Physica Fennica Vol. 9, 151—165, 1974

The eikonal problem and the asymptotic quasipotential

S. P. Kuleshov, V. A. Matveev, A. N. Sissakian, M. A. Smondyrev and
A. N. Tavkhelidze
Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
Dubna, USSR

Received 12 March 1974

Abstract

The paper is devoted to the investigation of the structure of »noneikonal» contributions to the scattering amplitude and its connection with the asymptotic quasipotential.

Introduction

In order to reproduce correctly the main features of high energy particle scattering at small angles one can begin from the investigation of the type of interaction potential. In this connection the study of the properties of the local quasipotential and the foundation of the eikonal representation in quantum field theory are of great interest [1, 2].

From studies of the eikonal problem, the Dubna group has formulated the »straight line path approximation» [1, 3]. The essence of this approximation lies in the assumption that the large momentum transfers are suppressed in each high energy particle interaction. Consequently, there is a tendency for the large momenta of the particles in the collision process to be conserved (»inertia» of large momenta).

The type of particles transferring large momenta may change during the interaction process according to the empirical regularities observed in the inclusive reactions. For example, in the collision of fast nucleons it is necessary to take into account the possibility of radiation of hard mesons which carry away the greatest part of the initial nucleon momenta.

Generally, to obtain the eikonal formula by summation of the perturbation theory series, one assumes that the initial particles predominate in transferring large momenta [4]. The results thus obtained are in essence equivalent to those of the »$k_i k_j$-approximation» [5]. However, the existence of virtual processes with the alteration of the »predominant» particle type must lead, in general, to the violation of the orthodox eikonal representation. The possibility that such extra terms might appear in the asymptotes of some diagrams was first noted by Tiktopoulos and Treiman [6].

In this paper we study the structure of the »noneikonal» contribution to the two nucleon scattering amplitude described by a ladder-type diagram sum, without taking into account radiative corrections and vacuum polarization effects in the scalar model. It will be shown in particular that there exist in the sum of all ladder-type graphs of the eighth order, terms which violate the orthodox eikonal formula but disappear in the limit $\mu/m \to 0$, where μ and m are meson and nucleon masses, respectively. These terms are associated with the contribution to the effective quasipotential corresponding to the nucleon-antinucleon pair exchange. We then study the so-called twisted graphs which violate the eikonal formula no further than in the fourth order. In addition the corresponding correction to the quasipotential is investigated.

1. High energy asymptotes of Feynman graphs and modification of the particle propagators

We now consider the scattering amplitude of two scalar nucleons in the model $\mathscr{L}_{\text{int}} = g\colon \psi^+\psi\phi$: neglecting the radiative corrections and closed nucleon loops. This amplitude is represented as the sum of the diagrams in Fig. 1 where p_1 and p_2

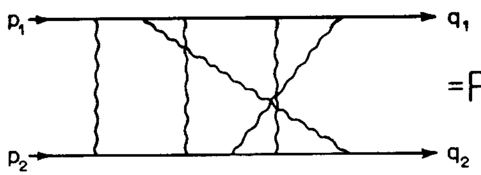


Fig. 1.

are the momenta of the incoming particles and q_1, q_2 are the momenta of the outgoing particles.

We shall use the representation for F in the Chisholm form [7], and Tiktopoulos' results and concept of the t-path [8]. Note that similar results have been obtained elsewhere [9].

When the momentum transfers in graph 1 are a zero, i.e. $p_1 = q_1$ and $p_2 = q_2$, we shall call a set of lines whose propagators depend on the momenta p_i a p-path. Thus in the graphs F there are two p-paths each forming a continuous arc. Note that each p-path is a t-path according to the definition. However the configurations of the t-paths depend on the concrete arrangement of the integration momenta while the t-paths are the topological characteristics of the given graph. In graphs being studied the integration momenta can be chosen so that the p-paths will coincide with any pair of t-paths not forming a closed loop.

The following statements can then be proved [10].

Statement 1

Let the given graph be such that the contribution to the leading asymptotes is due to the pair of $\bar{t}$-paths having no common line. Then the asymptotes of this graph will not be changed if the integration momenta are placed so that the $\bar{t}$-paths coincide with p-paths and the following modification of the propagators depending on external momenta is performed

$$\frac{1}{(\Sigma\, k_i)^2 + 2p\, \Sigma\, k_i - m^2 + i\varepsilon} \rightarrow \frac{1}{2p\, \Sigma\, k_i + i\varepsilon}, \tag{1.1}$$

i.e. we neglect masses and products of integration momenta.

Statement 2

Let the given graph be such that the contribution to the leading asymptotes is due to a pair of $\bar{t}$-paths having a common line. Also let the integration momenta be placed so that p-paths coincide with $\bar{t}$-paths. Then the asymptote of the graph is equal to the factor $(\pm 1/s)$ multiplied by the asymptote of the reduced graph obtained when we short-circuit the common line. We choose the plus sign when the external momenta in this line have the same direction; if they do not, we choose the minus sign. Dealing with the reduced graph we can use Statement 1. As is well known, the eikonal representation for the scattering amplitude

$$f \simeq \frac{is}{(2\pi)^4} \int d^2x_\perp e^{-ix_\perp \Delta_\perp} \left(e^{-\frac{ig^2}{4\pi s} K_0(\mu|x_\perp|)} - 1 \right), \tag{1.2}$$
$$s = (p_1 + p_2)^2 \rightarrow \infty, \quad t = (p_1 - q_1)^2 \quad \text{is fixed}$$

includes the contributions of the t-paths, coinciding with nucleon lines.

Previously [6] it has been pointed out that in diagrams of higher orders in powers of the coupling constant g (beginning from the 8-th) it is necessary to take into account other t-paths whose contributions may be comparable with those of the eikonal t-paths. We begin our study of the noneikonal contributions with the diagram shown in Fig. 2.

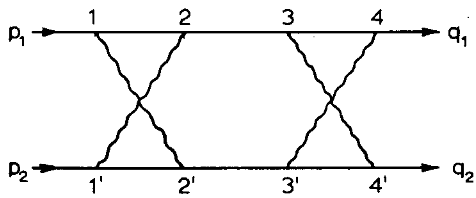


Fig. 2.

In this graph which we'll call the »xx-diagram» there exist four $\bar{t}$-paths, (1234), (1′2′3′4′), (1′234′) and (12′3′4), of the same length: three. It is then necessary to calculate a sum of contributions from the following pairs of $\bar{t}$-paths:

$$(1234;\ 1'2'3'4'),\ (1234;\ 1'234'),\ (12'3'4;\ 1'2'3'4'),\ (12'3'4;\ 1'234'). \tag{1.3}$$

Pairs (1234; 12'3'4) and (1'2'3'4'; 1'234') have no influence upon the asymptotes since these $\bar{t}$-paths form a closed loop. All pairs of $\bar{t}$-paths (1.3) lead to the same asymptotic dependence on s, namely $\ln s/s^3$. Thus, we are interested in dependence on t.

The contribution to the xx-diagram from the pair (1234; 1'2'3'4') is included in formula (1.2) and will be indicated

$$\frac{\ln s}{s^3}\, f_{\text{eik}}^{(\text{xx})}(t). \tag{1.4}$$

We will now find the contribution from the $\bar{t}$-paths (12'3'4) and (1'234'). Let us choose the integration momenta so that these paths coincide with the p-paths. Then, according to Statement 1, we can modify propagators of lines forming the $\bar{t}$-paths. We can also perform the substitution of integration momenta

$$k_i \to \frac{m}{\mu}\, k_i, \tag{1.5}$$

which results in the replacement of nucleon lines by meson ones

$$\begin{aligned} D_m\left(k\,\frac{m}{\mu}\right) &= \frac{1}{k^2\,\dfrac{m^2}{\mu^2} - m^2 + i\varepsilon} = \frac{\mu^2}{m^2}\, D_\mu(k), \\ D_m(p_1 - q_1 - k) &\to \frac{\mu^2}{m^2}\, D_\mu\left[(p_1 - q_1)\,\frac{\mu}{m} - k\right], \text{ i.e. } t \to t\,\frac{\mu^2}{m^2}. \end{aligned} \tag{1.6}$$

The propagators corresponding to the $\bar{t}$-paths will be multiplied by μ/m. Because of this fact we may consider all the lines of $\bar{t}$-paths as modified nucleon lines. As a result we obtain a diagram of the same type as in Fig. 2 but the p-paths are directed along nucleon lines.

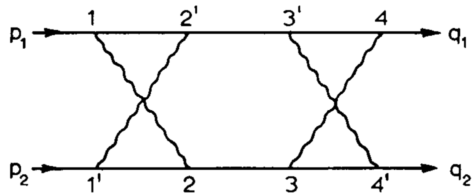


Fig. 3.

The contribution we are describing is therefore of the form

$$\frac{\ln s}{s^3} f^{(1)}_{\text{noneik}}(t),$$
$$f^{(1)}_{\text{noneik}}(t) = \frac{\mu^2}{m^2} f^{(\text{xx})}_{\text{eik}}\left(t \frac{\mu^2}{m^2}\right). \tag{1.7}$$

If the particle masses satisfy the condition

$$\frac{\mu^2}{m^2} \ll 1, \quad \frac{t}{m^2} \ll 1 \tag{1.8}$$

the contribution of noneikonal $\bar{t}$-paths will be less than that of the eikonal ones.

We now have only to consider the contribution to the asymptotes of the xx-diagram from the pair of $\bar{t}$-paths (1'2'3'4') and (12'3'4). The remaining pair (1234) and (1'234') (see (1.3)) evidently make the same contribution. The $\bar{t}$-paths (1'2'3'4') and (12'3'4) being short-circuited, we obtain the reduced graph in Fig. 4. It then

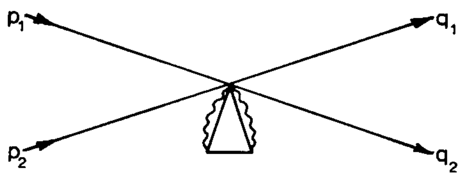


Fig. 4.

follows that the contribution of these $\bar{t}$-paths does not depend on momentum transfers, i.e. it can be represented in the form

$$\frac{\ln s}{s^3} \frac{1}{\mu^2} \phi\left(\frac{\mu^2}{m^2}\right). \tag{1.9}$$

Let us find the form of the function $\phi(\mu^2/m^2)$ if the condition (1.8) is satisfied. For this purpose choose the integration momenta in xx-diagram so that the p-paths coincide with the $\bar{t}$-paths (1'2'3'4') and (12'3'4). Then using Statement 2, we obtain the desired contribution which is equal to the reduced graph asymptotes (Fig. 5) multiplied by $1/s$.

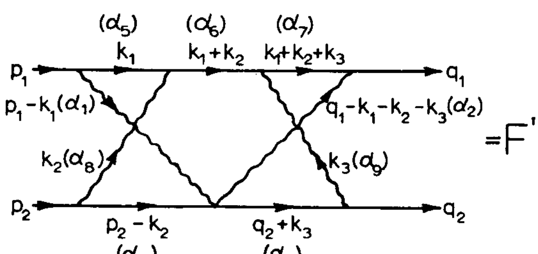


Fig. 5.

When $s \to \infty$, the asymptotes of F' will be of the form

$$F' \simeq \frac{\ln s}{s^2} \text{ const} \int d\alpha_1 \ldots d\alpha_9 \delta(1 - \alpha_1 - \alpha_2)\delta(1 - \alpha_3 - \alpha_4) \cdot \\ \cdot \delta(1 - \alpha_5 - \alpha_6 - \alpha_7 - \alpha_8 - \alpha_9) \frac{C_0}{(g_0 t + h_0)\tilde{f}_0^2}, \tag{1.10}$$

where

$$g_0 = 0 \\ h_0 = -\mu^2 \left[\frac{m^2}{\mu^2}(\alpha_5 + \alpha_6 + \alpha_7) + \alpha_8 + \alpha_9\right] C_0. \tag{1.11}$$

From eqs. (1.10) and (1.11) we get the expression for the function ϕ defined by the relation (1.9).

$$\phi\left(\frac{\mu^2}{m^2}\right) = \text{const} \int \{d\alpha\} \Pi\delta(1 - \Sigma\delta\alpha) \cdot \frac{\delta(1 - \alpha_5 - \alpha_6 - \alpha_7 - \alpha_8 - \alpha_9)}{\tilde{f}_0^2 \left[\frac{m^2}{\mu^2}(\alpha_5 + \alpha_6 + \alpha_7) + \alpha_8 + \alpha_9\right]} \tag{1.12}$$

from which follows

$$\phi\left(\frac{\mu^2}{m^2}\right) = \text{const} \frac{\mu^2}{m^2} \ln \frac{\mu^2}{m^2} \tag{1.13}$$

under condition (1.8). Note that const in eq. (1.13) now includes all the integrals over α_i. Taking into account the equality $f_{\text{eik}}(t = 0) = \text{const}/\mu^2$ and eqs. (1.4), (1.7), (1.9) and (1.13), we obtain the asymptotes of the xx-diagram

$$f^{(\text{xx})}(t) \simeq \frac{\ln s}{s^3} \{f^{(\text{xx})}_{\text{eik}}(t) + f^{(\text{xx})}_{\text{noneik}}(t)\}, \tag{1.14}$$

where

$$f^{(\text{xx})}_{\text{noneik}}(t) = \frac{\mu^2}{m^2} f^{(\text{xx})}_{\text{eik}}\left(t \frac{\mu^2}{m^2}\right) + \text{const} f^{(\text{xx})}_{\text{eik}}(t = 0) \frac{\mu^2}{m^2} \ln \frac{\mu^2}{m^2},$$

when $s \to \infty$, t is fixed and $\mu^2/m^2 \ll 1$.

2. Asymptotes of the nucleon-nucleon scattering amplitude. Other diagrams

In the previous section we have considered one of the eighth order diagrams. We now turn to all the remaining diagrams except for the twisted graphs discussed in section 3. In these diagrams there are three types of noneikonal $\bar{t}$-paths which can contribute to the leading asymptotes.

In the first type we include noneikonal $\bar{t}$-paths which have no common line. Except for the xx-diagram there is only one graph with such $\bar{t}$-paths (see Fig. 6)

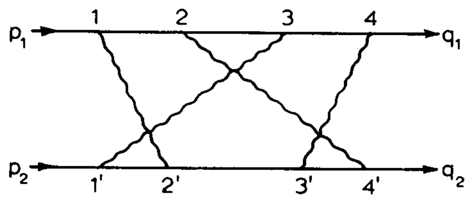


Fig. 6.

and two cross-symmetric diagrams. The contribution to the asymptotes of diagram 6 can be written in the same form as (1.7)

$$f^{(2)}_{\text{noneik}}(t) = \frac{\ln s}{s^3}\,\frac{\mu^2}{m^2}\,\text{cross}\,f^{(2)}_{\text{eik}}\left(t\,\frac{\mu^2}{m^2}\right). \tag{2.1}$$

If we add the eikonal contribution of diagrams xx and 6 to those of the cross-symmetric ones, then $\ln s$ cancels and we obtain the total eikonal contribution

$$\frac{1}{s^3}\,f_{\text{eik}}(t). \tag{2.2}$$

Then, according to eqs. (1.7) and (2.1), the noneikonal $\bar{t}$-path contribution to the same sum has the form

$$f_{\text{noneik}}(t) = \frac{1}{s^3}\,\frac{\mu^2}{m^2}\,f_{\text{eik}}\left(t\,\frac{\mu^2}{m^2}\right). \tag{2.3}$$

In the eighth order there are no other noneikonal contributions depending on the momentum transfers.

We attribute the noneikonal $\bar{t}$-paths, having a common nucleon line, to the second type. Their contribution does not depend on the momentum transfers and has been considered above for the xx-diagram (see eqs. (1.9)—(1.14)). However the similar contributions are cancelled in the sum of all diagrams with such $\bar{t}$-paths [10].

To the third type we attribute those $\bar{t}$-paths which have a common meson line. Their contribution to the leading asymptotes is also independent of the momentum transfers. In the eighth order there are some diagrams with the third type $\bar{t}$-paths. As an example, we consider one of these graphs (see Fig. 7), keeping in mind the validity of the results for other similar diagrams.

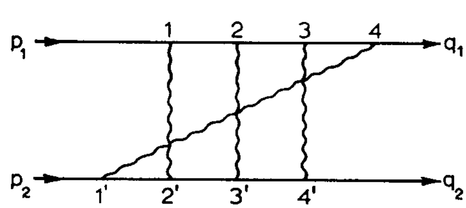


Fig. 7.

In this diagram the $\bar{t}$-paths (1'434') and (12'1'4) are noneikonal and belong to the third type. Their contribution may be written in the form (1.9)

$$\frac{\ln s}{s^3}\frac{1}{\mu^2}\Phi\left(\frac{\mu^2}{m^2}\right). \tag{2.4}$$

Using the same technique as in section 1 we obtain under condition (1.8)

$$\Phi\left(\frac{\mu^2}{m^2}\right) = \text{const}\,\frac{\mu^2}{m^2}. \tag{2.5}$$

This leads to the following expression for noneikonal contributions in the 8-th order

$$f^{(8)}_{\text{noneik}} = \frac{1}{s^3}\left\{\frac{\mu^2}{m^2} f_{\text{eik}}\left(t\,\frac{\mu^2}{m^2}\right) + \frac{\text{const}}{\mu^2}\,\Phi\left(\frac{\mu^2}{m^2}\right)\right\}. \tag{2.6}$$

The $f_{\text{eik}}(t)$ in eq. (2.6) denotes the t-dependent factor in the main asymptotic term of the sum of the diagram shown in Figs. 2 and 6 together with its cross-symmetric partners, when only the contributions of the eikonal paths are taken into account. The function $\Phi(\mu^2/m^2)$ goes as μ^2/m^2 at $\mu^2/m^2 \ll 1$. When the ratio μ^2/m^2 is small one can neglect the dependence on momentum transfers. Then if $t/m^2 \ll 1$

$$f_{\text{eik}}\left(t\,\frac{\mu^2}{m^2}\right) \simeq f_{\text{eik}}(0) = \frac{\text{const}}{\mu^2}, \tag{2.7}$$

which gives

$$f^{(8)}_{\text{noneik}}\Big|_{\substack{s\to\infty\\ t\text{-fixed}\\ \frac{\mu^2}{m^2}\ll 1}} \simeq \text{const}\,\frac{g^8}{s^3 m^2}. \tag{2.8}$$

Similar investigation of the diagrams of higher orders shows that only $\bar{t}$-paths of the first and third types contribute to the leading asymptotes. The result will be (compare with (2.8))

$$f^{(2l+2)}\Big|_{\substack{s\to\infty\\ t\text{-fixed}\\ \frac{\mu^2}{m^2}\ll 1}} \sim \frac{g^{2l+2}\text{const}}{s^3(m^2)^{l-2}}, \quad l \geq 3. \tag{2.9}$$

Note that when $t = 0$ the eikonal formula (1.2) gives the following result:

$$f^{(2l+2)}_{\text{eik}}(t=0) = \text{const}\,\frac{g^{2l+2}}{s^l\mu^2}. \tag{2.10}$$

Thus if one neglects twisted graphs one gets for the ratio of the noneikonal and eikonal contributions to the amplitude of the given order, the result:

$$\left.\frac{f^{(2l+2)}_{\text{noneik}}}{f^{(2l+2)}_{\text{eik}}}\right|_{\substack{s\to\infty\\ t\text{-fixed}\\ \frac{\mu^2}{m^2}\ll 1}} \simeq \text{const}\ \frac{\mu^2}{m^2}\left(\frac{s}{m^2}\right)^{l-3}, \quad l \geq 3. \tag{2.11}$$

From eq. (2.11) it follows that in the region

$$s \to \infty,\ \frac{\mu^2}{m^2} \ll 1,\ s \sim m^2,\ t = 0 \tag{2.12}$$

the eikonal part of the scattering amplitude dominates the noneikonal part, and eq. (1.2) gives the main asymptotic terms in each order in powers of g^2. On the other hand, it follows from eq. (2.11) that, in the region (2.12), but $s \gg m^2$, the noneikonal contributions dominate the eikonal contributions.

3. Twisted eikonal graphs and quasipotential structure

We have shown above that the investigation of the ladder type graphs in the scalar model demonstrates that the eikonal formula corresponds to the account of the $\bar{t}$-paths, coinciding with nucleon lines. The predominant particles, carrying large momenta, are nucleons in that case and do not change their type in virtual processes.

The noneikonal contributions to the amplitude are due to processes in which the predominant particle type is altered, i.e. in which large momenta are transferred from nucleons to mesons and vice versa. The important question then arises as to the significance of twisted graphs in which the final momenta q_1 and q_2 are exchanged (see Fig. 1). The possibility of large momentum being carried by the meson leads to the possibility that the corresponding contribution may dominate the eikonal one in the same order of coupling constant. In this section we shall consider these twisted diagrams and also study the reconstruction of the corresponding quasipotential.

To the second order of perturbation theory the only twisted graph which exists is shown in Fig. 8 with the known asymptote $1/s$.

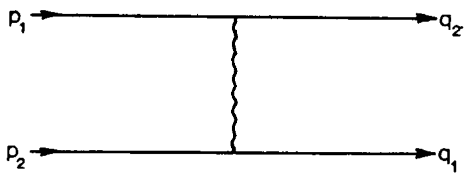


Fig. 8.

In the fourth order we already have two such diagrams. One of them possesses a weaker asymptote ($1/s^2$) than the corresponding nontwisted graph. The other (see Fig. 9) has the asymptote $\ln s/s$ that results in the breaking of the eikonal representation for the sum of generalized ladder graphs in the fourth order. (Recall that the noneikonal contributions of the previous sections appear only in the eighth order of perturbation theory).

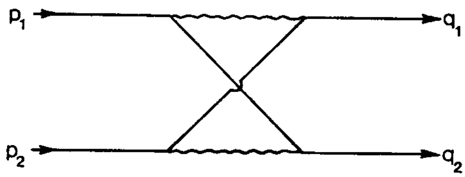


Fig. 9.

In the subsequent order we have six twisted diagrams. The diagram drawn in Fig. 10 possesses the strongest asymptotes [11] and behaves like $\ln^2 s/s$.

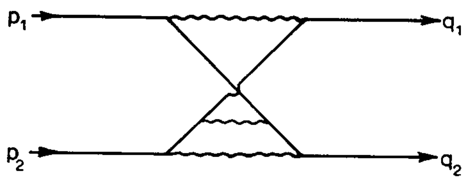


Fig. 10.

Consideration of the first six orders allows one to conjecture that in the higher orders the diagrams of the shape in Fig. 11 with asymptotes $g^{2n} \ln^{n-1} s/s$ will dominate.

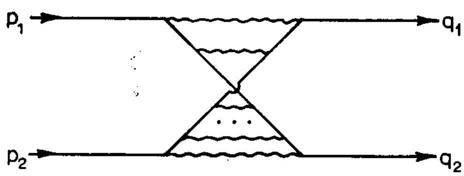


Fig. 11.

If only the leading asymptotic terms in each order of perturbation theory are summed, as is usual when deriving the eikonal representation, one obtains the following asymptotic expression for the sum F of twisted graphs:

$$F\Big|_{\substack{s\to\infty \\ t \text{ fixed}}} \simeq -i \frac{g^2}{(2\pi)^4} s^{\alpha(t)}, \tag{3.1}$$

$$\alpha(t) = -1 + \frac{g^2}{8\pi^2} \frac{1}{\sqrt{-t(4m^2-t)}} \ln \frac{\sqrt{1-\frac{4m^2}{t}}+1}{\sqrt{1-\frac{4m^2}{t}} -}. \tag{3.2}$$

With such a summation, the coefficient for $s^{\alpha(t)}$ and the expression for $\alpha(t)$ are computed to an accuracy of g^2 only. However, it already follows from (3.1) and (3.2) that within the framework of the scalar model the sum of ladder graphs leads by no means to the eikonal representation proper to the Yukawa potential scattering. Indeed, as has already been mentioned, the twisted graphs are due to the identity of scattering particles. An identity of particles implies in general the presence of an exchange force in two-particle quasipotential as well as in quantum mechanics.

The standard method of constructing the local quasipotential by perturbation theory [12] can be generalized in different ways when the exchange forces are present. Here we will briefly describe a method in which the normal and exchange interaction parts are introduced through the expression

$$V(s; p, k) = \mathcal{G}(s; p - k) + \mathcal{H}(s; p + k). \tag{3.3}$$

The quasipotential scattering amplitude is represented by the sum of two terms [13]

$$T(s; p, k) = G(s; p, k) + H(s; p, k), \tag{3.4}$$

satisfying the system of linear equations

$$\begin{pmatrix} G \\ H \end{pmatrix} = \begin{pmatrix} \mathcal{G} \\ \mathcal{H} \end{pmatrix} + \begin{pmatrix} \mathcal{G} & \mathcal{H} \\ \mathcal{H} & \mathcal{G} \end{pmatrix} \times \begin{pmatrix} G \\ H \end{pmatrix}, \tag{3.5}$$

where the symbol »×» means integration

$$\int \frac{dq}{\sqrt{m^2 + q^2}} \frac{1}{m^2 + q^2 - E^2}.$$

For a scattering of two identical particles we have

$$\begin{aligned} \mathcal{H}(s; p + k) &= \hat{P}\mathcal{G}(s; p - k) = \mathcal{G}(s; p + k), \\ H(s; p, k) &= \hat{P}G(s; p, k) = G(s; p, - k) = G(s; - p, k), \end{aligned} \tag{3.6}$$

where $\hat{P}$ is the transposition operator for the coordinates of the two particles. With this, the function G obeys the conventional equation by Logunov-Tavkhelidze[1]

$$G = \mathcal{G} + \mathcal{G} \times G. \tag{3.7}$$

Equation (3.7) can be used to construct the local quasipotential $\mathcal{G}$ over the given perturbation series defining the amplitude:

$$\begin{aligned} \mathcal{G}_2 &= [G_2], \\ \mathcal{G}_4 &= [G_4] - [\mathcal{G}_2 \times \mathcal{G}_2], \\ \mathcal{G}_6 &= [G_6] - [\mathcal{G}_2 \times \mathcal{G}_4] - [\mathcal{G}_4 \times \mathcal{G}_2] - [\mathcal{G}_2 \times \mathcal{G}_2 \times \mathcal{G}_2] \end{aligned} \tag{3.8}$$

and so on.

[1] Eq. (3.7) follows from (3.6) if one takes into account the fact that for identical particles the integration over intermediate two-particle states contains the statistical factor 1/2! and $\mathcal{G} \times G = \mathcal{H} \times H$, $\mathcal{G} \times H = \mathcal{H} \times G = \hat{P}(\mathcal{G} \times G)$.

The symbol »[. . .]« means the »local« continuation of the mass shell $E^2 = p^2 + m^2 = k^2 + m^2$ of an arbitrary function $A(E; p, k) = A(s, t, u, \delta)$, where

$$s = 4E^2,\ t = -(p - k)^2,\ u = -(p + k)^2,$$
$$\delta = p^2 - k^2.$$

In this notation we have

$$[A(s, t, u, \delta)] = A(s, t, 4m^2 - s - t, 0). \tag{3.9}$$

The quasipotential constructed in this way makes it possible, in turn, to reconstruct the initial scattering amplitude on the mass shell.

We should stress, however, that perturbation theory defines the amplitude T as a whole but not G and H parts separately, i.e.

$$T_{2n}(s, t) = [G_{2n}(E; p, k) + H_{2n}(E; p, k)]. \tag{3.10}$$

Defining

$$\begin{aligned} F_{2n}(s, t) &= [G_{2n}(E; p, k)], \\ B_{2n}(s, u) &= [H_{2n}(E; p, k)], \end{aligned} \tag{3.11}$$

which are connected in the case of scattering of identical particles by the symmetry relation

$$F_{2n}(s, t) \to B_{2n}(s, u) \quad \text{at} \quad t \to u, \tag{3.12}$$

we have

$$T_{2n}(s, t) = F_{2n}(s, t) + B_{2n}(s, u). \tag{3.13}$$

In general, the splitting (3.13) is not unique. As an additional condition fixing this splitting one may exmploy the analytical properties. In particular, one may assume that the quantities $F_{2n}(s, t)$ and $B_{2n}(s, u)$ are analytic functions of momentum transfer with singularities at $t > 0$ and $u > 0$ respectively, and obey the non-subtracted dispersion relation.

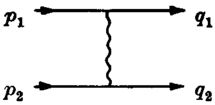
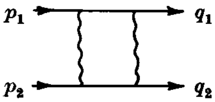
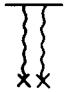
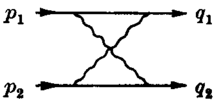
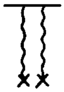
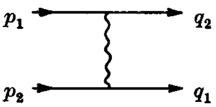
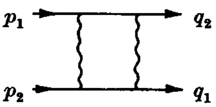
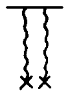
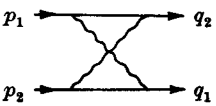
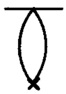
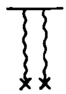
In this paper, with the main task of reconstructing the local quasipotential by perturbation theory in the region of asymptotically high energies, we will formulate the following condition:

$F_{2n}(s, t)$ is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s \to \infty$, t-fixed (the forward scattering).

$B_{2n}(s, u)$ is defined by the leading asymptotic term of the amplitude T_{2n} in the region $s \to \infty$, u-fixed (the backward scattering).

Table I illustrates the method of constructing the local quasipotential proceeding from the set of twisted and usual eikonal graphs on the basis of the conditions stated above.

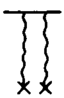
Table I.

Normal graphs	*F*-contributions	*B*-contributions
	g^2 	$O\left(\frac{1}{s}\right)$
	$-g^4\frac{\ln(-s)}{s}$ 	$O\left(\frac{1}{s^2}\right)$
	$g^4\frac{\ln s}{s}$ 	$g^4\frac{\ln s}{s}$ 
Twisted graphs	***F*-contributions**	***B*-contributions**
	$O\left(\frac{1}{s}\right)$	$\hat{P}$ 
	$O\left(\frac{1}{s^2}\right)$	$-g^4\frac{\ln(-s)}{s}\hat{P}$ 
	$g^4\frac{\ln s}{s}$ 	$g^4\frac{\ln s}{s}\hat{P}$ 

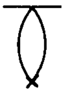
In Table I the following notation is used:

 $\sim \dfrac{1}{\Delta_\perp^2 + \mu^2}$

corresponding, in the language of quasipotential graphs, to the single scattering on the Yukawa potential at high energies and fixed momentum transfer;

 $\sim \displaystyle\int \frac{d^2 k_\perp}{(k_\perp^2 + \mu^2)[(\Delta_\perp + k_\perp)^2 + \mu^2]}$

is the two-dimensional contraction corresponding to the double scattering on the Yukawa potential;



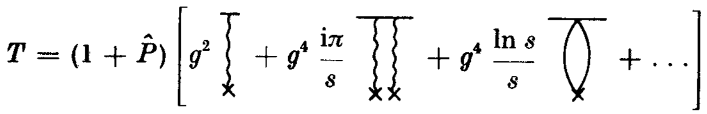
$$\sim \int \frac{d^2 k_\perp}{(k_\perp^2 + \mu^2)[(\Delta_\perp + k_\perp)^2 + m^2]},$$

which in turn corresponds to the contribution to scattering from the exchange by a nucleon-antinucleon pair.

The action of the operator $\hat{P}$ obviously turns into the substitution

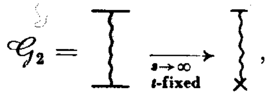
$$\hat{P}\Delta_\perp = \hat{P}(p - k)_\perp \to (p + k)_\perp \tag{3.14}$$

Summing the usual eikonal and twisted graphs we get for the scattering amplitude:

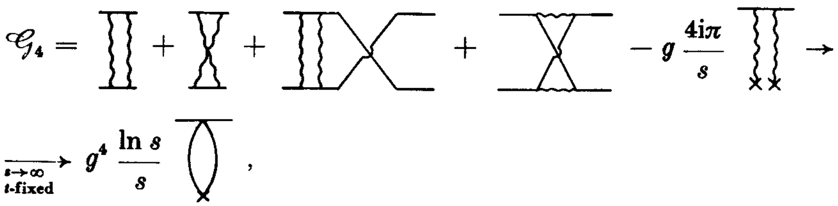


(3.15)

Making use of the above procedure the local quasipotential can now be reconstructed by perturbation theory



(3.16)



(3.17)

and so on.

As has been indicated above, $\mathcal{G}_2$ represents the conventional Yukawa potential in the phase of eikonal representation. The relation (3.17) defines the correction of the non-Yukawa type which originates from the graph in Fig. 9. In coordinate space this correction to the quasipotential is given by the formula [11]

$$\mathcal{G}_4(r) = \frac{g^4}{2(2\pi)^4} \frac{\ln s}{s} \frac{K_0(2mr)}{r}. \tag{3.18}$$

We see that $\mathcal{G}_4$ is asymptotically smaller than the leading term (Yukawa potential) of the quasipotential independent of s. However, even in the fourth order, $\mathcal{G}_4$ gives a larger contribution to the scattering amplitude than does the second iteration of the Yukawa potential that results in breaking of the eikonal formula. At short distances this potential behaves as $\ln r/r$ i.e. it is more singular than the Yukawa potential. The connection of non-eikonal terms with the increasing of singularity of the quasipotential corrections was previously pointed out [14].

Note that the method described above can be applied to calculations of the asymptotic quasipotential in higher orders of the perturbation theory, including corrections from the noneikonal terms of previous paragraphs.

Thus, noneikonal contributions to the amplitude appear when changing the sort of predominant particle, i.e. when allowing for the processes with large momenta transferred from nucleons to mesons and vice versa. Such a possibility results in the contribution from twisted graphs dominating the eikonal ones in the same order of the perturbation theory. This leads to the fact that the local quasipotential in the region of high energies will be represented by a power-series in the coupling constant, each term of which gives a correction to the Yukawa interaction. It should be stressed again that in spite of their smallness these corrections are necessary to find the correct asymptotes of amplitude.

Acknowledgements

We express our deep gratitude to N. N. Bogolubov for interest in the paper and valuable remarks and also to B. M. Barbashov, A. V. Efremov, A. A. Logunov, D. V. Shirkov, V. S. Vladimirov for fruitful discussions.

References

1. A. N. Tavkhelidze, in High Energy Physics, p. 367. Kiev, 1972.
2. M. M. Islam, Lectures in Theoretical Physics (eds. A. O. Barut and W. E. Brittin), vol. XIV A. Colorado Associated University Press, Boulder, 1972.
3. B. M. Barbashov, S. P. Kuleshov, V. A. Matveev, V. N. Pervushin, A. N. Sissakian and A. N. Tavkhelidze, Phys. Lett. **33 B**, 484 (1970).
4. M. Levy and J. Sucher, Phys. Rev. **186**, 1656 (1969); H. Cheng and T. Wu, XV Int. Conf. on High Energy Physics. Kiev, 1970.
5. E. S. Fradkin, Proc. Lebedev Phys. Inst. (USSR) **29**, 7 (1965).
   B. M. Barbashov, Zh. Eksp. & Teor. Fiz. (USSR) **48**, 607 (1965) [transl. Sov. Phys. – JETP **21**, 402 (1965)].
   H. D. I. Abarbanel and C. Itzykson, Phys. Rev. Lett. **23**, 53 (1969).
6. G. Tiktopoulos and S. Treiman, Phys. Rev. **3 D**, 1037 (1971).
7. J. Chisholm, Proc. Cambridge Phil. Soc. **48**, 300 (1953).
8. G. Tiktopoulos, Phys. Rev. **131**, 480 (1963).
9. A. V. Efremov, Joint Institute of Nuclear Research Preprint JINR-P-1242 (1963).
   O. I. Zavialov, Zh. Eksp. & Teor. Fiz. (USSR) **47**, 1103 (1964) [transl. Sov. Phys. – JETP **20**, 736 (1965)].
10. S. P. Kuleshov, V. A. Matveev, A. N. Sissakian, M. A. Smondyrev and A. N. Tavkhelidze, Joint Institute of Nuclear Research Preprint JINR-E2-7041 (1973).
11. S. P. Kuleshov, V. A. Matveev, A. N. Sissakian, M. A. Smondyrev and A. N. Tavkhelidze, Joint Institute of Nuclear Research Preprint JINR-E2-7720 (1974).
12. A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
    A. N. Tavkhelidze, Lectures on Quasipotential Method, Tata Institute, Bombay, 1963.
13. V. R. Garsevanishvili, S. V. Goloskokov, V. A. Matveev and L. A. Slepchenko, Yadernaya Fiz. (USSR) **10**, 627 (1969) [transl. Sov. J. Nucl. Phys. **10**, 361 (1970)].
14. S. P. Kuleshov, V. A. Matveev, A. N. Sissakian and M. A. Smondyrev, Joint Institute of Nuclear Research Preprint JINR-P2-6445 (1972); Joint Institute of Nuclear Research Preprint JINR-P2-6867 (1972).