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ON THE BEHAVIOUR OF INCLUSIVE HADRON CROSS SECTIONS AT LARGE TRANSFERRED MOMENTA AND CERN ISR AND SPS COLLIDER ENERGIES

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As is known, the perturbatuve quantum chromodynamics predicts the cross sections of the processes in which the main contribution comes from the quark-gluon interaction at small distances. The study of the inclusive processes with the production of an emitted particle with a large momentum perpendicular to the collision axis is most convenient. The data of the experiments [1-6] on measurement of the cross sections*

$$E\frac{d^3\sigma}{dp^3}(AB \to CX)\Big|_{\theta=90^\circ}$$

at the CERN ISR and SPS Collider are collected in table 1.

Table 1

Expt. N	Process AB→CX	$\sqrt{s}$ GeV	p_{min} GeV	p_{max}	$x_{T\,min}$	$x_{T\,max}$	Number of points
Ia	$p\bar{p} \to \pi^0$	540	I.5I	4.42	0.0056	0.0I60	I4
IB	$p\bar{p} \to \pi^0$	540	4.20	II.60	0.0I60	0.0430	4
Ic	$pp \to \frac{p+p}{2}$	540	0.55	I.35	0.0020	0.0050	9
2	$pp \to \pi^0$	52.7	3.05	II.00	0.II0	0.4200	23
	$pp \to \pi^0$	62.8	3.05	I3.50	0.097	0.43	I6
3	$pp \to p$	53.0	I.00	4.70	0.038	0.I80	II
	$pp \to p$	63.0	I.00	2.30	0.032	0.073	8
4	$pp \to \pi^0$	53.I	3.7I	I2.70	0.I4	0.48	I6
	$pp \to \pi^0$	62.4	3.72	I3.70	0.I2	0.44	2I
5	$pp \to \pi^0$	45.I	I.08	8.02	0.048	0.36	34
	$pp \to \pi^0$	53.2	I.28	7.8I	0.048	0.29	33
	$pp \to \pi^0$	62.7	I.08	6.42	0.034	0.2I	27
6	$pp \to \pi^0$	53.0	5.25	I4.30	0.20	0.50	I5
	$pp \to \pi^0$	63.0	5.25	I4.60	0.I7	0.46	I5
		$45.I \le \sqrt{s} \le 540$	0.55	I4.60	0.06	0.05	256

The aim of this paper is the description of the cross sections of these reactions within the quark counting rules of anomalous dimensions in QCD [7,8].

The inclusive cross section of hadron production with large p_T in the hard collision [9] has the form

* A,B.C are hadrons.

$$E \frac{d^3\sigma}{dp^3}(AB \to CX) = \sum_{a,b,c} \int_{x_a^{min}}^{1} dx_a \int_{x_b^{min}}^{1} dx_b F_{a/A}(x_a) \times$$

$$\times F_{b/B}(x_b) \int dx_c D_{c/C}(x_c)/x_c^2 \; \hat{s}/\pi \times$$

$$\times \delta(\hat{s} + \hat{t} + \hat{u}) \left(\frac{d\sigma}{dt}\right)_{ab} ,$$

where $\left(\frac{d\sigma}{dt}\right)_{ab}$ is the cross section of elementary parton subprocesses $a, b = q, \bar{q}, G$, $F_{a/A}(x_a)$ is the distribution function of partons a (b) in the hadron A(B) with momentum $x_{a,b} = \frac{2p_{a,b}}{\sqrt{s}}$, The variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ are related with the Mandelstam variables

$$s = (p_A + p_c)^2 ,$$

$$t = (p_A - p_C)^2 ,$$

$$u = (p_B - p_C)^2 ,$$

in the following way in the case of jet production have the form

$$\hat{s} \simeq x_a x_b s ,$$

$$\hat{t} \simeq x_a t/z ,$$

$$\hat{u} \simeq x_b u/z .$$

where

$$z = \frac{x_1}{x_a} + \frac{x_2}{x_b}, \quad x_1 = \frac{t}{s}, \quad x_2 = \frac{u}{s},$$

and

$$x_a^{min} = \frac{x_1}{1 - x_2}, \quad x_b^{min} = \frac{x_a x_2}{x_a - x_1} .$$

By using the quark counting rules the asymptotic formulae have been calculated [8] for the cross sections in the leading logarithmic approximation of QCD at large x_T and $\theta = 90°$

$$E\frac{d^2\sigma}{dp^3}(AB\to CX)\Big|_{\theta=90^\circ} = \left(\frac{\alpha_s}{p_T^2}\right)^2 \left(\frac{(1-x)^{n-1}}{(n-1)!}\alpha_s^{-2r\ln 2x + hd(n,x)}\right. , \quad (1)$$

where $\alpha_s = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}$, n_f is the number of quark flavours, Λ is the quantumchromodynamic scale, $r = \frac{16}{33-2n_f}$ and h is the number of hadrons.

The function $d(n,x)$ is

$$d(n,x) = -r\left[\frac{3}{4} + \frac{2}{n(n+1)} - \sum_{i=1}^{n}\frac{1}{i} + \left(\frac{1}{n} + \ln(1-x)\right)\right],$$

where n is the twice of the number of noninteracting quarks $-n = 2\sum_i(n_i-1)$ (n_i is the number of quarks in the hadron i).

The approach[7,8] uses essentially the solutions of the evolution equations[10] for the distribution functions and for the quark and gluon fragmentation with boundary conditions imposed by the quark counting rules[11]

$$F(x) = F(x, Q^2 = Q_0^2).$$

By representing $\sigma(p_T, s) \equiv E\frac{d^3\sigma}{dp^3}$ in the form of $\sigma(p_T, s) = c\left(\frac{p_T}{p_0}\right)^{-n_{eff}}$ and using the identity

$$\left(\frac{2p_{T_1}}{\sqrt{s_1}}\right)^{-n_{eff}} \equiv \left(\frac{2p_T}{\sqrt{s_2}}\right)^{-n_{eff}} \equiv x_T^{-n_{eff}},$$

one can define n_{eff} by

$$n_{eff}(x_T, s) = \lim_{s_2\to s_1\to s}\frac{\ln[\sigma(x, s_1)/\sigma(x, s_2)]}{\ln(s_2/s_1)}.$$

Substituting into this formula the cross section (1), we get

$$n_{eff}(x_T, s) = 4 - 2[2 - 2r\ln 2x_T + hd(n,x_T)]\ln(Q^2/\Lambda^2).$$

For the range $x_T \geq 2$ we have

$$n_{eff}(x_T, s) = 4 - 2[2 - 2r\ln 2x_T + hd(n,x_T) + a\ln x_T + b]\frac{1}{\ln(Q^2/\Lambda^2)},$$

where a and b are numerical parameters which can be estimated. It what follows we shall find their values solving the overdetermined algebraic system

$$\sigma^{expt}(x_i, s_j) = \sigma^{th}(x_i, s_j, A), \tag{3}$$

where

$$\sigma^{th}(x_i, s_j, A) = cp^{-4}(p/p_0)^{-(n_{eff}-4)} , \tag{4}$$

by the method of Gauss-Newton autoregularized iteration processes[12] (the COMPIL program in the JINR library of standard programs for the CDC 6500 computer -C401,F421). In this case the expression

$$\chi^2 = \sum_i \frac{(\sigma^{expt} - \sigma^{th})^2}{\Delta^2}$$

is minimized, where Δ is the measurement error of σ^{expt}. It has been found that (4) describes well the data at $x_T > 0{,}2$ and $\sqrt{s} \geq 40$ GeV. If $\Delta = \Delta^{stat.} + \Delta^{syst.}, \Delta^{syst.} = \sigma^{expt}/10$, then $\chi^2/DF = 1.04$. This can be seen from table 2 and fig.1, and

Table 2

$x \geq 0{,}2$

Expt. №		Number of points M_i	χ^2/M_i	Normalization coefficients
2	52.7	7	3.34/7	I.27 ± 0.I2
	62.8	8	5.08/8	I.32 ± 0.0II
4	53.I	I2	2.I5/2	I.II ±0.09
	62.[illegible]	I5	I2.4 /I5	0.95 ± 0.08
5	53.2	I3	20.6/I3	I.II ± 0.09
	45.I	I7	29.3/I7	0.72 ± 0.08
6	53.0	I4	I2.2/I4	0.94 ± 0.09
	63.0	I2	I3.3/I2	I.I8 ± 0.I0

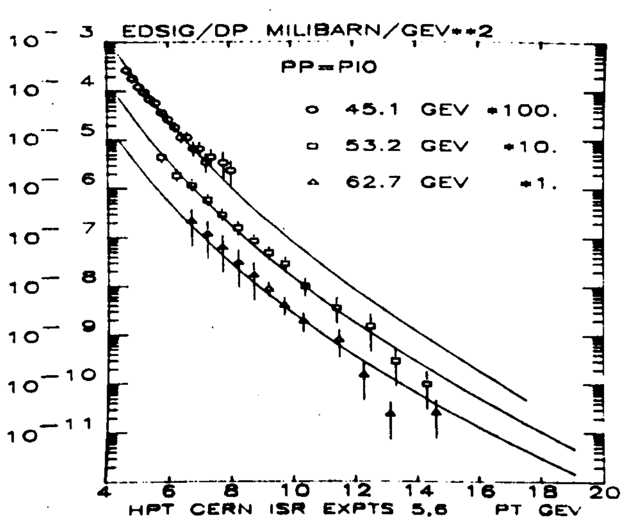

Fig.1. Description of the experimental data [5,6] for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^0 X)$ obtained by formula (4) at $x_T \geq 0.2$ and $\sqrt{s} = 45.1,\ 53.2,\ 62.7$ GeV. The values of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2,3$.

$\sigma(p_T, s)$ has the form

$$\sigma(p_T, s) = p_T^{-4}\,(p_T/1\,\mathrm{GeV})^{-(n_{eff}-4)},$$

where $a = 11.8 \pm 0.4$, $b = 30.3 \pm 0.4$, $\Lambda = 0.05$ GeV*, and

$$Q^2 = \frac{p_T^2}{1 - 4p_T^2/s} \equiv \frac{p_T^2}{1 - x_T^2}.$$

Thus, it is shown that in the leading logarithmic approximation of QCD, the quark counting rules of anomalous dimensions [7,8] enable the description of the experimental data at $x_T \geq 0.2$ and $\sqrt{s} \geq 40$ GeV.

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* The values of Λ in the internal $0.001 \leq 0.05$ GeV provide a good description of the experimental data, that is natural for the leading logarithmic approximation of QCD; at $\Lambda = 0.05$ the correlation dependences between the unknown parameters are minimal.

To describe the data at all experimentally available x_T (in particular, x_T at the SPS Collider energy), we use a modified formula (4) derived by changing the variable p_T by the rapidity χ_T, where

$$\chi_{p_T} = \ln\left(\sqrt{1 + \frac{p_T^2}{m^2}} + \frac{p_T}{m}\right). \qquad (5)$$

Here m has the meaning of the scale defining the boundary region of the transition of the differential cross section from the exponential to power regime.

This rule originated in certain consequences of quantum field theory with the momentum space of constant curvature[13]. It has been shown in paper[14] that it has a simple group-theoretical basis related with the harmonic analysis on the group of motion of the upper flap of the hyperboloid $p_0^2 - \vec{p}^{\,2} = m^2$, $p_0 = \sqrt{m^2 + \vec{p}^{\,2}}$ (harmonic analysis on the Lorentz group). In papers[15] this rule has been used for constructing the elastic scattering amplitude which is in agreement with the measured values $\sigma_t^{p\bar{p}}$ ($\sqrt{s}$ = = 540 GeV) and the behaviour of $\frac{d\sigma^{\vec{p}\vec{p}\to\vec{p}\vec{p}}}{dt}$ ($\sqrt{s}$ = 540 GeV)(see figs.2 and 3).

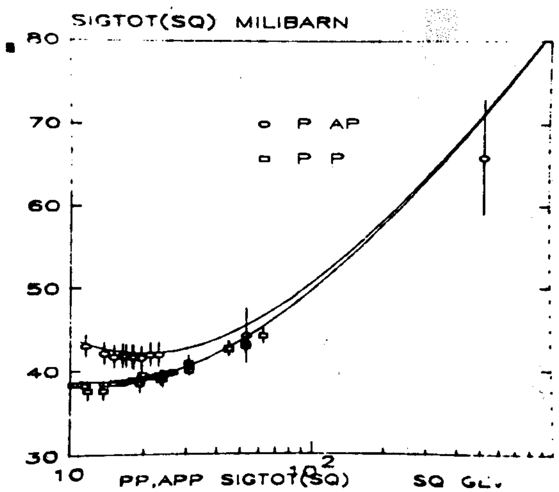
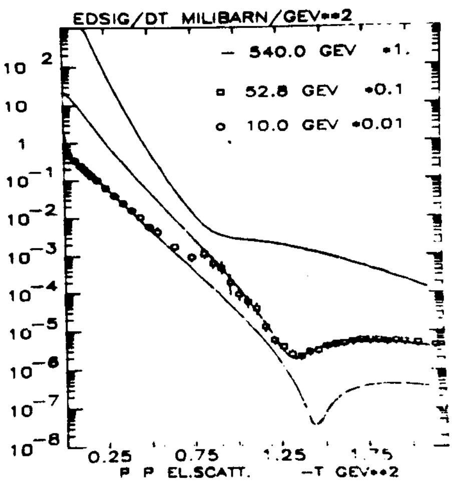


Fig.2. Experimental data for the total cross sections of $p\bar{p}$, pp-interactions at $\sqrt{s} \geq 10$ GeV and their description obtained in[15].



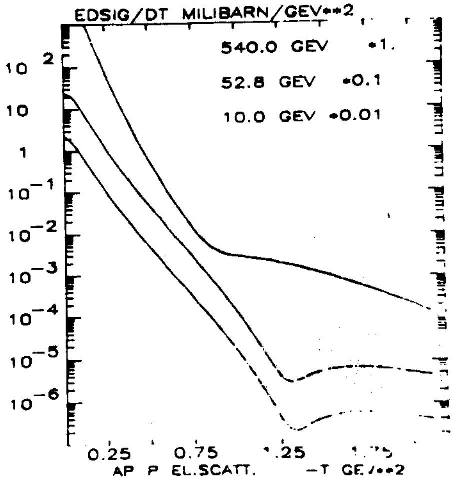


Fig.3a. Description[15] of the differential cross sections of the elastic pp-scattering at $\sqrt{s} = 10.0$, and 52.8 GeV. The upper curve is the prediction for the behaviour of $\frac{d\sigma}{dt}(s,t)$ of the elastic pp scattering at $\sqrt{s} = 540$ GeV. The values of $\frac{d\sigma}{dt}(s,t)$ for each curve (beginning with the upper one) are multipplied by $10^{(-n+1)}$, $n = 1,2,3$.

Fig.3b. Prediction made in ref.[15] for the behaviour of the elastic $p\bar{p}$-scattering at $\sqrt{s} = 10$, 52.8 and 540 GeV. The values of $\frac{d\sigma}{dt}(s,t)$ for each curve (beginning with the upper one) are multiplied by $10^{(-n+1)}$, $n=1,2,3$.

Let us illustrate the aforesaid by a simple example. Let us consider the function

$$f(p) = e^{-bm\chi} \equiv e^{-bm\ln\left(\sqrt{1+\frac{p^2}{m^2}}+\frac{p}{m}\right)} \equiv$$

$$\equiv \frac{1}{\left(\sqrt{1+\frac{p^2}{m^2}}+\frac{p}{m}\right)^{bm}}.$$

We evidently have

$$f(p) = \begin{cases} e^{-bp} & \text{at} \quad \frac{p}{m} \ll 1 , \\ \left(\frac{p}{m}\right)^{-bm} & \text{at} \quad \frac{p}{m} \gg 1 . \end{cases}$$

Note, that in ref.[15] it has been found that $m = \text{const}\,R(s)$, where

$$\sigma_t(s) = 2\pi R^2(s) .$$

Thus, we shall consider

$$\sigma(p_T, s) = a \exp(-bm\chi(p, m)\, n_{eff}(x_T, s) ,$$
$$n_{eff}(x_T, s) = 4 - 2[2 - 2r \ln 2x + hd(n, x) + c \ln x] / \ln(Q^2/\Lambda^2), \qquad (6)$$

where

$$m = m_0 n / \ln(s/\Lambda^2) .$$

The unknown parameters are estimated by solving the system (3).

It has been found that the solutions describing well the experimental data of table 1 -

$$\chi^2/DF = \frac{187.2}{256-5} = 0.75$$

are valid at*

$\Lambda = 0.05$ GeV,

$m_0 = 2.95 \pm 0.06$ GeV,

$b = 0.96 \pm 0.02$ GeV^{-1},

$c = 3.22 \pm 0.07$

$a = 4.45 \pm 0.65$ mb/GeV^2,

that is seen fron table 3 and figs.4-9.

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* See footnote in page 4.

Table 3

Expt. №	s GeV	Number of points μ_i	χ^2/μ_i	Normalization coefficients
Ia	540	I4	5.I6/I4	0.95±0.07
IB	540	4	3.32/4	0.42 ± 0.I4
Ic	540	9	4.08/9	I.82 ± 0.09
2	52.7	23	23.7/23	0.87 ± 0.06
	62.8	I6	26.2/I6	0.99 ± 0.05
3	53.0	II	8.8/II	0.90 ± 0.08
	54.0	8	I9.0/8	0.9I ∓ 0.I0
4	53.I	I6	6.3/I6	I.07 ± 0.07
	62.4	2I	I0.3/2I	0.95 ∓ 0.06
5	45.I	34	I9.I/34	0.84 ± 0.05
	53.2	33	I8.0/33	I.I0 ∓ 0.08
	62.7	27	I0.0/27	I.30 ± 0.05
6	53.0	I5	I3.I/I5	0.95 ± 0.07
	63.0	I5	I9.0/I5	I.06 ± 0.08

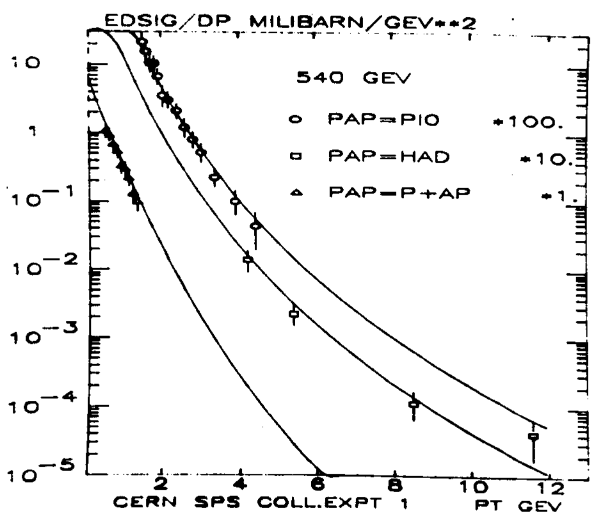

Fig.4. Description of the experimental data[1] for $E\frac{d^3\sigma}{dp^3}$ obtained by formula (6). The value of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2,3$.

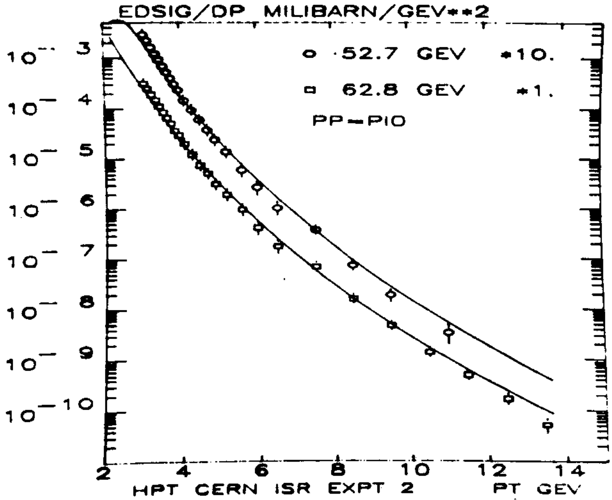

Fig.5. Description of the experimental data [2] for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^\circ X)$ obtained by formula (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2$.

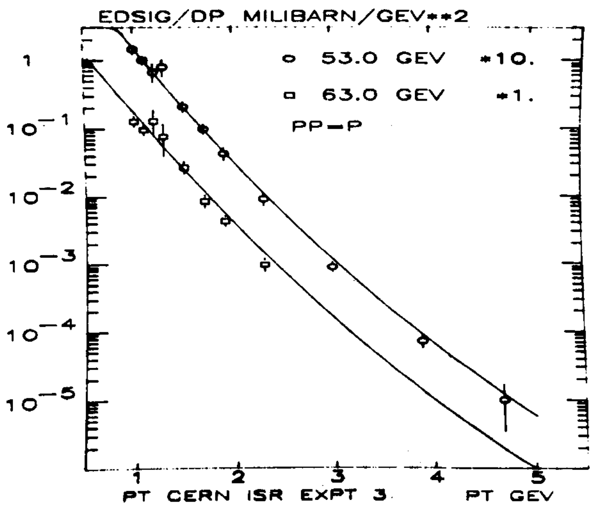

Fig.6. Description of the experimental data [3] for $E\frac{d^3\sigma}{dp^3}(pp \to pX)$, obtained by formula (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve (beginning with the lower one) are multiplied by $10^{(n-1)}$, $n = 1,2$.

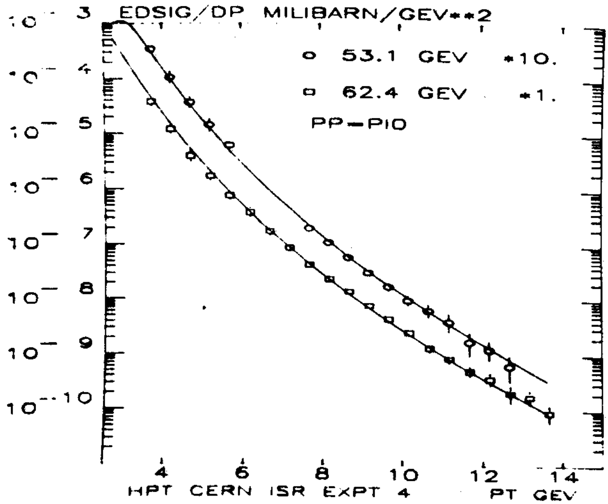

Fig.7. Description of the experimental data /4/ for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^o X)$, obtained ny (6). The values of $E\frac{d^3\sigma}{dp^3}$ for eacg curve are multiplied by $10^{(n-1)}$, $n = 1,2$.

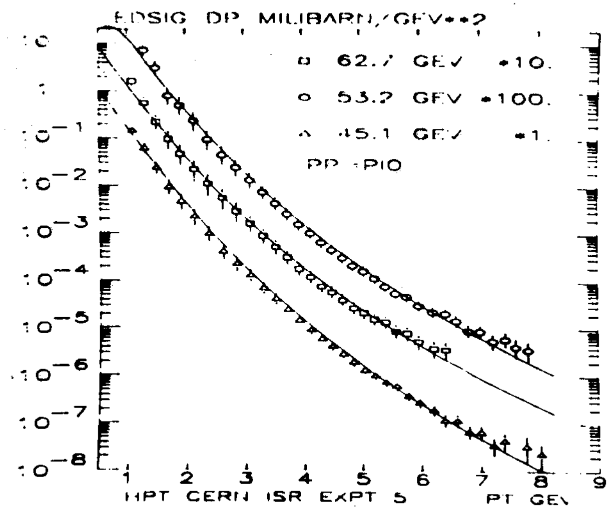

Fig.8. Description of the experimental data /5/ for $E\frac{d^3\sigma}{dp^3}(pp \to \pi^o X)$, obtained by (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve are multiplied by $10^{(n-1)}$, $n = 1,2,3$.

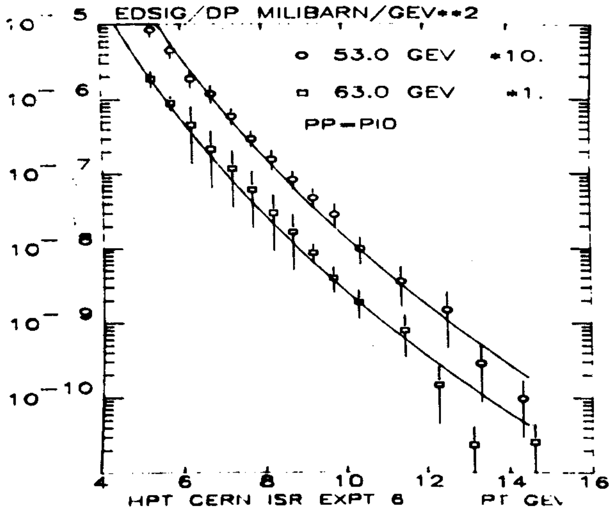

Fig.9. Description of the experimental data /6/ for $E\frac{d^3\sigma}{dp^3}$ obtained ny (6). The values of $E\frac{d^3\sigma}{dp^3}$ for each curve are multiplied by $10^{(n-1)}$, $n = 1,2$.

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Дренска С., Мавродиев С.Щ., Сисакян А.Н. E2-83-587
О поведении сечений инклюзивных адронных процессов
при больших переданных импульсах и энергиях CERN ISR и SPS Collider

В рамках правил кваркового счета аномальных размерностей для инклюзивных сечений в ведущем логарифмическом приближении квантовой хромодинамики получено описание экспериментальных данных CERN ISR и SPS Collider для сечения $E\frac{d^3\sigma}{dp^3}(AB \to CX)$ при $x_T = \frac{2p_T}{\sqrt{s}} > 0{,}2$ и $\sqrt{s} \geq 40$ ГэВ. При замене

$$p_T \to m\chi_T = m\ln\left(\sqrt{1 + \frac{p_T^2}{m^2}} + \frac{p_T}{m}\right)$$

приводящей к экспоненциальному поведению сечений с малыми p_T, сечения описываются во всем экспериментально доступном интервале x_T.

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Drenska S., Mavrodiev S.Cht., Sissakian A.N. E2-83-587
On the Behaviour of Inclusive Hadron Cross Sections
at Large Transferred Momenta and CERN ISR and SPS Collider Energies

The CERN ISR and SPS Collider experimental data for the $E\frac{d^3\sigma}{dp^3}(AB \to CX)$ cross section at $x_T \equiv \frac{2p_T}{\sqrt{s}} \geq 0{,}2$ and $\sqrt{s} \geq 40$ GeV have been described in the framework of quark counting rules of anomalous dimensions in the leading logarithmic approximation of quantum chromodynamics. A good description of the behaviour of the cross section in the whole exp rimentally possible interval is obtained under the change

$$p_T \to m\chi_{p_T} \equiv m\ln\left(\sqrt{1 + \frac{p_T^2}{m^2}} + \frac{p_T}{m}\right)$$

leading to the exponential behaviour of the cross section in the region of small p_T.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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