ function is 1 for combinations in the bin plotted and zero otherwise.

Such a histogram shows two peaks: the peak below $30^{\circ}$ corresponds to the angles between pairs of particles inside a jet, while the peak near $180^{\circ}$ corresponds to angle between particles in opposite jets. Gluon radiation causes an asymmetry around $90^{\circ}$. This can be seen easily at the parton level, where two large angles and one small angle in a qāg event give more entries at large angles than at small angles.

The asymmetry is defined as

$$
\begin{aligned}
& \operatorname{AEEC}(\chi)=\operatorname{EEC}\left(180^{\circ}-\chi\right)-\operatorname{EEC}(\chi) \\
& 0^{\circ}<\chi \leqslant 90^{\circ} .
\end{aligned}
$$

On average, the two-jet contribution to the EEC cancels in the asymmetry. This is a unique feature of the asymmetry: it is very insensitive to the tuning of the fragmentation parameters, which only change the EEC in a symmetric way. The weighting of the angles with the energy makes the EEC infrared stable, i.e. the contribution of soft gluons goes to zero as their energy goes to zero. The second order corrections are
about $30 \%$ for the EEC and $15 \%$ for the AEEC [ 2 5].

The coupling constant $\lambda_{\mathrm{s}}$ is defined as [11]
$\alpha_{\mathrm{s}}\left(Q^{2}\right)=\frac{1}{\beta_{0} L}\left[1-\frac{\beta_{1}}{\beta_{0}^{2}} \frac{\ln L}{L}+\mathrm{O}\left(\left(\frac{\ln L}{L}\right)^{2}\right)\right]$,
with
$L=\ln \left(Q^{2} / \Lambda^{(5)^{2}}\right)$,
$\beta_{0}=\left(33-2 n_{\mathrm{f}}\right) / 12 \pi$,
$\beta_{1}=\left(153-19 n_{\mathrm{f}}\right) / 24 \pi^{2}$.
Here $\Lambda_{\mathrm{MS}}^{(5)}$ is the QCD scale parameter in the $\overline{\mathrm{MS}}$ renormalization scheme [12] for $n_{f}=5$ quark flavours. For a choice of $Q$ below the b-mass $n_{f}=4$. In this case one should use either $\lambda \frac{(4)}{\mathrm{MS}}$ or apply a correction to $\alpha_{\mathrm{s}}$ [13]. For the small scale discussed below we have used $\Lambda_{\mathrm{MS}}^{(4)}$, but our final results are given as $\Lambda_{\mathrm{MS}}^{(5)}$. The simple relation between $\Lambda \frac{(4)}{\mathrm{MS}}$ and $\Lambda \frac{(5)}{\mathrm{MS}}$ can be found in ref. [13].

## 5. Choice of the scale of the coupling constant

The energy-energy correlation has been calculated using as argument of $\alpha_{\mathrm{s}}$ the scale $Q^{2}=s$. It is known that this choice of scale does not correctly describe the four-jet rate [7,8]. A different choice of scale
$Q^{2}=f s$
leads to a different coupling constant and different higher order contributions. Observables are independent of the choice of scale, if the calculations are done to all orders. However, in finite order there can be a dependence. Several "educated" guesses for the scale have been proposed [14-16].

Instead of studying such a specific scale, we decided to study the scale dependence for a large range of scales. We have generated the AEEC as function of the scale with the JETSET 7.2 Monte Carlo and found that the integral between $28.8^{\circ}$ and $90^{\circ}$ is rather stable. The observed variation corresponds to an increase in $\Lambda \frac{(5)}{\mathrm{MS}}$ of 30 MeV , if the scale factor $f$ is varied from 0.002 to 1 . With the lower scale factor of $f=0.002$ the contributions from the four-jet events are correctly taken into account [7,10]. Therefore we used this default value from JETSET 7.2 in our analysis.

As a second check we have calculated the scale dependence analytically at the parton level. The AEEC between $26^{\circ}$ and $90^{\circ}$ can be parametrized for a given scale $Q^{2}$ as
$\mathrm{AEEC}=C_{1} \alpha_{\mathrm{s}}+C \alpha_{\mathrm{s}}^{2}$.
If one chooses a different scale $Q^{\prime 2}=f Q^{2}$, one obtains from the renormalization group equation [or from its solution, eq. (1)] a change $\mathrm{d} \alpha_{\mathrm{s}}$ in the coupling constant:
$\mathrm{d} \alpha_{\mathrm{s}}=-\beta_{0} \alpha_{\mathrm{s}}^{2} \ln f+\mathrm{O}\left(\alpha_{\mathrm{s}}^{3}\right)$.
To keep the observable AEEC the same, one has to absorb the change in $\alpha_{\mathrm{s}}$ in different coefficients $C_{1}^{\prime}$ and $C_{2}^{\prime}$, i.e.

$$
\begin{align*}
& \mathrm{d} \mathrm{AEEC}=\alpha_{s} \mathrm{~d} C_{1}+C_{1} \mathrm{~d} \alpha_{\mathrm{s}}+\alpha_{\mathrm{s}}^{2} \mathrm{~d} C_{2}+\mathrm{O}\left(\alpha_{\mathrm{s}}^{3}\right) \\
& \quad=0, \tag{4}
\end{align*}
$$

which yields $C_{1}^{\prime}=C_{1}$ and $C_{2}^{\prime}=C_{2}+\beta_{0} C_{1} \ln f$, if $\alpha_{\mathrm{s}}$ is nonzero. For a given AEEC and scale one can solve eq. (2) for $\alpha_{\mathrm{s}}$ and calculate the corresponding $\Lambda \frac{(5)}{\mathrm{MS}}$; the result is shown in fig. 1 for $x=\sqrt{f}$ between 0.04 and 1 . This dependence is shown for two different parameters of the Sterman-Weinberg recombination scheme [17]. The value of the coefficients for $x=1$, as obtained from ref. [2], have been indicated too. For comparison, we show also the scale dependence in the $\Lambda_{\mathrm{MS}}^{(5)}$ values calculated from the three-jet rate $R_{3}$ for two recombination schemes, namely the $E$ and $E 0$ scheme. The coefficients for these schemes have been obtained from ref. [5] and the reference jet rate at $f=1$ has been estimated from Monte Carlo. One observes a stronger scale dependence for the jet rates, corresponding to the larger second order corrections. Note that these curves only indicate the scale dependence at the parton level. At the hadron level, where a large part of the higher order corrections are absorbed in the hadronization, the dependence may be different, especially if for each scale the fragmentation parameters are adjusted to the data in order to compensate for the different higher order corrections at the parton level corresponding to the different scales.


Fig. 1. Scale dependence of $A \frac{(5)}{M S}$ derived for the AEEC and the three-jet rate $R_{3} . R_{3}^{E 0}$ and $R_{3}^{E}$ are the three-jet rates for the $E 0$ and $E$ recombination schemes. The upper index of the AEEC indicates the energy cut in the Sternman-Weinberg recombination scheme. Note that these curves only indicate the scale dependence at the parton level. At the hadron level, where a large part of the higher order corrections are absorbed in the hadronization, the dependence may be different, especially if for each scale the fragmentation parameters are adjusted to the data in order to compensate for the different higher order corrections at the parton level.

## 6. Data correction

In order to correct for detector effects the measured distribution of the EEC was corrected by multiplying each bin of the histogram by a correction factor. The correction factor is constructed to contain all detector-specific characteristics and QED type radiative corrections, which contribute to the measurement process starting from the hadron level of the physical event up to the level of reconstructed tracks, energies and momenta including all applied kinematical cuts and selection criteria of the analysis. These corrections are obtained by comparing the distribution at the hadron level before detector simulation and radiative corrections with the outcome after a detailed simulation of the DELPHI detector including all steps of the analysis.

For each bin in the histogram we calculated
$c^{(i)}=\frac{N_{\mathrm{gn}}^{(i)}}{N_{\mathrm{sim}}^{(i)}}$,
where $N^{(i)}$ is the content of the $i$ th histogram bin either on the generator level (i.e. charged hadrons) or after the detailed detector simulation. The corrected data distribution is simply
$N_{\text {corr }}^{(i)}=c^{(i)} N_{\text {data }}^{(i)}$.
The corrections are small as can be seen in fig. 2 . They deviate by less than $20 \%$ from unity over the whole angular range. The corrected data are compared in fig. 3 to the PS model with default values and the ME Monte Carlo with optimized parameters to be discussed below. The PS model has too large an asymmetry, which implies too large a value of the default QCD scale parameter in the PS model \#3.

## 7. Determination of $\boldsymbol{\alpha}_{\mathbf{s}}$

There are several ways to extract the strong coupling constant from the corrected AEEC distribution. We shall consider only the large angle part, since the small angle region is dominated by angles within a jet, which are more sensitive to fragmentation effects. The simplest way to extract $\alpha_{\mathrm{s}}$ is to determine the integral of the AEEC in the Monte Carlo as a function of $A \frac{(5)}{M S}$ and to compare the resulting curve with the integral of the AEEC in the data. We have used the version JETSET 7.2 with the exact second order ERT matrix element [18] followed by string


Fig. 2. Correction factor for detector and QED radiative effects to the observed EEC as calculated from a detailed Monte Carlo simulation using the parton shower option in JETSET 6.3 with default parameters.


Fig. 3. The corrected EEC and AEEC compared with PS [(a) and (b) ], and ME [(c) and (d)] models. The disagreement seen in (b) implies a too large value of the QCD scale parameter in the PS model (default value). The first two bins of the AEEC are negative for the PS model and the data and therefore not shown.
fragmentation, an $\alpha_{s}$ scale of $0.002 s$ (the default in JETSET 7.2), and the fragmentation parameters tuned to $\sqrt{s}=91 \mathrm{GeV}$, as described in ref. [19]. This set of fragmentation parameters describes all aspects of multihadron production, so that one is able to compare data and theory (i.e. QCD-matrix element plus string fragmentation). Such a comparison is shown in fig. 4. The dashed horizontal line corresponds to the data:
$\int_{28.8^{\circ}}^{90^{\circ}} \operatorname{AEEC}(\chi) \mathrm{d} \chi=0.0246 \pm 0.0023$ (stat.) .
From the crossing with the Monte Carlo curve we find


Fig. 4. The AEEC for charged particles integrated between $28.8^{\circ}$ and $90^{\circ}$ as function of $\Lambda_{\mathrm{MS}}^{(5)}$, as calculated from the exact QCD matrix element model with an $\alpha_{s}$ scale of 0.002 s . The dashed horizontal line corresponds to the data, which interests the Monte Carlo curve at $\Lambda^{\left(\frac{(S)}{M S}\right.}=108 \mathrm{MeV}$. The shaded area indicates the errors.
$\Lambda_{\mathrm{MS}}^{(\mathrm{S})}=108 \pm 30$ (stat. $)_{-20}^{+25}$ (syst.) MeV,
which corresponds for $f=0.002$ to

$$
\begin{aligned}
& \alpha_{\mathrm{s}}(Q=4.1 \mathrm{GeV}) \\
& \quad=0.185 \pm 0.013 \text { (stat.) } \pm 0.010 \text { (syst.) }
\end{aligned}
$$

and for $f=1$ to

$$
\begin{aligned}
& \alpha_{s}(Q=91 \mathrm{GeV}) \\
& \quad=0.106 \pm 0.004 \text { (stat.) } \pm 0.003 \text { (syst.). }
\end{aligned}
$$

The first error is the statistical error, which was derived from the variance of the $\alpha_{\mathrm{s}}$ values determined from nine independent subsamples in the data. The second error is the systematic error as determined from the variation of several fragmentation parameters, especially the parameters determining the transverse and longitudinal momentum spectra, since these parameters change the angles and momenta of the particles and therefore change the EEC. However, these changes contribute mainly symmetrically to the EEC; therefore the asymmetry and $\Lambda \frac{(\mathcal{S})}{\mathrm{MS}}$ are rather insensitive to such parameter changes. The maximum
allowable variation of the fragmentation parameters has been estimated from a fit to the rapidity and aplanarity distributions.

The given systematic uncertainties are the experimental systematic uncertainties for the $\alpha_{\mathrm{s}}$ determination using string fragmentation. In addition, we have investigated what happens if one replaces the string fragmentation (SF) by independent fragmentation (IF). It is well known that at lower energies this can lead to appreciably different results [20]. Even at our energies we find differences between these fragmentation models: IF models as implemented as options in JETSET 7.2 yield $\Lambda \frac{(5)}{\text { MS }}$ values, which can be as much as 60 MeV lower than the SF model. However, it turns out to be difficult to tune the parameters in these IF models to get good agreement with the data, especially the rapidity distribution, which nearly always show a large excess at small rapidities. The reason is that in these IF models the soft and large angle gluons lead to more particles at large angles with respect to the thrust axis than in SF , since in SF particles follow the boost of the string, which especially affects the soft and large angle particles and tends to align them along the direction of the thrust axis, thus making the event more two-jet like. Therefore, SF always needs a larger $\Lambda_{\mathrm{MS}}^{(5)}$ value than IF in order to get the same kind of "three-jettiness" in the data. Since the IF models we used did not describe the data at all, we did not increase the systematic error for this additional model dependence.
In the previous method we have not taken into account any information on the shape of the AEEC. This can be done by fitting the complete distribution from the Monte Carlo to the corrected data: we generate for each iteration of the fit 20000 Monte Carlo events with the fast JETSET 7.2 generator and minimize the $\chi^{2}$ between the generated AEEC and the data by varying AMS in the generator. By this method we compare data and theory (i.e. QCD matrix element plus string fragmentation model) both at the hadron level and can easily repeat the fit for any value of the other fragmentation parameters or the choice of $Q^{2}$ in the definition of the coupling constant.
The result of the fit in the range of $\chi$ between $28.8^{\circ}$ and $90^{\circ}$ is
$\Lambda_{\mathrm{MS}}^{(\mathrm{S})}=104_{-20}^{+25}($ stat. $){ }_{-20}^{+25}($ syst. $){ }_{-00}^{+30}($ theor. $) \mathrm{MeV}$, which corresponds for $f=0.002$ to

$$
\begin{aligned}
& \left.\alpha_{\mathrm{s}}(Q=4.1 \mathrm{GeV})=0.184 \pm 0.010 \text { (stat. }\right) \\
& \quad \pm 0.010 \text { (syst. })_{-0.000}^{+0.012} \text { (theor.) }
\end{aligned}
$$

and for $f=1$ to

$$
\begin{aligned}
& \alpha_{s}(Q=91 \mathrm{GeV}) \\
& \quad=0.106 \pm 0.003(\text { stat. })_{-0.000}^{+0.003} \text { (theor.) }
\end{aligned}
$$

The first two errors have the same meaning as discussed above. The last error corresponds to the estimated error from the choice of scale for $\alpha_{s}$. The scale of $0.002 s$ was chosen in order to get a correct parametrization of the four-jet events in the Monte Carlo. Varying the scale factor $f$ between 0.004 and 1 change $\Lambda_{\mathrm{MS}}^{(5)}$ by about 30 MeV as estimated from the parton level calculation (see section 5 and fig. 1). In addition we fitted the AEEC distribution between $28.8^{\circ}$ to $90^{\circ}$ with a full Monte Carlo at the hadron level for three different scale factors $f$. The results are shown in table 1 and are qualitatively in agreement with the results at the parton level (see fig. 1), although the rise at small scales is not observed at the hadron level, indicating a more stable behaviour at the hadron level, especially if for each scale the fragmentation parameters are adjusted for the different higher orders at the parton level ${ }^{\# 4}$.

The result on $\Lambda_{\mathrm{MS}}^{\left(\frac{5}{\mathrm{MS}}\right)}$ from the shape fit is in good agreement with the one from the integral, as expected since the Monte Carlo describes well the shape of the AEEC in the angular range fitted (see fig. 3d).

From our analysis of jet production rates [20] we found

$$
\begin{aligned}
& \alpha_{\mathrm{s}}(Q=91 \mathrm{GeV})=0.114 \pm 0.003 \text { (stat.) } \\
& \quad \pm 0.004 \text { (syst.) } \pm 0.012 \text { (theor.) }
\end{aligned}
$$

in good agreement with the value give above. The larger theoretical error originates from the renormal-
\#4 We adjusted slightly the longitudinal and transverse fragmentation parameters to the aplanarity and rapidity distribution.

Table 1
$\Lambda_{\mathrm{MS}}^{(S)}$ and $\alpha_{\mathrm{s}}$ for three different choices of $Q^{2}=f s E_{\mathrm{CM}}=91 \mathrm{GeV}$.

| $f$ | $\Lambda_{\mathrm{MS}}^{(5)}[\mathrm{MeV}]$ | $\alpha_{\mathbf{s}}\left(\mathrm{Z}^{0}\right)$ |
| :--- | :--- | :--- |
| 1 | 133 | 0.109 |
| 0.0625 | 113 | 0.107 |
| 0.002 | 104 | 0.106 |

ization scale dependence for some of the recombination schemes (see section 5).

## 8. Conclusions

We have presented the first analysis of the energy weighted angular correlation in multihadronic events from $Z^{0}$ decays. A comparison with the exact second order QCD matrix element yields
$\Lambda_{\mathrm{MS}}^{(\mathrm{SS}}=104_{-20}^{+25}($ stat. $){ }_{-20}^{+25}($ syst. $){ }_{-00}^{30}$ (theor.) MeV ,
which corresponds to

$$
\begin{aligned}
& \alpha_{\mathrm{s}}(Q=91 \mathrm{GeV})=0.106 \pm 0.003 \text { (stat.) } \\
& \quad \pm 0.003 \text { (syst.) }{ }_{-0.000}^{+0.003} \text { (theor.) } .
\end{aligned}
$$

For this determination of $\ell \frac{(5)}{\mathrm{MS}}$, we have used the scale of the coupling constant which gives a good description of the jet multiplicities so the contribution from four-jet events is estimated correctly. Choosing $Q^{2}=s$ as scale increases $\Lambda \frac{(5)}{\mathrm{MS}}$ by 30 MeV as indicated by the theoretical error (see table 1).

The value of $\Lambda_{\mathrm{MS}}^{\mathrm{MS}}$ agrees well with our determination of $\alpha_{\mathrm{s}}$ from the jet multiplicities [8]. The fact that these methods with completely different contributions from fragmentation effects yield the same value of $\alpha_{s}$ gives confidence in our understanding of the event structure originating from perturbative QCD.

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[^0]:    \#1 Version JETSET 7.2.
    *2 Version JETSET 6.3.

