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Some mathematical methods of quantum field theory

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Abstract

In this paper some mathematical realizations of the straightline path concept at high energies are presented. Use is made of methods of functional integration and operator formalism which make it possible to provide a consistent account of the deflection of the particle paths from linear trajectories.

1. The methods of the theory of measure and integration in functional spaces are at present extensively employed for investigations in quantum field theory. This approach represents solutions to the exact field equations in the form of functional integrals [1,2]. However, as one has no technique on hand for calculations of rather general quadratures the functional integrals are »the things in themselves» in the sense that usually the necessary information has to be derived step by step, through the use of some approximation procedure. The simplest and most wellknown of these procedures are those which allow one to deal at every step of the calculations with the Gaussian quadratures only. Thus, when studying the infrared asymptotes of Green's functions in quantum electrodynamics some approximate methods were proposed [3,4]. In the language of Feynman diagrams, in particular, these correspond to the modification of nucleon propagators where terms of the type of $k_i k_j$ are neglected (k_i and k_j are momenta of various real and virtual mesons emitted by nucleons). For instance,

$$\frac{1}{\left(p - \sum_{i=1}^{n} k_i\right)^2 - m^2} \to \frac{1}{\sum_{i=1}^{n} k_i^2 - 2p \sum_{i=1}^{n} k_i} . \tag{1.1}$$

Later on [5—8] these methods were developed and successfully applied to investigations of high energy elastic and inelastic scattering of particles. Keeping the Feynman representation of the scattering amplitude as a sum over paths, it has been shown [3—8] that the approximation of the type of $k_i k_j = 0$ is the same as taking into account those paths which, for high energies, approach most closely the straight-line trajectories directed along the momenta of incoming and outgoing particles, respectively.

In this paper a number of approximation procedures are suggested. These procedures are connected in a general way with those referred to above [3—8]. Some applications of these approximate methods for high energies and fixed momentum transfers are also considered. The interest in this problem arises from the fact that there are strong arguments supporting the domination of straight-line particle trajectories in the asymptotic domain [6]. This statement can be illustrated by the coincidence of the exact solution with that taking into account only the straight-line trajectories, found in quantum field theory within the framework of the quasipotential equation [9] under the condition of quasipotential smoothness [10]. The analysis of the approximate methods considered here indicates that these are in fact realizations of the concept of straight-line paths. It should be emphasized that the approximate method must be chosen for each specific problem and definite kinematical domain.

2. We consider a functional integral over the Gaussian measure

$$\int \frac{\delta \nu}{\text{const}}\, e^{-i\int d\xi \nu^2(\xi)} e^{g\pi[\nu]}, \tag{2.1}$$

where $\pi[\nu]$ is a certain functional, and const means a normalization constant. As is well known, the calculation of (2.1) can be reduced to finding functional derivatives according to the formula

$$\int [\delta\nu] e^{g\pi[\nu]} = \exp\left\{\frac{1}{4i}\int d\xi\, \frac{\delta^2}{\delta\nu^2(\xi)}\right\} e^{g\pi[\nu]}\Big|_{\nu=0}. \tag{2.2}$$

In addition, in some quantum field theory problems [11] it is necessary to determine the differential operator $\exp\left\{\frac{i}{2}\int D\, \frac{\delta^2}{\delta\nu^2}\right\}$ where

$$\int D\, \frac{\delta^2}{\delta\nu^2} = \int d\xi_1 d\xi_2 D(\xi_1, \xi_2)\, \frac{\delta^2}{\delta\nu(\xi_1)\delta\nu(\xi_2)},$$

and $D(\xi_1, \xi_2)$ is a function of a propagator type. Bearing in mind further applications, we have united the problems as follows.

We have to find the functional $\Pi[\nu]$ from the relation

$$e^{\Pi[\nu]} = \exp\left\{\frac{i}{2}\int D\, \frac{\delta^2}{\delta\nu^2}\right\} e^{g\pi[\nu]} = \overline{e^{g\pi[\nu]}}, \tag{2.3}$$

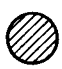
where $\pi[\nu]$ is a given functional and D is a function of two variables. When

$$D = -\tfrac{1}{2}\,\delta(\xi_1 - \xi_2)\,, \tag{2.4}$$

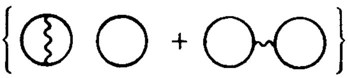
the value of the functional $\Pi[\nu]$ at $\nu = 0$ determines the functional integral according to (2.2). To simplify the formulae, the action of the differential operator will sometimes be denoted by the sign of averaging as in (2.3).

For graphic demonstration we introduce the notation

$\pi[\nu] \rightarrow$  , $\dfrac{\mathrm{i}}{2}\displaystyle\int D\,\frac{\delta^2}{\delta\nu^2}\,\pi \rightarrow$  ,

$\exp\left\{\dfrac{\mathrm{i}}{2}\displaystyle\int D\,\frac{\delta^2}{\delta\nu^2}\right\}\pi[\nu] = \bar{\pi} \rightarrow$  . (2.5)

In this notation, for example,

$\dfrac{\mathrm{i}}{2}\displaystyle\int D\,\frac{\delta^2}{\delta\nu^2}\,\pi^2[\nu] \rightarrow 2$ 

where, following the usual terminology, we call the first two terms the unconnected graphs. Let us stress that, in spite of the obvious analogy of the (2.5) graphs with the Feynman diagrams, in many cases their appearance has nothing to do with the usual Feynman graphs.

Assume now that the structure of the functional $\pi[\nu]$ is such that there exists a small parameter connected with a loop. In this case there is an approximation procedure which we call the correlative one, and according to which, we seek $\Pi[\nu]$ in the form of the series

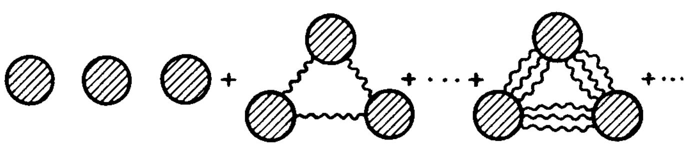
$$\Pi = \sum_{n=1}^{\infty} g^n \Pi_n\,. \tag{2.6}$$

Substituting (2.6) in (2.3), we immediately obtain

$\Pi_1 = \bar{\pi} \rightarrow$ 

$\Pi_2 = \dfrac{1}{2!}\,(\overline{\pi^2} - \bar{\pi}^2) \rightarrow$ 

$\Pi_3 = \dfrac{1}{3!}\,[\overline{\pi^3} - \bar{\pi}^3 - 3\bar{\pi}(\overline{\pi^2} - \bar{\pi}^2)] \rightarrow$

$\rightarrow$  (2.7)

. . . .

$$\Pi_n = \frac{1}{n!}\,\overline{\pi^n}\Big|_{\text{connected part}}$$

Considering graphs (2.7), we make sure that the correlative methods really correspond to the expansion in the number of loops, and that only the connected part of the sum of all graphs with n loops contributes to Π_n.

Truncating the series (2.6), we obtain the approximate expression for the functional Π. This approximation is valid when the inequality

$$\overline{\pi^n}\Big|_{\substack{\text{connected}\\ \text{part}}} \ll \overline{\pi^n}\Big|_{\substack{\text{unconnected}\\ \text{part}}} \tag{2.8}$$

is satisfied for any $n \geq 2$. In this case considering only Π_1 while expanding e^{Π} in a power series of g one obtains the leading terms of each order. The consideration of Π_2 gives us the corrections and so on.

The correlative procedure is closely connected with an expansion of the following type

$$\overline{e^{g\pi}} = e^{g\bar{\pi}}\left[1 + \sum_{n=2}^{\infty} \frac{g^n}{n!}\, \overline{(\pi - \bar{\pi})^n}\right]. \tag{2.9}$$

Such an expansion has been met previously [4,5]. It has in general the same domain of application as the correlative approximation and differs from it by giving the smaller number of correction terms in each order of g. Let us still note that the higher correction terms have, from our point of view, a more simple geometrical meaning (see (2.7)) in the correlative expansion which simplifies its usage to a certain extent.

As was mentioned above, the approximations under consideration are satisfactory when there exists a small parameter connected with a loop. But it may happen, that the theory contains a small parameter, connected with the line that arises when a functional $\pi[\nu]$ is varied. Then it is possible to make an expansion in the number of lines connecting the different loops. Representing π in the form

$$\pi[\nu] = \int \delta\eta\, \tilde{\pi}[\eta] e^{-i \int d\xi \eta(\xi)\nu(\xi)} \tag{2.10}$$

and substituting (2.10) in (2.3), we obtain

$$\begin{aligned} e^{\Pi_\varepsilon[\nu]} = 1 + \sum_{n=1}^{\infty} \frac{g^n}{n!} \int \prod_{j=1}^{n} \{\delta\eta_j \tilde{\pi}[\eta_j]\} \cdot \\ \cdot \exp\left\{- i \int \nu(\sum_{j=1}^{n} \eta_j) - \frac{i}{2} \int D(\sum_{j=1}^{n} \eta_j)^2\right\} \exp\left\{- i\varepsilon \int D(\sum_{i<j} \eta_i\eta_j)\right\}, \end{aligned} \tag{2.11}$$

where a small parameter ε is ascribed to the terms with different η and $\Pi[\nu] = \Pi_\varepsilon[\nu]$ at $\varepsilon = 1$.

We now seek the functional $\Pi_\varepsilon[\nu]$ in the form

$$\Pi_\varepsilon[\nu] = \sum_{n=0}^{\infty} \varepsilon^n \Pi_{n+1}. \tag{2.12}$$

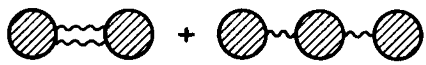
Confining ourselves to the first few terms of the series (2.12), we come to an approximation which we shall call the »$\eta_i\eta_j$-appproximation».

The calculations lead to the following expressions for the few first terms

$$\Pi_1 = g\bar{\pi} \rightarrow$$  ,

$$\Pi_2 = \frac{ig^2}{2} \int D \left(\frac{\delta\bar{\pi}}{\delta\nu}\right)^2 \rightarrow$$  ,

$$\Pi_3 = \frac{g^2}{2i} \int D_{13}D_{24} \frac{\delta^2\bar{\pi}}{\delta\nu_1\delta\nu_2} \left\{\frac{1}{2i} \frac{\delta^2\bar{\pi}}{\delta\nu_3\delta\nu_4} + g \frac{\delta\bar{\pi}}{\delta\nu_3} \frac{\delta\bar{\pi}}{\delta\nu_4}\right\} \qquad (2.13)$$

$\rightarrow$  ,

where figures indicate the order of the contraction, i.e.

$$\int D_{13}D_{24} \frac{\delta^2\bar{\pi}}{\delta\nu_1\delta\nu_2} \frac{\delta^2\bar{\pi}}{\delta\nu_3\delta\nu_4} = \int d\xi_1 d\xi_2 d\xi_3 d\xi_4 \cdot$$

$$D(\xi_1, \xi_3)D(\xi_2, \xi_4) \frac{\delta^2\bar{\pi}}{\delta\nu(\xi_1)\delta\nu(\xi_2)} \frac{\delta^2\bar{\pi}}{\delta\nu(\xi_3)\delta\nu(\xi_4)}$$

and so on.

Then we have found the expansion in the number of lines, connecting different loops. As we deal with the connected graphs the number of such lines k and of the loops n satisfy the inequality

$$k \geqq n - 1. \qquad (2.14)$$

This results in the inclusion of the sum of the first n terms of the $\eta_i\eta_j$-approximation in the analogical sum of the correlative approximation so that the domain of application of the former is not wider than that of the latter. However, its applications can simplify the calculation, because one can dispense with the sum

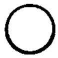
Note also that the first terms of all approximations considered above coincide, and the difference comes out only in calculations of the corrections. This is a reflection of the fact that the methods under consideration, when applied to the calculations of the high energy scattering amplitude, represent different versions of the straight-line path approximation [6].

Consider now the case when the expansion can be performed over the total number of lines involving those inside the loop. This means that a smallness parameter ε can be prescribed to all the terms of eq. (2.11) quadratic in η, or, which is the same, to the function D. Thus, $\Pi_\varepsilon[\nu]$ is investigated in the form of the series (2.12) using the equation

$$e^{\Pi_\varepsilon[\nu]} = \exp\left\{\frac{i\varepsilon}{2}\int D\,\frac{\delta^2}{\delta\nu^2}\right\} e^{g\pi[\nu]}. \tag{2.15}$$

From eq. (2.15) the relevant expression can be quite easily derived for the first terms Π_n

$$\Pi_1 = g\pi[\nu] \;\rightarrow$$



$$\Pi_2 = \frac{ig}{2}\int D\left[\frac{\delta^2\pi}{\delta\nu^2} + g\left(\frac{\delta\pi}{\delta\nu}\right)^2\right] \;\rightarrow \tag{2.16}$$

and so on.

In this way, we have indeed found the expansion in the total number of internal lines. The approximation obtained is less accurate than the $n_i n_j$ -approximation. Thus, applying eqs. (2.16) to the calculation of functional integrals gives as the first approximation

$$I = \int [\delta\nu] e^{g\pi[\nu]} \simeq e^{g\pi[0]}, \tag{2.17}$$

with the first correction

$$I \simeq e^{g\pi[0] - \frac{i}{4}\int d\xi\{g\pi^{(2)}(\xi,\xi) + g^2[\pi^{(1)}(\xi)]^2\}}, \tag{2.18}$$

where $\pi^{(2)}$ and $\pi^{(1)}$ are coefficient functions of the expansion of the functional $\pi[\nu]$ in powers of ν

$$\pi[\nu] = \pi[0] + \int d\xi\pi^{(1)}(\xi)\nu(\xi) + \tfrac{1}{2}\int d\xi_1 d\xi_2 \pi^{(2)}(\xi_1, \xi_2)\nu(\xi_1)\nu(\xi_2) + \dots \tag{2.19}$$

3. Let us consider a quasipotential equation with a local quasipotential for the scattering amplitude of scalar particles

$$T(\boldsymbol{p}, \boldsymbol{p}'; s) = gV(\boldsymbol{p} - \boldsymbol{p}'; s) + g\int d\boldsymbol{q} K(\boldsymbol{q}^2; s) V(\boldsymbol{p} - \boldsymbol{q}; s) T(\boldsymbol{q}, \boldsymbol{p}'; s), \tag{3.1}$$

where $\boldsymbol{p}$ and $\boldsymbol{p}'$ are the relative particle momenta in c.m.s. in initial and final states respectively, and $s = 4(\boldsymbol{p}^2 + m^2) = 4(\boldsymbol{p}'^2 + m^2)$.

To solve eq. (3.1) let us perform the Fourier transformation

$$V(\boldsymbol{p} - \boldsymbol{p}'; s) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\, e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{r}} V(\boldsymbol{r}; s), \tag{3.2}$$

$$T(\boldsymbol{p}, \boldsymbol{p}'; s) = \int d\boldsymbol{r}\, d\boldsymbol{r}'\, e^{i\boldsymbol{p}\cdot\boldsymbol{r} - i\boldsymbol{p}'\cdot\boldsymbol{r}'} T(\boldsymbol{r}, \boldsymbol{r}'; s). \tag{3.3}$$

Substituting (3.2) and (3.3) in (3.1), we obtain

$$T(\boldsymbol{r}, \boldsymbol{r}'; s) = \frac{g}{(2\pi)^3} V(\boldsymbol{r}; s)\delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') + \\ + \frac{g}{(2\pi)^3} \int \mathrm{d}\boldsymbol{q} \; K(\boldsymbol{q}^2; s) V(\boldsymbol{r}; s) \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \int \mathrm{d}\boldsymbol{r}'' \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}''} T(\boldsymbol{r}'', \boldsymbol{r}'; s). \tag{3.4}$$

Introducing the representation

$$T(\boldsymbol{r}, \boldsymbol{r}'; s) = \frac{g}{(2\pi)^3} V(\boldsymbol{r}; s) F(\boldsymbol{r}, \boldsymbol{r}'; s), \tag{3.5}$$

we have

$$F(\boldsymbol{r}, \boldsymbol{r}'; s) = \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') + \frac{g}{(2\pi)^3} \int \mathrm{d}\boldsymbol{q} K(\boldsymbol{q}^2; s) \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \cdot \\ \int \mathrm{d}\boldsymbol{r}'' \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}''} V(\boldsymbol{r}''; s) F(\boldsymbol{r}'', \boldsymbol{r}'; s). \tag{3.6}$$

Let us define the pseudo-differential operator

$$\hat{L}_{\boldsymbol{r}} = K(-\nabla_{\boldsymbol{r}}^2; s). \tag{3.7}$$

Then

$$K(\boldsymbol{r}; s) = \int \mathrm{d}\boldsymbol{q} \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}} K(\boldsymbol{q}^2; s) = \hat{L}_{\boldsymbol{r}} (2\pi)^3 \delta^{(3)}(\boldsymbol{r}). \tag{3.8}$$

Taking into account expression (3.8), eq. (3.6) may be written in the following symbolic form

$$F(\boldsymbol{r}, \boldsymbol{r}'; s) = \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}') + g\hat{L}_{\boldsymbol{r}}[V(\boldsymbol{r}; s) F(\boldsymbol{r}, \boldsymbol{r}'; s)]. \tag{3.9}$$

We shall seek the solution of this equation in the following form

$$F(\boldsymbol{r}, \boldsymbol{r}'; s) = \frac{1}{(2\pi)^3} \int \mathrm{d}\boldsymbol{k} \mathrm{e}^{W(\boldsymbol{r}; \boldsymbol{k}; s)} \mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}. \tag{3.10}$$

Substituting (3.10) in (3.9), we obtain the equation for the function $W(\boldsymbol{r}; \boldsymbol{k}; s)$

$$\mathrm{e}^{W(\boldsymbol{r}; \boldsymbol{k}; s)} = 1 + g\hat{L}_{\boldsymbol{r}}[V(\boldsymbol{r}; s) \mathrm{e}^{W(\boldsymbol{r}; \boldsymbol{k}; s) - i\boldsymbol{k}\cdot\boldsymbol{r}}] \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{3.11}$$

Expanding the function W in powers of the coupling constant g [3]

$$W(\boldsymbol{r}; \boldsymbol{k}; s) = \sum_{n=1}^{\infty} g^n W_n(\boldsymbol{r}; \boldsymbol{k}; s) \tag{3.12}$$

we immediately obtain from eq. (3.11) the following expressions for the functions

$$W_1(\boldsymbol{r}; \boldsymbol{k}; s) = \int \mathrm{d}\boldsymbol{q} V(\boldsymbol{q}; s) K[(\boldsymbol{k} + \boldsymbol{q})^2; s] \mathrm{e}^{-i\boldsymbol{q}\boldsymbol{r}}, \tag{3.13}$$

$$W_2(\boldsymbol{r}; \boldsymbol{k}; s) = -\frac{W_1(\boldsymbol{r}; \boldsymbol{k}; s)}{2} + \tfrac{1}{2} \int \mathrm{d}q_1 \mathrm{d}q_2 \mathrm{e}^{-i\boldsymbol{q}_1\boldsymbol{r} - i\boldsymbol{q}_2\boldsymbol{r}}$$
$$V(\boldsymbol{q}_1; s) V(\boldsymbol{q}_2; s) K[(\boldsymbol{q}_1 + \boldsymbol{q}_2 + \boldsymbol{k})^2; s]\{K[(\boldsymbol{q}_1 + \boldsymbol{k})^2; s] + K[(\boldsymbol{q}_2 + \boldsymbol{k})^2; s]\} \tag{3.14}$$

etc.

Considering only W_1 instead of W in formula (3.10), we obtain from (3.10), (3.5) and (3.3) the following approximate expression for the scattering amplitude [8]

$$T_1(\boldsymbol{p}, \boldsymbol{p}'; s) = \frac{g}{(2\pi)^3} \int \mathrm{d}\boldsymbol{r} \mathrm{e}^{i(\boldsymbol{p}-\boldsymbol{p}')\boldsymbol{r}} V(\boldsymbol{r}; s) \mathrm{e}^{gW_1(\boldsymbol{r}; \boldsymbol{p}'; s)} \tag{3.15}$$

The meaning of the approximation made above will be clear if we expand $T_1(\boldsymbol{p}, \boldsymbol{p}'; s)$ in powers of the coupling constant

$$T_1^{(n+1)}(\boldsymbol{p}, \boldsymbol{p}'; s) = \frac{g^{n+1}}{n!} \int \mathrm{d}\boldsymbol{q}_1 \ldots \mathrm{d}\boldsymbol{q}_n V(\boldsymbol{q}_1; s) \ldots V(\boldsymbol{q}_n; s) \cdot$$
$$V(\boldsymbol{p} - \boldsymbol{p}' - \sum_{i=1}^{n} \boldsymbol{q}_i; s) \prod_{i=1}^{n} K[(\boldsymbol{q}_i + \boldsymbol{p}')^2; s] \tag{3.16}$$

and compare it with the (n + 1)-th iteration of eq. (3.1)

$$T^{(n+1)}(\boldsymbol{p}, \boldsymbol{p}'; s) = \frac{g^{n+1}}{n!} \int \mathrm{d}\boldsymbol{q}_1 \ldots \mathrm{d}\boldsymbol{q}_n V(\boldsymbol{q}_1; s) \ldots V(\boldsymbol{q}_n; s) \cdot$$
$$V(\boldsymbol{p} - \boldsymbol{p}' - \sum_{i=1}^{n} \boldsymbol{q}_i; s) \sum_{P} K[(\boldsymbol{q}_1 + \boldsymbol{p}')^2; s] K[(\boldsymbol{q}_1 + \boldsymbol{q}_2 + \boldsymbol{p}')^2; s] \ldots \tag{3.17}$$
$$\ldots K[(\sum_{i=1}^{n} \boldsymbol{q}_i + \boldsymbol{p}')^2; s],$$

where $\sum_P$ denotes the sum over all possible permutations of momenta $\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_n$.

It is easy to see from the expressions (3.16) and (3.17) that in the case of the Lippman-Schwinger equation the approximation we have used coincides with the so-called »$\boldsymbol{q}_i\boldsymbol{q}_j = 0$ approximation» according to which terms of the type $\boldsymbol{q}_i\boldsymbol{q}_j (i = j)$ in the »nucleon propagator» are omitted.

Let us now establish the relation between the operator method and the Feynman path integration method. For this purpose we turn back to eq. (3.11) for the function W. The solution to this equation can be written symbolically as

$$\mathrm{e}^{W} = \frac{1}{1 - gK[(-i\nabla - \boldsymbol{k})^2] V(\boldsymbol{r})} \times 1 =$$
$$= -i \int_0^\infty \mathrm{d}\tau \mathrm{e}^{i\tau(1+i\varepsilon)} \mathrm{e}^{-i\tau g K[(-i\nabla - \boldsymbol{k})] V(\boldsymbol{r})} \times 1. \tag{3.18}$$

Following the Feynman parametrization [12] we introduce the ordering index η and rewrite (3.18) as

$$\mathrm{e}^{W} = -\mathrm{i} \int\limits_0^\infty \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\tau(1+\mathrm{i}\varepsilon)} \exp\left\{-\mathrm{i}g \int\limits_0^\tau \mathrm{d}\eta K[(-\mathrm{i}\nabla_{\eta+\varepsilon} - \boldsymbol{k})^2] V(\boldsymbol{r}_\eta)\right\}. \tag{3.19}$$

Making use of the Feynman transformation

$$\mathcal{F}[\mathcal{P}(\eta)] = \int \mathrm{D}\boldsymbol{p} \int\limits_{\boldsymbol{x}(0)=0} \mathrm{D}\, \frac{\boldsymbol{x}}{(2\pi)^3} \exp\left\{\mathrm{i} \int\limits_0^\tau \mathrm{d}\eta \dot{\boldsymbol{x}}(\eta)[\boldsymbol{p}(\eta) - \mathcal{P}(\eta)]\right\} \mathcal{F}[\boldsymbol{p}(\eta)], \tag{3.20}$$

the solution to eq. (3.11) is written as the functional integral

$$\mathrm{e}^{W} = -\mathrm{i} \int\limits_0^\infty \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\tau(1+\mathrm{i}\varepsilon)} \int \mathrm{D}\boldsymbol{p} \int\limits_{\boldsymbol{x}(0)=0} \mathrm{D}\, \frac{\boldsymbol{x}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\int_0^\tau \mathrm{d}\eta \dot{\boldsymbol{x}}(\eta)\boldsymbol{p}(\eta)}\, G(\boldsymbol{x}; \boldsymbol{p}; \tau) \times 1. \tag{3.21}$$

In the formula (3.21) $G(\boldsymbol{x}; \boldsymbol{p}; \tau)$ is given by

$$G(\boldsymbol{x}; \boldsymbol{p}; \tau) = \mathrm{e}^{-\int_0^\tau \mathrm{d}\eta \dot{\boldsymbol{x}}(\eta)\nabla_{\eta+\varepsilon}} \exp\left\{-\mathrm{i}g \int\limits_0^\tau \mathrm{d}\eta K[(\boldsymbol{p}(\eta) - \boldsymbol{k})^2] V(\boldsymbol{r}_\eta)\right\} \tag{3.22}$$

and obeys the equation

$$\begin{cases} \dfrac{\mathrm{d}G}{\mathrm{d}\tau} = \{-\mathrm{i}gK[(\boldsymbol{p}(\tau) - \boldsymbol{k})^2] V(\boldsymbol{r}) - \dot{\boldsymbol{x}}(\tau - \varepsilon)\nabla\} G \\ G(\tau = 0) = 1 \end{cases} \tag{3.23}$$

Deriving the operator function G from eq. (3.23) and substituting it in (3.21) we obtain the final expression for W

$$\mathrm{e}^{W} = -\mathrm{i} \int\limits_0^\infty \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\tau(1+\mathrm{i}\varepsilon)} \int \mathrm{D}\boldsymbol{p} \int\limits_{\boldsymbol{x}(0)=0} \mathrm{D}\, \frac{\boldsymbol{x}}{(2\pi)^3}\, \mathrm{e}^{\mathrm{i}\int_0^\tau \mathrm{d}\eta \dot{\boldsymbol{x}}(\eta)\boldsymbol{p}(\eta)}\, \mathrm{e}^{g\pi}, \tag{3.24}$$

where

$$\pi = -\mathrm{i} \int\limits_0^\tau \mathrm{d}\eta K[(\boldsymbol{p}(\eta) - \boldsymbol{k})^2] V\left[\boldsymbol{r} - \int\limits_0^\tau \mathrm{d}\xi \dot{\boldsymbol{x}}(\xi)\theta(\xi - \eta + \varepsilon)\right]. \tag{3.25}$$

Using the expansion [4]

$$\mathrm{e}^{W} = \overline{\mathrm{e}^{g\pi}} = \mathrm{e}^{g\bar{\pi}} \sum_{n=0}^\infty \frac{g^n}{n!}\, \overline{(\pi - \bar{\pi}^n)}, \tag{3.26}$$

where the symbol »—» means integration over τ , $x(\eta)$ and $p(\eta)$ with the appropriate measure (see eq. (3.24)), and performing the calculations we get

$$\pi = W_1, \quad \frac{\overline{\pi^2} - \overline{\pi}^2}{2} = W_2 \tag{3.27}$$

and so on.

Keeping only the first term ($n = 0$) in the expansion (3.26) we find the approximate expansion (3.15) for the scattering amplitude. This expression corresponds to the account in which the particle paths approach the classical ones most closely and coincide with straight-line paths in the case of highenergy, small-angle scattering. In other words, one can say that for high energies the operator method developed in this paper is the realization of the straight-line path concept [6].

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