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# SUPERSYMMETRIC TWO-PARTICLE EQUATIONS

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## 1. Introduction

In recent years supersymmetric quantum-field models have drawn great attention. We recall that a principal advantage of such models is the unification of bosonic and fermionic fields into one multiplet and a significant reduction of the number of divergences (up to a complete vanishing in models with N=4). Basic tool for studying the two-body problem in quantum field theory is two-particle dynamic equations (the Edwards equations for process $2 \mapsto 1$ and the Bethe-Salpeter, Logunov - Tavkhelidze equations for process $2 \mapsto 2$ ). A standard method for deriving these equations and their connection with the Lagrange formalism are provided by perturbation theory [1]. In the framework of this approach a supersymmetric Bethe-Salpeter equation has been obtained earlier [2].

A natural language exists for studying the properties of multiparticle equations in field theory - the method of higher-order Legendre transformations for the generating functional of Green functions [3]. With these transformations, multiparticle equations may be obtained as a consequence of the Schwinger equations. These multiparticle equations are model-independent and have been obtained beyond the scope of perturbation theory. The whole dynamic information on interaction is contained in the equation kernel. Note that the kernels obtained by the above method for multiparticle equations are expressed through the generating functional of the Legendre transformations. This provides new possibilities of studying kernels beyond the scope of perturbation theory and thus testifies once more in favour of this approach.

For the first time the Edwards and Bethe-Salpeter equations on the basis of the second Legendre transformation (for $\varphi^3$ theory) were obtained in ref.[4]. Then this method was further developed (for other quantum-field models and for a number of interacting particles larger than two) in refs.[5]. Our paper deals with derivation of the above equations by that method in a supersymmetric theory.

## 2. The Schwinger equation

We shall consider a theory of scalar chiral superfields $\varphi(x_L,\theta)$ and $\bar{\varphi}(x_R,\bar{\theta})$ with an arbitrary interaction functional $S^{int}[\varphi,\bar{\varphi}]$ (a generalization to the vector superfields is not, in principle, difficult). Then the total superfield action is written in the form[6]

$$S = S_0 + S_{int}, \tag{1}$$

where

$$\begin{aligned} S_0[\varphi, \bar{\varphi}] &= S_{kin}[\varphi, \bar{\varphi}] + S_{mass}[\varphi, \bar{\varphi}] = \\ &= \int d^4x\, d^2\theta\, d^2\bar{\theta}\; \varphi(x_L, \theta)\, \bar{\varphi}(x_R, \bar{\theta}) + \\ &+ \frac{m}{2}\int d^4x_L\, d^2\theta\; \varphi^2(x_L, \theta) + h.c. \end{aligned} \tag{2}$$

Using the connection between the left-hand and right-hand chiral bases we may rewrite the free part of the action (2) in the form (see A.1 ):

$$S_0 = \frac{1}{2}\big(\varphi(x,\theta)\; \bar{\varphi}(x\bar{\theta})\big)\begin{pmatrix} K_{11}(x\theta, x'\theta') & K_{12}(x\theta, x'\bar{\theta}') \\ K_{21}(x\bar{\theta}, x'\theta') & K_{22}(x\bar{\theta}, x'\bar{\theta}') \end{pmatrix} \times \begin{pmatrix} \varphi(x'\theta') \\ \bar{\varphi}(x'\bar{\theta}') \end{pmatrix}. \tag{3a}$$

where $\Phi \equiv \begin{pmatrix} \varphi \\ \bar{\varphi} \end{pmatrix}$ , $\Phi^T \equiv (\varphi, \bar{\varphi})$ is a transposed column, and $\mathcal{K}$ is an integral matrix operator with a kernel determined by the relation

$$\mathcal{K} = \begin{pmatrix} m\,\delta(\theta-\theta')\delta(x-x') & e^{-2i\theta\hat{\partial}\bar{\theta}'}\delta(x-x') \\ e^{2i\theta'\hat{\partial}\bar{\theta}}\delta(x-x') & m\,\delta(\bar{\theta}-\bar{\theta}')\delta(x-x') \end{pmatrix}. \tag{4}$$

Hereafter integration (summation) is done over twice repeated variables (indices).

For further convenience we introduce the following notation:

$$\begin{aligned} &\theta \equiv \theta^1, \; \bar{\theta} \equiv \theta^2 \\ &\varphi(x,\theta) \equiv \varphi^1(x\theta^1); \; \bar{\varphi}(x,\bar{\theta}) \equiv \varphi^2(x\theta^2). \end{aligned} \tag{5}$$

Then instead of (3a), we have

$$S_0 = \frac{1}{2}\varphi^\alpha(x_1\theta_1^\alpha)\,\mathcal{K}^{\alpha\beta}(x_1\theta_1^\alpha, x_2\theta_2^\beta)\,\varphi^\beta(x_2\theta_2^\beta), \tag{3b}$$

where $\alpha = 1;2$; $\beta = 1,2$ and the matrix elements $\mathcal{K}^{\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})$ of the kernel $\mathcal{K}$ are determined by relation (4). The kernel of the operator inverse to $\mathcal{K}$ is given by the relation

$$\mathcal{K}^{\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})\,\mathcal{K}^{-1\beta\gamma}(x_2\theta_2^{\beta}, x_3\theta_3^{\gamma}) = \tag{6}$$
$$= \mathcal{K}^{-1\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})\,\mathcal{K}^{\beta\gamma}(x_2\theta_2^{\beta}, x_3\theta_3^{\gamma}) = \delta_{\alpha\gamma}\,\delta(x_1 - x_3)\,\delta(\theta_1^{\alpha} - \theta_3^{\gamma}).$$

It represents a matrix of free propagators $D_0$ of the following form (see (A.2)):

$$\mathcal{K}^{-1} = D_0 = \begin{pmatrix} -m\,\delta(\theta_1-\theta_2)\Delta^c(x_1-x_2) & e^{-2i\theta_1\hat{\partial}\bar{\theta}_2}\Delta^c(x_1-x_2) \\ e^{2i\theta_2\hat{\partial}\bar{\theta}_1}\Delta^c(x_1-x_2) & -m\,\delta(\bar{\theta}_1-\bar{\theta}_2)\Delta^c(x_1-x_2) \end{pmatrix} \tag{7}$$

We shall define the generating functional of Green functions as a functional integral with two standard local sources $J^{\alpha}(x\theta^{\alpha})$ composing a column $\begin{pmatrix} J(x\theta) \\ \bar{J}(x\bar{\theta}) \end{pmatrix}$ and four bilocal sources $M^{\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})$ composing a matrix

$$M = \begin{pmatrix} M_{12}(x_1\theta_1, x_2\theta_2) & M_{12}(x_1\theta_1, x_2\theta_2) \\ M_{21}(x_1\bar{\theta}_1, x_2\theta_2) & M_{22}(x_1\bar{\theta}_1, x_2\bar{\theta}_2) \end{pmatrix}.$$

$$G[J, \bar{J}; M] = N\int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}\, \exp\Big[i\Big\{\tfrac{1}{2}\big(\varphi(x_1\theta_1)\,\bar{\varphi}(x_1\bar{\theta}_1)\big)\times$$
$$\times\left[\begin{pmatrix} K_{11}(x_1\theta_1, x_2\theta_2) & K_{12}(x_1\theta_1, x_2\theta_2) \\ K_{21}(x_1\bar{\theta}_1, x_2\theta_2) & K_{22}(x_1\bar{\theta}_1, x_2\bar{\theta}_2) \end{pmatrix} + \right. \tag{8}$$
$$\left. + \begin{pmatrix} M_{11}(x_1\theta_1, x_2\theta_2) & M_{12}(x_1\theta_1, x_2\theta_2) \\ M_{21}(x_1\bar{\theta}_1, x_2\theta_2) & M_{22}(x_1\bar{\theta}_1, x_2\bar{\theta}_2) \end{pmatrix}\right]\begin{pmatrix} \varphi(x_2\theta_2) \\ \bar{\varphi}(x_2\bar{\theta}_2) \end{pmatrix} +$$
$$+ \big(J(x\theta)\,\bar{J}(x\bar{\theta})\big)\begin{pmatrix} \varphi(x\theta) \\ \bar{\varphi}(x\bar{\theta}) \end{pmatrix} + S_{int}[\varphi, \bar{\varphi}]\Big\}\Big].$$

Here $N$ is a conventional normalization constant.

The Green functions are expressed through $G[J,\bar{J};M]$ as usual:

$$G_{(N)}^{\alpha_1\ldots\alpha_N}(x_1\theta_1^{\alpha_1},\ldots,x_N\theta_N^{\alpha_N}) = i\langle 0|T\varphi^{\alpha_1}(x_1\theta_1^{\alpha_1})\ldots\varphi^N(x_N\theta_N^{\alpha_N})|0\rangle$$

$$= i^{1-N}\,\frac{\delta^N G[J,\bar{J};M]}{\delta J^{\alpha_1}(x_1\theta_1^{\alpha_1})\ldots\delta J^{\alpha_N}(x_N\theta_N^{\alpha_N})}\Bigg|_{\substack{J=\bar{J}=0,\\ M=0.}} \qquad (9)$$

For simplicity we introduce the notation:

$$iJ = A_1,\quad i\bar{J} = \bar{A}_1,$$
$$i(M^{\alpha\beta} + D_0^{-1\,\alpha\beta}) = A_2^{\alpha\beta};\quad iS^{int} = A, \qquad (10)$$

in terms of which the generating functional is of the form

$$G[A_1^{\alpha}, A_2^{\alpha\beta}] = N\cdot\int\mathcal{D}\varphi^1\mathcal{D}\varphi^2\exp\{A_1^{\alpha}(x\theta^{\alpha})\varphi(x\theta^{\alpha}) + \frac{1}{2}\varphi^{\alpha}(x_1\theta_1^{\alpha})A_2^{\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})\varphi^{\beta}(x_2\theta_2^{\beta}) + A[\varphi_1,\varphi_2]\}. \qquad (11)$$

It is obvious that the limit of switched-off sources $J=\bar{J}=M=0$ corresponds to $A_1^1 = A_1^2 = 0$, $A_2^{\alpha\beta} = D_0^{-1\alpha\beta}$. From invariance of the measure of integration $\mathcal{D}\varphi^1$ with respect to translations $\varphi_1 \to \varphi_1 + \varepsilon^1$ we get

$$0 = N\cdot\int\mathcal{D}\varphi_1\mathcal{D}\varphi_2\,\frac{\delta}{\delta\varphi^1(x\theta^1)}\exp\{A_1^{\alpha}(x_1\theta_1^{\alpha})\varphi^{\alpha}(x_1\theta_1^{\alpha}) +$$

$$+ \frac{1}{2}\varphi^{\alpha}(x_1\theta_1^{\alpha})A_2^{\alpha\beta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta})\varphi^{\beta}(x_2\theta_2^{\beta}) + A[\varphi_1,\varphi_2]\} =$$

$$= N\cdot\int\mathcal{D}\varphi_1\mathcal{D}\varphi_2\{A_1^1(x\theta^1) + A_2^{11}(x\theta^1, x\theta_1^1)\varphi^1(x_1\theta_1^1) +$$

$$+ \frac{1}{2}[A_2^{12}(x\theta^1, x_1\theta_1^2)\varphi^2(x_1\theta_1^2) + \varphi^2(x_1\theta_1^2)A_2^{21}(x_1\theta_1^2, x\theta^1)] +$$

$$+ \frac{\delta A[\varphi_1,\varphi_2]}{\delta\varphi^1(x\theta^1)}\}\exp\{A_1^{\alpha}\varphi^{\alpha} + \frac{1}{2}\varphi^{\alpha}A_2^{\alpha\beta}\varphi^{\beta}\}.$$

As a result, we find the following equation in variational derivatives:

$$\Big\{A_2^1(x\theta^1) + A_2^{11}(x\theta^1, x_1\theta_1^1)\frac{\delta}{\delta A_1^1(x_1\theta_1^1)} + \frac{1}{2}\Big(A_2^{12}(x\theta^1, x_1\theta_1^1)\frac{\delta}{\delta A_1^2(x_1\theta_1^2)}$$

$$+ \frac{\delta}{\delta A_1^2(x_1\theta_1^2)} A_2^{21}(x_1\theta_1^2, x\theta^1)\Big) + \tag{12a}$$

$$+ \left(\frac{\delta A[\varphi_1, \varphi_2]}{\delta\varphi^1(x\theta^1)}\right)_{\varphi^1 = \frac{\delta}{\delta A_1^1},\ \varphi^2 = \frac{\delta}{\delta A_1^2}}\Big\} G[A_1, A_2] = 0,$$

which represents an analog of the Schwinger equation in a supersymmetric theory. Similarly, from invariance of the measure $\mathcal{D}\varphi^2$ we obtain the second Schwinger equation

$$\Big\{A_1^2(x\theta^2) + A_2^{22}(x\theta^2, x_1\theta_1^2)\frac{\delta}{\delta A_1^2(x_1\theta_1^2)} + \tag{12b}$$

$$+ \frac{1}{2}\Big(A_2^{21}(x\theta^2, x_1\theta_1^1)\frac{\delta}{\delta A_1^1(x_1\theta_1^1)} + \frac{\delta}{\delta A_1^1(x_1\theta_1^1)} A_2^{21}(x_1\theta_1^1, x\theta^2)\Big)$$

$$+ \left(\frac{\delta A[\varphi^1, \varphi^2]}{\delta\varphi^2(x\theta^2)}\right)_{\varphi^1 = \frac{\delta}{\delta A_1^1},\ \varphi^2 = \frac{\delta}{\delta A_1^2}}\Big\} G[A_1, A_2] = 0,$$

where $G[A_1, A_2]$ is the generating functional of Green functions.

The Schwinger equations (12a) and (12b) determine $G$ up to a factor, an arbitrary functional of $A_2^{\alpha\beta}$. The functional is fixed by constraints [3]. The constraints are obvious from (11):

$$\frac{\delta G[A_1, A_2]}{\delta A_2^{\alpha\beta}} = \frac{1}{2}\,\frac{\delta^2 G[A_1, A_2]}{\delta A_1^\alpha\,\delta A_1^\beta}\,. \tag{13}$$

As usual, it is necessary to pass from the generating functional of disconnected Green functions $G[A_1, A_2]$ to their logarithm if we want to get rid of nonphysical disconnected components:

$$W[A_1, A_2] = \ln G[A_1, A_2], \tag{14}$$

that represents the generating functional of connected Green functions. Then the connected Green functions $D_n^{\alpha_1 \ldots \alpha_n}$ are determined by the relation

$$D_n^{\alpha_1\ldots\alpha_n}(x_1\theta_1^{\alpha_1};\ldots,x_n\theta_n^{\alpha_n}) = \frac{\delta^n W}{\delta A_1^{\alpha_1}(x_1\theta_1^{\alpha_1})\ldots\delta A_n(x_n\theta_n^{\alpha_n})} =$$

$$=\frac{1}{G}\,\frac{\delta^n G}{\delta A_1^{\alpha_1}(x_1\theta_1^{\alpha_1})\ldots\,\delta A_n^{\alpha_n}(x_n\theta_n^{\alpha_n})}\,, \tag{15}$$

where the notation (5) is used, and indices $\alpha_1,\ldots,\alpha_n$ take values 1 and 2.

With the help of (14) and (15) it is not difficult to obtain, instead of (14), the equations for $W$ :

$$A_1^1(x\theta^1)+A_2^{11}(x\theta^1;x_1\theta_1^1)D_1^1(x_1\theta_1^1)+\frac{1}{2}\Big(A_2^{12}(x\theta^1;x_1\theta_1^2)D_1^2(x_1\theta_1^2)$$
$$+D_1^2(x_1\theta_1^2)A_2^{21}(x_1\theta_1^2,x\theta^1)\Big)+J^1(x\theta^1)=0, \tag{16a}$$

$$A_1^2(x\theta^2)+A_2^{22}(x\theta^2;x_1\theta_1^2)D_1^2(x_1\theta_1^2)+\frac{1}{2}\Big(D_1^1(x_1\theta_1^1)A_2^{12}(x_1\theta_1^1,$$
$$x\theta^2)+A_2^{21}(x\theta^2,x_1\theta_1^1)D_1^1(x_1\theta_1^1)\Big)+J^2(x\theta^2)=0, \tag{16b}$$

where

$$J^\alpha(x\theta^\alpha)=e^{-W}\left(\frac{\delta A[\varphi^1,\varphi^2]}{\delta\varphi^\alpha(x\theta^\alpha)}\right)_{\varphi^1=\frac{\delta}{\delta A_1^1},\ \varphi^2=\frac{\delta}{\delta A_1^2}} e^{W} . \tag{17}$$

From definition (10) it is obvious [1] that the symmetry properties with respect to the permutation of coordinates and indices of the matrix of sources $A_2^{\alpha\beta}$ are the same as the properties of the matrix $[D_0^{-1}]^{\alpha\beta}$ . With this fact taken into account and using expression (4 ) and antisymmetry of the derivative of $\delta$-function it may be shown that

$$A_2^{\alpha\beta}(x_1\theta_1^\alpha,x_2\theta_2^\beta)=A_2^{\beta\alpha}(x_2\theta_2^\beta,x_1\theta_1^\alpha). \tag{18}$$

With account of this relation we have, instead of (16a,b):

$$A_1^\alpha(x\theta^\alpha)+A_2^{\alpha\rho}(x\theta^\alpha;x_1\theta_1^\rho)D_1^\rho(x_1\theta_1^\rho)+J^\alpha(x\theta^\alpha)=0, \tag{19}$$

[1] We recall that the bilocal sources $M^{\alpha\beta}$ set zero at a final stage of calculations should possess the same symmetry properties, otherwise the properties of the free propagator will be distorted when differentiating the generating functional with respect to those sources.

or in the matrix form with notation (5):

$$\begin{pmatrix} A_1(x\theta) \\ \bar{A}_1(x\bar{\theta}) \end{pmatrix} + \begin{pmatrix} A_2^{11}(x\theta, x_1\theta_1) & A_2^{12}(x\theta, x_1\bar{\theta}_1) \\ A_2^{21}(x\bar{\theta}, x_1\theta_1) & A_2^{22}(x\bar{\theta}, x_1\bar{\theta}_1) \end{pmatrix} \begin{pmatrix} D_1(x\theta) \\ \bar{D}_1(x_1\bar{\theta}_1) \end{pmatrix}$$

$$+ \begin{pmatrix} \mathcal{J}(x\theta) \\ \bar{\mathcal{J}}(x\bar{\theta}) \end{pmatrix} = 0. \qquad (20)$$

## 3. The Edwards and Bethe-Salpeter equations

Before proceeding to derive two-particle equations let us show how Legendre transformations are introduced. The first Legendre transformation is determined as follows. From the relations

$$D_1^{\alpha}(x\theta^{\alpha}) = \frac{\delta W[A_1, A_2]}{\delta A_1^{\alpha}(x\theta^{\alpha})} \quad (\alpha = 1, 2) \qquad (21)$$

it follows that $A_1^1$ and $A_1^2$ are implicit functionals of $D_1^1$, $D_1^2$ and $A_2^{\alpha\beta}$. We introduce the following functional [2]:

$$W^{(1)}[D_1^1, D_1^2, A_2] = W - \frac{\delta W}{\delta A_1^{\alpha}} A_1^{\alpha} = W - D_1^{\alpha} A_1^{\alpha}. \qquad (22)$$

It can be shown /3/ that $W^{(1)}$ is a generating functional of irreducible amputated Green functions which are expressed through $W^{(1)}$

$$\Gamma_n^{\alpha_1 \ldots \alpha_n}(x_1\theta_1^{\alpha_1}, \ldots, x_n\theta_n^{\alpha_n}) = \frac{\delta^n W}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1}) \ldots \delta D_1^{\alpha_n}(x_n\theta_n^{\alpha_n})}. \qquad (23)$$

For further consideration we need the identity [3]:

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[2] Hereafter by this shortened notation we mean internal Grassmann variables of integration with the same indices having the meaning according to rules (5). In particular:

$$\frac{\delta W}{\delta A_1^{\alpha}} A_1^{\alpha} \equiv \frac{\delta W}{\delta A_1^{\alpha}(x\theta^{\alpha})} A_1^{\alpha}(x\theta^{\alpha}) \equiv \int d^4x\, d^2\theta \frac{\delta W}{\delta A_1(x\theta)} A_1(x\theta) +$$

$$+ \int d^4x\, d^2\bar{\theta} \frac{\delta W}{\delta \bar{A}_1(x\bar{\theta})} \bar{A}_1(x\bar{\theta}).$$

[3] The sign "$f$" means that when one differentiates over this source, all other sources are fixed.

$$\delta_{\alpha\beta}\,\delta(x-x')\,\delta(\theta^{\alpha}-\theta'^{\alpha}) = \left(\frac{\delta A_1^{\alpha}(x\theta^{\alpha})}{\delta A_1^{\beta}(x'\theta'^{\beta})}\right)_f = \tag{24}$$

$$= \left(\frac{\delta A_1^{\alpha}(x\theta^{\alpha})}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}\right)_f \frac{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}{\delta A_1^{\beta}(x'\theta'^{\beta})} = \left(\frac{\delta A_1^{\alpha}(x\theta^{\alpha})}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}\right)_f D_2^{\alpha_1\beta}(x_1\theta_1^{\alpha_1}, x'\theta'^{\beta}).$$

The left-hand side of this identity is a functional unity in the chiral basis (see (A.2)) . Consequently, the propagator inverse to the total one $D_2^{\alpha\beta}$ is given by the relation:

$$D_2^{-1\alpha\alpha_1}(x\theta^{\alpha}, x_1\theta_1^{\alpha_1})\, D_2^{\alpha_1\beta}(x_1\theta_1^{\alpha_1}, x'\theta'^{\beta}) = \tag{25}$$

$$= D_2^{\alpha\alpha_1}(x\theta^{\alpha}, x_1\theta_1^{\alpha_1})\, D_2^{-1\alpha_1\beta}(x_1\theta_1^{\alpha_1}, x'\theta'^{\beta}) = \delta_{\alpha\beta}\,\delta(x-x')\,\delta(\theta^{\alpha}-\theta'^{\alpha}).$$

With account of relations (24) and (25) it is not difficult to obtain the equality:

$$D_2^{-1\alpha\beta}(x\theta^{\alpha}, x'\theta'^{\beta}) = \frac{\delta A_1^{\alpha}(x\theta^{\alpha})}{\delta D_1^{\beta}(x'\theta'^{\beta})}\,. \tag{26}$$

Next, with the help of (22) and (23) at n=1 it can be shown that

$$\Gamma_1^{\alpha}(x\theta^{\alpha}) = -A_1^{\alpha}(x\theta^{\alpha}). \tag{27}$$

For $\Gamma_2^{\alpha\beta}$ using relationships (25) and (26) we get

$$\Gamma_2^{\alpha\beta}(x\theta^{\alpha}; x'\theta'^{\beta}) = -\frac{\delta A_1^{\alpha}(x\theta^{\alpha})}{\delta D_1^{\beta}(x'\theta'^{\beta})} = -D_2^{-1\alpha\beta}(x\theta^{\alpha}; x'\theta'^{\beta}). \tag{28}$$

Introduce a generating functional of the second Legendre transformation:

$$W^{(2)}[D_1^{\alpha}, D_2^{\alpha\beta}] = W - \frac{\delta W}{\delta A_1^{\alpha}} A_1^{\alpha} - \frac{\delta W}{\delta A_2^{\alpha\beta}} A_2^{\alpha\beta}. \tag{29}$$

From definition (29) and constraint equations (21) it follows, first, that

$$\frac{\delta W^{(2)}}{\delta D_2^{\rho\gamma}(x\theta^{\rho}, y\theta^{\gamma})} - \frac{1}{2} A_2^{\rho\gamma}(x\theta^{\rho}, y\theta^{\gamma}) \tag{30}$$

and second, that

$$\frac{\delta W^{(2)}}{\delta D_1^\rho(x\theta^\rho)} = -A_1^\rho(x\theta^\rho) - \frac{1}{2}[A_2^{\rho\alpha_2}(x\theta^\rho, x_2\theta_2^{\alpha_2}) D_1^{\alpha_2}(x_2\theta_2^{\alpha_2}) + D_1^{\alpha_1}(x_1\theta_1^{\alpha_1}) A_2^{\alpha_1\rho}(x_1\theta_1^{\alpha_1}; x\theta^\rho)].$$

With account of equality (18) this relationship can be rewritten in the form (as usual, the shortened notation is used, see [2]):

$$\frac{\delta W^{(2)}}{\delta D_1^\rho(x\theta^\rho)} = -A_1^\rho(x\theta^\rho) - A_2^{\rho\alpha_1}(x\theta^\rho, x_1\theta_1^{\alpha_1}) D_1^{\alpha_1}(x_1\theta_1^{\alpha_1}). \tag{31}$$

The third, fourth, etc., Legendre transformations can be defined analogously. For this it is necessary to add action (1) with terms with sources $A_3^{\alpha\beta\gamma}(x_1\theta_1^{\alpha_1}, x_2\theta_2^{\beta}, x_3\theta_3^{\gamma})$, $A_4^{\alpha\beta\gamma\delta}(x_1\theta_1^{\alpha}, x_2\theta_2^{\beta}, x_3\theta_3^{\gamma}, x_4\theta_4^{\delta})$ and so on. However, in this paper we are interested in the two-particle supersymmetric equations, and hence, for our purposes the second Legendre transformation is sufficient.

Now let us differentiate equation (19) with respet to $A_1^\beta(x'\theta'^\beta)$ assuming that all the other sources are fixed:

$$\delta_{\alpha\beta}\,\delta(x-x')\delta(\theta^\alpha-\theta'^\beta) + A_2^{\alpha\alpha_1}(x\theta^\alpha, x_1\theta_1^{\alpha_1}) \frac{\delta^2 W}{\delta A_1^\beta(x'\theta'^\beta)\,\delta A_1^{\alpha_1}(x_1\theta_1^{\alpha_1})} + \left(\frac{\delta J^\alpha(x\theta^\alpha)}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}\right)_f \left(\frac{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}{\delta A_1^\beta(x'\theta'^\beta)}\right)_f = 0.$$

With account of (15) this equation can be rewritten in the form

$$\delta_{\alpha\beta}\,\delta(x-x')\delta(\theta^\alpha-\theta'^\alpha) + A_2^{\alpha\alpha_1}(x\theta^\alpha, x_1\theta_1^{\alpha_1}) D_2^{\alpha_1\beta}(x_1\theta_1^{\alpha_1}, x'\theta'^\beta) + \left(\frac{\delta J^\alpha(x\theta^\alpha)}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})}\right)_f D_2^{\alpha_1\beta}(x_1\theta_1^{\alpha_1}, x'\theta'^\beta) = 0.$$

Using relationship (25) we get:

$$D_2^{-1\alpha\beta}(x\theta^\alpha; x'\theta'^\beta) + A_2^{\alpha\beta}(x\theta^\alpha; x'\theta'^\beta) + \Sigma^{\alpha\beta}(x\theta^\alpha; x'\theta'^\beta) = 0. \tag{32a}$$

Here the mass operator [4)]

$$\sum^{\alpha\beta}(x\theta^{\alpha}, x'\theta'^{\beta}) = \left(\frac{\delta \mathcal{J}^{\alpha}(x\theta^{\alpha})}{\delta D_1^{\beta}(x'\theta'^{\beta})}\right)_f . \tag{33}$$

is introduced, where $\mathcal{J}^{\alpha}(x\theta^{\alpha})$ is defined by relation (17). Coming back to the notation (5) we may rewrite equation (32) in the matrix form

$$\begin{pmatrix} D_2^{-1\,11}(x\theta, x'\theta'), & D_2^{-1\,12}(x\theta, x'\theta') \\ D_2^{-1\,21}(x\bar{\theta}, x'\theta'), & D_2^{-1\,22}(x\bar{\theta}, x'\bar{\theta}') \end{pmatrix} + \begin{pmatrix} A_2^{11}(x\theta, x'\theta'), & A_2^{12}(x\theta, x'\bar{\theta}') \\ A_2^{21}(x\bar{\theta}, x'\theta'), & A_2^{22}(x\bar{\theta}, x'\theta') \end{pmatrix} + \begin{pmatrix} \delta\mathcal{J}(x\theta)/\delta D_1(x'\theta'), & \delta\mathcal{J}(x\theta)/\delta \bar{D}_1(x'\theta') \\ \delta\mathcal{J}(x\bar{\theta})/\delta D_1(x'\theta'), & \delta\mathcal{J}(x\bar{\theta})/\delta D_1(x_1\bar{\theta}_1) \end{pmatrix}$$

$$= 0. \tag{32b}$$

To derive two-particle equations (the Edwards equation and the Bethe-Salpeter equation for the amputated connected three- and four-legged function, respectively), we carry out the second Legendre transformation (29) of the generating functional of the connected Green functions. In this case all the functionals of $A_1^{\alpha}$ and $A_2^{\alpha\beta}$ (including the mass operator (33) ) become functionals of $D_1^{\alpha}$ and $D_2^{\alpha\beta}$ . Differentiating $\sum_2^{\alpha\beta}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta})$ with respect to $A_1^{\gamma}(x_{III}\theta_{III}^{\gamma})$ with account of the above fact and relationships (15) we get:

$$\frac{\delta\sum^{\alpha\beta}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta})}{\delta A_1^{\gamma}(x_{III}\theta_{III}^{\gamma})} = \frac{\delta\sum^{\alpha\beta}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta})}{\delta D_1^{\alpha_1}(x_1\theta_1^{\alpha_1})} D_2^{\alpha_1\gamma}(x_1\theta_1^{\alpha_1}, x_{II}\theta_{II}^{\gamma}) + \frac{\delta\sum^{\alpha\beta}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta})}{\delta D_2^{\alpha_1\alpha_2}(x_1\theta_1^{\alpha_1}, x_2\theta_2^{\alpha_2})} D_3^{\alpha_1\alpha_2\gamma}(x_1\theta_1^{\alpha_1}; x_2\theta_2^{\alpha_2}; x_{III}\theta_{III}^{\gamma}) \tag{34a}$$

Note that further exposition may be much simplified if we define the rule according to which first external upper indices $\alpha, \beta, \gamma, \delta, \ldots$ denote the pairs of external variables $x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta}, x_{III}\theta_{III}^{\gamma}, x_{IV}\theta_{IV}^{\delta}, \ldots$

---

4) Indeed, in the limit of switched-off sources the third term of equation (32) is expressed through the inverse total and free propagators as the usual mass operator $\sum^{\alpha\beta} = D_2^{-1\alpha\beta} - D_{2(0)}^{-1\alpha\beta}$ which can be easily verified by applying definition (10).

(i.e., numeration of an external argument by a Roman number corresponds to a serial number of a Greek letter in the alphabet) and, second, internal upper indices $\alpha_1, \ldots, \alpha_n$ represent the pairs of internal variables $x_1\theta_1^{\alpha_1}, \ldots, x_n\theta_n^{\alpha_n}$.

According to this rule equality (34a) can be rewritten in a shortened form

$$\frac{\delta\Sigma^{\alpha\beta}}{\delta A_1^{\gamma}} = \frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}} D_2^{\alpha_1\gamma} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} D_3^{\alpha_1\alpha_2\gamma} . \tag{34b}$$

We differentiate equation (32) with respect to $A_1^{\gamma}(x_{III}\theta_{III}^{\gamma})$ setting all the other sources fixed. Then, with account of (25), relation (34) and the identity

$$\left(\frac{\delta}{\delta A_1^{\gamma}} D_2^{-1\alpha\alpha_1}\right) D_2^{\alpha_1\alpha_2} = - D_2^{-1\alpha\alpha_1}\left(\frac{\delta}{\delta A_1^{\gamma}} D_2^{\alpha_1\alpha_2}\right),$$

we obtain the equation

$$-D_2^{-1\alpha\alpha_1} D_3^{\alpha_1\alpha_2\gamma} D_2^{-1\alpha_2\beta} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}} D_2^{\alpha_1\gamma} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} D_3^{\alpha_1\alpha_2\gamma} . \tag{35}$$

Let us go over to the amputated functions

$$\Gamma^{\alpha\beta\gamma} = D_2^{-1\alpha\alpha_1} D_2^{-1\beta\alpha_2} D_2^{-1\gamma\alpha_3} D_3^{\alpha_1\alpha_2\alpha_3} . \tag{36}$$

Then, instead of equation (35) we easily get the Edwards equation [5) ] for a connected amputated three-legged function

$$\Gamma^{\alpha\beta\gamma} = \frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\gamma}} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} D_2^{\alpha_1\alpha_3} D_2^{\alpha_2\alpha_4} \Gamma^{\alpha_3\alpha_4\gamma} . \tag{37a}$$

or in a detailed form

$$\Gamma^{\alpha\beta\gamma}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta}, x_{III}\theta_{III}^{\gamma}) = \frac{\delta\Sigma^{\alpha\beta}(x_I\theta_I^{\alpha}, x_{II}\theta_{II}^{\beta})}{\delta D_1^{\gamma}(x_{III}\theta_{III}^{\gamma})} +$$

---

5) When we write "equation", we imply fixed values of indices. Equation (37), obviously represents a set of eight Edwards equation corresponding to eight combinations of two-valued external indices $\alpha, \beta, \gamma$. This comment also concerns the Bethe-Salpeter equation to be obtained below, (42), which is a system of sixteen equations.

$$+\sum_{\alpha_1\ldots\alpha_4=1,2}\int\prod_{i=1}^{4}(d\theta_i^{\alpha_i}dx_i)\,\frac{\delta\Sigma^{\alpha\beta}(x_I\theta_I^{\alpha},\, x_{II}\theta_{II}^{\beta})}{\delta D_2^{\alpha_1\alpha_2}(x_1\theta_1^{\alpha_1},\, x_2\theta_2^{\alpha_2})}\,\times$$

$$\times\, D_2^{\alpha_1\alpha_3}(x_1\theta_1^{\alpha_1},\, x_3\theta_3^{\alpha_3})\, D_2^{\alpha_2\alpha_4}(x_2\theta_2^{\alpha_2},\, x_4\theta_4^{\alpha_4})\,\Gamma^{\alpha_3\alpha_4\gamma}. \qquad (37b)$$

A graphic scheme for the Edwards equation (37) is shown in Fig.1.

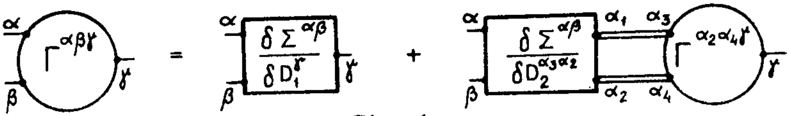


Fig. 1

Let us proceed to derive the Bethe–Salpeter equations in supersymmetry theory. To this end, let us differentiate eq. (32) with respect to $A_2^{\gamma\delta}$ with account of the relation [6)]

$$\frac{\delta}{\delta A_2^{\gamma\delta}}A_2^{\alpha\beta}=\frac{1}{2}[\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}]=sym[\delta_{\alpha\gamma}\delta_{\beta\delta}]. \qquad (38)$$

As a result,

$$-D_2^{-1\alpha\alpha_1}\left(\frac{\delta}{\delta A_2^{\gamma\delta}}D_2^{\alpha_1\alpha_2}\right)D_2^{-1\alpha_2\beta}+sym[\delta_{\alpha\gamma}\delta_{\beta\delta}]+ \qquad (39)$$

$$+\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}}\left(\frac{\delta D_2^{\alpha_1\alpha_2}}{\delta A_2^{\gamma\delta}}\right)+\frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}}\left(\frac{\delta D_1^{\alpha_1}}{\delta A_2^{\gamma\delta}}\right)=0.$$

From constraint equation (18) and relation (15) it follows, first, that

$$\left(\frac{\delta D_1^{\alpha_1}}{\delta A_2^{\gamma\delta}}\right)_f=\frac{\delta}{\delta A_2^{\gamma\delta}}\frac{\delta W}{\delta A_1^{\alpha_1}}=\frac{1}{2}\frac{\delta}{\delta A_1^{\alpha_1}}[D_2^{\gamma\delta}+D_1^{\alpha}D_1^{\delta}]= \qquad (40a)$$

$$=\frac{1}{2}[D_3^{\alpha_1\gamma\delta}+D_2^{\alpha_1\gamma}D_1^{\delta}+D_1^{\gamma}D_2^{\alpha_1\delta}]$$

and second, that

---

6) Here we have made use of relation (18) written in a shortened form $A_2^{\alpha\beta}=A_2^{\beta\alpha}$. Therefore, in relation (38), alongside with the permutation of upper indices, the permutation of the corresponding arguments is meant.

$$\left(\frac{\delta D_2^{\alpha_1\alpha_2}}{\delta A_2^{\gamma\delta}}\right)_f = \frac{\delta^2}{\delta A_1^{\alpha_1}\delta A_1^{\alpha_2}}\frac{1}{2}[D_2^{\gamma\delta} + D_1^{\gamma} D_1^{\delta}] = \tag{40b}$$

$$= \frac{1}{4}[D_4^{\alpha_1\alpha_2\gamma\delta} + D_3^{\alpha_1\alpha_2\gamma} D_1^{\delta} + D_2^{\alpha_1\gamma} D_2^{\alpha_2\delta} + D_2^{\alpha_1\delta} D_2^{\alpha_2\gamma} + D_3^{\alpha_1\alpha_2\delta} D_1^{\gamma}].$$

Substitution of relation (40) into (39), first, cancels a number of terms owing to eq. (35)

$$-\frac{1}{2} D_2^{-1\alpha\alpha_1}[D_3^{\alpha_1\alpha_2\gamma} D_1^{\delta} + D_3^{\alpha_1\alpha_2\delta} D_1^{\gamma}] D_2^{-1\alpha_2\beta} +$$

$$+\frac{1}{2}\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}}[D_3^{\alpha_1\alpha_2\gamma} D_1^{\delta} + D_3^{\alpha_1\alpha_2\delta} D_1^{\gamma}] + \frac{1}{2}\frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}}[D_2^{\alpha_1\gamma} D_1^{\delta}$$

$$+ D_1^{\gamma} D_2^{\alpha_1\delta}],$$

and second, upon that substitution the term in the left-hand side of eq. (39) contains the term

$$-\frac{1}{2} D_2^{-1\alpha\alpha_1}[D_2^{\alpha_1\gamma} D_2^{\alpha_2\delta} + D_2^{\alpha_1\delta} D_2^{\alpha_2\delta}] D_2^{-1\alpha_2\beta},$$

that cancels out with the second term. Consequently, eq. (3) assumes the form

$$-\frac{1}{2} D_2^{-1\alpha\alpha_1} D_4^{\alpha_1\alpha_2\gamma\delta} D_2^{-1\alpha_2\beta} + \frac{1}{2}\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}}[D_4^{\alpha_1\alpha_2\gamma\delta} + \tag{41}$$

$$+ D_2^{\alpha_1\delta} D_2^{\alpha_2\gamma}] + \frac{1}{2}\frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}} D_3^{\alpha_1\gamma\delta} = 0.$$

Proceed to the amputated functions $F^{\alpha\beta\gamma\delta} = D_2^{-1\alpha\alpha_1} D_2^{-1\beta\alpha_2} D_2^{-1\gamma\alpha_3} \times D_2^{-1\delta\alpha_4} D_4^{\alpha_1\alpha_2\alpha_3\alpha_4}$. Considering that

$$-\frac{1}{2}\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} D_2^{-1\alpha'\gamma} D_2^{-1\beta'\delta}[D_2^{\alpha_1\gamma} D_2^{\alpha_2\delta} + D_2^{\alpha_1\delta} D_2^{\alpha_2\gamma}] =$$

$$= \frac{1}{2}\left(\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha'\beta'}} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\beta'\alpha'}}\right) = \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha'\beta'}},$$

we get

$$F^{\alpha\beta\gamma\delta} = 2\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} + \frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\alpha_1}} D_2^{\alpha_1\alpha_2}\Gamma_3^{\alpha_2\gamma\delta} +$$

$$+ \frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\alpha_1\alpha_2}} D_2^{\alpha_1\alpha_3} D_2^{\alpha_2\alpha_4} F^{\alpha_3\alpha_4\gamma\delta}. \tag{42}$$

This equation represents a system of the Bethe-Salpeter [7] equations (see footnote [5]). It is plotted in Fig.2.

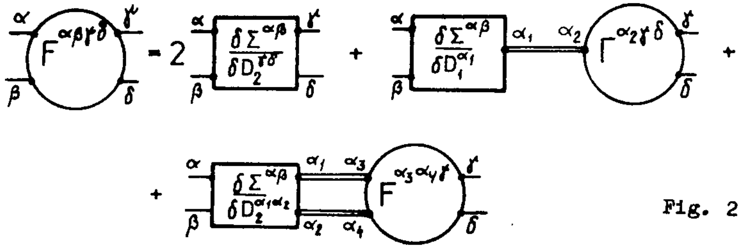


Fig. 2.

Let us find a connection of the Edwards equation (37) and Bethe-Salpeter eq. (42) with generating functional of the second Legendre transformation $W^{(2)}$. With account of (32) and (30), (31) we get

$$\frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\gamma}}\frac{\delta}{\delta D_1^{\gamma}}\left[-D_2^{-1\alpha\beta}+\frac{\delta W^{(2)}}{\delta D_2^{\alpha\beta}}\right]=2\frac{\delta W^{(2)}}{\delta D_1^{\gamma}\delta D_2^{\alpha\beta}}, \tag{43}$$

and

$$\frac{\delta\Sigma^{\alpha\beta}}{\delta D_2^{\gamma\delta}}=\frac{\delta}{\delta D_2^{\gamma\delta}}\left[-D_2^{-1\alpha\beta}+2\frac{\delta W^{(2)}}{\delta D_2^{\alpha\beta}}\right]= \tag{44}$$

$$=\frac{1}{2}\left[D_2^{-1\alpha\gamma}D_2^{-1\delta\beta}+D_2^{-1\alpha\delta}D_2^{-1\gamma\beta}\right]+2\frac{\delta W^{(2)}}{\delta D_2^{\gamma\delta}\delta D_2^{\alpha\beta}}.$$

As will be exemplified below with the Wess-Zumino model, these relations play an important role since they provide new possibilities for investigating kernels and inhomogeneous terms of the Edwards and Bethe-Salpeter equations.

## 4. The Wess-Zumino model

Note that if the interaction term $S_{int}[\varphi,\overline{\varphi}]$ in (1) is taken in the form

$$S_{int}[\varphi,\overline{\varphi}]=\frac{g}{3}\Big\{\int d^4x_L\, d^2\theta\, \varphi^3(x_L,\theta)+ \tag{45}$$

$$+\int d^4x_R\, d^2\overline{\theta}\,\overline{\varphi}^3(x_R,\overline{\theta})\Big\}\equiv\int d^4x\, d^2\theta^{\alpha}\varphi^{\alpha}(x,\theta^{\alpha}),$$

---

7) Eqs. (37), (42) have been obtained in the supersymmetric approach in a matrix form. A component form can be easily found by a standard procedure /6/: One should choose concrete values for the indices $\alpha,\beta,\gamma,\delta$ ( thus choosing a particular channel) and then differentiate with respect to $\theta$ and $\overline{\theta}$ and put them zero.

we shall arrive at the well-known Wess-Zumino model. Let us express $\mathcal{J}^{\alpha}(x\,\theta^{\alpha})$ that enter into inhomogeneous terms, and the kernel of equations (37) and (42) in terms of the above expression for $S_{int}[\varphi,\bar{\varphi}]$ Formulae (10), (17), (43) give us

$$\mathcal{J}^{\alpha} = i e^{-W}\left(\frac{\delta S_{int}[\varphi^1,\varphi^2]}{\delta\varphi^{\alpha}}\right)e^{W}\Bigg|_{\varphi^1=\frac{\delta}{\delta A_1^1},\ \varphi^2=\frac{\delta}{\delta A_1^2}} = \tag{46}$$

$$=\frac{ig}{3}e^{-W}\left\{3\left(\frac{\delta}{\delta A_1^{\alpha}}\right)^2 e^{W}\right\} = ig\left[D_2^{\alpha\alpha} + D_1^{\alpha}D_1^{\alpha}\right].$$

Let us determine the inhomogeneous term of the Edwards equation (37) from its connection with the generating functional of the second Legendre transformation (43).

The Schwinger equation (19) and relation (31) give

$$\frac{\delta W^{(2)}}{\delta D_1^{\alpha}} = -A_1^{\alpha} - A_2^{\alpha\alpha_1}D_1^{\alpha_1} = \mathcal{J}^{\alpha}.$$

Insert it into (43) and taking account of (46) one easily obtains

$$\frac{\delta\Sigma^{\alpha\beta}}{\delta D_1^{\gamma}} = 2\frac{\delta}{\delta D_2^{\alpha\beta}}\left(\frac{ig}{2}D_2^{\gamma\gamma} + D_1^{\gamma}D_1^{\gamma}\right) = ig\,\delta_{\alpha\gamma}\delta_{\gamma\beta}\,, \tag{47}$$

$$\frac{\delta\Sigma^{\alpha\beta}(x_I\theta_I^{\alpha},\, x_{II}\theta_{II}^{\beta})}{\delta D_1^{\gamma}(x_{III}\theta_{III}^{\gamma})} = ig\,\delta_{\alpha\gamma}\delta_{\gamma\beta}\,\delta(x_I - x_{III}).$$

Relation (47) is exact, i.e., it holds valid beyond the scope of perturbation theory.

## Conclusion

We considered a scalar superfield model, a particular case of which is the well-known Wess-Zumino model. The supersymmetric Schwinger equations are found, and on their basis with the use of the second Legendre transformation, the two-particle Edwards and Bethe-Salpeter equations are derived. It seems reasonable to extend the formalism we have developed to more complicated models, including the Yang-Mills supersymmetric theory. It is also of interest to study the Logunov--Tavkhelidze equations which may be useful for examining bound states of particles occurring in the supersymmetric QCD (for instance, a gluon-gluino ( $g\tilde{g}$ ) exotic bound state). Note that the connection of the kernel of the Bethe-Salpeter equation with the second Legendre transformation provides new possibilities for studying both the kernel of the Bethe-Salpeter equation and the quasipotential.

These problems seem to be important, specifically, in view of elaboration of the scientific program of experiments at the constructed colliders LEP (CERN), SSC (USA).

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## Appendix 1

Chiral superfields are determined by the equalities

$$D_\alpha \varphi(x,\theta,\bar\theta)=0, \qquad \bar D_{\dot\alpha}\bar\varphi(x,\theta,\bar\theta)=0, \tag{A.1.1}$$

where $D_\alpha$ is a covariant derivative (see ref. /6/). In the chiral basis

$$L:(x^\mu_L,\theta,\bar\theta)=(x^\mu-i\theta\sigma^\mu\bar\theta,\ \theta,\ \bar\theta); \qquad R:(x^\mu_R,\theta,\bar\theta)=(x^\mu+i\theta\sigma^\mu\bar\theta,\ \theta,\ \bar\theta); \tag{A.1.2}$$

we have

$$\varphi=\varphi(x_L,\theta);\quad \bar\varphi=\bar\varphi(x_R,\bar\theta). \tag{A.1.3}$$

Allowing for relations (A.1.2), (A.1.3) and using a Taylor expansion we may obtain the following relations

$$\varphi(x_L,\theta)=e^{-i\theta\hat\partial\bar\theta}\varphi(x,\theta), \qquad \varphi(x_R,\bar\theta)=e^{i\theta\hat\partial\bar\theta}\varphi(x,\theta), \tag{A.1.4}$$

where the notaion is used:

$$\theta\hat\partial\bar\theta=\theta\left(\sigma^\mu\frac{\partial}{\partial x^\mu}\right)\bar\theta=\frac{\partial}{\partial x^\mu}\theta^\alpha \tag{A.1.5}$$

in which $\sigma_\mu=(1,\vec\sigma)$ are Pauli matrices, and $\theta^\alpha,\ \bar\theta^{\dot\alpha}$ are Weyl spinors $(\theta^\alpha=\varepsilon^{\alpha\beta}\theta_\beta;\ \bar\theta^{\dot\alpha}=\bar\theta_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha};\ \varepsilon^{11}=\varepsilon^{\dot1\dot1}=-\varepsilon_{11}=1)$, which represent Grassmann coordinates of superfields.

## Appendix 2

For the superfield action (1) there exist four free propagators corresponding to four possible pairs of superfields $\varphi$ and $\bar{\varphi}$ sandwiched between vacuum expectations. In the chiral basis the matrix of free propagators is of the form:

$$D_0^{ch.B} = -i\begin{pmatrix} \langle 0|T\varphi(x\theta)\varphi(x'\theta')|0\rangle & \langle 0|T\varphi(x\theta)\bar{\varphi}(x'\bar{\theta}')|0\rangle \\ \langle 0|T\bar{\varphi}(x\bar{\theta})\varphi(x'\theta')|0\rangle & \langle 0|T\bar{\varphi}(x\bar{\theta})\bar{\varphi}(x'\theta')|0\rangle \end{pmatrix}$$

$$= \begin{pmatrix} -m\,\delta(\theta-\theta')\Delta_c(x-x') & e^{-2i\theta\hat{\partial}\bar{\theta}'}\Delta_c(x-x') \\ e^{2i\theta\hat{\partial}\bar{\theta}'}\Delta_c(x-x') & -m\,\delta(\bar{\theta}-\bar{\theta}')\Delta_c(x-x') \end{pmatrix}, \qquad \text{(A.2.1)}$$

where $\Delta^c(x) = \frac{1}{(2\pi)^4}\int dp\, e^{ipx}\frac{1}{m^2-p^2-i\varepsilon}$ .

We shall show that this matrix is a kernel of the operator inverse to the integral matrix operator $\mathcal{K}$ (7) with the functional unity

$$\mathcal{J} = \begin{pmatrix} \delta(\theta-\theta')\delta(x-x') & 0 \\ 0 & \delta(\bar{\theta}-\bar{\theta}')\delta(x-x') \end{pmatrix}. \qquad \text{(A.2.2)}$$

From anticommutation of the Grassmann variables $\theta$ and $\bar{\theta}$ it follows that

$$e^{-2i\theta\hat{\partial}\bar{\chi}} = 1 - 2i\theta\hat{\partial}\bar{\chi} + 2(\theta\hat{\partial}\bar{\chi})^2, \qquad \text{(A.2.3)}$$

where $\theta$ and $\chi$ are arbitrary Grassmann coordinates. Using the definition of the Grassmann $\delta$-function we arrive at the following equality

$$(\theta\hat{\partial}\bar{\chi})^2 \equiv \left(\theta^\alpha(\sigma_\mu)_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}\right)\left(\theta^\beta(\sigma_\nu)_{\beta\dot{\beta}}\bar{\chi}^{\dot{\beta}}\frac{\partial}{\partial x^\nu}\right) =$$

$$= -\delta(\theta)\delta(\bar{\chi})\left\{\varepsilon^{\alpha\beta}\varepsilon^{\dot{\alpha}\dot{\beta}}(\sigma_\mu)_{\alpha\dot{\alpha}}(\sigma_\nu)_{\beta\dot{\beta}}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\right\} =$$

$$= -\delta(\theta)\delta(\bar{\chi})\left\{(\sigma_\mu)_{\alpha\dot{\alpha}}(\sigma_\nu)^{\alpha\dot{\alpha}}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\right\}. \qquad \text{(A.2.4)}$$

With the rule of lifting (lowering) indices with or without the dot and the explicit form of $\sigma$ matrices being used we may find that the expression in braces in the r.h.s. of (A.2.4) equals

$$\begin{cases} 0, & \mu\neq\nu \\ -Sp\,I = -2, & \mu=\nu=0 \\ Sp\,I = 2, & \mu=\nu\neq 0. \end{cases}$$

As a result, instead of (A.2.4) we get

$$(\theta\hat{\partial}\bar{x})^2 = 2\delta(\theta)\delta(\bar{x})g^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu} = \tag{A.2.5}$$
$$= 2\delta(\theta)\delta(\bar{x})\Box \,.$$

Obviously, applying to the latter we may rewrite (A.2.3) as follows:

$$e^{-2i\theta\hat{\partial}\bar{x}} = 1 - 2i\theta\hat{\partial}\bar{x} + \delta(\theta)\delta(\bar{x})\Box \,.$$

It is easy to obtain with account of (A.2.5), that

$$\mathcal{K} D_0 = D_0 \mathcal{K} = I \,,$$

where the kernels of operators $\mathcal{K}$ and $D_0$ are defined by relations (7) and (A.2.1), respectively, and the functional unity is given by (A.2.2).

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Сисакян А.Н., Скачков Н.Б., Шевченко О.Ю. E2-86-635
Суперсимметричные двухчастичные уравнения

В рамках скалярной суперполевой модели, частным случаем которой является хорошо известная модель Весса-Зумино, получены суперсимметричные уравнения Швингера. На их основе с использованием второго преобразования Лежандра получены двухчастичные суперсимметричные уравнения Эдвардса и Бете-Солпитера. Найдена связь ядер и неоднородных членов этих уравнений с производящим функционалом второго преобразования Лежандра.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Sissakian A.N., Skachkov N.B., Shevchenko O.Yu. E2-86-635
Supersymmetric Two-Particle Equations

In the framework of the scalar superfield model, a particular case of which is the well-known Wess-Zumino model, the supersymmetric Schwinger equations are found. On their basis with the use of the second Legendre transformation the two-particle supersymmetric Edwards and Bethe-Salpeter equations are derived. A connection of the kernels and inhomogeneous terms of these equations with generating functional of the second Legendre transformation is found.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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