UNIVERSITY OF HELSINKI

REPORT SERIES IN PHYSICS

No 80

THE STRAIGHT-LINE PATH METHOD IN QUANTUM FIELD THEORY

S. P. Kuleshov*, V. A. Matveev*, A. N. Sissakian*, M. A. Smondyrev* and A. N. Tavkhelidze*

1974

DEPARTMENT OF NUCLEAR PHYSICS
SILTAVUORENPENGER 20 · HELSINKI 17 · FINLAND

No 80

THE STRAIGHT-LINE PATH METHOD IN QUANTUM FIELD THEORY

S. P. Kuleshov*, V. A. Matveev*, A. N. Sissakian*, M. A. Smondyrev* and A. N. Tavkhelidze*

1974

* Joint Institute for Nuclear Research, Dubna

ISBN 951-45-0444-5

C o n t e n t s

Abstract

This survey is an attempt to familiarise the reader with the mathematical formulation which underlies the straight-line path method in quantum field theory. The suggested approach is based on the representation of Green's functions and the scattering amplitudes in the form of continuous integrals over the particle paths.

Some applications of the method to solving problems of high-energy particle interactions are considered, and its connection with the quasipotential description in quantum field theory is studied.

§1. INTRODUCTION

The study of strong interaction processes at high energies is one of the fundamental problems of present-day elementary particle physics. The general principles of quantum field theory constitute the theoretical basis for the interpretation of the laws, which govern these processes.

The study of high-energy hadron interaction is based on the fruitful idea of the scattering amplitude as a unified analytic function of physical variables, which was suggested by N.N. Bogolubov in his fundamental work on the theory of dispersion relations[1,2].

This concept expresses the important requirement of a mutual relationship between different physical processes and is a corner-stone of the majority of theoretical and phenomenologic approaches to high-energy strong interactions which are being extensively developed. Some of the most useful are the dispersion relations and equations, dispersion sum rules, asymptotic approach, phenomenological Regge, eikonal and quasipotentional approaches. Recently the quasipotential method has proved to be rather fruitful in describing processes of the high-energy particle interactions.

The quasipotential method[3-5] was suggested in 1963. Over the past years it has been extensively developed and applied to various branches of quantum field theory.

This approach has made great progress because it combines the rigorous bases of quantum field theory and clear physical interpretation which allow the use of both empirical and heuristic considerations about the nature of high-energy particle interactions.

In a number of recent studies devoted to the description of various high-energy processes, attention has been paid to the importance of heuristic ideas concerning the smoothness of the local quasipotential which originated in the pioneer studies of D.I. Blokhintsev[/6/], S.P. Alliluev, S.S. Gershtein and A.A. Logunov[/7/]. On the one hand, this hypothesis makes it possible to reproduce the main features of high-energy hadron scattering and, on the other hand, it leads to a simple qualitative picture of the interaction of particles at asymptotically high energies. Within this picture, hadrons, when scattering at high energies, conserve their large longitudinal momenta (in the c.m.s.) in each act of interaction and undergo small momentum transfers.

In a certain sense, this behaviour is close to the classical picture of the scattering of the fast particles which move along approximately straight-line paths and undergo only small angular deflections. However, the important difference is that the high-energy hadron scattering is of an absorptive, i.e. essentially inelastic, nature with approximately constant total cross section and diffraction behaviour at small momentum transfers.

This qualitative picture of the approximately straight-line motion of interacting particles can be extended to high-energy inelastic processes, since one of the most important empirical laws of the latter is the limited nature of the transverse momenta and the predominance of the longitudinal (along the collision axis) components of the momenta of the secondary particles.

It seems therefore especially important to develope methods based on the straight-line path concept, i.e. on the assumption that the of momentum transfers are small in elastic and inelastic interactions of particles at asymptotically high energies. We shall call all these methods the straight-line path method (SLPM). This is the topic of the present survey.

The relativistic formulation of SLPM was first presented by the Dubna group (see e.g.[/8,9/]) on the basis of the functional integration methods in quantum field theory. This choice was not accidental. Firstly, the method of functional integration was shown in the fundamental work of R. Feynman[/10/] and N.N. Bogolubov[/11/] to be very convenient for finding closed expressions for complete Green's functions. Secondly, in the framework of the continuous representation of the amplitudes for various processes as a sum over the paths of colliding particles, the straight-line path concept is realized in a simple and clear manner. Straight-line path approximation consists here in accounting for particle paths, which approach most closely the classical ones and coincide approximately with straight-line trajectories in the case of high-energy small-angle scattering. The methods of approximate calculation of functional quadratures used in this case, are close to the approximation procedures proposed by E.S. Fradkin[/12/] and B.M. Barbashov[/13/] for investigating the infrared asymptotic of the Green's functions in quantum electrodynamics.

Section two is devoted to the description of the general method of constructing the two-particle Green's functions and the scattering amplitudes in the form of continuous integrals over the particle paths. As the object of our investigation, we chose the standard models of field theory: the model of scalar "nucleons" which exchange scalar "mesons" and the model of scalar "nucleons" which exchange vector "mesons". The closed functional expressions obtained for the two-particle Green's functions contain contributions from different graphs which take into account radiative corrections, closed nucleon loops, and so on. The important stage in the construction of the scattering amplitude is the developement of the method of calculating the correct transition to the mass shell and the analysis of the problem of renormalization. An interesting aspect of the general expression found for the scattering amplitude written in the form of the functional integral is a specific factorization in the amplitude of the contributions describing self-action nucleon effects, exchange effects and vacuum polarization. The expression for the two-particle Green's function is then used to construct the amplitude for the processes with the production of some number of meson quanta. The general properties of the amplitudes for inelastic processes are considered.

In section three the relativistic formulation of straight-line path approximation is given for the expression for elastic and inelastic amplitudes written in the form of continuous integrals. This approximation leads to a modification of nucleon propagators for which the interference combinations of the virtual meson momenta are not taken into account.

Sometimes a simpler version of the approximation is used in which the quadratic terms in the propagators vanish. However, in a number of cases such a method is unacceptable due mainly to difficulties associated with the divergences of the Feynman integrals.

As an example of the application of straight-line path approximation, the amplitude for two-particle scattering at high energies and fixed momentum transfers is considered. Of importance is the factorization of the radiative corrections to the scattering amplitude which, in this approximation, have the form of an eikonal representation with a Yukawa interaction potential. The final result does not show retardation effects. It is interesting to note that the sum of the ladder graphs with meson line crossing reduces in straight-line path approximation to the sum of ladder graphs of the quasipotential type. It is shown that the diffraction behaviour of the scattering amplitude is due to the form of radiative corrections, which lead naturally to a smooth complex potential.

As another example of application of straight-line path approximation, the processes of multiple particle production are considered. Among different approaches developed along this line the idea of interpreting the meson production in strong interactions by analogy with the bremsstrahlung of "soft" particles in electrodynamics is closest to the straight-line path approach. The feature common to both approaches is the assumption that the recoil of "leading" particles in the emission of secondaries can be partially or completely neglected. The main results of this part of the survey are as follows: prediction concerning the Poisson character of the distribution over multiplicity at fixed t; observation of the region of the automodel behaviour of the cross sections summed over the number of secondaries; and approximate linearity of the average multiplicity over t.

Straight-line path approximation has recently been extensively developed. In particular, in this approximation some other models of field theory have been studied, a wider class of diagrams has been taken into account and different asymptotic domains have been considered. The present survey is not intended to provide a wide discussion of all the related problems for which the reader is referred to a recent survey[14], in which there are some other results and an extensive bibliography.

In section four some mathematical realizations of the straight-line concept, which use methods of functional integration and allow one to perform a consistent consideration of the particle path deflections from linear trajectories, are formulated. The study of this problem is of great importance since it may lead to a self-consistent resolution of the problem of the foundation of straight-line path approximation and may extend the range of its applicability. It is shown that accounting for the correction terms results in the appearance of retardation effects. Owing to the fact that the correction terms are singular at small distances it is difficult to provide a definite to the question of the behaviour of the correction series as a whole.

In section five an operator method for solving the quasi-potential equations is formulated, and its relationship with the approximate methods of functional integration is established. It is shown that under the condition of smoothness of the local quasi-potential the operator method makes it possible to give a consistent foundation for the eikonal representation of the scattering amplitude and to find to its corrections.

In section six the structure of the "noneikonal" contribution to the two nucleon scattering amplitude is studied. It is shown in particular that in the sum of all ladder-type graphs of eighth order, there exist terms which violate the orthodox eikonal formula but disappear in the limit $\frac{\mu}{m} \to 0$ where μ and m are meson and nucleon masses respectively. These terms are associated with the contribution to the effective quasipotential corresponding to the nucleon-antinucleon pairs exchange. Further, the asymptotics are studied for the twisted eikonal graphs. The results obtained are employed in the reconstruction of the asymptotic quasipotential.

§2. REPRESENTATION OF SCATTERING AMPLITUDES AS CONTINUOUS INTEGRALS OVER PATHS

2.1. The Construction of the Two-Particle Green's Function

For simplicity we consider first the model of scalar nucleons interacting with a scalar meson, with the interaction Lagrangian of the form $\mathcal{L}_{int} = g : \psi^+\psi\phi :$

The results will later be generalization to the model of scalar nucleons interacting with a neutral vector field.

The one-particle Green's function of the nucleon in the given external scalar field ϕ satisfies the equation

$$\left[\Box + m^2 - g\varphi(x) \right] G(x,y|\varphi) = \delta(x-y) \quad (2.1.1)$$

The formal solution of eq. (2.1.1) can be represented by means of the functional integral[13]

$$G(x,y/\varphi) = i\int_0^\infty d\tau\, e^{-i\tau m^2} \int [\delta\nu]_0^\tau \exp\{ig\int_0^\tau d\xi \cdot \varphi[x + 2\int_0^\xi \nu(\eta)\, d\eta]\}\, \delta[x-y+2\int_0^\tau \nu(\eta)\, d\eta], \tag{2.1.2}$$

where

$$[\delta\nu]_{\tau_1}^{\tau_2} = \frac{\delta\nu \exp[-i\int_{\tau_1}^{\tau_2} \nu^2(\eta)\, d\eta]}{\int \delta\nu \exp[-i\int_{\tau_1}^{\tau_2} \nu^2(\eta)\, d\eta]} \tag{2.1.3}$$

and $\delta\nu$ is a volume element of the functional space of the four-dimensional functions $\nu(\eta)$ defined on the interval $\tau_1 \leq \eta \leq \tau_2$.

The Fourier transform of the Green's function (2.1.2) has the following form

$$G(p,q/\varphi) = \int d^4x\, d^4y\, e^{ipx - iqy}\, G(x,y/\varphi) = i\int_0^\infty d\tau\, e^{i\tau(p^2-m^2)} \int d^4x\, e^{ix(p-q)} \int [\delta\nu]_0^\tau \cdot \exp\{ig\int_0^\tau d\xi\, \varphi[x+2p\xi + 2\int_0^\xi \nu(\eta)\, d\eta]\}. \tag{2.1.4}$$

Using expression (2.1.4), we can find the two-particle Green's function of nucleons in the form

$$G(p_1,p_2;q_1,q_2) = [\exp \frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\varphi^2}]\, G(p_1,q_1/\varphi)\, G(p_2,q_2/\varphi)\, S_0(\varphi)\Big|_{\varphi=0} \tag{2.1.5}$$

where

$$[\exp\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\varphi^2}] \equiv [\,\frac{i}{2}\int d^4x_1\, d^4x_2\, \mathcal{D}(x_1-x_2)\, \frac{\delta^2}{\delta\varphi(x_1)\,\delta\varphi(x_2)}\,] \tag{2.1.6}$$

and $S_0(\phi)$ is the S-matrix averaged over the nucleon vacuum fluctuations in the presence of the external field ϕ. $S_0(\phi)$ can be written as

$$S_0(\varphi) = \exp\left[i\, \Pi(\varphi) \right], \tag{2.1.7}$$

where the functional $\pi(\phi)$ in the models considered here corresponds to the sum of the connected diagrams with one closed nucleon loop and an arbitrary number of external meson tails.

Introducing the notation

$$\int j_i \varphi \equiv \int dz\, \varphi(z)\, j(x_i - z;\, p_i;\, \tau_i / \nu_i), \quad i = 1, 2, \tag{2.1.8}$$

where

$$j(x_i - z;\, p_i;\, \tau_i/\nu_i) = \int_0^{\tau_i} d\xi\, \delta\left[x_i - z + 2p_i \xi + 2\int_0^{\xi} \nu_i(\eta)\, d\eta\right] \tag{2.1.9}$$

We can rewrite the expression for the two-particle Green's function (2.1.5) in the following simple form

$$G(p_1, p_2; q_1, q_2) = i^2 \int_0^{\infty} d\tau_1\, d\tau_2\, e^{i\tau_1(p_1^2 - m^2) + i\tau_2(p_2^2 - m^2)} \int dx_1^4\, dx_2^4 \cdot$$
$$e^{ix_1(p_1 - q_1) + ix_2(p_2 - q_2)} \int [\delta\nu_1]_0^{\tau_1} [\delta\nu_2]_0^{\tau_2}\, \mathcal{E}(x_{1,2};\, p_{1,2};\, \tau_{1,2}/\nu_{1,2}), \tag{2.1.10}$$

where

$$\mathcal{E} = \left[\exp \frac{i}{2} \int \mathcal{D} \frac{\delta^2}{\delta\varphi^2}\right] e^{ig\int \varphi(j_1 + j_2)}\, S_0(\varphi)\Big|_{\varphi=0} \tag{2.1.11}$$

We will now examine in detail the structure of the quantity $\mathcal{E}$. Let us determine for each functional $A(\phi)$ the quantity

$$\bar{A}(\varphi) = \left[\exp \frac{i}{2} \int \mathcal{D} \frac{\delta^2}{\delta\varphi^2}\right] A(\varphi), \tag{2.1.12}$$

so, that the average value $A(\phi)$ over the meson vacuum fluctuation is given by $\bar{A} = \bar{A}(\phi)\Big|_{\phi=0}$.

From the average value of the product of two functionals

$$\overline{A\cdot B} = \left[\exp \frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\varphi^2}\right] A(\varphi)\, B(\varphi)\Big|_{\varphi=0} \qquad (2.1.13)$$

one can easily show that the following identity holds

$$\begin{aligned}\overline{A\cdot B} &= \left[\exp \frac{i}{2}\int \mathcal{D}\left(\frac{\delta}{\delta\varphi_1}+\frac{\delta}{\delta\varphi_2}\right)^2\right] A(\varphi_1)\, B(\varphi_2)\Big|_{\varphi_1=\varphi_2=0} = \\ &= \left[\exp i\int \mathcal{D}\frac{\delta^2}{\delta\varphi_1\,\delta\varphi_2}\right] \bar{A}(\varphi_1)\, \bar{B}(\varphi_2)\Big|_{\varphi_1=\varphi_2=0} = \qquad (2.1.14) \\ &= \bar{A}\left(i\int \mathcal{D}\frac{\delta}{\delta\varphi_2}\right) \bar{B}(\varphi_2)\Big|_{\varphi_2=0}\end{aligned}$$

Choosing

$$A(\varphi) = e^{ig\int \varphi(j_1+j_2)}, \quad B(\varphi) = S_0(\varphi), \qquad (2.1.15)$$

we have

$$\bar{A}(\varphi) = \exp\left[ig\int\varphi(j_1+j_2) - \frac{ig^2}{2}\int \mathcal{D}(j_1+j_2)^2\right], \qquad (2.1.16)$$

$$\bar{S}_0(\varphi) = \exp\left[i\,\Pi(\varphi)\right], \qquad (2.1.17)$$

where the quantity $\Pi(\phi)$ corresponds to the sum of all the connected Feynman diagrams with an arbitrary number of closed nucleon loops and internal meson lines (bearing in mind, that the nucleons interact with the external field ϕ).

Using the identity (2.1.14) and eqs. (2.1.15-17), we find the following expression for the quantity (2.1.11)

$$\mathcal{E} = \exp\left[-\frac{ig^2}{2}\int\mathcal{D}(j_1+j_2)^2 - g\int\mathcal{D}(j_1+j_2)\frac{\delta}{\delta\varphi}\right] e^{i\Pi(\varphi)}\Big|_{\varphi=0} =$$
$$= \exp\left\{-\frac{ig^2}{2}\int\mathcal{D}(j_1+j_2)^2 + i\Pi\left[-g\int\mathcal{D}(j_1+j_2)\right]\right\} \qquad (2.1.18)$$

Expanding the quantity (2.1.18) in powers of the coupling constant and substituting the series in eq. (2.1.10), we get, after performing simple functional integrations over ν_i, just the usual non-renormalized series of perturbation theory for the two-particle Green's function.

We now stress an important fact which will be used below. The expression (2.1.18) allows us to separate, in a general form, the contributions to an interaction between two nucleons (exchange effects), the self-interaction of the nucleons through the meson field (radiative corrections), and the vacuum renormalization.

The first term in the exponential in eq. (2.1.18) can be rewritten as

$$\frac{ig^2}{2}\int\mathcal{D}(j_1+j_2)^2 = ig^2\int\mathcal{D}j_1j_2 + \frac{ig^2}{2}\int\mathcal{D}j_1^2 + \frac{ig^2}{2}\int\mathcal{D}j_2^2, \qquad (2.1.19)$$

where the first term on the right hand side corresponds to one-meson exchange between the nucleons, and the rest of the terms lead to the radiative corrections.

Correspondingly, the second term in the exponential in eq. (2.1.18) can be represented in the form

$$\Pi = \Pi_{12} + \Pi_1 + \Pi_2 + \Pi(0), \qquad (2.1.20)$$

where

$$\Pi_{12} = \Pi\left[-g\int\mathcal{D}(j_1+j_2)\right] - \Pi\left(-g\int\mathcal{D}j_1\right) - \Pi\left(-g\int\mathcal{D}j_2\right) + \Pi(0) \qquad (2.1.21)$$

$$\Pi_i = \Pi\left(-g\int\mathcal{D}j_i\right) - \Pi(0), \quad i = 1,2. \qquad (2.1.22)$$

It can be shown that the quantities (2.1.21-22) may be expressed in terms of the polarization operator of the meson field

$$P(x_1, x_2 | \varphi) = - \frac{\delta^2}{\delta\varphi(x_1)\,\delta\varphi(x_2)} \Pi(\varphi) \tag{2.1.23}$$

or in terms of the full Green's function of the meson field

$$\mathcal{D}(x_1, x_2 | \varphi) = \mathcal{D}(x_1 - x_2) + \int d^4y_1\, d^4y_2\, \mathcal{D}(x_1 - y_1)\, P(y_1, y_2 | \varphi)\, \mathcal{D}(y_2 - x_2) \tag{2.1.24}$$

in the presence of external sources.

As a result we get for the quantity $\mathcal{E}$ the following expression[/15/]

$$\mathcal{E} = e^{i\overline{\Pi(0)}} \cdot \mathcal{E}^{(1)} \cdot \mathcal{E}^{(2)}\, \mathcal{E}^{(12)}, \tag{2.1.25}$$

where

$$\mathcal{E}^{(i)} = \exp\left[-\frac{ig^2}{2} \int \mathcal{D}_i^* j_i^2\right], \quad i = 1, 2, \tag{2.1.26}$$

$$\mathcal{E}^{(12)} = \exp\left[-ig^2 \int \mathcal{D}_{12}^* j_1 j_2\right]. \tag{2.1.27}$$

and we have used the notation

$$\mathcal{D}_i^* = 2 \int_0^1 d\sigma \int_0^\sigma d\lambda\, \mathcal{D}\left(x_1, x_2 \middle| -g\lambda \int \mathcal{D} j_i\right), \quad i = 1, 2, \tag{2.1.28}$$

$$\mathcal{D}_{12}^* = \int_0^1 d\lambda_1 \int_0^1 d\lambda_2\, \mathcal{D}\left(x_1, x_2 \middle| -g\lambda_1 \int \mathcal{D} j_1 - g\lambda_2 \int \mathcal{D} j_2\right). \tag{2.1.29}$$

We note that the $\mathcal{D}_i^*$ are connected to the Green's function of the scalar meson, interacting with the external sources, associated with the i-th nucleon, and $\mathcal{D}_{12}^*$ corresponds to the Green's function of the scalar meson, interacting simultaneously with the sources of both nucleons.

Thus, the quantity $\mathcal{E}$ which determines the two-particle Green's function for nucleons is factorized into the terms which describe the interaction between two nucleons, the radiative connections, and the vacuum renormalization respectively.

2.2. The Representation for the Two-Nucleon Scattering Amplitude

The two-nucleon scattering amplitude is determined through the two-particle Green's function (2.1.5) by

$$(2\pi)^4 \delta(p_1+p_2-q_1-q_2)\; i F(p_1,p_2;q_1,q_2) = \\ = \lim_{p_i^2, q_i^2 \to m^2} \prod_{i=1,2} (p_i^2-m^2)(q_i^2-m^2)\; G(p_1,p_2;q_1,q_2) \qquad (2.2.1)$$

We will ignore the renormalization problem which will be discussed in the following section and add on the right hand side of eq. (2.2.1) the factor $\exp i\Pi(0)$, so that it does not contribute to scattering processes.

As was mentioned in the Introduction, it is very important to develope the correct procedure for passing to the mass shell in constructing the scattering amplitude in general form, before any approximations are made. Many approximations, being reasonable from a physical point of view, when applied before the transition to the mass shell, disturb the positions of poles of the Green function and invalidate the whole procedure mathematically.

In this paper we develope the method for extracting poles of the Green's function. This method is a generalization of that used previously[8,9,16,17] to find a scattering amplitude for the model of scalar nucleons interacting with a scalar meson where the contributions of the closed nucleon loops are neglected.

Using the expression for a two-particle Green's function (2.1.5) and eqs. (2.1.9) and (2.1.25-29), we put the scattering amplitude (2.2.1) in the form

$$(2\pi)^4 \delta(p_1+p_2-q_1-q_2)\, F(p_1,p_2;q_1,q_2) = \lim_{p_i^2, q_i^2 \to m^2} \prod_{i=1,2} (p_i^2-m^2)(q_i^2-m^2) \cdot$$
$$\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \int_0^{\tau_1} d\xi_1 \int_0^{\tau_2} d\xi_2\, e^{i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)} \int d^4x_1\, d^4x_2 \int d^4z_1\, d^4z_2 \cdot$$
$$e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)} \int [\delta\nu_1]_0^{\tau_1} [\delta\nu_2]_0^{\tau_2}\, i\mathcal{F}(z_1, z_2 | j_1, j_2) \cdot$$
$$\delta\Big(x_1 - z_1 + 2p_1\xi_1 + 2\int_0^{\xi_1} \nu_1\, d\eta\Big)\, \delta\Big(x_2 - z_2 + 2p_2\xi_2 + 2\int_0^{\xi_2} \nu_2\, d\eta\Big), \tag{2.2.2}$$

where

$$\mathcal{F}(z_1, z_2 | j_1, j_2) = g^2\, \mathcal{E}^{(1)} \mathcal{E}^{(2)} \mathcal{D}_{12}^{*} \int_0^1 d\gamma\, e^{-i\gamma g^2 \int \mathcal{D}_{12}^{*} j_1 j_2} \tag{2.2.3}$$

To obtain eqs. (2.2.2-3) we have used the fact that the free part of the Green's function which is not connected with an interaction between nucleons, can be subtracted using the formula

$$\mathcal{E}^{(12)} \to \mathcal{E}^{(12)} - 1 = ig^2 \int \mathcal{D}_{12}^{*} j_1 j_2 \int_0^1 d\gamma\, e^{-i\gamma g^2 \int \mathcal{D}_{12}^{*} j_1 j_2} \tag{2.2.4}$$

Using the identity

$$\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \int_0^{\tau_1} d\xi_1 \int_0^{\tau_2} d\xi_2 \ldots \equiv \int_0^\infty d\xi_1 \int_0^\infty d\xi_2 \int_{\xi_1}^\infty d\tau_1 \int_{\xi_2}^\infty d\tau_2 \ldots \tag{2.2.5}$$

and changing the ordinary and the functional variables

$$\begin{aligned} &\tau_i \to \tau_i + \xi_i, \\ &x_i \to x_i - 2p_i\xi_i - 2\int_0^{\xi_i} \nu_i\, d\eta, \\ &\nu_i(\eta) \to \nu_i(\eta-\xi_i) - (p_i - q_i)\, \vartheta(\eta-\tau_i), \end{aligned} \tag{2.2.6}$$

we get

$$(2\pi)^4 \delta(p_1+p_2-q_1-q_2)\, F(p_1,p_2;q_1,q_2) = \lim_{p_i^2,q_i^2\to m^2} \prod_{i=1,2} (p_i^2-m^2)(q_i^2-m^2)\cdot$$
$$\int_0^\infty d\tau_1\, d\tau_2 \int_0^\infty d\xi_1\, d\xi_2\, \exp[\,i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)+i\xi_1(q_1^2-m^2)+i\xi_2(q_2^2-m^2)]\cdot$$
$$\int d^4x_1\, d^4x_2\, e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)} \int[\delta\nu_1]_{-\xi_1}^{\tau_1}[\delta\nu_2]_{-\xi_2}^{\tau_2}\, i\,\mathcal{F}(x_1,x_2|j_1,j_2) \qquad (2.2.7)$$

Going to the limit in eq. (2.2.7) and taking into account the translation symmetry of the quantity $\mathcal{F}$ x/, we get the final result for the scattering amplitude

$$F(p_1,p_2;q_1,q_2) = \int[\delta\nu_1]_{-\infty}^{\infty}\, \mathcal{E}^{(1)}(p_1,q_1|\nu_1) \int[\delta\nu_2]\, \mathcal{E}^{(2)}(p_2,q_2|\nu_2)\cdot$$
$$ig^2\int d^4x\, e^{ix\Delta}\, \mathcal{D}_{12}^*(x;p_i,q_i|\nu_i) \int_0^1 d\gamma\, e^{-i\gamma g^2 \int \mathcal{D}_{12}^* j_1 j_2}, \qquad (2.2.9)$$

where

$$\Delta = (p_1-q_1) = -(p_2-q_2),\ x = x_1-x_2$$

and all the quantities in this expression are the functionals of the limiting sources

$$j_i \equiv \int_{-\infty}^{\infty} d\zeta\, \delta[x_i - z + 2p_i\zeta\,\vartheta(\zeta) + 2q_i\zeta\,\vartheta(-\zeta) + 2\int_0^\zeta \nu_i\, d\eta] \qquad (2.2.10)$$

Note that the expression (2.2.10) determines the scalar density of the point-like particle, moving along the classical path $x_i(s)$ which depends on the proper time $s = 2m\zeta$ and satisfies the equation

$$m\frac{dx_i(s)}{ds} = p_i\vartheta(\zeta) + q_i\vartheta(-\zeta) + \nu_i(\zeta) \qquad (2.2.11)$$

---

x/ We remind the reader that under translations $x_i \to x_i + h$ the functional variables of $\mathcal{F}$ that is the "current densities"

$$j_i = \int_{-\xi_i}^{\tau_i} d\zeta\, \delta[x_i - z + 2p_i\zeta\,\vartheta(\zeta) + 2q_i\zeta\,\vartheta(-\zeta) + 2\int_0^\zeta \nu_i\, d\eta], \qquad (2.2.8)$$

also undergo the transformation.

under the condition $\mathcal{X}_i(0) = \mathcal{X}_i$, i = 1,2.

For this reason the representation (2.2.9) for the scattering amplitude can be considered as a continuous summation over all possible nucleon paths in a scattering process.

Now we generalize the above consideration to the case of the vector-exchange model with the interaction Lagrangian

$$L_{int} = ig : A_\mu \psi^+ \overleftrightarrow{\partial}_\mu \psi : + \frac{g^2}{2} : A_\mu^2 \psi^+ \psi : \qquad (2.2.12)$$

We shall not go into all the details, but only summarize briefly the final results.

The scattering amplitude in this model is given by

$$F(p_1,p_2;q_1,q_2) = \int [\delta\nu_1]_{-\infty}^{\infty} \mathcal{E}^{(1)}(p_1,q_1|\nu_1) \int [\delta\nu_2]_{-\infty}^{\infty} \mathcal{E}^{(2)}(p_2,q_2|\nu_2) \cdot$$
$$\ell_\alpha^{(1)} \ell_\beta^{(2)} ig^2 \int d^4x\, e^{ix\Delta} \mathcal{D}_{12}^{\alpha\beta *}(x;p_i,q_i|\nu_i) \cdot \qquad (2.2.13)$$
$$\cdot \int_0^1 d\gamma\, e^{-i\gamma g^2 \int \mathcal{D}_{12}^{\alpha\beta *} j_\alpha^{(1)} j_\beta^{(2)}},$$

where

$$\ell_\alpha^{(i)} = [p_i + q_i + 2\nu_i(0)]_\alpha, \quad i = 1,2. \qquad (2.2.14)$$

As in the previous case, all the quantities in eq. (2.2.13) are expressed through the Green's function of the vector meson field interacting with the external sources

$$j_\alpha^{(i)} = \int_{-\infty}^{\infty} d\xi\, [2p_i \vartheta(\xi) + 2q_i \vartheta(-\xi) + 2\nu_i(\xi)]_\alpha \cdot$$
$$\delta[x_i - z + 2p_i \xi \vartheta(\xi) + 2q_i \vartheta(-\xi) + 2\int_0^\xi \nu_i(\eta) d\eta] \qquad (2.2.15)$$

It is easy to see that eq. (2.2.15) determines the current density of the point-like particle moving along the classical path (2.2.11) and obeys the condition

$$\partial_\alpha j_\alpha^{(i)} = 0, \quad i = 1,2. \qquad (2.2.16)$$

2.3. Discussion of the Renormalization Problems

It is evident that the one-particle Green's function of interacting nucleons G(p), which is determined by

$$(2\pi)^4 \delta(p-q) G(p) = \left[\exp \frac{i}{2}\int \mathcal{D} \frac{\delta^2}{\delta\varphi^2}\right] G(p,q|\varphi) S_0(\varphi)\Big|_{\varphi=0} \tag{2.3.1}$$

has in general different positions for the pole and the value of the residues than the Green's function of free nucleons, i.e.

$$G(p) = \frac{1}{m^2 - p^2 + \Sigma(p^2)}\Big|_{p^2 \sim m^2_{phys}} \sim \frac{z^{-1}}{m^2_{phys} - p^2}, \tag{2.3.2}$$

where

$$\begin{aligned} m^2_{phys} &= m^2 + \Sigma(m^2_{phys}) = m^2 + \delta m^2, \\ z &= 1 - \frac{\partial \Sigma}{\partial p^2}(m^2_{phys}). \end{aligned} \tag{2.3.3}$$

For this reason in defining the scattering amplitude (2.2.1) as the residue of the two-particle Green's function at the poles, associated with the external nucleon tails, we should write, for example, $z(p_i^2 - m^2_{phys})$ instead of $(p_i^2 - m^2)$.

Moreover, it can be shown that due to the renormalization of the mass and the wave functions, the functional integrals in eq. (2.2.7) diverge, or more precisely

$$\begin{aligned} \int [\delta\nu_1]^{\tau_1}_{-\xi_1} [\delta\nu_2]^{\tau_2}_{-\xi_2} \mathcal{F} \xrightarrow[\tau_i, \xi_i \to \infty]{} & \exp\Big\{-i \sum_{k=1,2} \tau_k [\delta m^2 + (1-z)(p_k^2 - m^2_{phys})] \\ & - i \sum_{k=1,2} \xi_k [\delta m^2 + (1-z)(q_k^2 - m^2_{phys})]\Big\} \int_R [\delta\nu_1]^{\infty}_{-\infty} [\delta\nu_2]^{\infty}_{-\infty} \mathcal{F}, \end{aligned} \tag{2.3.4}$$

where the symbol $\int\limits_R$ denotes the renormalized value of the functional integral which is finite after extracting the divergent exponential factors.

Thus, we obtain for the scattering amplitude, defined as a residue of the two-particle Green's function at the physical poles, the same expressions (2.2.13), the only difference being that instead of $\int[\delta\nu_1]_{-\infty}^{\infty}[\delta\nu_2]_{-\infty}^{\infty}$ we should write more correctly $\int\limits_R[\delta\nu_1]_{-\infty}^{\infty}[\delta\nu_2]_{-\infty}^{\infty}$.

In a general investigation of the structure of scattering amplitudes the procedure of regularization of the functional integrals can be considerably simplified, if one assumes that the following limits exist

$$\frac{\mathcal{E}^{(i)}(p,q;\tau,\xi|\nu)}{\int[\delta\nu]_{-\xi}^{\tau}\mathcal{E}^{(i)}(p,q;\tau,\xi|\nu)} \xrightarrow[\tau,\xi\to\infty]{} e^{(i)}(p,q|\nu) \qquad (2.3.5)$$

where the momenta p and q are on the mass shell and the quantities $\mathcal{E}^{(i)}(p,q,\tau,\xi|\nu)$ are determined by eq. (2.1.26) with the nucleon current, given by (2.2.8).

These limits exist in the sense that the following "improper" functional integrals exist

$$\int[\delta\nu]_{-\infty}^{\infty} e^{(i)}(p,q|\nu = 1 \qquad (2.3.6)$$

and

$$\int[\delta\nu]_{-\infty}^{\infty} e^{(i)}(p,q|\nu)A(\nu) = [A]_{e^{(i)}} \qquad (2.3.7)$$

for appropriate functionals $A(\nu)$.

Using eqs. (2.3.5-7), we get for the two-nucleon scattering amplitude (2.2.9) the expression

$$F(p_1,p_2;q_1,q_2) = r^{(1)}(t)\, r^{(2)}(t)\, f(p_1,p_2;q_1,q_2), \qquad (2.3.8)$$

where

$$f(p_1,p_2;q_1,q_2) = \int [\delta\nu_1]_{-\infty}^{\infty} e^{(1)}(p_1,q_1/\nu_1) \int [\delta\nu_2]_{-\infty}^{\infty} e^{(2)}(p_2,q_2/\nu_2) \cdot$$
$$ig^2 \int d^4x\, e^{ix\Delta}\, \mathcal{D}_{12}^{*}(x;p_i,q_i/\nu_i) \int_0^1 d\gamma\, e^{-i\gamma g^2 \int \mathcal{D}_{12}^{*} j_1 j_2} \qquad (2.3.9)$$

and

$$r^{(i)}(t) = \int_R [\delta\nu_i]_{-\infty}^{\infty}\, \mathcal{E}^{(i)}(p_i,q_i/\nu_i),\quad t=(p_i-q_i)^2 \qquad (2.3.10)$$

It can be shown that $r^{(i)}(t=0) = 1$.

One can see from eq. (2.3.8) that part of the radiative correction is factorized in the scattering amplitude in the form of terms which depend only on the square of momentum transfers.

These radiative factors have a simple physical meaning: they describe an interaction of the asymptotically free nucleons in the initial and final states with the fluctuations of the meson vacuum.

The representation (2.3.8) may be useful in studying asymptotic behaviour of scattering amplitudes at high energies, as it extracts, in a consistent form, the factors independent of energy.

2.4. Construction of the Inelastic Processes Amplitudes

In this section we consider the generalization of the methods described above to the construction of amplitudes of the inelastic processes. We shall consider such inelastic processes when some number of secondary mesons is produced in two-nucleon collisions.

These processes can be described by means of the two-particle Green's function of nucleons in the presence of the external meson field φ^{ext}

$$G(p_1,p_2;q_1,q_2/\varphi^{ext}) = i^2\int_0^\infty d\tau_1\, d\tau_2\, e^{i\tau_1(p_1^2-m^2)+i\tau_2(p_2^2-m^2)} \int d^4x_1\, d^4x_2\, e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)} \int [\delta\nu_1]_0^{\tau_1}[\delta\nu_2]_0^{\tau_2}\,\mathcal{E}(\varphi^{ext}), \tag{2.4.1}$$

where

$$\mathcal{E}(\varphi^{ext}) = \left[\exp\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\varphi^2}\right] e^{ig\int\varphi(j_1+j_2)}\, S_0(\varphi)\Big|_{\varphi=\varphi^{ext}} = \exp\Big\{-\frac{ig^2}{2}\int\mathcal{D}(j_1+j_2)^2 + ig\int\varphi^{ext}(j_1+j_2) + i\Pi\big[\varphi^{ext} - g\int\mathcal{D}(j_1+j_2)\big]\Big\} \tag{2.4.2}$$

It is convenient to rewrite (2.4.2) as follows

$$\mathcal{E}(\varphi^{ext}) = e^{i\Pi(\varphi^{ext})}\,\mathcal{E}\cdot R(\varphi^{ext}), \tag{2.4.3}$$

where

$$R(\varphi^{ext}) = \exp\Big\{ig\int\varphi^{ext}(j_1+j_2) + i\Pi\big[\varphi^{ext} - g\int\mathcal{D}(j_1+j_2)\big] - i\Pi(\varphi^{ext}) - i\Pi\big[-g\int\mathcal{D}(j_1+j_2)\big]\Big\} \tag{2.4.4}$$

and $\mathcal{E} = \mathcal{E}(\varphi^{ext}=0)$ is defined by eq. (2.1.18) and corresponds to the pure elastic scattering processes.

Introduce now the quantities

$$\Gamma_i(z/\varphi^{ext}) = j_i + \int_0^1 d\sigma \int_0^1 d\lambda \int dy\,(\mathcal{D}j_i)_y\, P(z,y/\sigma\varphi^{ext} - \lambda g\int\mathcal{D}j_i), \quad i=1,2 \tag{2.4.5}$$

and

$$\Gamma_{12}(z/\varphi^{ext}) = \int_0^1 d\sigma\int_0^1 d\lambda_1\int_0^1 d\lambda_2 \int dy_1\, dy_2\, (\mathcal{D}j_1)_{y_1}(\mathcal{D}j_2)_{y_2}\cdot \Gamma[z,y_1,y_2/\sigma\varphi^{ext} - g\int\mathcal{D}(\lambda_1 j_1+\lambda_2 j_2)), \tag{2.4.6}$$

where

$$\Gamma(z, y_1, y_2 | \varphi) = \frac{\delta^3}{\delta\varphi(z)\,\delta\varphi(y_1)\,\delta\varphi(y_2)} \Pi(\varphi) \qquad (2.4.7)$$

is the generalized vertex operator of three-meson coupling in the presence of an external field, and the polarization $P(x,y|\phi)$ is determined by eq. (2.1.23).

The functional $R(\phi^{ext})$ in terms of the quantities (2.4.5-6) has the following form

$$R(\varphi^{ext}) = e^{-i\Pi(0)} \exp[ig\int \varphi^{ext}(\Gamma_1 + \Gamma_2) + ig^2 \int \varphi^{ext} \Gamma_{12}]. \qquad (2.4.8)$$

The generating function for amplitudes of the inelastic processes is determined as the residue of the two-particle Green's function of nucleons in the presence of an external field (2.4.1)

$$iF(p_1, p_2; q_1, q_2 | \varphi^{ext}) = \lim_{p_i^2, q_i^2 \to m^2} \prod_{i=1,2} (p_i^2 - m^2)(q_i^2 - m^2)\, G(p_1, p_2; q_1, q_2 | \varphi^{ext}) \qquad (2.4.9)$$

Adding to the right hand side (2.4.9) the factor $\exp i\Pi(\phi^{ext})$ which does not contribute to the particle interaction, and performing changes of the variables (2.2.6), we get the following expression for the generating functional (2.4.9)

$$F(p_1, p_2; q_1, q_2 | \varphi^{ext}) = \int d^4x_1\, d^4x_2\, e^{ix_1(p_1 - q_1) + ix_2(p_2 - q_2)}$$
$$\int [\delta \nu_1]_{-\infty}^{\infty} \mathcal{G}^{(1)}(p_1, q_1 | \nu_1)\, e^{ig\int \varphi^{ext}\Gamma_1} \int [\delta \nu_2]_{-\infty}^{\infty} \mathcal{G}^{(2)}(p_2, q_2 | \nu_2)\, e^{ig\int \varphi^{ext}\Gamma_2} \cdot$$
$$i\tilde{\mathcal{F}}(x_1, x_2 | j_1, j_2; \varphi^{ext}), \qquad (2.4.10)$$

where

$$\tilde{\mathcal{F}}(x_1, x_2 | j_1, j_2; \varphi^{ext}) = g^2 \tilde{\mathcal{D}}^*_{12} \int_0^1 d\gamma \, e^{-i\gamma g^2 \int \tilde{\mathcal{D}}^*_{12} j_1 j_2} \quad (2.4.11)$$

Here the quantity $\tilde{\mathcal{D}}^*_{12}$ is defined by

$$\tilde{\mathcal{D}}^*_{12} = \mathcal{D}^*_{12}(x_1, x_2; p_i \cdot q_i | \nu_i) - \int_0^1 d\sigma \int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dz \, dy_1 dy_2 \cdot$$
$$\varphi^{ext}(z) \mathcal{D}(x_1 - y_1) \Gamma_{12}[z, y_1, y_2 | \sigma \varphi^{ext} - g \int \mathcal{D}(\lambda_1 j_1 + \lambda_2 j_2)] \mathcal{D}(y_2 - x_2) \quad (2.4.12)$$

and corresponds to the full Green's function of the meson field interacting simultaneously with an external field ϕ^{ext} and the sources j_1 and j_2.

The amplitudes of the processes in which secondary mesons are produced in two-nucleon collision are determined by the functional derivatives of the generating functional (2.4.10)

$$(2\pi)^4 \delta(q_1 + q_2 - p_1 - p_2 - \sum_{i=1}^N k_i) \, F(p_1, p_2; q_1, q_2; k_1, k_2, \ldots, k_N) =$$
$$= \prod_{i=1}^N \int dy_i \, e^{i y_i k_i} \frac{\delta}{\delta \varphi^{ext}(y_i)} F(p_1, p_2; q_1, q_2 | \varphi^{ext}) \Big|_{\varphi^{ext} = 0} . \quad (2.4.13)$$

For example, the amplitude of production of one secondary meson with the momentum k has the form

$$F(p_1, p_2; q_1, q_2; k) = ig \int [\delta \nu_1]_{-\infty}^{\infty} \mathcal{E}^{(1)}(p_1, q_1 | \nu_1) \int [\delta \nu_2]_{-\infty}^{\infty} \mathcal{E}^{(2)}(p_2, q_2 | \nu_2) \cdot$$
$$\int d^4x \, e^{ix\Delta} \{ [\Gamma_1(p_1, q_1; k | \nu_1) + \Gamma_2(p_2, q_2; k | \nu_2)] \, g^2 \mathcal{D}^*_{12} \cdot$$
$$\int_0^1 d\gamma \, e^{-i\gamma g^2 \int \mathcal{D}^*_{12} j_1 j_2} + \tilde{\Gamma}_{12}(x; p_i, q_i; k | \nu_i) \, e^{-ig^2 \int \mathcal{D}^*_{12} j_1 j_2} \} \quad (2.4.14)$$

Here we have used the notation

$$\Gamma_i(p_i, q_i; k|\nu_i) = \int dz\, e^{izk}\, \Gamma_i(z|\varphi^{ext})\Big|_{\substack{x_i=0\\ \varphi^{ext}=0}}, i=1,2; \tag{2.4.15}$$

and

$$\tilde{\Gamma}_{12}(x; p_i, q_i; k|\nu_i) = -g\int_0^1 d\lambda_1 \int_0^1 d\lambda_2 \int dz\, dy_1\, dy_2\, e^{izk} \cdot$$
$$\mathcal{D}(y_1 - \tfrac{x}{2})\,\mathcal{D}(y_2 + \tfrac{x}{2})\,\Gamma[y_1, y_2, z| -g\int \mathcal{D}(\lambda_1 j_1 + \lambda_2 j_2)] \tag{2.4.16}$$

Similar expressions can be obtained for the production amplitudes of two or more secondary mesons.

§3. STRAIGHT-LINE PATH APPROXIMATION AND ASYMPTOTIC BEHAVIOUR OF HIGH-ENERGY SCATTERING AMPLITUDES

In this section we use the continuous representation of the scattering amplitudes, obtained above, to formulate straight-line path approximation and then to investigate, by means of straight-line path approximation, the asymptotic behaviour of the elastic and inelastic high-energy nucleon amplitudes.

We take, ~~as~~ an example, a model of scalar nucleons interacting with the vector field and neglecting the vacuum polarization effects i.e. the contributions of closed nucleon loops.

3.1. Elastic Scattering

When the vacuum polarization effects are neglected $\pi = 0$, the two-nucleon elastic amplitude is determined by eq. (2.2.13), in which

$$\mathcal{D}_{12}^{\alpha\beta *} = \mathcal{D}_{i}^{\alpha\beta *} = \mathcal{D}^{\alpha\beta} = \frac{1}{(2\pi)^4}\int d^4k \frac{e^{ikx}}{\mu^2 - k^2}\left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}\right). \tag{3.1.1}$$

As a result, we obtain the following expression for the two-particle scattering amplitude[18]

$$F(p_1, p_2; q_1, q_2) = \frac{ig^2}{(2\pi)^4}\int d^4x\, e^{ix\Delta}\mathcal{D}^{\alpha\beta}(x)\int [\delta\nu_1]_{-\infty}^{\infty}[\delta\nu_2]_{-\infty}^{\infty}\cdot$$
$$[2\nu_1(0) + p_1 + q_1]_\alpha [2\nu_2(0) + p_2 + q_2]_\beta \int_0^1 d\gamma\, \exp - \left\{\frac{ig^2}{2}\int d^4k\, \mathcal{D}^{\alpha\beta}(k)\cdot\right.$$
$$\Big[\sum_{i=1,2} j_\alpha^{(i)}(k; p_i, q_i|\nu_i)\, j_\beta^{(i)}(-k; p_i, q_i|\nu_i) +$$
$$\left. + \gamma e^{ikx} j_\alpha^{(1)}(k; p_1, q_1|\nu_1)\, j_\beta^{(2)}(-k; p_2, q_2|\nu_2) + (k \to -k)\Big]\right\}, \tag{3.1.2}$$

where

$$j_\alpha^{(i)}(k; p_i, q_i|\nu_i) = 2i\int_{-\infty}^{\infty} d\xi\, [\nu_i(\xi) + p_i\vartheta(\xi) + q_i\vartheta(-\xi)]_\alpha\cdot$$
$$\exp\left\{2ik\left[p_i\xi\vartheta(\xi) + q_i\xi\vartheta(-\xi) + \int_0^\xi \nu_i d\eta\right]\right\} \tag{3.1.3}$$

is the transition current satisfying the continuity equation

$$k_\alpha j_\alpha^{(i)}(k; p_i, q_i|\nu_i) = 0 \tag{3.1.4}$$

We note, that the terms

$$j^{(i)} j^{(i)}, \quad i = 1, 2 \tag{3.1.5}$$

in eq. (3.1.2) describe the radiative corrections to each of the nucleon lines, and the terms

$$j^{(1)} j^{(2)} \tag{3.1.6}$$

describe the interaction between two nucleons.

Let us now consider in more detail the physical meaning of the functional variables ν_1 and ν_2. These variables, formally introduced for obtaining the solution for the Green's function, describe the deviation of a particle trajectory from the straight-line path. In fact, if we put $\nu=0$ in formula (3.1.3) for transition current, we obtain

$$j_\alpha(\kappa; p, q|0) = -\left(\frac{2p_\alpha}{2p\kappa+i0} - \frac{2q_\alpha}{2q\kappa-i0}\right). \qquad (3.1.7)$$

This corresponds to the classical current of the nucleon, moving with momentum p at $\zeta>0$ and with momentum q at $\zeta<0$.

We note, however, that the approximation $\nu=0$ is known to be inapplicable at values of the proper time s of the particle, close to zero, when the particle classical trajectory changes its direction. In the language of Feynman diagrams the approximation assumes that the quadratic κ-dependence in the nucleon propagator can be neglected, i.e.

$$\frac{1}{m^2-(p+\kappa)^2} \to -\frac{1}{2p\kappa}$$

This can cause the integrals over $d^4\kappa$ to diverge at the upper limit.

A better approximation to the nucleon current taking into account the recoil effects, is given by the average current value (3.1.3) over the functional variable ν, i.e.

$$\begin{aligned}\bar{j}_\alpha(\kappa; p, q|\nu) &= \int[\delta\nu]_{-\infty}^{\infty} j_\alpha(\kappa; p, q|\nu) = \\ &= i\int_{-\infty}^{\infty} d\xi\,[\kappa\,\varepsilon(\xi) + 2p\,\vartheta(\xi) + 2q\,\vartheta(-\xi)]_\alpha \cdot \\ &\quad \exp\{2i\kappa[p\xi\,\vartheta(\xi) + q\xi\,\vartheta(-\xi) + i\kappa^2|\xi|\} = \qquad (3.1.8) \\ &= -\left(\frac{2p_\alpha+\kappa_\alpha}{2p\kappa+\kappa^2+i0} - \frac{2q_\alpha-\kappa_\alpha}{2q\kappa-\kappa^2-i0}\right).\end{aligned}$$

Straight-line path approximation, which is used to find the elastic amplitude, consists in substituting in the exponent in eq. (3.1.2) the current products averaged over the functional variables ν_1 and ν_2

$$\overline{j_\alpha^{(1)}(k;p_1,q_1)\, j_\beta^{(2)}(-k;p_2,q_2)} =$$
$$= \left(\frac{2p_{1\alpha}+k_\alpha}{2p_1k+k^2+i0} - \frac{2q_{1\alpha}-k_\alpha}{2q_1k-k^2-i0}\right)\left(\frac{2p_{2\beta}-k_\beta}{2p_2k-k^2+i0} - \frac{2q_{2\beta}+k_\beta}{2q_2k+k^2-i0}\right) \tag{3.1.9}$$

and

$$\overline{j_\alpha^{(i)}(k;p_i,q_i)\, j_\beta^{(i)}(k;p_i,q_i)} =$$
$$= \left(\frac{2p_{i\alpha}+k_\alpha}{2p_ik+k^2+i0} - \frac{2q_{i\alpha}+k_\alpha}{2q_ik+k^2-i0}\right)\left(\frac{2p_{i\beta}+k_\beta}{2p_ik+k^2+i0} - \frac{2q_{i\beta}+k_\beta}{2q_ik+k^2-i0}\right);\quad i=1,2 \tag{3.1.10}$$

Consequently, in straight-line path approximation the expression for the elastic amplitude with the term $(q_1 \leftrightarrow q_2)$ being neglected takes the form

$$f_{el}(p_1,p_2;q_1,q_2) = \frac{ig^2}{(2\pi)^4}\, \tau^{(1)}(t)\, \tau^{(2)}(t)\, (p_1+q_1)_\alpha\, (p_2+q_2)_\beta \tag{3.1.11}$$
$$\int d^4x\, e^{ix(p_1-q_1)}\, D^{\alpha\beta}(x) \int_0^1 d\gamma\, e^{-i\gamma\chi^{(0)}(x;p_i,q_i)},$$

where

$$\chi^{(0)}(x;p_i,q_i) = \frac{g^2}{(2\pi)^4}\int d^4x\, e^{ikx}\, D^{\alpha\beta}(k)\,\cdot \tag{3.1.12}$$
$$\left(\frac{2p_1+k}{2p_1k+k^2+i0} - \frac{2q_1-k}{2q_1k-k^2-i0}\right)_\alpha \left(\frac{2p_2-k}{2p_2k-k^2+i0} - \frac{2q_2+k}{2q_2k+k^2-i0}\right)_\beta$$

and

$$\tau^{(1)}(t) = \exp\left[\frac{g^2}{2i}\int\frac{d^4k}{(2\pi)^4}\, D(k)\left(\frac{2p_1+k}{2p_1k+k^2} - \frac{2q_1+k}{2q_1k+k^2}\right)_\alpha^2\right], \tag{3.1.13}$$
$$\tau^{(2)}(t) = \exp\left[\frac{g^2}{2i}\int\frac{d^4k}{(2\pi)^4}\, D(k)\left(\frac{2p_2+k}{2p_2k+k^2} - \frac{2q_2+k}{2q_2k+k^2}\right)_\alpha^2\right] \tag{3.1.14}$$

It is interesting to note that the contribution of the radiative corrections to the ladder type diagrams in the framework of straight-line path approximation is extracted as a factor $r^{(1)}r^{(2)}$ depending only on the square of the momentum transfer $t = (p_1-q_1)^2$. A phenomenon analogous to the factorization of the radiative correction contribution in quantum electrodynamics, was found by Jennie et al[19].

In the high-energy limit $s\to\infty$ and at fixed momentum transfers t, limited by the condition $\left|\frac{t}{m^2}\right| \ll g^2$ the quantities $\chi^{(0)}$ and $r^{(i)}$ take the form

$$\chi^{(0)} = \frac{g^2}{8\pi}\int \frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2+\mu^2}\, e^{-i\vec{k}_\perp\vec{x}_\perp} = \frac{g^2}{4\pi}K_0(\mu|\vec{x}_\perp|), \tag{3.1.15}$$

$$r^{(1)}r^{(2)} = e^{at}, \tag{3.1.16}$$

where K_0 is the Macdonald function of zero order, and

$$a = \frac{g^2}{3(2\pi)^2 m^2}\left[\ln\frac{m^2}{\mu^2} + \frac{1}{2} + O\left(\frac{\mu^2}{m^2}\right)\right] \tag{3.1.17}$$

Thus, in this asymptotic limit the expression for the elastic amplitude of the two scalar nucleon, interacting with the vector field has the form[x]

$$f_{el}(s,t) = f^{(0)}(s,t)\, e^{at}, \tag{3.1.18}$$

where

$$f^{(0)}(s,t) = \frac{i(s-u)}{2}\int d^2\vec{x}_\perp\, e^{i\vec{x}_\perp\vec{\Delta}_\perp}\left(e^{-\frac{ig^2}{4\pi}K_0(\mu|\vec{x}_\perp|)} - 1\right) \tag{3.1.19}$$

---

[x] Taking into account the identity of nucleons, we are led in symmetrizing eq. (3.1.18) to terms vanishing in the limit $s\to\infty$ with fixed t.

$$t = -\vec{\Delta}_{\perp}^{2} \tag{3.1.20}$$

It is clear from formula (3.1.18), that the consideration of radiative effects leads to the diffraction behaviour of the high-energy, small-angle scattering amplitude which corresponds to the Gaussian form of the local quasipotential of the elastic scattering with an interaction radius of the order $g\frac{\hbar}{mc}$. The forces which are due to the exchange of mesons between the nucleons obviously have a radius $\frac{\hbar}{\mu c}$ and it is assumed that $g\frac{\hbar}{mc} << \frac{\hbar}{\mu c}$. Thus, in the region of momentum transfers $\mu^2 \lesssim |t| < g^2 m^2$ it is very important to take into account the multiple meson exchange which leads to the eikonal structure of the quantity $f^{(0)}(s,t)$.

As was shown by Barbashow et al[20], the consideration of the interaction of nucleons with vacuum fluctuation of the meson field allows one to get the qualitative explanation of the origin of smoothness for the smoothness of the local quasipotential.

Writing the amplitude (3.1.18) in the eikonal form, we find the equation for the corresponding eikonal phase

$$e^{2i\chi(\vec{x}_\perp)} = \int \frac{d^2\vec{\rho}}{4\pi a}\, e^{-\frac{\rho^2}{4a}}\, e^{2i\chi^{(0)}(\vec{x}_\perp - \vec{\rho})} \tag{3.1.21}$$

It is easy to show from eq. (3.1.21), that $\chi(\vec{\kappa}_\perp)$ is a complex quantity with the positively defined imaginary part, i.e. $|e^{2if}|<1$ in correspondence with the unitarity.

Expanding the exponential in eq. (5.2.6) in powers of $\chi^{(0)}$, the phase χ is given by the series

$$\chi(\vec{\rho}) = \frac{g^2}{8\pi}\int \frac{d^2\vec{k}_\perp\, e^{-i\vec{k}_\perp\vec{x}_\perp}}{\vec{k}_\perp^{2} + \mu^2}\, e^{-a\vec{k}_\perp^{2}} + i\left(\frac{g^2}{8\pi}\right)\int \frac{d^2\vec{k}_\perp\, d^2\vec{k}'_\perp\, e^{-i\vec{x}_\perp(\vec{k}_\perp+\vec{k}'_\perp)}}{(\vec{k}_\perp^{2}+\mu^2)(\vec{k}_\perp'^{2}+\mu^2)}\left[e^{-a(\vec{k}_\perp+\vec{k}'_\perp)^2} - e^{-a\vec{k}_\perp^{2}-a\vec{k}_\perp'^{2}}\right] + \ldots \tag{3.1.22}$$

The first term in eq. (3.1.22) is pure real and corresponds to the scattering on the Yukawa potential with the centre of force being stochastically distributed by the Gauss low. The second term in eq. (3.1.22) contributes to the imaginary part of the quasipotential.

Thus the account of the radiative effects in two-particle scattering leads naturally to the smooth complex local quasipotential, the imaginary part of which is a positive-defined quantity in correspondence with the unitarity requirement.

3.2. Inelastic Processes

The amplitudes of inelastic processes describing the production of a certain number of the vector field quanta at high-energy two-nucleon collision can be determined by means of a generating function $f(p_1,p_2;q_1,q_2|A^{ext})$.

In the framework of straight-line path approximation the quantity $f(p_1,p_2;q_1q_2|A^{ext})$ takes the form

$$
\begin{aligned}
f(p_1,p_2;q_1,q_2|A^{ext}) &= g^2\int d^4x_1\,d^4x_2\, e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)}(p_1+q_1)_\alpha(p_2+q_2)_\beta \\
&\mathcal{D}^{\alpha\beta}(x_1-x_2)\exp\Big\{ig\int d^4k\, A_\gamma^{ext}\Big[\overline{j_\gamma^{(1)}(k,p_1,q_1)}e^{ikx}+\overline{j_\gamma^{(2)}(k;p_2,q_2)}e^{-ikx}\Big]\cdot \\
&\int_0^1 d\lambda\,\exp\Big\{\frac{ig^2\lambda}{2}\int d^4k\,\mathcal{D}^{\alpha\beta}(k)\Big[\sum_{i=1,2}\overline{j_\alpha^{(i)}(k;p_i,q_i)\,j_\beta^{(i)}(-k;p_i,q_i)}+ \\
&+\lambda e^{ik(x_1-x_2)}\overline{j_\alpha^{(1)}(k;p_1,q_1)\,j^{(2)}(-k;p_2,q_2)}+(\vec{k}\to-\vec{k})\Big]\Big\}
\end{aligned}
\tag{3.2.1}
$$

where the functional average values of the currents and their bilinear combinations are defined by eqs. (3.1.8), (3.1.9) and (3.1.10), respectively.

The amplitude for production of N vector quanta can be found by functional derivatives with respect to the field A^{ext}

$$(2\pi)^4 \delta(p_1+p_2-q_1-q_2-\sum_{i=1}^{N} k_i) f(p_1,p_2;q_1,q_2;k_1,k_2,\dots,k_N) =$$
$$= \prod_{i=1}^{N} e_\alpha^*(k_i) \frac{\delta}{\delta A_\alpha^{ext}(k_i)} f(p_1,p_2;q_1,q_2 | A^{ext}) \Big|_{A^{ext}=0} =$$
$$= g^2 \int d^4x_1\, d^4x_2\, e^{ix_1(p_1-q_1)+ix_2(p_2-q_2)} \prod_{i=1}^{N} e_\alpha^*(k_i)\cdot$$
$$\overline{\left[j_\alpha^{(1)}(k_i;p_1,q_1) e^{ik_ix_1} + j_\alpha^{(2)}(k_i,p_2,q_2) e^{ik_ix_2}\right]} (p_1+q_1)_\sigma (p_2+q_2)_\rho \cdot$$
$$\mathcal{D}^{\sigma\rho}(x_1-x_2) \int_0^1 d\lambda \exp\left\{ \frac{ig^2\lambda}{2} \int d^4k\, \mathcal{D}^{\mu\delta}(k) \left[\sum_{i=1,2} \overline{j_\mu^{(i)}(-k;p_i,q_i) j_\delta^{(i)}(k;p_i,q_i)} \right.\right.$$
$$\left.\left. + \lambda \overline{j_\mu^{(1)}(k;p_1,q_1) j_\delta^{(2)}(-k;p_2,q_2)} e^{ik(x_1-x_2)} + (k \to -k) \right]\right\}, \tag{3.2.2}$$

where $e_\alpha(\kappa)$ is the polarization vector of a meson with momentum κ.

Further, we consider the case in which the momenta of the secondary mesons satisfy the requirement of "softness"

$$\frac{1}{\sqrt{s}} \sum_{i=1}^{N} k_{0i} \ll 1\,, \left|\sum_{i=1}^{N} \vec{k}_{i\perp}\right| \ll |\vec{p}_{1\perp} - \vec{q}_{1\perp}| \approx |\vec{p}_{2\perp} - \vec{q}_{2\perp}|\,, \tag{3.2.3}$$

where the components of the particle momenta are given in the c.m.s. $\vec{p}_1+\vec{p}_2=0$, and the momenta of the initial nucleons are chosen along the z-axis.

With these requirements the amplitude of N-meson production is factorized and can be written in the following form

$$f_{inel}(N) = f(p_1,p_2;q_1,q_2;k_1,k_2,\dots,k_N) = \tag{3.2.4}$$
$$= f_{el} \prod_{i=1}^{n_1} g\, e_{i\alpha}^*(k_i) \overline{j_\alpha^{(1)}(k_i;p_1,q_1)} \prod_{l=1}^{n_2} g\, e_{l\beta}^*(k_l') \overline{j_\beta^{(2)}(k_l';p_2,q_2)},$$

where

$$j_\alpha^{(i)}(k;p_i,q_i) = \left(\frac{2p_i+k}{2p_ik+k^2} - \frac{2q_i-k}{2q_ik-k^2}\right)_\alpha, \quad i=1,2$$

$$t=\Delta^2=\left(q_1-p_1+\sum_{i=1}^{n_1}k_i\right)^2=\left(q_2-p_2+\sum_{\ell=1}^{n_2}k'_\ell\right)^2, \quad n_1+n_2=N \tag{3.2.5}$$

The differential cross section of N mesons production in two-nucleon collision is defined by

$$d\sigma_N = \frac{1}{2\sqrt{s(s-4m^2)}}\,|f_{inel}(N)|^2(2\pi)^4\,\delta\left(p_1+p_2-q_1-q_2-\sum_{i=1}^{N}k_i\right)\cdot$$
$$\frac{1}{(2\pi)^6}\,\frac{d\vec{q}_1\,d\vec{q}_2}{2q_{10}\cdot 2q_{20}}\,\frac{1}{N!}\prod_{i=1}^{N}\frac{d\vec{k}_i}{2k_{0i}}\,\frac{1}{(2\pi)^3}, \tag{3.2.6}$$

where $s=(p_1+p_2)^2$.

Using (3.2.6) and making the transformation

$$\delta\left(p_1+p_2-q_1-q_2-\sum_{i=1}^{n}k_i-\sum_{\ell=1}^{n_2}k'_\ell\right)=$$
$$=\int d^4\Delta\,\delta\left(p_1-q_1-\sum_{i=1}^{n_1}k_i+\Delta\right)\delta\left(p_2-q_2-\sum_{\ell=1}^{n_2}k'_\ell-\Delta\right) \tag{3.2.7}$$

we obtain the expression for the plural meson production differential cross section [18]

$$(d\sigma)_{n_1,n_2}\xrightarrow[\Delta-\text{fixed}]{s\to\infty}\frac{1}{2s}\,\frac{d^4\Delta}{(2\pi)^4}\,|f_{el}(s,t)|^2\,W_{n_1}(p_1,\Delta)\,W_{n_2}(p_2,-\Delta), \tag{3.2.8}$$

where

$$W_{n_1}(p_1,\Delta)=\frac{2\pi}{n_1!}\int\frac{d^4q_1}{2q_{10}}\,\delta\left(p_1-q_1-\sum_{i=1}^{n_1}k_i+\Delta\right)\cdot$$
$$\prod_{i=1}^{n_1}\frac{d\vec{k}_i}{2k_{0i}}\,\frac{(-g^2)}{(2\pi)^3}\,\left|j_\alpha^{(1)}(k;p_1,q_1)\right|^2 \tag{3.2.9}$$

and we have an analogous equation for $W_{n_2}(p_2,-\Delta)$.

The quantities $W_{n_1}(p_1,\Delta)$ and $W_{n_2}(p_2,-\Delta)$ depend on the variables

$$t=\Delta^2, \quad \nu_1=p_1\Delta \quad \text{and} \quad t=\Delta^2,\ \nu_2=-p_2\Delta \tag{3.2.10}$$

respectively.

Using the variables (3.2.10), we rewrite the volume element $d^4\Delta$ in the following form

$$d^4\Delta = \frac{4\pi}{\sqrt{s(s-4m^2)}}\, dt\, d\nu_1\, d\nu_2\, \frac{d\phi}{2\pi} \tag{3.2.11}$$

where ϕ is an azimuthal angle, and the physical domain of integration is determined by the inequalities.

$$\begin{aligned} -t \le 2\nu_i &\le s \quad , \quad i = 1,2 \\ -s \le t &\le 0 \quad , \quad m^2 \ll s . \end{aligned} \tag{3.2.12}$$

Further, we find the asymptotic behaviour of the differential cross section $(\frac{d\sigma}{dt})_{n_1 n_2}$ at $s\to\infty$ and fixed t. Integrating eq. (3.2.9) over $d\nu_1$ and $d\nu_2$ and using eq. (3.1.18) we get at $\left|\frac{t}{m^2}\right| \ll g^2$

$$\left(\frac{d\sigma}{dt}\right)_{n_1,n_2} \xrightarrow[t-\text{fixed}]{s\to\infty} \frac{1}{4\pi}\, v^2(t)\, w_{n_1}(s,t)\, w_{n_2}(s,t) \tag{3.2.13}$$

where

$$w_n(s,t) = \frac{e^{at}}{\pi} \int d\nu\, W_n(t,\nu)$$

$$= e^{at} \frac{1}{n!} \int_{\Omega_p} \prod_{i=1}^{n} \frac{d\vec{k}_i}{2k_{0i}} \frac{(-g^2)}{(2\pi)^3} \left| \overline{j_\alpha^{(\ell)}(k_i; p_\ell, q_\ell)} \right|^2 , \quad \ell = 1,2 \tag{3.2.14}$$

The integration domain Ω_p over the secondary meson momenta is determined by the condition

$$-t \le 2p\sum_{i=1}^{n} k_i - (\Delta - \sum_{i=1}^{n} k_i)^2 \le s \tag{3.2.15}$$

or, taking into account, that in the case considered here $(\Delta - \sum_{i=1}^{n} k_i)^2 \approx \Delta^2$ by the condition

$$0 \le 2p\sum_{i=1}^{n} k_i \le s + t \tag{3.2.16}$$

Consider now an approximation in which the total momentum of the secondary mesons can be neglected in accordance with the requirement of "softness" (3.2.3). In this approximation, eq. (3.2.14) takes the form of the Poisson-type distribution [21]

$$w_n(s,t) = \frac{1}{n!}\, e^{at} \left[\bar{n}(s,t) \right]^n \tag{3.2.17}$$

where the quantity[x]

$$\bar{n}(s,t) = -\frac{g^2}{(2\pi)^3} \int \frac{d\vec{k}}{2k_0} \left| \overline{j_\alpha^{(\ell)}(k; p,q)} \right|^2 , \quad \ell = 1,2 \tag{3.2.18}$$

plays the role of the average number of secondary particles produced in the two-nucleon collision at $s \to \infty$ with fixed t.

Using eq. (3.2.5) for $\bar{j}_\alpha$, we find for $|t| \ll g^2 m^2$, that

$$\bar{n}(s,t) = -bt \tag{3.2.19}$$

The quantity b generally speaking depends on a special form of the cut-off of the integrals over the emission meson momentum. In the particular case when

$$R_\perp^2 \sim m^2$$
$$1 \gg \alpha^2 \gg \frac{\mu^2}{m^2} , \tag{3.2.20}$$

where $\alpha = \frac{R_z}{p_0}$, we get

$$b = \frac{2g^2}{3(2\pi)^2 m^2} \left[\ln \frac{m^2}{\mu^2} + \frac{1}{2} + O\left(\frac{\mu}{m}\right) \right] , \tag{3.2.21}$$

---

[x] The integration in eq. (3.2.18) is effectively limited by $|K_z| \le R_z$, $|\vec{K}| \le R$.

which coincides with the doubled slope parameter of the diffractional exponential (3.1.17). Note also that the equality $2a=b$ holds in the infrared asymptotic limit $\mu\to 0$. In this case after summing eq. (3.2.13) over the number of secondary mesons we find that the dependence of the variable t cancels, and the diffraction peak in the total differential cross section disappears. This regularity was mentioned in paper /22/ and has some analogy with the automodel behaviour of deep-inelastic hadron interactions at high energies /23,24/.

§4. SOME MATHEMATICAL REALIZATION OF THE STRAIGHT-LINE PATH CONCEPTION IN THE FRAMEWORK OF FUNCTIONAL INTEGRATION METHOD

In the previous paragraph some applications of SLPM in its simple form have been considered. Roughly speaking, we used the approximation

$$\int[\delta\nu]\, e^{F} \approx e^{\int[\delta\nu] F} \quad \text{or even} \quad \approx e^{F_{\nu=0}}$$

In other words, it was assumed that in particle scattering at asymptotically high energies and fixed momentum transfers the main contribution to the Feynman path integral is given by those paths which are the least deflected from the classical particle trajectories.

We consider below a number of approximation systems which are different mathematical realizations of the physical straight-line path concept within the scope of functional integration.

Note that the methods of the theory of measure and integration in functional spaces have lately been extensively used in papers on quantum field theory. This approach is based on the representation of the exact theory equations in the form of the functional integrals. However, because of the absence of a well-developed technique of calculation of general quadratures the functional integral is a "thing in itself" in the sense, that the extraction of necessary information is usually performed in stages, using some approximation procedure. The simplest and best known are the approximation procedures for which we are dealing only with the Gaussian quadratures at each stage of calculation. The "$\kappa_i \kappa_k = 0$ -approximation" discussed above and straight-line path approximation are among just this type of approximation.

The procedures developed below originate from the concept of a straight-line path and in particular, allow us to estimate consistently the effects of particle path deflection from the straight-line trajectories in scattering processes at high energies.

4.1. Formulation of Approximation

We consider a functional integral over the Gaussian measure

$$\int \frac{\delta \nu}{const.} \, e^{-i \int d\xi \, \nu^2(\xi)} \, e^{g \, \pi[\nu]} \qquad (4.1.1)$$

where $\pi[\nu]$ is a certain functional, and const is a normalization constant. As is well known, the calculation of (4.1.1) can be reduced to finding functional derivatives according to the formula

$$\int [\delta\nu] e^{g\pi[\nu]} = \exp\left\{\frac{1}{4i}\int d\xi \frac{\delta^2}{\delta\nu^2(\xi)}\right\} e^{g\pi[\nu]}\Big|_{\nu=0} \quad (4.1.2)$$

In addition, in some quantum field theory problems (see, for example, Kuleshov et al[25]) it is required to determine the differential operator $\exp\left\{\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\nu^2}\right\}$ where

$$\int \mathcal{D}\frac{\delta^2}{\delta\nu^2} = \int d\xi_1 d\xi_2 \mathcal{D}(\xi_1,\xi_2)\frac{\delta^2}{\delta\nu(\xi_1)\delta\nu(\xi_2)}$$

and $D(\xi_1,\xi_2)$ is a function of a propagator type. With a view to further applications, we unite both problems as follows.

We have to find the functional $\Pi[\nu]$ from the relation

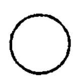
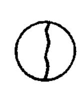
$$e^{\Pi[\nu]} = \exp\left\{\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\nu^2}\right\} e^{g\pi[\nu]} \equiv \overline{e^{g\pi[\nu]}} \quad (4.1.3)$$

where $\pi|\nu|$ is a given functional and D is a function of two variables. When

$$\mathcal{D} = -\frac{1}{2}\delta(\eta_1-\eta_2) \quad (4.1.4)$$

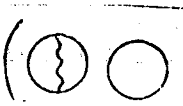
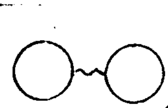
the value of the functional $\Pi|\nu|$ at $\nu=0$ determines the functional integral according to (4.1.2). To simplify the formulae the action of the differential operator will sometimes be denoted by the sign of averaging as in (4.1.3).

For graphic demonstration we introduce the notation

$\pi[\nu] \Rightarrow$  , $\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\nu^2}\pi \Rightarrow$  , (4.1.5)

$\exp\left\{\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\nu^2}\right\}\pi[\nu] = \overline{\pi} \Rightarrow$  .

In this notation, for example,

$\frac{i}{2}\int \mathcal{D}\frac{\delta^2}{\delta\nu^2}\pi^2[\nu] \Rightarrow 2\Big($  $+$  $\Big)$, where, following the ordinary terminology, we call the two first terms the unconnected graphs. Let us stress that in spite of the obvious analogy of the (4.1.5) graphs with the Feynman diagrams, in many cases their appearance has nothing to do with the usual Feynman graphs.

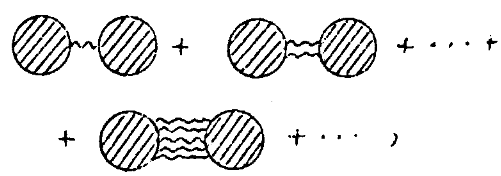
Assume now, that the structure of the functional $\pi|\nu|$ is such that there exists a small parameter connected with a loop. In this case there is an approximation procedure which we call the correlative one, and according to which we seek $\Pi|\nu|$ in the form of the series

$$\Pi = \sum_{n=1}^{\infty} g^n \Pi_n \tag{4.1.6}$$

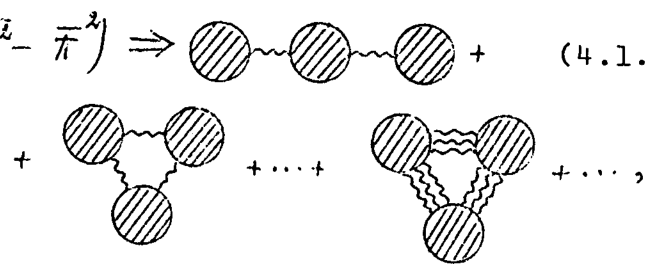
Substituting (4.1.6) in (4.1.3), we immediately obtain

$$\Pi_1 = \overline{\pi} \Rightarrow$$

$$\Pi_2 = \frac{1}{2!}\left(\overline{\pi^2} - \overline{\pi}^2\right) \Rightarrow$$



$$\Pi_3 = \frac{1}{3!}\left[\overline{\pi^3} - \overline{\pi}^3 - 3\pi\left(\overline{\pi^2} - \overline{\pi}^2\right) \Rightarrow \right. \tag{4.1.7}$$



. . . .

$$\Pi_n = \frac{1}{n!} \overline{\pi^n}\Big|_{\text{connected part}}$$

Considering graphs (4.1.7), we make sure that the correlative method really corresponds to the expansion in the number of loops, and only the connected part of the sum of all graphs with loops contributes to .

Truncating the series (4.1.6), we obtain the approximate expression for the functional Π. This approximation is valid when inequality

$$\overline{\Pi^n}\Big|_{\substack{\text{connected}\\ \text{part}}} \ll \overline{\Pi^n}\Big|_{\substack{\text{unconnected}\\ \text{part}}} \qquad (4.1.8)$$

is satisfied for any $n \geq 2$. In this case considering only Π_1 while expanding e^{Π} in a power series of g one obtains the leading terms in each order, the consideration of Π_2 gives us the corrections to them, etc.

The correlative procedure is closely connected with an expansion of the following type

$$\overline{e^{g\Pi}} = e^{g\overline{\Pi}}\left[1 + \sum_{n=2}^{\infty} \frac{g^n}{n!}\,\overline{(\Pi - \overline{\Pi})^n}\,\right] \qquad (4.1.9)$$

Such an expansion has been met previously[9,13]. It has in general the same domain of application as the correlative approximation and differs from it by giving the smaller number of correction terms in each order of g. Let us still note that the higher correction terms have, from our point of view, a more simple geometrical meaning (see (4.1.7)) in the correlative expansion which simplifies its usage to a certain extent.

As was mentioned above, the approximations considered are satisfactory when there exists a small parameter connected with a loop. But it may happen, that the theory contains a small parameter, connected with a line that is the one arising when a functional $\pi[\nu]$ is varied. Then it is possible to make an expansion in the number of lines connecting the different loops. Representing π in the form

$$\pi[\nu] = \int d\eta\, \tilde{\pi}[\eta]\, e^{-i\int \eta(\xi)\nu(\xi)\,d\xi} \tag{4.1.10}$$

and substituting (4.1.10) in (4.1.3), we obtain

$$e^{\Pi_\varepsilon[\nu]} = 1 + \sum_{n=1}^{\infty} \frac{g^n}{n!} \int \prod_{j=1}^{n} \{\delta\eta_j\, \tilde{\pi}[\eta_j]\} \cdot \exp\Big[-i\int \nu\Big(\sum_{j=1}^{n}\eta_j\Big) - \frac{i}{2}\int \mathcal{D}\Big(\sum_{j=1}^{n}\eta_j^2\Big) - i\varepsilon\int \mathcal{D}\Big(\sum_{i<j}\eta_i\eta_j\Big)\Big], \tag{4.1.11}$$

where a small parameter ε is ascribed to the terms with different η and $\Pi|\nu| = \Pi_\varepsilon|\nu|$ at $\varepsilon=1$.

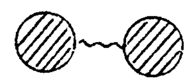
Now we seek the functional $\Pi_\varepsilon|\nu|$ in the form

$$\Pi_\varepsilon = \sum_{n=0}^{\infty} \varepsilon^n\, \Pi_{n+1} \;. \tag{4.1.12}$$

If we confine ourselves to the first few terms of the series (4.1.12), we obtain the "$\eta_i\eta_j$ -approximation".

The calculations lead to the following expressions for the first few terms

$$\Pi_1 = g\bar{\pi} \quad \Rightarrow$$  ,

$$\Pi_2 = \frac{ig^2}{2}\int \mathcal{D}\Big(\frac{\delta\bar{\pi}}{\delta\nu}\Big)^2 \quad \Rightarrow$$  , (4.1.13)

$$\Pi_3 = \frac{g^2}{2i}\int \mathcal{D}_{13}\,\mathcal{D}_{24}\, \frac{\delta^2\bar{\pi}}{\delta\nu_1\,\delta\nu_2}\Big(\frac{1}{2i}\,\frac{\delta^2}{\delta\nu_3\,\delta\nu_4} + g\,\frac{\delta\bar{\pi}}{\delta\nu_3}\,\frac{\delta\bar{\pi}}{\delta\nu_4}\Big) \quad \Rightarrow$$  ,

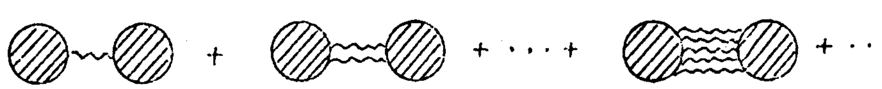
Where the figures indicate the order of the contraction, i.e.

$$\int \mathcal{D}_{13}\,\mathcal{D}_{24}\,\frac{\delta^2\bar{\pi}}{\delta\nu_1\,\delta\nu_2}\,\frac{\delta^2\bar{\pi}}{\delta\nu_3\,\delta\nu_4} \equiv \int d\xi_1\, d\xi_2\, d\xi_3\, d\xi_4 \cdot$$
$$\mathcal{D}(\xi_1,\xi_3)\,\mathcal{D}(\xi_2,\xi_4)\,\frac{\delta^2\bar{\pi}}{\delta\nu(\xi_1)\delta\nu(\xi_2)}\,\frac{\delta^2\bar{\pi}}{\delta\nu(\xi_3)\delta\nu(\xi_4)}$$

and so on. This is the expansion in the number of lines, connecting different loops. As we deal with the connected graphs the number of such lines and the number of loops satisfy the inequality

$$\kappa \geq n - 1 \tag{4.1.14}$$

This results in the inclusion of the sum of the first n terms of the $\eta_i\eta_j$ -approximation in the analogical sum of the correlative approximation so that the application domain of the former is not wider than that of the latter. However, its application might simplify the calculations because one can dispense with the sum



Note also that the first terms of all the approximations considered above coincide and the difference only comes out in calculating the corrections. This reflects the fact that the methods under consideration, when applied to the calculation of the high-energy scattering amplitude, represent different versions of the straight-line path approximation.

4.2. Corrections to the Eikonal Formula

Let us consider an application of the methods developed above to the concrete example of the two-scalar "nucleon" scattering amplitude in the model $\mathcal{L}_{int} = g:\psi^+\psi\phi:$. Neglecting the radiative corrections and the contribution of the vacuum polarization, this amplitude can be represented as follows[9]

$$f_{ee}(p_1, p_2; q_1, q_2) = \frac{ig^2}{(2\pi)^4} \int d^4x\, \mathcal{D}(x)\, e^{-ix(p_1-q_1)} \cdot \int_0^1 d\lambda\, S_\lambda(x; p_1, p_2; q_1, q_2) + (q_1 \leftrightarrow q_2)\,, \tag{4.2.1}$$

where

$$S_\lambda = \int [\delta\nu_1]_{-\infty}^{\infty} [\delta\nu_2]_{-\infty}^{\infty} \exp\Big\{ ig^2\lambda \int_0^{\infty} d\xi \int_{-\infty}^{\infty} d\tau\, \mathcal{D}\Big[-x + 2\xi a_1(\xi) - 2\tau a_2(\tau) - 2\int_0^{\xi} \nu_1(\eta)\, d\eta + 2\int_{-\tau}^{0} \nu_2(\eta)\, d\eta\Big]\Big\} \overset{def}{=} \overset{def}{=} \int [\delta\nu_1]_{-\infty}^{\infty} [\delta\nu_2]_{-\infty}^{\infty} e^{ig^2\lambda\pi - \xi}\,, \tag{4.2.2}$$

$$a_{1,2}(\xi) = p_{1,2}\,\vartheta(\xi) + q_{1,2}\,\vartheta(-\xi)\,. \tag{4.2.3}$$

We shall seek the asymptotics of the functional integral S_λ at high energies $s = (p_1+p_2)^2$ and fixed momentum transfers $t = (p_1-q_1)^2$. The calculations performed in this case show that there are $\frac{1}{s}$ parameters connected with a loop and $\frac{1}{\sqrt{s}}$ connected with a line[x]. Consequently, as a result of relation (4.1.14) in the n-th order of g^2 at fixed $x \neq 0$ the leading term of S_λ has the asymptotics $\frac{1}{s^n}$, and the following correction - $\frac{1}{s^n\sqrt{s}}$

---

[x] Note that we are considering the loops and lines defined by rules (4.1.5).

If we want to calculate only the first two asymptotic terms in each order of g^2, it is convenient to apply the "$\eta_i\eta_j$ -approximation", generalized for two functional variables ν_1 and ν_2, and to use the approximation $e^{\Pi_2} = 1 + \Pi_2 + \ldots$ of the (4.1.2) expansion type. Then the approximate formula for S_λ has the form

$$S_\lambda \simeq e^{ig^2\lambda\bar{\pi}}\Big[1 + \frac{ig^4\lambda^2}{4}\int d\xi \sum_{i=1,2}\Big(\frac{\delta\bar{\pi}}{\delta\nu_i(\eta)}\Big)^2\Big]\Big|_{\nu=0} \qquad (4.2.4)$$

Proceeding from (4.2.2), we obtain the following expression for $\bar{\pi}$

$$\bar{\pi}\big|_{\nu=0} = \frac{1}{(2\pi)^4}\int d^4k\, \mathcal{D}(k) e^{-ikx}\int_{-\infty}^{\infty} d\xi d\tau \exp\{2ik[\xi a_1(\xi) - \tau a_2(\tau)] + ik^2(|\xi| + |\tau|) = \frac{1}{(2\pi)^4 s}\int d^4k\, \mathcal{D}(k) e^{-ikx} \cdot \qquad (4.2.5)$$

$$\cdot \int_{-\infty}^{\infty} d\xi d\tau \exp\Big\{2ik\Big[\xi\frac{a_1(\xi)}{\sqrt{s}} - \tau\frac{a_2(\tau)}{\sqrt{s}}\Big] + i\frac{k^2}{\sqrt{s}}(|\xi| + |\tau|)\Big\}$$

In formula (4.2.5) we have replaced the variables ξ,τ by $\frac{\xi}{\sqrt{s}},\frac{\tau}{\sqrt{s}}$. Similarly we obtain

$$\frac{i\lambda^2 g^4}{4}\int d\eta\Big[\Big(\frac{\delta\bar{\pi}}{\delta\nu_1(\eta)}\Big)^2 + \Big(\frac{\delta\bar{\pi}}{\delta\nu_2(\eta)}\Big)^2\Big]\Big|_{\nu=0} =$$

$$= \frac{i\lambda^2 g^4}{(2\pi)^8 s^2}\int d^4k_1 d^4k_2\, \mathcal{D}(k_1)\mathcal{D}(k_2)\, e^{-ix(k_1+k_2)}\,(k_1k_2)\cdot$$

$$\cdot\int_{-\infty}^{\infty} d\xi_1 d\tau_1 d\xi_2 d\tau_2 \exp\Big\{2ik_1\Big[\xi_1\frac{a_1(\xi_1)}{\sqrt{s}} - \tau_1\frac{a_2(\tau_1)}{\sqrt{s}}\Big] + \qquad (4.2.6)$$

$$+ i\frac{k_1^2}{\sqrt{s}}(|\xi_1| + |\tau_1|)\Big\}\exp\Big\{2ik_2\Big[\xi_2\frac{a_1(\xi_2)}{\sqrt{s}} - \tau_2\frac{a_2(\tau_2)}{\sqrt{s}}\Big] +$$

$$+ i\frac{k_2^2}{\sqrt{s}}(|\xi_2| + |\tau_2|)\Big\}\frac{1}{\sqrt{s}}\big[\Phi(\xi_1,\xi_2) + \Phi(\tau_1,\tau_2)\big],$$

where

$$\Phi(\xi_1,\xi_2) = \vartheta(\xi_1,\xi_2)\left[|\xi_1|\,\vartheta(|\xi_2|-|\xi_1|)+|\xi_2|\,\vartheta(|\xi_1|-|\xi_2|)\right]. \qquad (4.2.7)$$

Let us find the asymptotics of the expressions (4.2.5) and (4.2.6) for s and t fixed. The expression (4.2.5) must be calculated to within $0(\frac{1}{s^2})$, and (4.2.6) - to within $0(\frac{1}{s^3})$.

For this purpose let us choose in the c.m.s. the z-axis along the initial particle momenta. Then

$$p_{1,2} = \left\{ \frac{\sqrt{s}}{2},\ 0,\ 0,\ \pm\frac{\sqrt{s-4m^2}}{2} \right\},$$

$$q_{1,2} = \left\{ \frac{\sqrt{s}}{2},\ \pm\vec{\Delta}_\perp\sqrt{1+\frac{t}{s-4m^2}};\ \pm\frac{\sqrt{s-4m^2}}{2}\left(1+\frac{2t}{s-4m^2}\right)\right\}, \qquad (4.2.8)$$

$$\vec{\Delta}_\perp^2 = -t$$

and substituting (4.2.8) in (4.2.3), we obtain asymptotically

$$\frac{a_1(\xi)}{\sqrt{s}} \simeq \frac{1}{2}n^+ + \frac{\vec{\Delta}_\perp}{\sqrt{s}}\vartheta(-\xi) + O\left(\frac{1}{s}\right),$$

$$\frac{a_2(\xi)}{\sqrt{s}} \simeq \frac{1}{2}n^- - \frac{\vec{\Delta}_\perp}{\sqrt{s}}\vartheta(-\xi) + O\left(\frac{1}{s}\right), \qquad (4.2.9)$$

$$n^\pm = (1,0,0,\pm 1).$$

Using (4.2.9), we obtain asymptotic expressions for (4.2.5) and (4.2.6). Namely,

$$\overline{\Pi} = \frac{1}{(2\pi)^4 s}\int d^4k\,\mathcal{D}(k)\,e^{-ikx}\int_{-\infty}^{\infty} d\xi\,d\tau\,e^{i\xi(k_0-k_z)-i\tau(k_0+k_z)}\cdot$$

$$\left\{1-2i\frac{\vec{k}_\perp\vec{\Delta}_\perp}{\sqrt{s}}\left[\xi\vartheta(-\xi)+\tau\vartheta(-\tau)\right]+\frac{ik^2}{\sqrt{s}}\left(|\xi|+|\tau|\right)\right\}+O\left(\frac{1}{s^2}\right)\simeq$$

$$\simeq -\frac{1}{8\pi^2 s}\int\frac{d^2\vec{k}_\perp}{\vec{k}_\perp^2+\mu^2}\,e^{i\vec{k}_\perp\vec{x}_\perp} + \frac{i\vec{\Delta}_\perp}{s\sqrt{s}\,8\pi^2}\left[(x_0+x_z)\,\vartheta(-x_0-x_z)\right. +$$

$$+ (x_z - x_0)\,\vartheta(x_0 - x_z)\Big] \int d^2\vec{k}_\perp\, e^{i\vec{k}_\perp \vec{x}_\perp} \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + \mu^2} +$$

$$+ \frac{i}{16\pi^2 s\sqrt{s}} \left(|x_0 + x_z| + |x_0 - x_z|\right) \int d^2\vec{k}_\perp\, e^{i\vec{k}_\perp \vec{x}_\perp} \frac{\vec{k}_\perp}{\vec{k}_\perp^2 + \mu^2} + O\left(\frac{1}{s^2}\right) =$$

$$= -\frac{1}{4\pi s} K_0(\mu|\vec{x}|) - \frac{\mu}{4\pi s\sqrt{s}} \frac{\vec{\Delta}_\perp \vec{x}_\perp}{|\vec{x}_\perp|} \Big[(x_0 + x_z)\,\vartheta(-x_0 + x_z) +$$

$$+ (x_z - x_0)\,\vartheta(x_0 - x_z)\Big] K_1(\mu|\vec{x}_\perp|) - \frac{i\mu^2}{8\pi s\sqrt{s}} \Big(|x_0 + x_z| + \quad (4.2.10)$$

$$+ |x_0 - x_z|\Big) K_0(\mu|\vec{x}_\perp|) + O\left(\frac{1}{s^2}\right);$$

$$\frac{i\lambda^2 g^4}{4} \int d\eta \left[\left(\frac{\delta \overline{\pi}}{\delta \nu_1(\eta)}\right)^2 + \left(\frac{\delta \overline{\pi}}{\delta \nu_2(\eta)}\right)^2\right]\Bigg|_{\nu=0} \simeq$$

$$\simeq -\frac{i\lambda^2 g^4}{(2\pi)^8 s^2\sqrt{s}} \int d^4k_1\, d^4k_2\, \mathcal{D}(k_1)\,\mathcal{D}(k_2)\, e^{-ix(k_1 + k_2)}\,(k_1 k_2)\cdot$$

$$\int_{-\infty}^{\infty} d\xi_1\, d\tau_1\, d\xi_2\, d\tau_2 \,\exp\Big[i\xi_1(k_{10} - k_{1z}) - i\tau(k_{10} + k_{1z}) + \quad (4.2.11)$$

$$+ i\xi_2(k_{20} - k_{2z}) - i\tau_2(k_{20} + k_{2z})\Big]\Big[\Phi(\xi_1, \xi_2) + \Phi(\tau_1, \tau_2)\Big] + O\left(\frac{1}{s^3}\right) =$$

$$= -\frac{i\lambda^2 g^4 \mu^2}{32\pi^2 s^2\sqrt{s}} \left(|x_0 + x_z| + |x_0 - x_z|\right) K_1^2(\mu|\vec{x}_\perp|).$$

In formulae (4.2.10) and (4.2.11) we consider $|\vec{x}_\perp| \neq 0$, which guarantees convergence of all integrals. K_0 and K_1 are the Macdonald functions of zero and first orders defined by the expressions

$$K_0(\mu|\vec{x}_\perp|) = \frac{1}{2\pi} \int d^2\vec{k}_\perp \frac{e^{i\vec{k}_\perp \vec{x}_\perp}}{\vec{k}_\perp^2 + \mu^2}; \quad K_1(\mu|\vec{x}_\perp|) = -\frac{\partial K_0(\mu|\vec{x}_\perp|)}{\partial(\mu|\vec{x}_\perp|)}.$$

Now substituting (4.2.10) and (4.2.11) in (4.2.4), we obtain the desired formula for S_λ /26/

$$S_\lambda \simeq e^{-\frac{ig^2\lambda}{4\pi s} K_0(\mu|\vec{x}_\perp|)} \Big\{ 1 - \frac{ig^2\lambda\mu}{4\pi s\sqrt{s}} \frac{\vec{\Delta}_\perp \vec{x}_\perp}{|\vec{x}_\perp|} \cdot \Big[(x_0 + x_z)\,\vartheta(-x_0 - x_z) + (x_z - x_0)\,\vartheta(x_0 - x_z) \Big] K_1(\mu|\vec{x}_\perp|) + \\ + \frac{g^2\lambda\mu^2}{8\pi s\sqrt{s}} \Big(|x_0 + x_z| + |x_0 - x_z| \Big) K_0(\mu|\vec{x}_\perp|) - \\ - \frac{ig^4\lambda^2\mu^2}{32\pi^2 s^2\sqrt{s}} \Big(|x_0 + x_z| + |x_0 - x_z| \Big) K_1(\mu|\vec{x}_\perp|) \Big\} \tag{4.2.12}$$

In expression (4.2.12) the factor in front of the curly brackets corresponds to the eikonal behaviour of the scattering amplitude, and the terms inside define the corrections of the relevant value $\frac{1}{\sqrt{s}}$.

As is known from investigations of the scattering amplitude Using the scope of Feynman diagram technique, the high energy asymptotics can only contain logarithms and integral powers of S. In this case there is an analogous phenomenon, as long as the integration of the expression (4.2.12) for quantity S_λ according to formula (4.2.1) leads to the disappearance of the coefficients for half-integer power of S. None the less, it is necessary to take into account the terms containing the half-integral powers of S if we want to calculate the next corrections to the scattering amplitude.

It is worth noting the emergence of the dependence on x_0 and x_z in the correction terms, that is the so called retardation effects, which are absent in the leading asymptotic term.

Carrying out analogous calculations, we can convince ourselves that the following corrections decrease sufficiently fast by .

comparison with the ones we have written out. However, it should be stressed that this does not imply proof of the validity of the eikonal representation of the scattering amplitude in the framework under consideration. The fact of the matter is that the coefficient functions in the asymptotic expansion, which are expressed through the Macdonald functions, are singular at small distances. Hence, when calculating the scattering amplitude, the integration of the quantity S_λ according to the formula (4.2.1) may lead to the emergence of terms violating the eikonal series in the high order of g^2. The possibility of the appearance of such extra-terms in some order of perturbation theory in the ϕ^3-type models has been indicated previously /27,28/.

In this connection, it should be noted that in the framework of the quasipotential approach in quantum field theory there exist strict grounds for the eikonal representation based on the assumption of quasipotential smoothness.

In the example considered above we dealt with the singular interaction which leads to the Yukawa-type quasipotential when the radiative effects are neglected, and which requires particular caution.

§5. THE OPERATOR METHOD AND STRAIGHT-LINE PATH APPROXIMATION

In this paragraph we consider the operator method of finding the approximate solutions of quasipotential equations. This method is sufficiently general to be applied to other equations of quantum field theory.

As a concrete application of the operator method the asymptotic behaviour of the scattering amplitude at high energies and fixed momentum transfers is considered. Within the limitations of this method and using the essential assumption of nonsingular interaction one gets the possibility of corroborating the concept of straight-line paths for smooth effective quasipotentials.

5.1. Formation of the Operator Method

Let us consider a quasipotential equation with a local quasipotential for the scattering amplitude of scalar particles

$$T(\vec{p},\vec{p}';s)=gV(\vec{p}-\vec{p}';s)+g\int d\vec{q}\,K(\vec{q}^{\,2};s)\,V(\vec{p}-\vec{q};s)\,T(\vec{q},\vec{p}';s) \tag{5.1.1}$$

where $\vec{p}$ and $\vec{p}'$ are the relative particle momenta in c.m.s. in initial and final states, respectively, and $s = 4(\vec{p}^{\,2}+m^2) = 4(\vec{p}'^{\,2}+m^2)$.

To solve eq. (5.1.1) let us perform the Fourier transformation

$$V(\vec{p}-\vec{p}';s) = \frac{1}{(2\pi)^3}\int d\vec{r}\, e^{i(\vec{p}-\vec{p}')\vec{r}}\, V(\vec{r};s), \tag{5.1.2}$$

$$T(\vec{p},\vec{p}';s) = \int d\vec{r}\,d\vec{r}'\, e^{i\vec{p}\vec{r}-i\vec{p}'\vec{r}'}\, T(\vec{r},\vec{r}';s) \tag{5.1.3}$$

Substituting (5.1.2) and (5.1.3) in (5.1.1), we obtain

$$T(\vec{r},\vec{r}';s) = \frac{g}{(2\pi)^3}\, V(\vec{r};s)\,\delta^{(3)}(\vec{r}-\vec{r}') +$$

$$+\frac{g}{(2\pi)^3}\int d\vec{q}\,K(\vec{q}^{\,2};s)\,V(\vec{r};s)\,e^{-i\vec{q}\vec{r}}\int d\vec{r}''\,e^{i\vec{q}\vec{r}''}\,T(\vec{r}'',\vec{r}';s) \tag{5.1.4}$$

Introducing the representation

$$T(\vec{r},\vec{r}\,';s) = \frac{g}{(2\pi)^3} V(\vec{r};s)\, F(\vec{r},\vec{r}\,';s), \tag{5.1.5}$$

we have

$$F(\vec{r},\vec{r}\,';s) = \delta^{(3)}(\vec{r}-\vec{r}\,') +$$

$$+ \frac{g}{(2\pi)^3} \int d\vec{q}\, K(\vec{q}^{\,2};s) e^{-i\vec{q}\vec{r}} \int d\vec{r}\,'' e^{i\vec{q}\vec{r}\,''} V(\vec{r}\,'';s)\, F(\vec{r}\,',\vec{r}\,'';s) \tag{5.1.6}$$

Let us define the pseudo-differential operator

$$\hat{L}_{\vec{r}} = K(-\vec{\nabla}_{\vec{r}}^{\,2};s). \tag{5.1.7}$$

Then

$$K(\vec{r};s) = \int d\vec{q}\, e^{-i\vec{q}\vec{r}} K(\vec{q}^{\,2};s) = \hat{L}_{\vec{r}}\, (2\pi)^3\, \delta^{(3)}(\vec{r}). \tag{5.1.8}$$

Taking into account the expression (5.1.8), eq. (5.1.6) may be written in the following symbolic form

$$F(\vec{r},\vec{r}\,';s) = \delta^{(3)}(\vec{r}-\vec{r}\,') + g\,\hat{L}_{\vec{r}} \left[V(\vec{r},s)\, F(\vec{r},\vec{r}\,';s) \right]. \tag{5.1.9}$$

We shall seek the solution of this equation in the following form

$$F(\vec{r},\vec{r}\,';s) = \frac{1}{(2\pi)^3} \int d\vec{k}\, e^{W(\vec{r};\vec{k};s)} e^{-i\vec{k}(\vec{r}-\vec{r}\,')} \tag{5.1.10}$$

Substituting (5.1.10) in (5.1.9), we obtain the equation for the function

$$e^{W(\vec{r};\vec{k};s)} = 1 + g\,\hat{L}_{\vec{r}} \left[V(\vec{r};s) e^{W(\vec{r};\vec{k};s) - i\vec{k}\vec{r}} \right] e^{i\vec{k}\vec{r}} \tag{5.1.11}$$

Expanding the function W in powers of the coupling constant g[12]

$$W(\vec{r};\vec{k};s)=\sum_{n=1}^{\infty} g^n W_n(\vec{r};\vec{k};s), \qquad (5.1.12)$$

we immediately obtain from eq. (5.1.11) the expressions for the functions

$$W_1(\vec{r};\vec{k};s)=\int d\vec{q}\, V(\vec{q};s)\, K[(\vec{k}+\vec{q})^2;s]\, e^{-i\vec{q}\vec{r}}, \qquad (5.1.13)$$

$$W_2(\vec{r};\vec{k};s)=-\frac{W_1^2(\vec{r};\vec{k};s)}{2}+\frac{1}{2}\int d\vec{q}_1 d\vec{q}_2\, e^{-i\vec{q}_1\vec{r}-i\vec{q}_2\vec{r}}\cdot$$
$$V(\vec{q}_1;s)\,V(\vec{q}_2;s)\,K[(\vec{q}_1+\vec{q}_2+\vec{k})^2;s]\{K[(\vec{q}_1+\vec{k})^2;s]+K[(\vec{q}_2+\vec{k})^2;s]\} \qquad (5.1.14)$$

etc. Confining ourselves only to a consideration of W_1 instead of W in formula (5.1.10), we obtain from (5.1.10), (5.1.5) and (5.1.3) the following approximate expression for the scattering amplitude[29]

$$T_1(\vec{p},\vec{p}';s)=\frac{g}{(2\pi)^3}\int d\vec{r}\, e^{i(\vec{p}-\vec{p}')\vec{r}}\, V(\vec{r};s)\, e^{g W_1(\vec{r};\vec{p}';s)} \qquad (5.1.15)$$

The meaning of the approximation made above is clear, if we expand $T_1(\vec{p},\vec{p}';s)$ in powers of the coupling constant g

$$T_1^{(n+1)}(\vec{p},\vec{p}';s)=\frac{g^{n+1}}{n!}\int d\vec{q}_1\cdots d\vec{q}_n\, V(\vec{q}_1;s)\cdots V(\vec{q}_n;s)\cdot$$
$$V(\vec{p}-\vec{p}'-\sum_{i=1}^{n}\vec{q}_i;s)\prod_{i=1}^{n}K[(\vec{q}_i+\vec{p}')^2;s] \qquad (5.1.16)$$

and compare it with the (n+1)-th iteration terms of eq. (5.1.1)

$$T^{(n+1)}(\vec{p},\vec{p}';s)=\frac{g^{n+1}}{n!}\int d\vec{q}_1\cdots d\vec{q}_n\, V(\vec{q}_1;s)\cdots V(\vec{q}_n;s)\cdot$$
$$V(\vec{p}-\vec{p}'-\sum_{i=1}^{n}\vec{q}_i;s)\sum_{P}K[(\vec{q}_1+\vec{p}')^2;s]\cdot$$
$$K[(\vec{q}_1+\vec{q}_2+\vec{p}')^2;s]\cdots K[(\sum_{i=1}^{n}\vec{q}_i+\vec{p}')^2;s], \qquad (5.1.17)$$

where $\sum_P$ denotes the sum ovar all possible permutations of momenta $\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_n$.

It is easy to see from the expressions (5.1.16) and (5.1.17) that in the case of the Lippman-Schwinger equation the approximation coincides with the so-called "$\vec{q}_i\vec{q}_j=0$ approximation" according to which the terms of $\vec{q}_i\vec{q}_j$ $(i\neq j)$ -type in the "nucleon propagator" are omitted.

5.2. Operator Method and Asymptotic Behaviour of the Scattering Amplitude

In this section, taking as an example the Logunov-Tavkhelidze quasipotential equation, we consider the case when the above approximate expressions of the scattering amplitude can be used to find the asymptotics, when s tends to infinity and t is fixed. In asymptotic expressions we shall take into account not only the leading term, but also the next correction, using the formula

$$e^{W(\vec{r};\vec{p}';s)} = e^{gW_1(\vec{r};\vec{p}';s)}\left[1+g^2W_2(\vec{r};\vec{p}';s)+\cdots\right], \tag{5.2.1}$$

where W_1 and W_2 are defined by (5.1.13) and (5.1.14).

Let us choose the z-axis along the $(\vec{p}+\vec{p}\,')$ vector. Then

$$\vec{p}-\vec{p}\,' = \vec{\Delta}_\perp \;, \quad \vec{\Delta}_\perp\vec{n}_z = 0 \;, \quad t = -\vec{\Delta}_\perp^2 \tag{5.2.2}$$

Noting that

$$K[(\vec{q}+\vec{p}\,')^2;s] = \frac{1}{\sqrt{(\vec{q}+\vec{p}\,')^2+m^2}}\;\frac{1}{(\vec{q}+\vec{p}\,')^2-\frac{s}{4}+m^2-i\varepsilon} \underset{\substack{s\to\infty\\ t\text{-fixed}}}{=}$$

$$= \frac{2}{s(q_z-i\varepsilon)}\left[1-\frac{3q_z+\vec{q}_\perp^{\,2}-\vec{q}_\perp\vec{\Delta}_\perp}{\sqrt{s}\,(q_z-i\varepsilon)}\right]+O\left(\frac{1}{s^2}\right), \tag{5.2.3}$$

we obtain from eqs. (5.1.13) and (5.1.15)

$$W_1 = \frac{W_{10}}{s} + \frac{W_{11}}{s\sqrt{s}} + O\left(\frac{1}{s^2}\right), \tag{5.2.4}$$

$$W_2 = \frac{W_{20}}{s^2\sqrt{s}} + O\left(\frac{1}{s^3}\right), \tag{5.2.5}$$

where

$$W_{10} = 2\int d\vec{q}\, V(\vec{q};s)\frac{e^{-i\vec{q}\vec{r}}}{q_z - i\varepsilon} = 2i\int_z^\infty dz'\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z'^2};\, s\right) \tag{5.2.6}$$

$$W_{11} = -2\int d\vec{q}\, V(\vec{q};s)\, e^{-i\vec{q}\vec{r}}\, \frac{3\vec{q}_z^{\,2} + \vec{q}_\perp^{\,2} - \vec{q}_\perp\vec{\Delta}_\perp}{(q_z - i\varepsilon)^2} =$$
$$= 6V\left(\sqrt{\vec{r}_\perp^{\,2} + z^2};\, s\right) + 2\left(\vec{\nabla}_\perp^{\,2} + i\vec{\Delta}_\perp\vec{\nabla}_\perp\right)\int_z^\infty dz'(z-z')\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z'^2};\, s\right), \tag{5.2.7}$$

$$W_{20} = -4\int d\vec{q}_1 d\vec{q}_2\, e^{-i(\vec{q}+\vec{q}_2)\vec{r}}\, V(\vec{q}_1;s)\, V(\vec{q}_2;s)\cdot$$
$$\frac{3q_{1z}q_{2z} + \vec{q}_{1\perp}\vec{q}_{2\perp}}{(q_{1z}-i\varepsilon)(q_{2z}-i\varepsilon)(q_{1z}+q_{2z}-i\varepsilon)} = -4i\int_z^\infty dz'\left\{3V^2\left(\sqrt{\vec{r}_\perp^{\,2} + z'^2};\, s\right) + \right. \tag{5.2.8}$$
$$\left. + \left[\vec{\nabla}_\perp \int_{z'}^\infty dz''\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z''^2};\, s\right)\right]^2\right\}$$

To obtain the desired asymptotics with the accuracy mentioned above it is sufficient to write the scattering amplitude as follows

$$T(\vec{p},\vec{p}';s) = \frac{g}{(2\pi)^3}\int d\vec{r}_\perp dz\, e^{i\vec{\Delta}_\perp\vec{r}_\perp}\, V\left(\sqrt{\vec{r}^{\,2} + z^2};\, s\right) e^{g\frac{W_{10}}{s}}$$
$$\left(1 + g\frac{W_{11}}{s\sqrt{s}} + g^2\frac{W_{20}}{s^2\sqrt{s}} + \ldots\right) \tag{5.2.9}$$

Then substituting eqs. (5.2.6-8) in (5.2.9), we obtain for smooth potentials the expression [30]

$$T(t,s) \underset{\substack{s\to\infty \\ t\text{-fixed}}}{=} \frac{s}{(2\pi)^3}\int d^2\vec{r}_\perp\, e^{i\vec{\Delta}_\perp\vec{r}_\perp}\, \frac{e^{\frac{2ig}{s}\int_{-\infty}^{\infty} dz\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z^2};\, s\right)} - 1}{2i} -$$
$$-\frac{6g^2}{(2\pi)^3 s\sqrt{s}}\int d^2\vec{r}_\perp\, e^{i\vec{\Delta}_\perp\vec{r}_\perp}\, e^{\frac{2ig}{s}\int_{-\infty}^{\infty} dz'\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z'^2};\, s\right)}\int_{-\infty}^{\infty} dz\, V\left(\sqrt{\vec{r}_\perp^{\,2} + z^2};\, s\right) -$$

$$-\frac{ig}{(2\pi)^3\sqrt{s}}\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp\vec{r}_\perp}\int_{-\infty}^{\infty}dz\left\{e^{\frac{2ig}{s}\int_z^\infty dz' V(\sqrt{\vec{r}_\perp^{\,2}+z'^2};\,s)}-\right.$$
$$\left.-e^{\frac{2ig}{s}\int_{-\infty}^{\infty}dz' V(\sqrt{\vec{r}_\perp^{\,2}+z'^2};\,s)}\right\}\left\{\int_z^\infty dz'\,\vec{\nabla}_\perp^2 V(\sqrt{\vec{r}_\perp^{\,2}+z'^2};\,s)+\right.$$
$$\left.+\frac{2ig}{s}\left[\int_z^\infty dz'\,\vec{\nabla}_\perp V(\sqrt{\vec{r}_\perp^{\,2}+z'^2};\,s)\right]^2\right\}-\frac{ig}{(2\pi)^3\sqrt{s}}\,\vec{\Delta}_\perp^2\cdot$$
$$\int d^2\vec{r}_\perp e^{i\vec{\Delta}_\perp\vec{r}_\perp}\int_{-\infty}^{\infty}z\,dz\,V(\sqrt{\vec{r}_\perp^{\,2}+z^2};\,s)\,e^{\frac{2ig}{s}\int_z^\infty dz' V(\sqrt{\vec{r}_\perp^{\,2}+z'^2};\,s)} \qquad (5.2.10)$$

It is easy to see that the first term in eq. (5.2.10) describes the eikonal behaviour of the scattering amplitude and all others define the corrections of the relative value $\frac{1}{\sqrt{s}}$.

5.3. The Connection between Operator Method and Path Integration

In order to discover what physical picture corresponds to the results obtained above, let us ascertain the connection of the operator method with the Feynman method of integration over particle trajectories. To do that we come back to eq. (5.1.11) for the function W. The solution of this equation can be written symbolically as

$$e^W=\frac{1}{1-gK[(-i\vec{\nabla}-k)^2]V(\vec{r})}\times 1=$$
$$=-i\int_0^\infty d\tau\,e^{i\tau(1+i\varepsilon)}\,e^{-i\tau gK[(-i\vec{\nabla}-\vec{k})^2]V(\vec{r})}\times 1 \qquad (5.3.1)$$

According to the Feynman parameterization [31], we introduce an ordering index η and rewrite (5.3.1) as follows

$$e^W=-i\int_0^\infty d\tau\,e^{i\tau(1+i\varepsilon)}\exp\left\{-ig\int_0^\tau d\eta\,K[(-i\vec{\nabla}_{\eta+\varepsilon}-k)^2]V(\vec{r}_\eta)\right\} \qquad (5.3.2)$$

Using the Feynman transformation

$$\mathcal{F}[\vec{\mathcal{P}}(\eta)] = \int \mathcal{D}\vec{p} \int\limits_{\vec{x}(0)=0} \mathcal{D}\frac{\vec{x}}{(2\pi)^3} \exp\left\{ i\int_0^\tau d\eta\, \dot{\vec{x}}(\eta)\left[\vec{p}(\eta) - \vec{\mathcal{P}}(\eta)\right]\right\} \mathcal{F}[\vec{p}(\eta)], \qquad (5.3.3)$$

the solution of eq. (5.1.11) can be written in the form of a functional integral

$$e^{W} = -i\int_0^\infty d\tau\, e^{i\tau(1+i\varepsilon)} \int \mathcal{D}\vec{p} \int\limits_{\vec{x}(0)=0} \mathcal{D}\frac{\vec{x}}{(2\pi)^3}\, e^{i\int_0^\tau d\eta\, \dot{\vec{x}}(\eta)\vec{p}(\eta)}\, G(\vec{x};\vec{p};\tau) \times 1 \qquad (5.3.4)$$

In the formula (5.3.4)

$$G(\vec{x};\vec{p};\tau) = e^{-\int_0^\tau d\eta\, \dot{\vec{x}}(\eta)\,\vec{\nabla}_{\eta+\varepsilon}}\, \exp\left\{-ig\int_0^\tau d\eta\, K[(\vec{p}(\eta)-\vec{k})^2]\, V(\vec{r}_\eta)\right\} \qquad (5.3.5)$$

and G satisfies the equation

$$\begin{cases} \dfrac{dG}{d\tau} = \left\{-ig\, K[(\vec{p}(\tau)-\vec{k})^2]\, V(\vec{r}) - \dot{\vec{x}}(\tau-\varepsilon)\,\vec{\nabla}\right\} G \\ G(\tau=0) = 1 \end{cases} \qquad (5.3.6)$$

Using eq. (5.3.6) to obtain the operator function G and substituting it in formula (5.3.4), we obtain the final expression for W [32]

$$e^{W} = -i\int_0^\infty d\tau\, e^{i\tau(1+i\varepsilon)} \int \mathcal{D}\vec{p} \int\limits_{\vec{x}(0)=0} \mathcal{D}\frac{\vec{x}}{(2\pi)^3}\, e^{i\int_0^\tau d\eta\, \dot{\vec{x}}(\eta)\vec{p}(\eta)}\, e^{g\pi}, \qquad (5.3.7)$$

where

$$\pi = -i\int_0^\tau d\eta\, K[(\vec{p}(\eta)-\vec{k})^2]\, V\left[\vec{r} - \int_0^\tau d\xi\, \dot{\vec{x}}(\xi)\,\vartheta(\xi-\eta+\varepsilon)\right] \qquad (5.3.8)$$

Writing down the expression

$$e^{W} = \overline{e^{g\pi}} = e^{g\bar{\pi}} \sum_{n=0}^{\infty} \frac{g^n}{n!}\overline{(\pi-\bar{\pi})^n}, \qquad (5.3.9)$$

in which the sign of averaging means integration over $\tau, \vec{x}(\eta)$ and $\vec{p}(\eta)$ with the appropriate measure (see (5.3.7)), and carrying out the calculations, we find that

$$\overline{\Pi} = W_1 , \quad \frac{\overline{\Pi^2} - \overline{\Pi}^2}{2} = W_2 \quad \text{and so on,} \tag{5.3.10}$$

i.e. the expansions (5.3.9) and (5.2.1) coincide completely.

Confining ourselves to the first term in expansion (5.3.9) ($n=0$), we obtain the approximate expression (5.1.15) for the scattering amplitude, which corresponds to taking into account only those particle paths closest to the classical one and coinciding with the straight-line trajectories in the case of high energy particle scattering at small angles. In other words, one can say that the operator method at high energies is the realization of the concept of straight-line paths.

§6. STRAIGHT-LINE PATH AND EIKONAL PROBLEM

As it was shown above, the essence of SLPM lies in the assumption that the large momentum transfers are suppressed in each high energy particle interaction. Such large momenta, when carried by the particles in the collision process, tend to be conserved ("inertia" of large momenta). The type of the particles transferring large momenta may change during the interaction process according to the empirical regularities observed in the inclusive reactions. Thus, for example, in the collision of fast nucleons it is necessary to take into account the possibility of radiation of hard mesons which take away the greatest part of the initial nucleon momenta. Generally, in order to obtain the eikonal formula by summation of the perturbation series, one takes the initial particles as the leading ones transferring large momenta. However, the existence of virtual processes with the alteration of particles of the "leading" type must generally speaking lead to the violation of the orthodox eikonal representation. The possibility that such an extra-term in the asymptotics of the sum of diagrams may appear was first noted by Tiktopolous et al[27].

In this section we have studied a structure of the "non-eikonal" contribution to the two nucleon scattering amplitude described by a sum of ladder-type diagrams without taking into account radiative corrections and vacuum polarization effects in the scalar model.

6.1. High Energy Asymptotics of Feynman Graphs and Modification of the Particles Propagators

We now choose to study the scattering amplitude of two scalar nucleons in the model $\mathcal{L}_{int} = g{:}\psi^{+}\psi\phi{:}$ neglecting the radiative corrections and closed nucleon loops. This amplitude is represented as the sum of the following diagrams

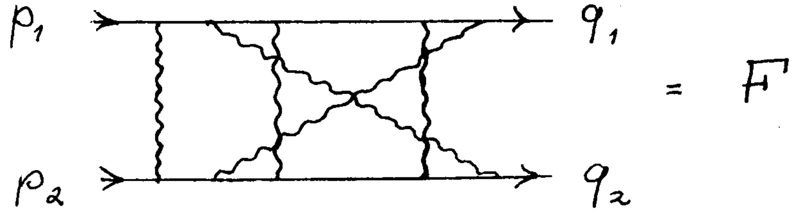


Fig. 1.

where p_1 and p_2 are the momenta of the in-particles and q_1 and q_2 the momenta of the out-particles. If the number of integration momenta is ℓ and the number of internal lines is I (for the diagrams of Fig. 1 type $I = 3\ell + 1$)

$$F = \int dk_1 \cdots dk_\ell \prod_{i=1}^{I} \frac{1}{r_i^2 - m^2 + i\varepsilon} , \tag{6.1.1}$$

where r_i are linear combinations of integration momenta k_j.

Using the Feynman parameterization we have

$$F = (I-1)! \int_0^1 d\alpha_1 \cdots d\alpha_I \, \delta\Big(1 - \sum_{i=1}^{I} \alpha_i\Big) \int \frac{dk_1 \cdots dk_\ell}{[\Psi(k,\alpha,s,t)]^I} , \tag{6.1.2}$$

where

$$\Psi = \sum_{i=1}^{I} \alpha_i (r_i^2 - m_i^2 + i\varepsilon) = \sum_{i,j=1}^{\ell} a_{ij} k_i k_j + 2 \sum b_i k_i + c \tag{6.1.3}$$

Following this procedure it is possible to obtain a representation for F in the Chisholm form[34], integrating over k_i in eq. (6.1.3)

$$F = (i\pi)^\ell (I-2\ell-1)! \int_0^1 d\alpha_1 \cdots d\alpha_I \, \delta\Big(1 - \sum_{i=1}^{I} \alpha_i\Big) \frac{[C(\alpha)]^{I-2\ell-2}}{[\mathcal{D}(\alpha,s,t)]^{I-2\ell}} \tag{6.1.4}$$

In the formula (6.1.4)

$$C = \det \| a_{ij} \| , \quad \mathcal{D} = \det \left\| \begin{array}{c|c} a_{ij} & \begin{matrix} b_1 \\ \vdots \\ b_\ell \end{matrix} \\ \hline b_1 \cdots b_\ell & c \end{array} \right\| \tag{6.1.5}$$

and the Chisholm determinant D can be represented in the following form

$$\mathcal{D}(\alpha, s, t) = f(\alpha) s + g(\alpha) t + h(\alpha) \tag{6.1.6}$$

We will now give a brief account of the results obtained in[35] which we will then use to study the asymptotic behaviour of expression (6.1.4)[x].

D e f i n i t i o n

A t-path is a set of lines forming a continuous arc, such that

a) If we short-circuit these lines, the entire graph is split into two parts having no common line and only one common vertex (to which these lines have been reduced). The p_1 and q_1 external lines of the graph are attached to one of the two parts and the p_2 and q_2 lines to the other.

b) None of its subsets has property a). A t-path is a $\bar{t}$-path of minimum length (i.e. number of lines).

R u l e

If the graph F is such that there exist M $\bar{t}$-paths of lenth ρ its asymptotics are

$$F \simeq (i\pi^2)^{\ell} \frac{(I-2\ell-1-\rho)!\,\rho!}{(M-1)!} \frac{(\ln s)^{M-1}}{s^{\rho}} \int \frac{[C_0(\alpha)]^{I-2\ell-2}}{(g_0 t + h_0)^{I-2\ell-\rho} \tilde{f}_0^{\rho}} \cdot \prod_{j=1}^{M} \delta\Big(\sum_{\nu=1}^{\rho} \alpha_\nu^{(j)} - 1\Big)\, \delta\Big(\sum_{\nu \notin p} \alpha_\nu - 1\Big) \{d\alpha\} \tag{6.1.7}$$

In the formula (6.1.7)

$$g_0 t + h_0 = \mathcal{D}(\alpha, s, t)\Big|_{\alpha_\nu^{(j)} = 0}\,, \qquad C_0(\alpha) = C(\alpha)\Big|_{\alpha_\nu^{(j)}}\,, \tag{6.1.8}$$

$\alpha_\nu^{(j)}$ are parameters of those lines which belong to the j-th

---

[x] Results similar to those of ref.[35] have been obtained also in paper[36,37].

$\bar{t}$-path, $\alpha_\nu (\nu \not\subset p)$ are the remaining parameters and the quantity $\tilde{f}_0$ is obtained from f (see (6.1.6)) as follows. Let us perform the replacement

$$\alpha_\nu^{(j)} \rightarrow \lambda_j \alpha_\nu^{(j)} , \tag{6.1.9}$$

then

$$f \rightarrow \lambda_1 \lambda_2 \ldots \lambda_M \tilde{f}(\lambda) \quad \text{and} \quad \tilde{f}_0 = \tilde{f}\Big|_{\lambda_j=0} \tag{6.1.10}$$

Having written out the formulae we need we can proceed further.

If the momentum transfers in graph 1 are zero, i.e. $p_1 = q_1$ and $p_2 = q_2$, we shall call a set of lines whose propagators depend on the momentum a p-path. Thus, in the graphs F there are two p-paths, each forming a continuous arc. Note that each p-path is a t-path according to the definition. However, the configurations of the p-paths depend on the concrete arrangement of the integration momenta while the t-paths are the topological characteristics of the given graph. In the graphs under consideration the interation momenta can be chosen so that the p-paths will coincide with any pair of t-paths not forming a closed loop.

S t a t e m e n t 1

Let the given graph be such that the contribution to the leading asymptotics is due to the pair of $\bar{t}$-paths which have no common line. Then the asymptotics of this graph will not be changed if the integration momenta are chosen so that the p-paths coincide with $\bar{t}$-paths and the following modification of the propagators depending on external momenta p is performed

$$\frac{1}{(\sum k_i)^2 + 2p\sum k_i - m_j^2 + i\varepsilon} \longrightarrow \frac{1}{2p\sum k_i + i\varepsilon} , \tag{6.1.11}$$

i.e. we neglect masses and products of integration momenta.

P r o o f

The propagator modification (6.1.11) results in the following alteration of determinants C and D. In determinant C the parameters corresponding to the $\bar{t}$-paths become equal to zero, i.e. C becomes C_0. In the determinant D the quantity C in which the same parameters become equal to zero is also changed. As a result, the quantity $f(\alpha)$ (see (6.1.6)) conserves its main properties which determine the asymptotic dependence on s. Determinants C_0, $g_0 t + h_0$ and $\tilde{f}_0$, calculated according to eqs. (6.1.8) and (6.1.10) are also unchanged. Thus we make sure that whenever the propagator modification (6.1.11) is performed, the expression (6.1.7) is the correct asymptotic form of our Feynman integral.

S t a t e m e n t 2

Let the given graph be such that the contribution to the leading asymptotics is due to a pair of $\bar{t}$-paths having a common line. Also let the integration momenta be placed to that p-paths coincide with $\bar{t}$-paths. Then the asymptotics of the graph are equal to the factor $(\pm \frac{1}{s})$ multiplied by the asymptotics of the reduced graph obtained when we short-circuit the common line. We choose the plus sign when external momenta in this line have the same direction if they do not, we choose the minus sign. Dealing with the reduced graph we can use Statement 1.

Proof

Let a parameter β be associated with the common line to which corresponds the propagator

$$\frac{1}{(\sum k_i)^2 + 2(p_1 \pm p_2)(\sum k_i) - M^2 \pm s + i\varepsilon} \qquad (6.1.12)$$

It is sufficient to show that the propagator (6.1.12) can be replaced by $(\pm \frac{1}{s})$. In fact the quantities C_0 and $g_0 t + h_0$ are not changed as a consequence of the arguments used in the proof of Statement 1. The quantity f has the structure

$$f = \beta C' + \left| \begin{array}{c|c} C & (\alpha^{(1)} + \beta) \\ \hline (\alpha^{(2)} + \beta) & 0 \end{array} \right| \qquad (6.1.13)$$

where $\alpha^{(1)}$ and $\alpha^{(2)}$ are sets of parameters corresponding to the two $\bar{t}$-paths. It is now evident that instead of f we can use the quantity

$$f^{(1)} = \beta C'(\beta = 0) + \left| \begin{array}{c|c} C(\beta = 0) & (\alpha^{(1)}) \\ \hline \underbrace{\alpha^{(2)}} & 0 \end{array} \right| \qquad (6.1.14)$$

which proves our statement.

6.2. Eikonal and Noneikonal Contributions to the Scattering Amplitude

As is known (see 4.2), the scattering amplitude for two scalar nucleons, with the radiative corrections and the contribution of the vacuum polarization being neglected, can be represented in the form (4.2.1).

Putting the variables ν_1 and ν_2 equal to zero, i.e. neglecting the terms of $\kappa_i\kappa_j$-type in nucleon propagators, we obtain according to Statement 1 a sum of contributions in each diagram of those t-paths which coincide with nucleon lines. Note that for the present twisted graphs corresponding to the term $(q_1 \leftrightarrow q_2)$ in eq. (4.2.1) are not under consideration. As a result we have the well-known eikonal representation for the scattering amplitude

$$f \simeq \frac{2s}{(2\pi)^4} \int d^2\vec{x}_\perp e^{-i\vec{x}_\perp \vec{\Delta}_\perp} \left(e^{-\frac{ig^2}{4\pi s} K_0(\mu|\vec{x}_\perp|)} - 1 \right) \qquad (6.2.1)$$

when $s = (p_1+p_2)^2 \to \infty$ and $t = (p_1-q_1)^2$ is fixed. According to this fact we shall call the contributions of the t-paths coinciding with nucleon lines the eikonal ones.

In a paper by Tiktopoulos and Treiman/27/ it was pointed out that in diagrams of higher orders (namely, beginning from the 8-th) in powers of the coupling constant g it is necessary to take into account other t-paths whose contributions may be comparable with those of the eikonal t-paths. We begin our study of the noneikonal contributions with the diagram shown in Fig. 2.

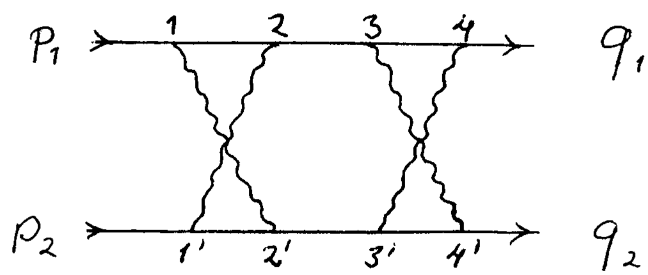


Fig. 2

In this graph which we shall call "XX-diagram" there exist four $\bar{t}$-paths, (1234), (1'2'3'4'), (1'234') and (12'3'4') all of length three. A formal account of all the $\bar{t}$-paths leads us to the asymptotic $\frac{\ln^3 s}{s^3}$. However, this corresponds to the conversion zero of all line parameters which is impossible due to the factor

$\delta(1-\Sigma_i\alpha_i)$. Using any three paths should lead to the asymptotic $\frac{\ln^2 s}{s^3}$, but in that case the coefficient including determinant C_0 also becomes equal to zero so long as these three paths from a closed loop. It is then necessary to calculate a sum of contributions from the following pairs of $\bar{t}$-paths:

$$\begin{array}{ll} (1234,\ 1'2'3'4'), & (1234,\ 1'234'), \\ (12'3'4,\ 1'2'3'4'), & (12'3'4,\ 1'234') \end{array} \tag{6.2.2}$$

Pairs (1234, 12'3'4) and (1'2'3'4', 1'234') have no influence on the asymptotics since these $\bar{t}$-paths form a closed loop. All pairs of $\bar{t}$-paths (6.2.2) lead to the same asymptotic dependence on s, namely $\frac{\ln s}{s}$. Thus we shall be interested in coefficients.

The contribution to the XX-diagram from the pair (1234, 1'2'3'4') is included in formula (6.2.1) and will be indicated

$$\frac{\ln s}{s^3} \int_{eik}^{(XX)} (t) \tag{6.2.3}$$

Now we shall obtain the contribution from the $\bar{t}$-paths (12'3'4) and (1'234'). Let us choose the integration momenta so that these paths coincide with the p-paths. Then, according to Statement 1 we can modify propagators of lines forming the $\bar{t}$-paths. After that, perform the substitution of integration momenta

$$k_i \rightarrow \frac{m}{\mu} k_i \tag{6.2.4}$$

which results in the replacements of nucleon lines by meson lines

$$\mathcal{D}_m\left(k\frac{m}{\mu}\right) = \frac{1}{k^2\frac{m^2}{\mu^2} - m^2 + i\varepsilon} = \frac{\mu^2}{m^2}\mathcal{D}_\mu(k),$$
$$\mathcal{D}_m(p_1-q_1-k) \rightarrow \frac{\mu^2}{m^2}\mathcal{D}_\mu\left[(p_1-q_1)\frac{\mu}{m}-k\right], \text{ i.e. } t \rightarrow t\frac{\mu^2}{m^2} \tag{6.2.5}$$

The propagators corresponding to the $\bar{t}$-paths will be multiplied by $\frac{\mu}{m}$. Because of this fact we may consider all the lines of $\bar{t}$-paths as modified nucleon lines. As a result we obtain a diagram of the same type (Fig. 2) but with the p-paths being directed along nucleon lines

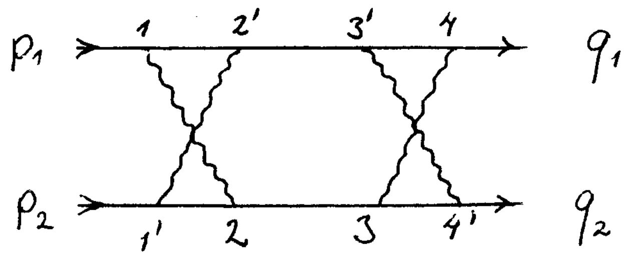


Fig. 3

Thus the desired contribution is of the form

$$\frac{\ln s}{s^3} f^{(1)}_{noneik}(t),$$

$$f^{(1)}_{noneik}(t) = \frac{\mu^2}{m^2} f^{(XX)}_{eik}\left(t \frac{\mu^2}{m^2}\right) . \qquad (6.2.6)$$

If the particle masses satisfy the condition

$$\frac{\mu^2}{m^2} \ll 1 \quad , \quad \frac{t}{m^2} \ll 1 \qquad (6.2.7)$$

the contribution of the noneikonal $\bar{t}$-paths will be less than that of the eikonal ones.

Now we have only to consider the contribution to the asymptotics of the XX-diagram from the pair of $\bar{t}$-paths, (1'2'3'4') and (12'3'4). The remaining pair, (1234) and (1'234') (see (6.2.2)), evidently lead to the same contribution. The $\bar{t}$-paths (1'2'3'4') and (12'3'4') being short-circuited, we obtain the reduced graph

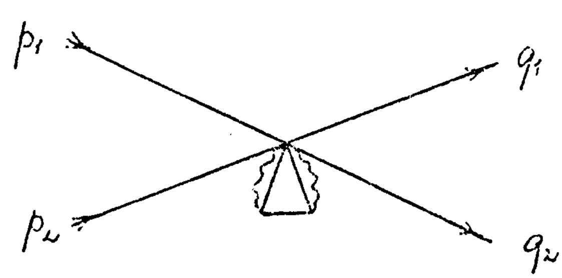


Fig. 4

Then it follows that the contribution of these $\bar{t}$-paths does not depend on momentum transfers, i.e. it can be represented in the form

$$\frac{\ln s}{s^3}\,\frac{1}{\mu^2}\,\varphi\left(\frac{\mu^2}{m^2}\right) \tag{6.2.8}$$

Let us find the form of the function $\phi(\frac{\mu^2}{m^2})$ if the condition (6.2.7) is satisfied. For this purpose we choose the integration momenta in the XX-diagram so that the p-paths coincide with the $\bar{t}$-paths (1'2'3'4') and (12'3'4). Then using Statement 2 we find that the desired contribution will be equal to the reduced graph asymptotics multiplied by $\frac{1}{s}$ (Fig. 5).

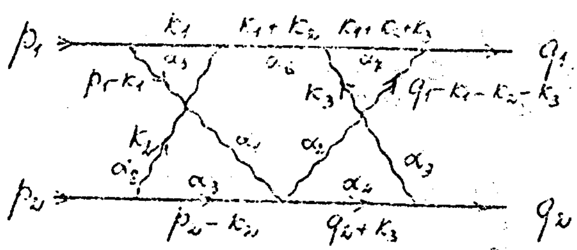


Fig. 5

When $s \to \infty$ the asymptotics of F' will, according to the formula (6.1.5), be of the form

$$F' \simeq \frac{\ln s}{s^2}\,\mathrm{const}\int d\alpha_1 \dots d\alpha_9\,\delta(1-\alpha_1-\alpha_2)\,\delta(1-\alpha_3-\alpha_4)\,\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)\,\frac{C_0}{(g_0 t+h_0)\,\tilde{f}_0^{\,2}}, \tag{6.2.9}$$

where

$$g_0 = 0,\quad h_0 = -\mu^2\left[\frac{m^2}{\mu^2}(\alpha_5+\alpha_6+\alpha_7)+\alpha_8+\alpha_9\right]C_0 . \tag{6.2.10}$$

From eqs. (6.2.9) and (6.2.10) we get the expression for the function ϕ defined by the relation (6.2.8)

$$\varphi\left(\frac{\mu^2}{m^2}\right) = \mathrm{const}\int\{d\alpha\}\prod\delta\left(1-\sum\alpha\right)\frac{\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)}{\tilde{f}_0^{\,2}\left[\frac{m^2}{\mu^2}(\alpha_5+\alpha_6+\alpha_7)+\alpha_8+\alpha_9\right]} \tag{6.2.11}$$

At large $\frac{m^2}{\mu^2}$ the main contribution comes from the domain $\alpha_5 + \alpha_6 + \alpha_7 = 0$ and we can again use the Tiktopoulos method, performing the substitution $\alpha_{5,6,7} \to \lambda\alpha_{5,6,7}$. As a result $d\alpha_5 d\alpha_6 d\alpha_7 \to \lambda\delta(1 - \alpha_5 - \alpha_6 - \alpha_7) d\alpha_5 d\alpha_6 d\alpha_7 d\lambda$,

$$\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9) \to \delta(1-\alpha_8-\alpha_9) \tag{6.2.12}$$

$$\tilde{f}_0 \longrightarrow \lambda \tilde{\tilde{f}}_0$$

from which follows

$$\varphi\left(\frac{M^2}{m^2}\right) = const \int_0^1 \frac{d\lambda}{\lambda \frac{m^2}{\mu^2} + 1}, \quad \text{i.e.}$$

$$\varphi\left(\frac{M^2}{m^2}\right) = const \frac{M^2}{m^2} \ln \frac{M^2}{m^2} \tag{6.2.13}$$

under condition (6.2.7). Note that const in eq. (6.2.13) now includes all the integrals over α_i. Taking into account the equality $f_{eik}(t=0) = \frac{const}{\mu^2}$ and eqs. (6.2.3), (6.3.6), (6.2.8), (6.2.13) we obtain the asymptotics of the XX-diagram

$$f^{(XX)}(t) \simeq \left\{ f^{(XX)}_{eik}(t) + f^{(XX)}_{noneik}(t) \right\}, \tag{6.2.14}$$

where

$$f^{(XX)}_{noneik}(t) = \frac{M^2}{m^2} f^{(XX)}_{eik}\left(t\frac{M^2}{m^2}\right) + const\, f^{(XX)}_{eik}(t=0) \frac{M^2}{m^2} \ln \frac{M^2}{m^2} \tag{6.2.15}$$

when $s\to\infty$, t is fixed and $\frac{\mu^2}{m^2} << 1$.

6.3. Asymptotics of the Nucleon-Nucleon Scattering Amplitude. Eighth Order

In the previous section we have considered one of the eighth order diagrams. We now turn to the remaining diagrams except for the twisted graphs described by the term $(q_1 \leftrightarrow q_2)$ in formula (4.2.1). In these diagrams there are three types of noneikonal $\bar{t}$-paths which can contribute to the leading asymptotics.

In the first type we include noneikonal $\bar{t}$-paths which have no common line. Except for the XX-diagram there is only one graph with such $\bar{t}$-paths (see Fig. 6) and two cross-symmetric diagrams

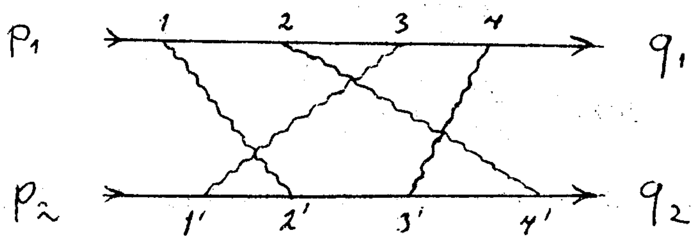


Fig. 6

The contribution to the asymptotics of diagram 6 can be written in the same form as (6.2.6)

$$f^{(2)}_{noneik}(t) = \frac{\ln s}{s^3}\,\frac{\mu^2}{m^2}\;cross\, f^{(2)}_{eik}\left(t\,\frac{\mu^2}{m^2}\right) \tag{6.3.1}$$

If we add the eikonal contribution of diagram XX to that of the cross-symmetric diagrams, then $\ln s$ cancels and we obtain the total eikonal contribution

$$\frac{1}{s^3}\,f_{eik}(t) \tag{6.3.2}$$

Then, according to the eqs. (6.2.6) and (6.3.1) the noneikonal $\bar{t}$-path contribution to the same sum has the form

$$f_{noneik}(t) = \frac{\mu^2}{m^2}\,f_{eik}\left(t\,\frac{\mu^2}{m^2}\right) \tag{6.3.3}$$

In the eighth order there are no other noneikonal contributions depending on the momentum transfers.

The noneikonal $\bar{t}$-paths have a common nucleon line which we attribute to the second type. Its contribution does not depend on the momentum transfers and has been considered above for the XX-diagram (see eqs. (6.2.8)-(6.2.14)). However, the equivalent contributions are cancelled in the sum of all diagrams with such $\bar{t}$-paths.

Consider, for example, the diagram

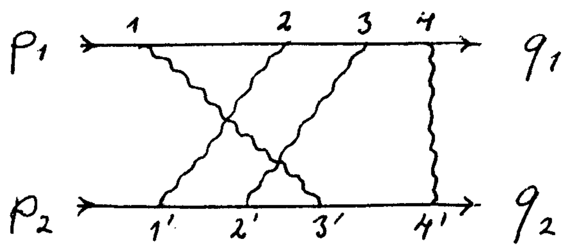
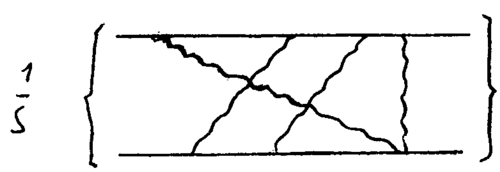


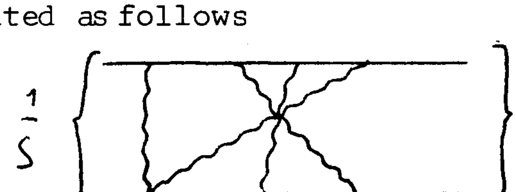
Fig. 7

whose paths (1'2'3'4') and (13'4'4) belong to the second type. Its contributions may be taken into account with the help of Statement 2 since, in this diagram the asymptotics may be graphically represented in the form



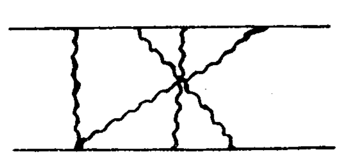
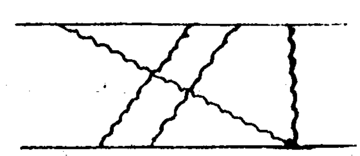
(6.3.4)

The asymptotics of the graph which appears as a result of mirror reflection of 1 and 2 vertices relative to the vertex, may be represented as follows



(6.3.5)

Now let us consider cross-symmetric graphs. According to Statement 2 we have to replace the common lines by the factors $(-\frac{1}{s})$ to obtain

$-\frac{1}{s}$ {  +  } (6.3.6)

The first term in eq. (6.3.6) corresponds to the noneikonal contribution to the diagram which is cross-symmetric to the graph shown in Fig. 7.

Summing expressions (6.3.4), (6.3.5) and (6.3.6) we convince ourselves of the cancellation of the contributions from the non-eikonal second type $\bar{t}$-paths.

Evidently, the same arguments hold for the other similar diagrams. To the third type we attribute those of $\bar{t}$-paths which have a common meson line. Its contribution to the leading asymptotics also does not depend on the momentum transfers. In the eighthorder there are the same diagrams with the third type $\bar{t}$-paths. As an example we consider only one of these graphs (see Fig. 8), keeping in mind the validity of the results for other similar diagrams.

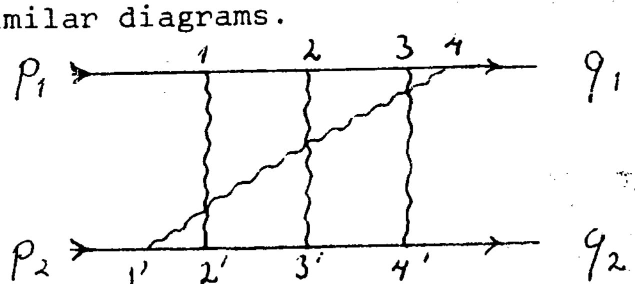


Fig. 8

In this diagram, the $\bar{t}$paths (1'434') and (12'14') are non-eikonal and belong to the third type. Their contribution may be written down in the form (6.2.8):

$$\frac{\ln s}{s^3}\,\frac{1}{\mu^2}\,\Phi\left(\frac{\mu^2}{m^2}\right) \qquad (6.3.7)$$

We shall examine the behaviour of function Φ under condition (6.2.7). Let us choose the integration momenta so that the p-paths coincide with $\bar{t}$-paths (1'434') and (12'1'4) (see Fig. 8). According to Statement 2 the desired contribution will be equal to the asymptotes of the reduced graph (Fig. 9), multiplied by $\frac{1}{s}$

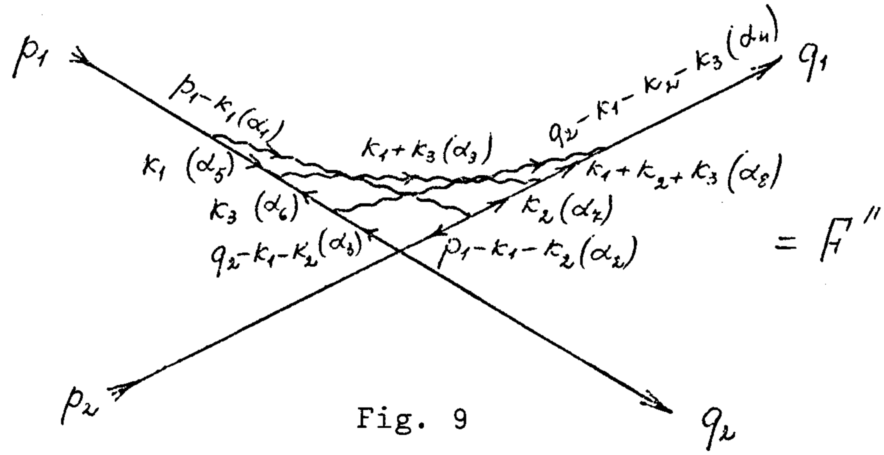


Fig. 9

Using eq. (6.1.7) when $s\to\infty$ we get the the asymptotics

$$F'' \simeq \frac{\ln s}{s^2}\, const \int d\alpha_1 \ldots d\alpha_9\, \delta(1-\alpha_1-\alpha_2)\, \delta(1-\alpha_3-\alpha_4)\cdot$$
$$\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)\, \frac{C_0}{(g_0 t + h_0)\, \tilde{f}_0^{\,2}}\,, \qquad (6.3.8)$$

where

$$g_0 = 0, \quad h_0 = -\mu^2 C_0 \left[\frac{m^2}{\mu^2}(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+\alpha_9\right]. \qquad (6.3.9)$$

From eqs. (6.3.8) and (6.3.9) we obtain the expression for the function $\Phi(\frac{\mu^2}{m^2})$, defined by the relation (6.3.7)

$$\Phi\left(\frac{\mu^2}{m^2}\right) = const \int \{d\alpha\} \prod \delta(1-\sum\alpha)\, \frac{\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9)}{\tilde{f}_0^{\,2}\left[\frac{m^2}{\mu^2}(\alpha_5+\alpha_6+\alpha_7+\alpha_8)+\alpha_9\right]} \qquad (6.3.10)$$

At large $\frac{m^2}{\mu^2}$ the main contribution is due to the region $\alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 = 0$. By substituting

$$\alpha_{5,6,7,8} \longrightarrow \lambda\alpha_{5,6,7,8}\,, \qquad (6.3.11)$$

we get
$$d\alpha_5 \cdots d\alpha_8 \to \lambda^3 \delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8)\, d\alpha_5 \cdots d\alpha_8\, d\lambda,$$
$$\delta(1-\alpha_5-\alpha_6-\alpha_7-\alpha_8-\alpha_9) \to \delta(1-\alpha_9), \qquad (6.3.12)$$
$$\tilde{f}_0 \to \lambda \tilde{\tilde{f}}_0 .$$

It then follows, under condition (6.2.1), that
$$\Phi\left(\frac{\mu^2}{m^2}\right) = const \int_0^1 d\lambda \frac{\lambda}{\lambda \frac{m^2}{\mu^2}+1}, \quad \text{i.e.}$$
$$\Phi\left(\frac{\mu^2}{m^2}\right) = const \frac{\mu^2}{m^2}. \qquad (6.3.13)$$

The results of the second and third sections can be expressed by a single formula in which the cancellation of ℓns in the cross-symmetric sum of diagrams is taken into account. In fact, at large s the asymptotic behaviour of the nucleon-nucleon scattering amplitude in the eighth order of the perturbation theory has the form
$$f^{(8)} \simeq \frac{g^8}{s^3}\left\{ \frac{1}{8\cdot 4!(2\pi)^8} \int d^2\vec{x}_\perp e^{-i\vec{x}_\perp \vec{\Delta}_\perp} K_0^4(\mu|\vec{x}_\perp|) + f^{(8)}_{noneik}(t) \right\} \qquad (6.3.14)$$
where
$$f^{(8)}_{noneik}(t) = \frac{\mu^2}{m^2} f_{eik}\left(t \frac{\mu^2}{m^2}\right) + \frac{const}{\mu^2} \Phi\left(\frac{\mu^2}{m^2}\right) \qquad (6.3.15)$$

The $f_{eik}(t)$ in eq. (6.3.14) denotes the t-dependent factor in the main asymptotic term of the sum of the diagram shown in Fig. 2 and 6, together with its cross-symmetric partners, when only the contributions of eikonal $\bar{t}$-paths are taken into account the function $\Phi(\frac{\mu^2}{m^2})$ goes as $\frac{\mu^2}{m^2}$ at $\frac{\mu^2}{m^2} << 1$. The first term in curly bracket belongs to the sum of the eikonal contributions from all the graphs of the eighth order (compare with eq. (6.2.1)). If the ratio $\frac{\mu^2}{m^2}$ is small, one can neglect the dependence on momentum transfers, and if $\frac{t}{m^2} << 1$

$$f_{eik}\left(t\,\frac{\mu^2}{m^2}\right) \simeq f_{eik}(0) = \frac{const}{\mu^2}\,, \tag{6.3.16}$$

gives the result

$$f^{(8)}\Big|_{\substack{s\to\infty \\ t\text{ - fixed} \\ \frac{\mu^2}{m^2} << 1}} \simeq \frac{g^8}{s^3}\left\{\frac{1}{8\cdot 4!\,(2\pi)^8}\int d^2\vec{x}_\perp\, e^{-i\vec{x}_\perp\vec{\Delta}_\perp} K_0^4(\mu|\vec{x}_\perp|) + \frac{const}{m^2}\right\} \tag{6.3.17}$$

To conclude this section it should be stressed that only the contributions of various $\bar{t}$-paths corresponding to zeros of the function $f(\alpha)$ in eqs. (6.1.4)-(6.1.7) were taken into account.

6.4. Asymptotics of the Nucleon-Nucleon Scattering Amplitude. Higher Orders

In sec. 6.3 we considered the high-energy behaviour of the scattering amplitude in the eighth order in powers of g. We showed that in this order there exist graphs which give the noneikonal contributions to the asymptotics of the amplitude of the same order in s as the eikonal one. However, it has been shown /27/ that higher orders in powers of g there exist graphs in which the noneikonal asymptotic term dominates the eikonal one. A typical example of these graphs with noneikonal paths of the first type (see. sec. 6.3) is illustrated in Fig. 10.

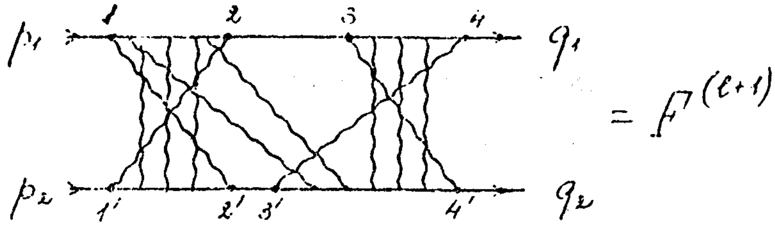


Fig. 10

In this diagram, just as in the XX-diagram there are two noneikonal $\bar{t}$-paths with length equal three: (12'3'4) and (1'234'). To study its asymptotic behaviour we use the same method as in sec. 6.2, i.e. we direct the p-paths along the $\bar{t}$-paths and replace the momenta as in (6.2.4). The asymptotics of the graph with $(\ell+1)$ meson lines (of the $(2\ell+2)$ order in powers of g) shown in Fig. 10 coincide with the asymptotics of the graph shown in Fig. 11 up to a factor $(\frac{\mu^2}{m^2})^{\ell-2}$

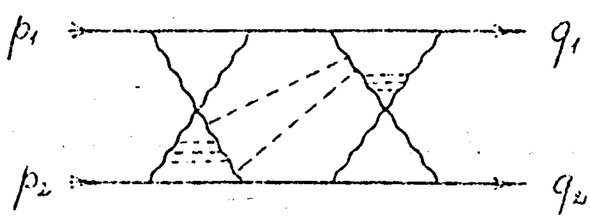


Fig. 11

Moreover, the substitution $t \to t\,\frac{\mu^2}{m^2}$ in the graph in Fig. 11 should be done (c.f. eq. (6.2.5)). The dotted-lines of this reduced graph correspond to the virtual particles with mass $\frac{\mu^2}{m^2}$. These lines are due to the meson lines (see Fig. 10), which do not belong to the $\bar{t}$-paths

$$\mathcal{D}_\mu(k) \to \mathcal{D}_\mu\left(k\,\frac{m}{\mu}\right) = \frac{1}{k^2\frac{m^2}{\mu^2} - \mu^2 + i\varepsilon} = \frac{\mu^2}{m^2}\,\mathcal{D}_{\mu\frac{\mu}{m}}(k) \qquad (6.4.1)$$

Under the condition (6.2.7) one may put $t=0$ in the asymptotics of this diagram. Thus, using eq. (6.1.7) for the main asymptotic term of the graph of order $(2\ell+2)$ considered above, we get the following expression

$$F^{(\ell+1)} \simeq \frac{\ln s}{s^3}\frac{const}{\mu^{2(\ell-2)}}\int\{d\alpha\}\{d\beta\}\{d\gamma\}\,\Pi\,\delta(1-\Sigma\gamma)\cdot$$
$$\delta(1-\Sigma\alpha-\Sigma\beta)\frac{C_0}{\tilde{f}_0^{3}\left(\frac{m^2}{\mu^2}\Sigma'\alpha_i+\Sigma\beta_i\right)^{\ell-2}},\quad \ell\geq 3. \qquad (6.4.2)$$

In formula (6.4.2) the parameters α_i correspond to the wavy meson lines, β_i to the dotted-lines and γ_i to the nucleon lines. Apparently the region $\sum\alpha_i=0$ does not give the essential contribution to the integral (6.4.2) at $\frac{m^2}{\mu^2}\gg 1$ x/. Hence

$$F^{(\ell+1)} \simeq \frac{\ln s}{s^3}\,\frac{const}{(m^2)^{\ell-2}},\quad \ell\geq 3 \qquad (6.4.3)$$

In the case of order $(2\ell+2)$ in powers of g whcih we consider above, there exist the graphs with noneikonal $\bar{t}$-paths of the third type, which have the form

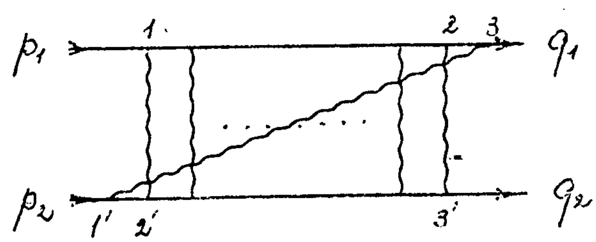


Fig. 12

---

x/ This can be shown by calculating the power of λ appearing in the numerator when the substitution is performed (c.f. eqs. (6.3.12), (6.3.13)).

In Fig. 12 there exist two $\bar{t}$-paths of length three, namely (12'1'3) and (1'323'), which lead to the asymptotics $\frac{\ell n s}{s^3}$. The method used above in sec. 6.3 for the eighth order graphs gives in this case an equation similar to eq. (6.4.3). The noneikonal $\bar{t}$-paths of the second type, whose contributions cancel in the sum of the eighth order graphs, now give the nonleading asymptotic terms.

All the graphs of order $(2\ell+2)$ belong either to the type described in this section and lead to the asymptotics of the form (6.4.3.), or have $\bar{t}$-paths of length greater than three and consequently do not dominate in the asymptotic region $s\to\infty$. Taking into account the cancellation of $\ell n s$ when graphs are summed with their cross-symmetric partners, we get the following asymptotic expression for the amplitude $f^{(2\ell+2)}$ in order $(2\ell+2)$ in power of g

$$f^{(2\ell+2)}\Bigg|_{\substack{s\to\infty \\ t\text{-fixed} \\ \frac{\mu^2}{m^2}\ll 1}} \simeq \frac{1}{s^3}\,\frac{const}{(m^2)^{\ell-2}}\,, \quad \ell\geq 3. \tag{6.4.4}$$

Note that the eikonal formula (6.2.1) when $t=0$ in the same order of g gives the following result

$$f_{eik}^{(2\ell+2)}(t=0) = \frac{const}{s^{\ell}\mu^2} \tag{6.4.5}$$

Thus, if one neglects the twisted graphs one gets for the ratio of the noneikonal to the eikonal contributions, to the amplitude of the given order, the result

$$\frac{f_{noneik}^{(2\ell+2)}}{f_{eik}^{(2\ell+2)}}\Bigg|_{\substack{s\to\infty \\ t\text{-fixed} \\ \frac{\mu^2}{m^2}\ll 1}} \simeq const\,\frac{\mu^2}{m^2}\left(\frac{s}{m^2}\right)^{\ell-3}, \quad \ell\geq 3 \tag{6.4.6}$$

From eq. (6.4.6) it follows that in the region

$$s \to \infty \, , \quad \frac{\mu^2}{m^2} \ll 1 \, , \quad s \sim m^2 \, , \quad t = 0 \tag{6.4.7}$$

the eikonal part of scattering amplitude dominates the noneikonal one, and eq. (6.2.1) gives the main asymptotic terms in each order in powers of g^2. On the other hand, in the region (6.4.7) it follows from eq. (6.4.6) that when $s \gg m^2$ the noneikonal contributions dominate the eikonal ones.

Thus the investigation of the ladder type graphs in the scalar model demonstrates that the eikonal formula corresponds to the account of the $\bar{t}$-paths, coinciding with nucleon lines. In this case the "leading" particles, carrying large momenta, are nucleons and do not change their type in virtual processes.

The noneikonal contributions to the amplitude are due to processes with alteration of the leading particles type, i.e. with the large momenta transfer from nucleons to mesons and vice versa. Then the important question arises about the significance of twisted graphs in which the final momenta q_1 and q_2 are exchanged (compare Fig. 1 and eq. (4.2.1)).

The possibility of large momentum being carried by a meson establishes the fact that the corresponding contribution may dominate the eikonal one in the same order of the coupling constant.

6.5. Twisted Eikonal Graphs and Quasipotential Structure

In this section we shall consider these diagrams in detail and will also study the reconstruction of the asymptotic quasipotential from them[38]. To the second order of perturbation theory only the twisted graph

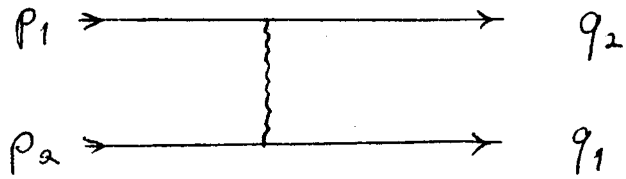


Fig. 13

exists with the known asymptotics $\frac{1}{s}$.

To the fourth order we have already two such diagrams. One of them (see Fig. 14)

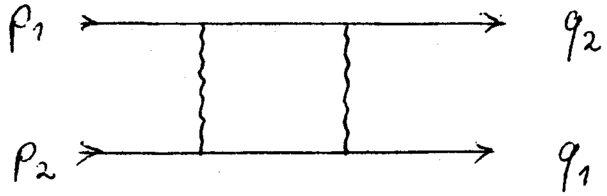


Fig. 14

possesses weaker asymptotics $\frac{1}{s^2}$ than the corresponding nontwisted graph. The other (see Fig. 15)

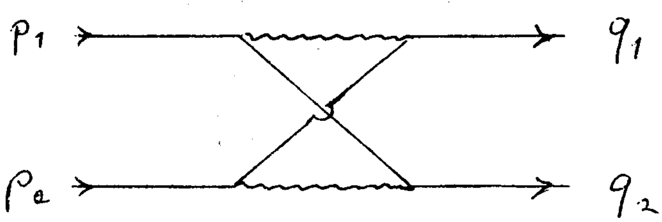


Fig. 15

has the asymptotics $\frac{\ln s}{s}$ that have already resulted in the breaking of the eikonal representation for the sum of generalized ladder graphs in the fourth order[33]. We recall that noneikonal contributions (the possibility of which was pointed out previously[27]) appear only in the eighth order of perturbation theory.

In the subsequent order we have six twisted diagrams. The diagram drawn in Fig. 16

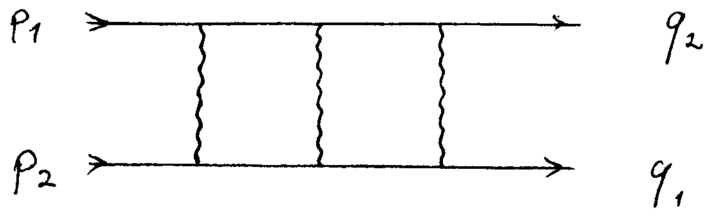


Fig. 16

possesses the weakest asymptotics.

The next pair of graphs

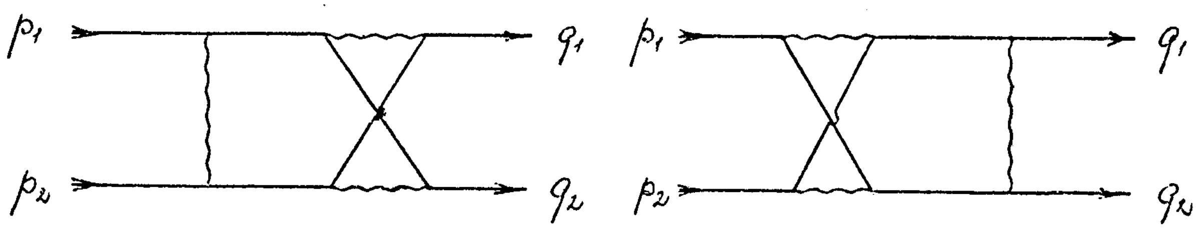


Fig. 17

in the limit $s\to\infty$ with t fixed, behaves as $\frac{\ln s}{s^2}$ i.e. they have the same asymptotics as nontwisted graphs of this order. The diagrams in Fig. 18

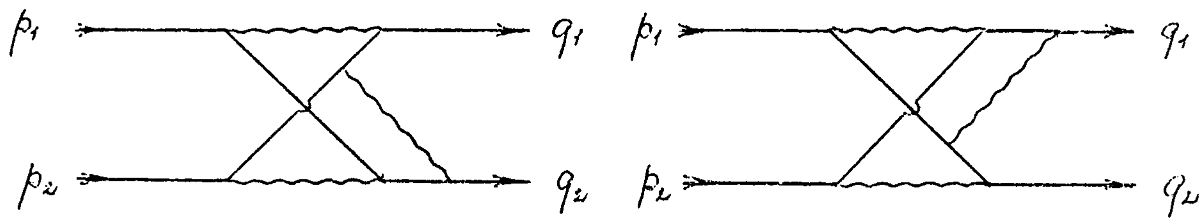


Fig. 18

have the asymptotics $\frac{1}{s}$ which is stronger than that given by the eikonal formula.

Finally, the last graph in the sixth order

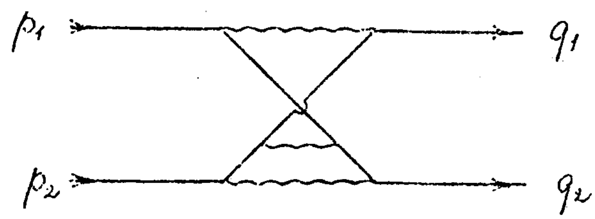


Fig. 19

behaves like $\frac{\ln^2 s}{s}$.

The consideration of the first six orders allows one to conjecture that in the higher orders the diagrams of the shape illustrated in Fig. 20

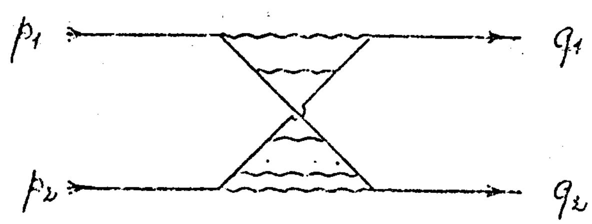


Fig. 20

with asymptotics $g^{2n}\frac{\ln^{n-1}s}{s}$ will dominate. Only if the leading asymptotic terms in each order of perturbation theory are summed up, as is usually done when deriving the eikonal representation, does one get the following asymptotic expression for the sum F of twisted graphs:

$$F\Big|_{\substack{s\to\infty\\ t\text{-fixed}}} \simeq -i\frac{g^2}{(2\pi)^4}\, s^{\alpha(t)}, \tag{6.5.1}$$

$$\alpha(t) = -1 + \frac{g^2}{8\pi^2}\frac{1}{\sqrt{-t(4m^2-t)}}\ \ln\frac{\sqrt{1-\frac{4m^2}{t}}+1}{\sqrt{1-\frac{4m^2}{t}}-1} . \tag{6.5.2}$$

With such a summation, the coefficient for $s^{\alpha(t)}$ and the expression for $\alpha(t)$ are computed only to an accuracy of g^2. However, from (6.5.1) and (6.5.2) it does not follow that within the framework of the scalar model the sum of ladder graphs leads to the proper eikonal representation of the Yukawa potential scattering. Indeed, as has already been mentioned, the twisted graphs are due to the identity of scattering particles. Within the framework of quasipotential scattering theory the particle identity necessarily implies the exchange forces in two-particle interaction as it holds in quantum mechanics.

The standard method of constructing the local quasipotential by perturbation theory [3] can be generalized in different ways when the exchange forces are present. Here we will briefly describe a method based on introducing the normal and exchange interaction parts through the expression

$$V(s;p;k) = \mathcal{G}(s;p;-k) + \mathcal{H}(s;p;k) \tag{6.5.3}$$

Here the quasipotential scattering amplitude is represented by the sum of two terms [39]

$$T(s;p;k) = G(s;p;k) + H(s;p;k) \tag{6.5.4}$$

satisfying the system of linear equations,

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \mathcal{G} \\ \mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathcal{G} & \mathcal{H} \\ \mathcal{H} & \mathcal{G} \end{pmatrix} \times \begin{pmatrix} G \\ H \end{pmatrix} \tag{6.5.5}$$

where the symbol "x" means integration

$$\int \frac{dq}{\sqrt{m^2+q^2}} \, \frac{1}{m^2+q^2-E^2}$$

For a scattering of two identical particles we have

$$\mathcal{H}(s;p;k) = \hat{P}\mathcal{G}(s;p-k) = \mathcal{G}(s;p+k)$$
$$H(s;p;k) = \hat{P}G(s;p;k) = G(s;p;-k) = G(s;-p;k) \tag{6.5.6}$$

where $\hat{P}$ is the transposition operator for coordinates of the two particles. With this, the function G obeys the conventional equation by Logunow-Tavkhelidze[x/]

$$G = \mathcal{G} + \mathcal{G} \times G \tag{6.5.7}$$

[x/] Eq. (6.5.7) follows from (6.5.6) if one takes into account the fact that in the case of identical particles the integration over intermediate two-particle states contains the statistical factor $\frac{1}{2!}$, and $\mathcal{G}\times G = \mathcal{H}\times H$, $\mathcal{G}\times H = \mathcal{H}\times G = \hat{P}(\mathcal{G}\times G)$.

Equation (6.5.7) can be used to construct the local quasipotential $\mathcal{V}$ over the given perturbation series defining the amplitude G:

$$\begin{aligned}
\mathcal{V}_2 &= [G_2], \\
\mathcal{V}_4 &= [G_4] - [\mathcal{V}_2 \times \mathcal{V}_2], \\
\mathcal{V}_6 &= [G_6] - [\mathcal{V}_2 \times \mathcal{V}_4] - [\mathcal{V}_4 \times \mathcal{V}_2] - [\mathcal{V}_2 \times \mathcal{V}_2 \times \mathcal{V}_2]
\end{aligned} \tag{6.5.8}$$

and so on.

The symbol $[\ldots]$ means the "local" continuation of the mass shell $E^2 = p^2+m^2 = k^2+m^2$ of an arbitrary function $A(E;p,k)=A(s,t,u,\delta)$ where

$$s = 4E^2, \quad t = -(p-k)^2, \quad u = -(p+k)^2,$$

$$\delta = p^2 - k^2$$

In this notation we have

$$[A(s,t,u,\delta)] = A(s,t,4m^2-s-t,0) \tag{6.5.9}$$

The quasipotential constructed in this way makes it possible, in turn, to reconstruct the initial scattering amplitude on the mass shell. We should stress, however, that perturbation theory defines the amplitude T as a whole but not the G and H parts separately, i.e.

$$T_{2n}(s,t) = [G_{2n}(E;p,k) + H(E;p,k)] \tag{6.5.10}$$

Defining

$$F_{2n}'(s,t) = \left[G_{2n}(E;p,k)\right]$$
$$B_{2n}(s,u) = \left[H_{2n}(E;p,k)\right] \qquad (6.5.11)$$

which, in the case of scattering of identical particles, are connected by the symmetry relation

$$F_{2n}(s,t) \longleftrightarrow B_{2n}(s,u) \quad \text{at} \quad t \longleftrightarrow u \qquad (6.5.12)$$

we have

$$T_{2n}(s,t) = F_{2n}(s,t) + B_{2n}(s,u) \qquad (6.5.13)$$

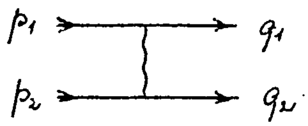
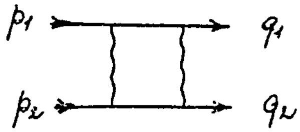
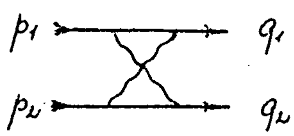
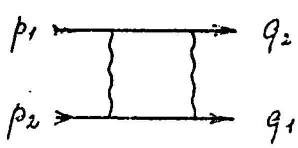
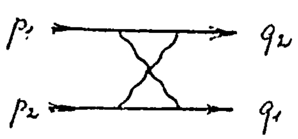
In general, the splitting (6.5.13) is not unique. As an additional condition determining this splitting one may employ the analytical properties. In particular, one may assume that the quantities $F_{2n}(s,t)$ and $B_{2n}(s,u)$ are analytic functions of momentum transfer with singularities at $t>0$ and $u>0$ respectively, and obey the nonsubtracted dispersion relation.

In this paper, in which the main task is to reconstruct the local quasipotential by perturbation theory in the region of asymptotically high energies, we will formulate the following condition:

$F_{2n}(s,t)$ - is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s\to\infty$, t-fixed (the forward scattering).

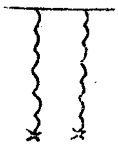
$B_{2n}(s,u)$ - is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s\to\infty$, u-fixed (the backward scattering).

The following Table is an example of the method of constructing the local quasipotential prodeeding from the set of twisted and usual eikonal graphs on the basis of the conditions stated above.

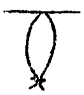
Normal graphs	F-contributions	B-contributions
	g^2	$O\left(\frac{1}{s}\right)$
	$-g^4\frac{\ln(-s)}{s}$	$O\left(\frac{1}{s^2}\right)$
	$g^4\frac{\ln s}{s}$	$g^4\frac{\ln s}{s}$
Twisted graphs	**F-contributions**	**B-contributions**
	$O\left(\frac{1}{s}\right)$	$\hat{P}$
	$O\left(\frac{1}{s^2}\right)$	$-g^4\frac{\ln(-s)}{s}\hat{P}$
	$g^4\frac{\ln s}{s}$	$g^4\frac{\ln s}{s}\hat{P}$

Here the following notation is used:

 $\sim \dfrac{1}{\Delta_\perp^2 + \mu^2}$ corresponds, in the language of quasipotential graphs, to the single scattering on Yukawa potential at high energies and fixed momenta transfer


$$\sim \int \frac{d^2 k_\perp}{(k_\perp^2 + \mu^2)\left[(\Delta_\perp + k_\perp)^2 + \mu^2\right]}$$

is the two-dimensional contraction corresponding to the double scattering on Yukawa potential


$$\sim \int \frac{d^3 k_\perp}{(k_\perp^2 + m^2)\left[(\Delta_\perp + k_\perp)^2 + m^2\right]}$$

that corresponds to the contribution to scattering from the exchange by nucleon-antinucleon pairs.

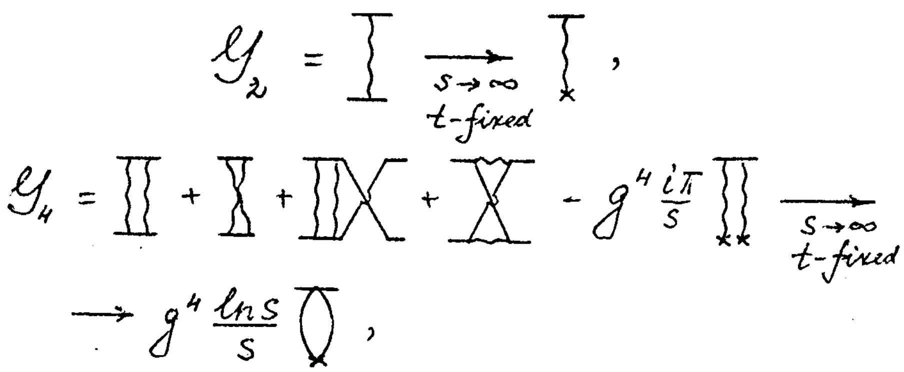
The action of operator $\hat{P}$ turns, obviously, into the substitution

$$\hat{P}\Delta_\perp = \hat{P}(p-k)_\perp \longrightarrow (p+k)_\perp \tag{6.5.14}$$

Summing the usual eikonal and twisted graphs we get for the scattering amplitude

$T = (1 + \hat{P})\Big[g^2$  $+\, g^4 \dfrac{i\pi}{s}$  $+\, g^4 \dfrac{\ln s}{s}$  $+ \dots \Big]$ (6.5.15)

Making use of the above procedure, the local quasipotential can now be reconstructed according to perturbation theory



(6.5.16)

(6.5.17)

and so on.

As has been indicated above, Y_2 represents the conventional Yukawa potential in the phase of eikonal representation. The relation (6.5.17) define the corrections of non-Yukawa type which orginate from the graph in Fig. 15. In momentum space this correction to the quasipotential is given by the formula

$$\mathcal{Y}_4(q^2) = \frac{\ln s}{s} \frac{g^4}{2(2\pi)^7} \int \frac{d^2 k_\perp}{(k_\perp^2 + m^2)[(\Delta_\perp + k_\perp)^2 + m^2]} \tag{6.5.18}$$

where the replacement $\Delta^2 = -t \to g^2$ should be performed after integrating.

Introducing α-representation we obtain from (6.5.18)

$$\mathcal{Y}_4(q^2) = \frac{g^4}{4(2\pi)^6} \frac{\ln s}{s} \int_0^1 \frac{d\alpha}{\alpha(1-\alpha)q^2 + m^2} \tag{6.5.19}$$

The representation (6.5.19) allows one to calculate the quasipotential in the coordinate representation

$$\mathcal{Y}_4(r) = \frac{g^4}{4(2\pi)^6} \frac{\ln s}{s} \int_0^1 d\alpha \int dq \frac{e^{iqr}}{\alpha(1-\alpha)q^2 + m^2} = \frac{g^4}{2(2\pi)^4} \frac{\ln s}{s} \frac{K_0(2mr)}{r} \tag{6.5.20}$$

We see that Y_4 is asymptotically smaller than the leading term (Yukawa potential) of quasipotential independent of s.

However, even in the fourth order Y_4 gives larger contribution to the scattering amplitude than the second iteration of the Yukawa potential that results in the breaking of the eikonal formula. At short distances this potential behaves as $\frac{\ln r}{r}$, i.e. it is more singular than the Yukawa potential. The connection of noneikonal terms with the increasing of singularity of the quasi-potential corrections was pointed out previously /26,40/.

We note that the method described above can be applied to calculations of the asymptotical quasipotential in higher orders of perturbation theory.

§7. CONCLUSION

In this survey we have attempted to familiarise the reader with the main ideas and elements of the mathematical formalism, which underlie straight-line path method in quantum field theory. There are many separate papers devoted to the discussion of the result obtained with the aid of this method. It is hoped that presentating them in the form of a survey has helped the reader to understand more clearly the essence of the straight-line path concept.

One may believe that further development of the straight-line path method will allow one to enlarge the range of its applications and make it one of the effective tools of quantum field theory.

The authors are deeply indebted to N.N Bogolubov for his interest in this work and for numerous fruitful discussions. One of us (A.N.S.) express his thanks to Prof. Laurikainen for hospitality and stimulating conversations.

REFERENCES

1. N.N. Bogolubov, D.V. Shirkov. "Introduction to the Theory of Quantized Fields". New York-London, Interscience Publ., 1959.
2. N.N. Bogolubov, B.V. Medvedev, M.K. Polyvanov. "Questions of the Theory of Dispersion Relations". Moscow, Fizmatgiz, 1958 (In Russian).
3. A.A. Logunov, A.N. Tavkhelidze. Nuovo Cim., 89, 380, 1963.
4. V.G. Kadyshevsky, A.N. Tavkhelidze. "Problems of Theoretical Physics", p.261, "Nauka", Moscow, 1969 (In Russian).
5. V.R. Garsevanishvili, V.A. Matveev, L.A. Slepchenko. "Problems of Elementary Particles and Atomic Nuclear Physics". "Atomizdat", 1, 92, Moscow, 1970 (In Russian).
6. D.I. Blokhintsev. Nucl. Phys., 31, 628, 1962.
7. S.P. Alliluyev, S.S. Gershtein, A.A. Logunov. Phys. Lett., 18, 195, 1965.
8. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, V.N. Pervushin, A.N. Sissakian, A.N. Tavkhelidze. Phys. Lett., 18, 195, 1965.
9. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, A.N. Sissakian. Theor. and Mat. Phys., 3, 342, 1970 (In Russian).
10. R.P. Feynman. Rev. Mod. Phys., 20, 376, 1947.
11. N.N. Bogolubov, Doklady Akademii Nauk SSSR, 99, 225, 1954. (In Russian).
12. E.S. Fradkin. Lebedev Institute Proceedings, 29, 7, 1965 (In Russian).
13. B.M. Barbashov. JETP, 48, 607, 1965 (In Russian).
14. B.M. Barbashov, D.I. Blokhintsev, V.V. Nesterenko, V.N. Pervushin. "Problems of Elementary Particle and Atomic Nucleus Physics", vol. 4, part 3, "Atomizdet", Moscow, 1973 (In Russian).

15. V.A. Matveev, A.N. Tavkhelidze. JINR, E2-5851, Dubna, 1971.

16. G.D. Milekhin, E.S. Fradkin. JETP, 45, 1926, 1963 (In Russian).

17. B.M. Barbashov, M.K. Volkov. JETP, 50, 13, 1966 (In Russian).

18. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, V.N. Pervushin, A.N. Sissakian, A.N. Tavkhelidze. Theor. and Mat. Phys., 5, 330, 1970 (In Russian).

19. D. Jennie, S. Frautschi, H. Suura. Ann. of Phys., 13, 379, 1961.

20. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, A.N. Tavkhelidze. Phys. Lett., 33B, 419, 1970.

21. B.M. Barbashov, S.P. Kuleshov, V.A. Matveev, V.N. Pervushin, A.N. Sissakian. Nuovo Cim., 4A, 731, 1971.

22. V.A. Matveev, A.N. Tavkhelidze. JINR, E2-5141, Dubna, 1971.

23. M.A. Markov. "Neutrino". "Nauka", Moscow, 1964 (In Russian).

24. V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze. "Problems of Elementary Particles and Atomic Nuclear Physics". "Atomizdat", vol. 3, part 1, Moscow, 1971. (In Russian).

25. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev. JINR, E2-5897, Dubna, 1971.

26. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev. JINR, P2-6445, Dubna, 1972 (In Russian).

27. G. Tiktopoulos, S. Treiman. Phys. Rev., D2, 805, 1970.

28. B.M. Barbashov, V.V. Nesterenko. Theor. and Mat. Phys., 4, 293, 1970 (In Russian).

29. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev. JINR, E2-5833, Dubna, 1971.

30. V.R. Garsevanishvili, S.V. Goloskokov, V.A. Matveev, L.A. Slepchenko, A.N. Tavkhelidze. Theor. and Mat. Phys., 6, 36, 1971. (In Russian).

31. R.P. Feynman. Phys. Rev., 84, 108, 1951.

32. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev. JINR, P2-6437, Dubna, 1972 (In Russian).

33. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev, A.N. Tavkhelidze. JINR, E2-7041, Dubna, 1973.

34. J. Chisholm. Proc. Cambridge Phil. Soc., 48, 300, 1952.

35. G. Tiktopoulos. Phys. Rev., 131, 480, 1963.

36. A.V. Efremov, JINR, P-1242, Dubna, 1963 (In Russian).

37. O.I. Zavialov. JETP, 47, 1103, 1964 (In Russian).

38. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev, A.N. Tavkhelidze. JINR, E2-7720, Dubna, 1974.

39. V.R. Garsevanishvili, S.V. Goloskokov, V.A. Matveev, L.A. Slepchenko. Nuclear Physics, 10, 627, 1969 (In Russian).

40. S.P. Kuleshov, V.A. Matveev, A.N. Sissakian, M.A. Smondyrev. "High Energy Physics", P2-6867, p.183, Dubna, 1972 (In Russian).