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Analysis of Experimental Semi-Inclusive Distributions at the SPS-Collider Energies

YA. Z. DARBAIDZE, A. N. SISSAKIAN, L. A. SLEPCHENKO, H. T. TOROSIAN

Joint Institute for Nuclear Research, Laboratory of Theoretical Physics, Dubna, USSR

Abstract

The experimental data on hadron-hadron interactions at high up to SPS-collider energies are analysed carefully within a multicomponent approach based on the renormalization group method. In particular, the behaviour of total cross sections and forward-backward correlations of scattered particles is described as well as the broadening of p_T-spectra with increasing multiplicity of charged particles and the "sea-gull" effect for η-distributions observed at small multiplicities.

1. Introduction

With the CERN collider put into operation, it became possible to check up the predictions of various models and approaches to the description of high energy processes [1, 2]. In particular, the experimental data on $p\bar{p}$-interaction at $\sqrt{s} = 540$ GeV confirm an asymptotic increase in the total cross sections σ_{tot} [3, 4], which does not contradict the Froissart bound, an increase in mean multiplicity of neutral particles $\langle n_0 \rangle_{n_c}$ at large values of n_c, etc.

In this paper the experimental data on hadron-hadron collisions at the SPS collider are analysed within the renormalisation group approach [5, 6].

The second section is devoted to the study of the behaviour of total cross sections σ_{tot} as a function of collision energies. In the third section it is shown that the experimental data on forward-backward correlations of scattered particles confirm an automodel relation of the type [7, 8]

$$\frac{\langle n_B(n_F) \rangle}{\langle n_B \rangle} = L(z_F, k) \tag{1}$$

which is obtained under the assumption of the multidimensional KNO scaling [9, 10]. The properties of $L(z_F, k)$ as a function of the number k of correlated components and the values of $z_F = n_F/\langle n_F \rangle$ are discussed. The experimental results at the SPS collider for $p\bar{p}$ interaction show that the semi-inclusive p_T spectra at large values of momentum transferred broaden with increasing multiplicity of charged particles n_c [11]. Moreover, the distribution densities over pseudorapidity η have dips at $\eta \sim 0$ and $n_c < 10$ of the "sea-gull" effect-type [12]. The inclusive and semi-inclusive distributions over p_T and η are uniquely described in the fourth section [13].

2. Increase in Total Cross Sections with Energy

In the considered approach for the cross sections $\sigma_{n_1\cdots n_k}$ of the exclusive reaction $ab \to n_1 + \cdots + n_k$ we proceed from the following equation for the renormalization group given in the characteristic form [5].

$$\frac{d\sigma_{n_1\cdots n_k}}{dt} = \left(\sum_{i=1}^{k} \gamma_i n_i\right) \sigma_{n_1\cdots n_k}, \tag{2}$$

where $t = \ln \mu$, μ is the normalization mass, and γ_i are anomalous dimensions $i = 1, \ldots, k$ of fields, respectively. Averaging (2) over $n_1, \ldots, n_k$ we get equations for the total cross section σ_{tot} and mean multiplicities $\langle n_i\rangle$ ($i = 1, \ldots, k$):

$$\frac{d\sigma_{\text{tot}}}{dt} = \left(\sum_{i=1}^{k} \gamma_i \langle n_i\rangle\right) \sigma_{\text{tot}}, \tag{3a}$$

$$\frac{d\langle x\rangle}{dt} = D^2(x), \tag{3b}$$

where

$$\langle x\rangle = \sum_{i=1}^{k} \gamma_i \langle n_i\rangle, \quad D(x) = (\langle x^2\rangle - \langle x\rangle^2)^{1/2}.$$

Excluding dt from (3a) and (3b) we find the following equation:

$$\frac{d\sigma_{\text{tot}}}{d\langle x\rangle} = \frac{\langle x\rangle}{D^2(x)}\, \sigma_{\text{tot}}. \tag{4}$$

We simplify eq. (4) assuming the linear dependence (see refs. [6, 14])

$$D(x) = \frac{1}{\sqrt{a}} \langle x\rangle, \tag{5}$$

where a is the parameter defining the strength of correlation between the considered hadron systems (components).

The solution of equation (4) with condition (5) is

$$\sigma_{\text{tot}} = \sigma_{\text{tot}}^{0} \left(\frac{\langle x\rangle}{\langle x\rangle_0}\right)^a, \tag{6}$$

where σ_{tot}^0 and $\langle x\rangle_0$ are the values of σ_{tot} and $\langle x\rangle$ at $\mu = 1$ (i.e., $t = 0$).

Solutions for $\langle n_i\rangle$ satisfy the condition

$$\gamma_i\langle n_i\rangle = \gamma_j\langle n_j\rangle; \qquad i, j = 1, \ldots, k. \tag{7}$$

Then from relation (6) it follows that

$$\sigma_{\text{tot}} = A\langle n_c\rangle^a \tag{8}$$

where $A = \sigma_{\text{tot}}^0/\langle n_c\rangle_0^a$ is independent of $\langle n_c\rangle$ (i.e. of energy) [15].

The results of analysis of the experimental data on $p\bar{p}$, pp, $K^{\pm}p$, and $\pi^{\pm}p$ interactions [3, 4, 16, 17] with the help of (8) are shown in table 1 and fig. 1. The mean multiplicities

Table 1

	$p\bar{p}$	pp	K^+p	K^-p	π^+p	π^-p
A	16.68	15.46	8.8	7.48	10.19	9.73
B	16.16	4.298	5.26	3.94	14.15	8.38
C	2.58	1.924	2.75	1.69	2.85	2.3
F	2.17	1.92	0.92	2.56	1.47	2.05
M	1.6	0.2	1.34	−0.74	1.82	1.0
N	15.14	14.36	8.51	6.9	6.56	8.82
a^{as}			0.37			

are parametrized in the form given in ref. [18]. For instance, for $\langle n_c\rangle_{p\bar{p}}$ and $\langle n_c\rangle_{pp}$ we have

$$\langle n_c\rangle_{p\bar{p}} = 0.18(\ln s)^2 - 0.25 \ln s + 2.9, \tag{9a}$$

$$\langle n_c\rangle_{pp} = 0.13(\ln s)^2 + 0.3 \ln s + 1.17. \tag{9b}$$

Note, that an adequate description can also be obtained in parametrizing the mean multiplicity within the multicomponent model of two mechanisms [19]

$$\langle n_c\rangle_{p\bar{p}} = 0.34 \left(\ln \frac{s}{s_0}\right)^{1.7} + 5, \tag{10a}$$

$$\langle n_c\rangle_{pp} = 0.34 \left(\ln \frac{s}{s_0}\right)^{1.65} + 5, \tag{10b}$$

where $s_0 = (m_1 + m_2)^2$.

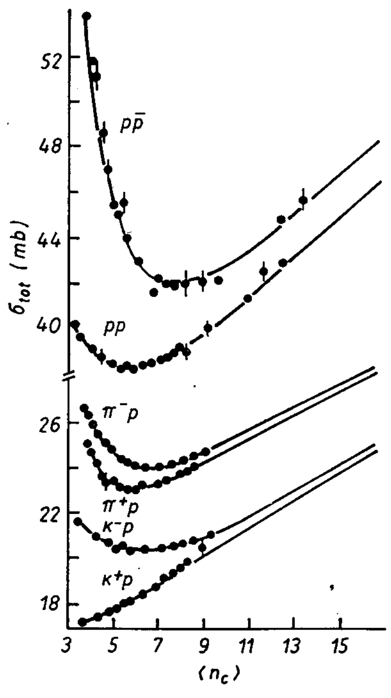

Fig. 1. Description of $p\bar{p}$, pp, $K^{\pm}p$ and $\pi^{\pm}p$ total cross sections by formula (6)

Table 2

$\langle n_c/\Delta y\rangle$	$\varkappa_{n_c}^{p_T}$	G^{p_T}/mb. GeV^{-2}	$\gamma_{\mathrm{PH}}^{p_T}$	a^{as}
2.4	26.86	311.58	4	0.37
5.7	29.16	452.29		
10.2	28.14	474.82		
Inclus.	29.16	452.29		

Intervals of n_c	$\varkappa_{n_c}^{\eta}$	G^{η}	$\gamma_{\mathrm{PH}}^{\eta}$	a^{as}
(1–5)	0.00005	0.053	0	0.35
(6–10)	0.0037	0.34		
(11–20)	0.42	1.05		
(21–30)	0.47	2.94		
(31–40)	0.74	5.18		
Inclus.	0.33	1.89		

The parameter a is slowly decreasing with energy, that corresponds to an enhancement of the correlation between multiplicities of various components, and it becomes almost constant beginning with $\sqrt{s} = 20 \div 25$ GeV (saturation of correlations). We have used the following parametrization:

$$a = a^{\mathrm{as}} + \frac{B}{\langle n_c\rangle^C}. \tag{11}$$

The values of the parameters a^{as}, B and C are given in table 1. It is seen from fig. 2 that for incident antiparticles value of a is larger than for the relevant particles. This leads to differences in the behaviour of the corresponding σ_{tot}.

The parameter a is thought to be constant, if one used by analogy with ref. [20] a modified condition (5) of the following type:

$$D(x) = \frac{1}{\sqrt{a^{\mathrm{as}}}}\left(\langle x\rangle - \alpha\right), \tag{12}$$

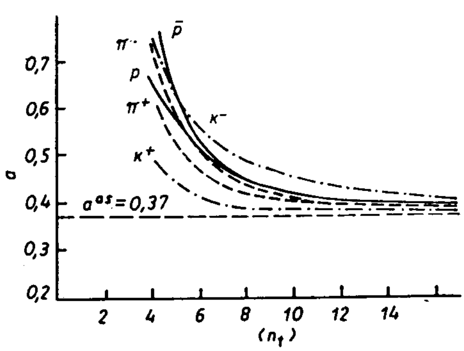

Fig. 2. a is the correlation intensity for multiplicities of hadron systems as a function of $\langle n_c\rangle$ for p$\bar{\mathrm{p}}$, pp, K$^{\pm}$p and $\pi^{\pm}$p interactions

that corresponds to the separation of contribution from leading particles. As a result we have

$$\frac{d\sigma_{\text{tot}}}{d\langle x\rangle} = \frac{a^{as}(\langle x\rangle - \beta)}{(\langle x\rangle - \alpha)^2}\,\sigma_{\text{tot}}, \tag{13}$$

where the constants α and β depend only on the type of leading hadrons. Hence, for σ_{tot} we have

$$\sigma_{\text{tot}} = N(\langle n_c\rangle - M)^{a^{as}} \exp\left(\frac{F}{\langle n_c\rangle - M}\right), \tag{14}$$

where

$$M = \frac{\alpha}{k\gamma_c}, \qquad F = \frac{a^{as}(\beta - \alpha)}{k\gamma_c},$$
$$N = \sigma^0_{\text{tot}} \exp\left[-F/(\langle n_c\rangle_0 - M)\right]/(\langle n_c\rangle_0 - M)^{a^{as}}. \tag{15}$$

The comparison of (14) with the experimental data also provides a satisfactory result (see table 1).

3. Forward-Backward-Correlations at $k \gg 1$

The solution of equation (see sect. 2)

$$\frac{d\sigma_{n_1\cdots n_k}}{d\langle x\rangle} = \frac{ax}{\langle x\rangle^2}\,\sigma_{n_1\cdots n_k} \tag{16}$$

leads to the following relation [6,7]:

$$\langle x\rangle^k \frac{\sigma_{n_1\cdots n_k}}{\sigma_{\text{tot}}} = \psi(z) \sim z^{a-k} \exp(-az), \tag{17}$$

where

$$z = x/\langle x\rangle, \qquad x = \sum_{i=1}^{k} \gamma_i n_i.$$

Taking the relation (7) into account in (17) we have

$$\left(\prod_{i=1}^{k} \langle n_i\rangle\right) \frac{\sigma_{n_1\cdots n_k}}{\sigma_{\text{tot}}} = C_k \tilde{\psi}(\tilde{z}). \tag{18}$$

Here the value of $C_k = \Gamma(k)/\Gamma(a)\,(k/a)^{-a}$ is found from the normalization condition,

$$\tilde{z} = \sum_{i=1}^{k} z_i, \qquad z_i = n_i/\langle n_i\rangle \quad \text{and}$$

$$\tilde{\psi}(\tilde{z}) = \tilde{z}^{a-k} \exp\left(-\frac{a}{k}\,\tilde{z}\right). \tag{19}$$

Proceeding from (17) one can easily find for $L(z_F, k) = \dfrac{\langle n_B(Z_F)\rangle}{\langle n_B\rangle}$:

$$L(z_F, k) = \frac{1}{k-1} \frac{\int_{z_F}^{\infty} (t - z_F)^{k-1}\, \tilde{\psi}(t)\, dt}{\int_{z_F}^{\infty} (t - z_F)^{k-2}\, \tilde{\psi}(t)\, dt}. \tag{20}$$

Substituting expression (19) into (20) we get

$$L(z_F, k) = z_F \frac{\psi\left(k, a+1, \frac{a}{k} z_F\right)}{\psi\left(k-1, a, \frac{a}{k} z_F\right)}, \tag{21}$$

where $\psi(\alpha, \beta, x)$ is the confluent hypergeometric function.

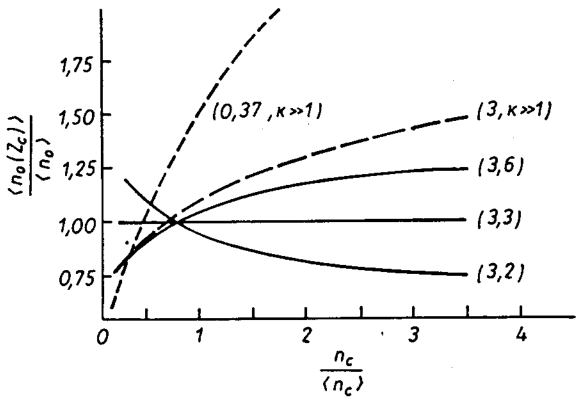

Fig. 3. $L = \langle n_B(n_F)\rangle/\langle n_B\rangle$ as a function of z_F for the following values of the parameters $(a, k) = (3,2), (3,3), (3,6), (3, k \gg 1)$, and $(0.37, k \gg 1)$

The form of this dependence $L(z_F, k)$ on z_F at the following values of the parameters: $(a, k) = (3.2), (3.3), (3.6)$ is shown in fig. 3 (solid lines). The dashed lines correspond to the limit $k \gg 1$ when the function (21) is transformed into the one-parametric function

$$L(z_F, k \gg 1) = \left(\frac{z_F}{a}\right)^{1/2} \frac{K_a\left(2\sqrt{a z_F}\right)}{K_{a-1}\left(2\sqrt{a z_F}\right)}, \tag{22}$$

where $K_a(x)$ is the modified Bessel function.

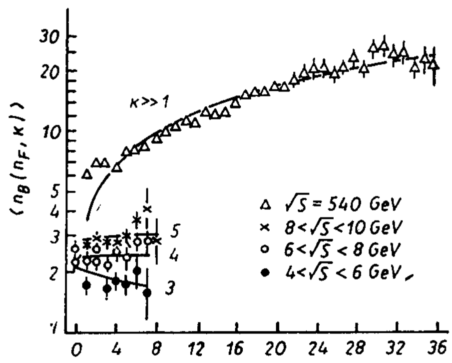

Fig. 4. Description of the dependence of $\langle n_B\rangle$ on n_F at $\sqrt{s} = 540$ GeV by formula (22) (upper curve)

The analysis of the experimental data on charge-neutral and forward-backward-correlations in hadron-hadron processes in a wide range of accelerator energies (up to $\sqrt{s} = 540$ GeV) [21—23] leads to the conclusion that the number of correlated components k increases with energy, and beginning from $\sqrt{s} = 10 \div 15$ GeV an automodel behaviour (22) is achieved ($k \gg 1$).

The description of the data on forward-backward correlations of scattered particles in p$\bar{\text{p}}$-interaction at $\sqrt{s} = 540$ GeV is given in fig. 4. For comparison this figure also shows the description by formula (21) of the corresponding data at relatively low energies [24] with the values of the parameters $a = 4$ and $k = 3, 4, 5$, respectively.

4. A Joint Analysis of Longitudinal and Transversal Distributions at the SPS energies

Let us consider the solutions of eq. (2) taking into account of the "maximal" automodelity [25, 26]

$$E\frac{d\sigma_{n_c}}{d\vec{p}} = \left(E\frac{d\sigma}{d\vec{p}}\Big/\langle n_c(\vec{p})\rangle\right)\phi\big(k, z_c(\vec{p})\big), \tag{23}$$

where

$$\phi\big(k, z_c(\vec{p})\big) = \frac{k-1}{\Gamma(a)}\left(\frac{a}{k}\right)^a [z_c(\vec{p})]^{a-1}\exp\left(-\frac{a}{k}z_c(\vec{p})\right)\psi\left(k-1, a, \frac{a}{k}z_c(\vec{p})\right). \tag{24}$$

Here the inclusive cross section of the process is [13]

$$E\frac{d\sigma}{d\vec{p}} = E\frac{d\sigma}{d\vec{p}_0}\left[1 + \frac{k\gamma_c}{a}\langle n_c(\vec{p}_0)\rangle\,\tau\right]^{-a}\exp\left[-\gamma_{\text{PH}}\tau\right] \tag{25}$$

and the associative multiplicity of charged particles is

$$\langle n_c(\vec{p})\rangle = \frac{\langle n_c(\vec{p}_0)\rangle}{1 + \frac{k\gamma_c}{a}\langle n_c(\vec{p}_0)\rangle\cdot\tau}, \tag{26}$$

where $\vec{p}_0$ is the fixed initial value of $\vec{p}$, $\tau = \ln(pp_0/p_0^2)$ is the "time" variable, and $z_c(\vec{p}) = n_c/\langle n_c(\vec{p})\rangle$.

In the case of a large number of correlated components $k \gg 1(\langle n(\vec{p}_0)\rangle \gg 1$ and $a \simeq a^{as})$ we have

$$E\frac{d\sigma_{n_c}}{d\vec{p}} = G\tau^{-(a^{as}-1)/2}\exp\left[-\gamma_{\text{PH}}\tau\right]K_{a^{as}-1}\big(2\sqrt{\varkappa_{n_c}\tau}\big), \tag{27}$$

where G is the normalization constant, $\varkappa_{n_c} = \gamma_c k n_c$.

Using a convenient parametrization $(pp_0/p_0^2) = (m_T/m)\,\text{ch}\,(\eta - \eta_0)$, where $m_T = (p_T^2 + m^2)^{1/2}$, $\eta = (1/2)\ln\left[(E + p_\parallel)/(E - p_\parallel)\right]$, and the formulae (23)—(27) we analyse the hadron-hadron experimental data on p_T and η-spectra of secondaries in the p$\bar{\text{p}}$-interaction at $\sqrt{s} = 540$ GeV [11—12]. The results of comparison with experiment are given in table 2 and in figs. 5—6. The p_T spectra were considered for the values of $\langle n_c/\Delta y\rangle = 2.4, 5.7, 10.2$, whereas the η-spectra for five multiplicity intervals of charged particles $n_c = (1—5), (6—10), (11—12), (21—30), (31—40)$.

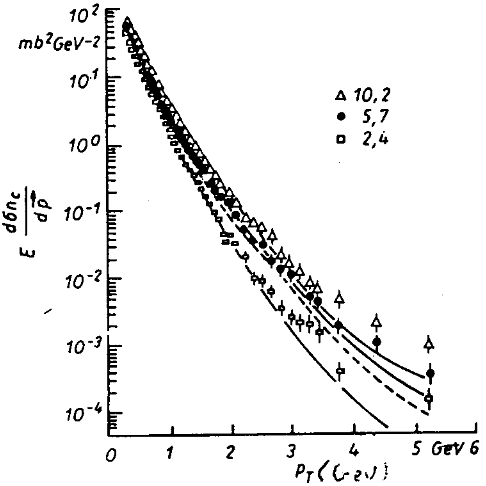

Fig. 5. Description of the invariant cross section $E(d\sigma_{n_c}/d\vec{p})$ as a function of p_T by formula (27) at $\eta = \eta_0$, $\gamma_{PH} = 4$ at three values of $\langle n_c/\Delta y \rangle$ (solid lines). The dashed line corresponds to the inclusive spectrum

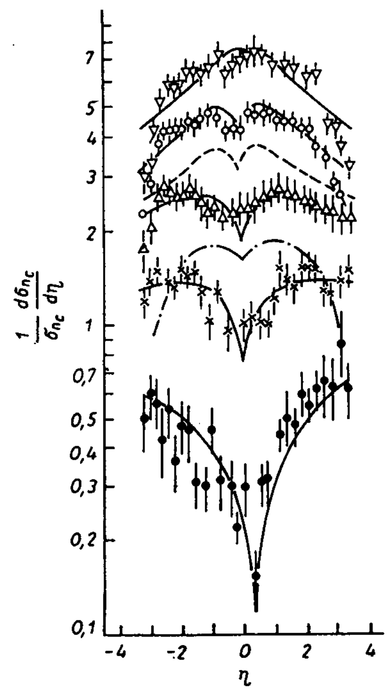

Fig. 6. Normalized $dN/d\eta$-density of charged particles as a function of η. The solid lines correspond to formula (27) at $p_T = 0$, $\gamma_{PH} = 0$ for five intervals of n_c : (6—5), (6—10), (11—20), (21—30), (31—40). The dashed line is the inclusive spectrum

It is seen from formulae (23)—(27) that at small values of n_c the parameter $\varkappa^{\eta}_{n_c}$ is small and

$$\frac{1}{\sigma_{n_c}} \frac{d\sigma_{n_c}}{d\eta} = \frac{dN}{d\eta} \sim \big(\ln \operatorname{ch} (\eta - \eta_0)\big)^{-(\alpha^{**}-1)/2} = [\ln \operatorname{ch} (\eta - \eta_0)]^{0.34}, \tag{28}$$

that allows a description of the "sea-gull" effect for the η-spectra observed at the ISR and SPS energies at small multiplicity values [12]. At large multiplicity values $\varkappa^{\eta}_{n_c}$ becomes large and the effect is smoothed (see fig. 6).

5. Conclusion

In conclusion we should like to note that the scheme proposed provides a possibility for describing the observed experimental regularities and effects up to the SPS-collider energies. It turns out that with increasing energy the number of correlated components contributing to the production of secondaries increases.

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