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The Poisson Distribution of Secondary Particles in Straight-Line Paths. Approximation in Quantum Field Theory (*).

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Summary. — The problem of multiple production of « soft » mesons in high-energy two-nucleon collision is considered in the framework of standard quantum field theory models. The straight-line-path approximation is used to obtain the Poisson distribution of the secondary-particle number and to investigate the average particle number as a function of the choice of the cut-off parameters for momenta of the secondary particles in various asymptotic regions.

1. – Introduction.

In recent papers [1-5] a straight-line-path approximation (SLPA) has been formulated which then has been applied to considering problems of high-

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energy particle scattering in the framework of standard models of quantum field theory. This approximation is closely associated with the methods which are extensively being developed by many authors (6-9) for the so-called eikonal summation of the Feynman graphs. The eikonal approximation which is used for studying the asymptotic behaviour of the sum of a definite class of perturbation diagrams is based on the modification of the nucleon propagator according to which the bilinear terms on the mesons momenta are rejected. This modification has been studied well and proven in the infra-red region (10-13). By the present time we have already some grounds for its validity for a certain class of s-channel diagrams (14,15) in the asymptotic domain of high energies and fixed momentum transfers.

In formulating SLPA it is convenient to start from the Feynman interpretation of the scattering amplitude as a sum over the paths. In so doing, the method of averaging over the functional variable used in ref. (1-5) and in the present paper is equivalent to taking account of paths which approach most closely the straight-line ones. In the case of high-energy and fixed-momentum-transfer scattering the particle trajectories are approximately intercepts of straight lines having the directions of the particle momenta before and after collision, respectively. Such a physical picture permits us to give to the method used the name of the straight-line-path approximation.

The present paper is devoted to the investigation of the behaviour of the distribution of secondary « mesons » produced in high-energy « nucleon » collisions on the SLPA basis. It is useful to study different aspects of this problem in order to clarify the hadron interaction mechanism in the asymptotic domain in question.

For the sake of simplicity, we shall consider the field theory models in which scalar nucleons (*) exchange scalar and vector mesons. The inelastic amplitude and the n-particle production cross-section are factorized in these models under the condition that the components of the emitted-meson momenta be restricted by the « softness » conditions.

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(*) In the framework of this method a generalization may be made to the case of spinor particles, too.

It is interesting to study the differential cross-section of inelastic processes, which in the SLPA has the shape of the Poisson distribution over the secondary-particle number, as a function of the choice of the integral cut-off parameters in various asymptotic regions.

It should be noted that the Poisson distribution was considered earlier in various phenomenological models ([16]), electrodynamics ([17-20]), as well as in other approaches. Recently it has been shown ([1,21]) that such a distribution corresponds to the physical picture where the nucleons interact via their meson « clouds ». It is interesting to note that the total differential cross-section summed up over all the emitted mesons may have no pronounced diffraction peak in a certain domain of momentum transfers. In this connection an analogy should be indicated with the automodel behaviour of the cross-sections of high-energy deep inelastic interactions of hadrons with leptons ([22]).

The problems formulated above are studied first for a simple example of potential scattering. Further the consideration is made in the framework of ordinary quantum field theory models.

2. – Secondary-particle distribution in nucleon-potential scattering.

We consider a potential scattering with multiple production of secondary particles. The field with which the scattered nucleon interacts is represented as the sum of the external (classical) V and quantized φ fields. The amplitude of the scattering of a nucleon in the external field with n-meson production is found according to the well-known formula

$$f_n(p, q; k_1, \dots, k_n) = \langle 0|T_\varphi F(p, q|g\varphi + V)|k_1, \dots, k_n\rangle , \tag{2.1}$$

where T_φ is the sign of the T-product of the operators φ, g is a constant of interaction.

The generating functional $F(p, q|g\varphi + V)$ is expressed through the Fourier transform of the nucleon Green's function $G(p, q|g\varphi + V)$ as follows

$$F(p, q|g\varphi + V) = \lim_{(p^2, q^2)\to m^2} (p^2 - m^2)(q^2 - m^2)\, G(p, q|g\varphi + V) . \tag{2.2}$$

([16]) V. SAVRIN, N. TYURIN and O. KHRUSTALEV: preprint IHEP, 70-62, Serpukhov (1970).
([17]) R. GLAUBER: *Phys. Rev.*, **84**, 395 (1951).
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The transition to the mass shell is easily performed using the method of ref. ([2]). Thus, after extracting the poles, eq. (2.2) takes the form (*)

$$(2.3)\qquad F(p, q|g\varphi + V) = \int d^4x \exp[ix(p-q)]\langle \Gamma(x;\nu)\exp[iq]\int d^4y\,\varphi(y)J(x,y;\nu)\rangle_\nu ,$$

where the nucleon current is

$$(2.4)\qquad J(x, y; \nu) = \int d\xi\,\delta^4\left[x + a(\xi) + 2\int_0^\xi \nu(\eta)\,d\eta - y\right], \qquad a(\xi) = 2p\nu(\xi) + 2q\nu(\xi),$$

$$(2.5)\qquad \Gamma(x;\nu) = V(x)\int_0^1 d\lambda \exp\left[i\lambda\int d^4z\,V(z)J(x,z;\nu)\right]$$

and the functional averaging over paths ν means

$$(2.6)\qquad \langle\varphi(\nu)\rangle_\nu = \frac{\int\delta^4\nu\exp\left[-i\int\nu^2(\eta)\,d\eta\right]\varphi(\nu)}{\int\delta^4\nu\exp\left[-i\int\nu^2(\eta)\,d\eta\right]}.$$

Let us assume that all the particles produced (both real and virtual) are « soft » mesons ([1]), namely that the region of change of their momenta D is limited so that they do not lead to a strong change of the momentum transfer of nucleons. It is easily seen that the x-dependence in eq. (2.4) can be neglected.

We neglect the functional variable $\nu(\xi)$ which describes the deviation of particles from the straight-line paths.

Under these assumptions the functional F takes the form

$$(2.7)\qquad F(p, q|g\varphi + V) = f_{el}(p, q;\nu)\exp\left[ig\int_{\mathscr{D}} d^4y\,\varphi(y)J(y)\right],$$

where the elastic scattering amplitude ([23]) is expressed as follows:

$$(2.8)\qquad f_{el}(p, q;\nu) = \int d^4x \exp[ix(p-q)]\langle\Gamma(x;\nu)\rangle_\nu .$$

Hence, it is not difficult to get ([17]) that the cross-section of scattering on a potential with n-« soft »-meson production has the shape of the Poisson

(*) In the transition to the mass shell the terms corresponding to the diagrams without interaction of the nucleon with the external field V are eliminated.

([23]) V. PERVUSHIN: *Teor. Math. Fiz.*, **4**, 22 (1970).

distribution. Indeed,

$$f_n = f_{el}\langle 0|T_\varphi \exp\left[ig\int_{\mathscr{D}} d^4x\,\varphi(x)J(x)\right]|n\rangle = \tag{2.9}$$

$$= f_{el}\frac{1}{\sqrt{n!}}\left(\prod_{i=1}^{n}\frac{J(k_i)}{(2\pi)^{\frac{3}{2}}\sqrt{2k_{0i}}}\right)\exp\left[\frac{ig^2}{2}\int_{\mathscr{D}} d^4k\,J(k)\,\mathscr{D}(k)\,J(-k)\right],$$

where $\mathscr{D}(k)$ is the meson propagator.

Consequently,

$$d\sigma_n = d\sigma_{el}\frac{1}{n!}\left(\frac{1}{(2\pi)^3}\int_{\mathscr{D}}\frac{d^3k|J(k)|^2}{2k_0}\right)^n. \tag{2.10}$$

Now taking into account that

$$\mathscr{D}(k) - \mathscr{D}^*(k) = \frac{i}{(2\pi)^3}\,\delta(k^2 - m^2), \tag{2.11}$$

we finally get

$$d\sigma_n = d\sigma_0\frac{a^n}{n!}\exp[-a], \tag{2.12}$$

where a is defined by the following formula:

$$a = \frac{g^2}{(2\pi)^3}\int_{\mathscr{D}}\frac{d^3k|J(k)|^2}{2k_0} \tag{2.13}$$

and the current $J(k)$ is of the form

$$J(k) = \frac{1}{2pk + i\varepsilon} - \frac{1}{2qk - i\varepsilon} = -J^*(-k). \tag{2.14}$$

If the momentum transfer is in the region $\mathscr{D}_t$, $S \gg t \gg m^2$, $\mu^2 \neq 0$, then

$$a \sim g^2\frac{\ln \alpha t}{t}, \tag{2.15}$$

where α depends only on the masses m and μ.

Note that in electrodynamics, in eq. (2.15), there appears an infra-red singularity and the factor $\alpha \to 0$.

3. – Poisson distribution of secondary particles in quantum field theory models.

Now we proceed to the case of multiple production of « soft » particles in the model $L_{int} = g:\psi^2\varphi:$, where, for simplicity, the fields ψ and φ are assumed to be scalar. The vector-exchange model was considered earlier [1]. Here we present only some results of this model.

The amplitude of scattering of two nucleons (field ψ) with meson production (field φ) is determined by the formula [24]

$$(3.1)\qquad (2\pi)^4\delta\Big(p_1+p_2-q_1-q_2-\sum_{i=1}^{n}k_i\Big)f(p_1,p_2;q_1,q_2;k_1,\dots,k_n)=$$
$$=\langle 0|T_{\varphi_1}T_{\varphi_2}F(p_1,p_2;q_1,q_2;\varphi_1,\varphi_2)|n\rangle\,,$$

where the functional F, after the transition to the mass shell, is of the form [5]

$$(3.2)\qquad F(p_1,p_2;q_1,q_2;\varphi_1,\varphi_2)=\int \mathrm{d}x_1\,\mathrm{d}x_2\,\langle\langle A(x_1,x_2;\nu_1,\nu_2)\cdot$$
$$\cdot\exp\Big[ig\int \mathrm{d}^4x\sum_{i=1}^{2}\varphi_i(x)J_i(x;x_i;\nu_i)\Big]\rangle_{\nu_1}\rangle_{\nu_2}\,.$$

The following notation is introduced:

$$(3.3)\quad A(x_1,x_2;\nu_1,\nu_2)=\exp[i(p_1-q_1)x_1+i(p_2-q_2)x_2]\mathscr{D}(x_1-x_2)\cdot$$
$$\cdot\int_0^1 \mathrm{d}\lambda\exp\Big[i\lambda g^2\int \mathrm{d}\xi_1\,\mathrm{d}\xi_2\,\mathscr{D}\Big[x_1-x_2+a_1(\xi_1)-a_2(\xi_2)+\int_0^{\xi_1}\nu_1(\eta)\,\mathrm{d}\eta-\int_0^{\xi_2}\nu_2(\eta)\,\mathrm{d}\eta\Big]\Big]\,,$$

$$(3.4)\quad J_i(x;x_i;\nu_i)=\int \mathrm{d}\xi\,\delta^4\Big[x_i-x+a_i(\xi)+\int_0^{\xi}\nu_i(\eta)\,\mathrm{d}\eta\Big]$$

and the symbol $\langle\langle\dots\rangle_{\nu_1}\rangle_{\nu_2}$ means the averaging over the both functional variables ν_1 and ν_2 (see eq. (2.6)).

We consider the case when the produced meson momenta are very small compared with the nucleon momenta. This makes it possible to neglect the x_1 and x_2 dependence in eq. (3.4) for the nucleon current J_i. In other words, we consider the production of the so-called «soft» mesons which do not very much affect the motion of scattered high-energy particles.

According to the method developed and employed in refs. [1-5] for the estimate of the functional integrals over ν we use the following approximation:

$$(3.5)\qquad \langle\langle A(x_1,x_2;\nu_1,\nu_2)\exp\Big[ig\int \mathrm{d}^4x\sum_{i=1}^{2}\varphi_i(x)J_i(x;\nu_1,\nu_2)\Big]\rangle_{\nu_1}\rangle_{\nu_2}\simeq$$
$$\simeq\langle\langle A(x_1,x_2;\nu_1,\nu_2)\rangle_{\nu_1}\rangle_{\nu_2}\exp\Big[ig\int \mathrm{d}^4x\sum_{i=1}^{2}\varphi_i(x)J_i(x)\Big]\,,$$

([24]) N. BOGOLIUBOV and D. SHIRKOV: *Introduction to the Theory of Quantized Fields* (New York and London, 1959).

where

$$J(x) = \langle\langle J(x; \nu_1, \nu_2)\rangle_{\nu_1}\rangle_{\nu_2}\,.$$

As a first simple example we consider the case when both the real and virtual mesons are « soft ». In order to take into account the « softness » of mesons we restrict the region of integration over the meson momenta. Owing to the fact that now the integrals are no longer divergent in the upper limit, the averaging over ν, according to eq. (3.5), may be replaced by another more rough approximation when the D dependence in quantum currents is eliminated. As was already noted in the case of potential scattering, this approximation means an entire neglect of the nucleon deviation from straight-line paths.

Thus, in this approximation the functional F takes the form

$$\text{(3.6)}\qquad F(p_1, p_2; q_1, q_2; \varphi_1, \varphi_2) = f_{el}(p_1, p_2; q_1, q_2) \exp\left[i\int_{\mathscr{D}} \mathrm{d}^4k \sum_{i=1}^{2} \varphi_i(k) J_i(k)\right],$$

where $J_i(k) = \big(1/(2p_i k + i\varepsilon)\big)\big(1/(2q_i k - i\varepsilon)\big)$; $\mathscr{D}$ is the region of integration over the meson momenta limited by the « softness » conditions; f_{el} is the elastic-scattering amplitude without taking into account radiation corrections.

We note that, in principle, for f_{el} it is possible to use the eikonal representation obtained in refs. (1-3,6-9).

Using (3.6) for the cross-section for inelastic process with n-« soft »-meson production, it is not difficult to get the expression (1)

$$\text{(3.7)}\qquad \mathrm{d}\sigma_n = \mathrm{d}\sigma_0 \frac{a_1^n}{n!} \exp[-a]\,,$$

where

$$\text{(3.8)}\qquad a_1 = \frac{g^2}{(2\pi)^3}\int_{\mathscr{D}} \frac{\mathrm{d}^3k}{2k_0}\,|J_1 + J_2|^2$$

and

$$\text{(3.9)}\qquad a_2 = \frac{g^2}{(2\pi)^3}\int_{\mathscr{D}} \frac{\mathrm{d}^3k}{2k_0}\,[|J_1|^2 + |J_2|^2]$$

are the contributions of the real and virtual meson, respectively, and

$$\mathrm{d}\sigma_0 = |f_{el}|^2\,\mathrm{d}\Omega\,.$$

Since the interference term $J_1 J_2$ in the domain $t/s \ll 1$ gives a smaller contribution than the quadratic terms, we have $a_1 = a_2 = a$ and the distribution of the number of secondary particles takes the simple form:

$$d\sigma_n = d\sigma_0 \frac{a^n}{n!} \exp[-a]\,. \tag{3.10}$$

We notice that eq. (3.7) holds in a more general case [1]. But in this case, it is necessary to make the following substitutions:

$$a_1 \to \bar{n}_{m.s.} = \frac{g^2}{(2\pi)^3}\int \frac{d^3k}{2k_0} |J_1 + J_2|^2\,, \tag{3.11}$$

where

$$J_i = \frac{1}{\mu^2 + 2kp_i} - \frac{1}{-\mu^2 + 2kq_i}\,, \tag{3.12}$$

$$a_2 \to \bar{n}_{r.c.} = 2\,\mathrm{Re}\left\{\frac{g^2}{(2\pi)^4}\frac{i}{2}\int \frac{d^4k}{k^2 - \mu^2 + i\varepsilon}\left[|\tilde{\tilde{J}}_1|^2 + |\tilde{\tilde{J}}_2|^2\right]\right\}, \tag{3.13}$$

where

$$\tilde{\tilde{J}}_i = \frac{1}{k^2 + 2kp_i} - \frac{1}{k^2 + 2kq_i}\,. \tag{3.14}$$

When the mesons are vector particles the quantities $\tilde{J}$ and J read

$$\tilde{\tilde{J}}^v_{i\mu} = \frac{k_\mu + 2p_{i\mu}}{2p_ik + k^2} + \frac{k^2 - 2q_{i\mu}}{2q_ik + k^2}\,, \tag{3.15}$$

$$\tilde{\tilde{J}}^v_{i\mu} = \frac{k_\mu + 2p_{i\mu}}{2p_ik + \mu^2} - \frac{k_\mu + 2q_{i\mu}}{2q_ik - \mu^2}\,. \tag{3.16}$$

Thus, see that in quantum field models the n-s soft s-meson production cross-section has the behaviour of the Poisson distribution.

4. – Investigation of the radiative correction of real meson contributions to the cross-section.

We first consider the integral (3.13) with currents (3.14) and (3.15), which corresponds to the radiative-correction contribution. In the models in question the integration over the functional variables ν_1 and ν_2, according to eq. (3.5), leads to a factorization of the radiative corrections in the scattering amplitude and the cross-section. Then in the nucleon propagators the k^2-dependence is kept and the integrals converge in the upper limit.

It is not difficult to see that the integral (3.13) in this approximation does not contain the s-dependence [4].

In the asymptotic domain $|t| \ll m^2$,

$$\bar{n}_{\text{r.c.}} = t\,\frac{g^2}{24(2\pi)^2 m^4}\, l_n\left(\frac{m^2}{\mu^2}\right) + O\left(\frac{t^2}{m^4}\right), \tag{4.1}$$

if the scalar current (3.14) is inserted in (3.13) and

$$\bar{n}^v_{\text{r.c.}} = t\,\frac{2g^2}{3(2\pi)^2 m^2}\left[\ln\left(\frac{m^2}{\mu^2}\right) + \frac{1}{2}\right] + O\left(\frac{t^2}{m^4}\right), \tag{4.2}$$

if the vector current (3.15) is used.

As was already noted in ref. [1], the quantity $\bar{n}^v_{\text{r.c.}}/t$ is the width of the diffraction peak.

The softness conditions imposed on the meson momenta are as follows [1-5]:

$$\frac{1}{\sqrt{S}}\sum_{i=1}^{n} k_{0i} \ll 1\,, \qquad \left|\sum_{i=1}^{n} \boldsymbol{k}_{i\perp}\right| \ll |\boldsymbol{p}_{1\perp} - \boldsymbol{q}_{1\perp}| = |\boldsymbol{p}_{2\perp} - \boldsymbol{q}_{2\perp}|\,, \tag{4.3}$$

where the particle momentum components are given in the c.m.s., the initial nucleon momenta being chosen along the z-axis.

In order to cut off the integrals (3.11) with currents (3.12) and (3.16) in the upper single limit it is natural to single out in the whole momentum space a cylindrical domain oriented along the z-axis

$$|k_z| < \alpha p_0 = \varepsilon_z\,, \tag{4.4}$$

$$|\boldsymbol{k}_\perp| < \varepsilon_\perp\,, \tag{4.5}$$

since in the majority of cases the experimentally observed secondary mesons are emitted forward.

As was shown in ref. [1] in the infra-red asymptotic limit $\mu \to 0$ as well as in a broader region defined by the conditions

$$\left|\begin{array}{l} \varepsilon_\perp \sim m^2\,, \\[2ex] 1 \gg \alpha^2 \gg \dfrac{\mu^2}{m^2}\,, \\[2ex] \ln\left(\dfrac{\mu^2}{m^2}\right) \gg \ln\left(\dfrac{1}{\alpha^2}\right), \end{array}\right. \qquad \text{where } \alpha \equiv \frac{\varepsilon_z}{p_0}, \tag{4.6}$$

the absolute contributions from the radiative corrections and emitted mesons coincide (*). In this case, in summing in the expression for $d\sigma_n$ over the number

(*) The results of estimate of the integrals for different domains are given in the Appendix.

of all the emitted mesons the t-dependence vanishes. This leads to the peak being also vanishing in the diffraction cross-section. A similar feature has been noted in ref. [21] and is analogous to the automodel behaviour of deep inelastic interactions of hadrons with leptons at high energies [22].

Note that the quantity $\bar{n}_{\text{m.e.}}$ in the Poisson distribution has a simple physical meaning of the average number of emitted particles, *i.e.* multiplicity. Thus, in the region indicated, under the condition that the cut-off boundaries are energy independent, the multiplicity of « soft » particles turns out to be independent of the incident nucleon energy. As a result, the n-soft-particle production cross-section at high energies is also independent of S.

As was noted in the Introduction, in the framework of the present method there is a certain ambiguity in choosing the cut-off parameters and the result is sensitive to this choice. For example, if in the infra-red domain the cut-off parameter $\varepsilon_\perp$ is chosen to be increasing with energy $\varepsilon_\perp = \beta S$ ($\beta \ll 1$ and fixed), then *the multiplicity increases logarithmically with energy.*

Let us consider the case when the scalar meson-mass μ is fixed and is not small as compared with the nucleon mass m and $S \to \infty$. The average number of emitted particles in this domain is expressed as

$$\bar{n}_{\text{m.e.}} \sim \alpha^4 \int \mathrm{d}^2 k_\perp \frac{(\boldsymbol{k}_\perp \boldsymbol{\Delta}_\perp)}{(\mu^2 + \boldsymbol{k}_\perp^2)^4} . \tag{4.7}$$

It is seen that at a fixed cut-off parameter ε, the multiplicity decreases with increasing energy $S \to \infty$. This means that with increasing energy the number of « soft » mesons, the momenta of which lie in a given interval defined by the fixed parameter ε_*, decreases.

In addition, the condition may be imposed that the maximum longitudinal component of the meson momenta increases linearly with energy. In so doing, the average particle multiplicity tends with increasing energy to a finite limit. A similar result is obtained for the vector mesons.

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APPENDIX

Estimates of integrals, corresponding to real and virtual « mesons ».

		Scalar « mesons »			
$n_{r.c.}$	$2\,\mathrm{Re}\left\{\frac{g^2}{(2\pi)^4}\frac{i}{2}\int\frac{d^4k}{k^2-\mu^2+i\varepsilon}\cdot\left[\frac{1}{k^2+2kp_1}-\frac{1}{k^2+2kq_1}\right]^2\right\}$	$t\,\frac{g^2}{24(2\pi)^2m^4}\ln\left(\frac{m^2}{\mu^2}\right)+O\left(\frac{t^2}{m^4}\right)$	$t\,\frac{g^2}{24(2\pi)^2m^4}\ln\left(\frac{m^2}{\mu^2}\right)$	$\bar{n}_{r.c.}=\bar{n}_{r.c.}(t)$	$\bar{n}_{r.c.}$
$\bar{n}_{m.c.}$	$\frac{g^2}{(2\pi)^3}\int_{\lvert k_z\rvert\leqslant\alpha p_0}\frac{dk_z}{2k_0}\int_{\lvert k_\perp\rvert\leqslant\varepsilon_\perp}d^2k_\perp\cdot\left[\frac{1}{2kp_1}-\frac{1}{2kq_1}\right]^2$	$-t\,\frac{g^2}{24(2\pi)^2m^4}(A+B)$	$-t\,\frac{g^2}{24(2\pi)^2m^4}\ln\left(\frac{\varepsilon_\perp^2}{\mu^2}\right)$	$\frac{\alpha^4g^2}{(2\pi)^3}\int_{\lvert k_\perp\rvert\leqslant\varepsilon_\perp}d^2k_\perp\frac{(k_\perp\Delta_\perp)^2}{(\mu^2+k_\perp^2)^4}$	$\bar{n}_{m.c.}$
	General form of the integral taking into account only J_1	Asymptotic at $\lvert t\rvert\ll m^2$, $(t<0)$, $\alpha\gg\frac{\varepsilon_\perp}{p_0}$	Asymptotic at $\lvert t\rvert\ll m^2$, $\alpha\gg\frac{\varepsilon_\perp}{p_0}$, $\frac{\mu^2}{m^2}\ll\frac{m^2}{s}$, $\mu\to0$	Asymptotic at $\lvert t\rvert\ll m^2$, $\alpha\gg\frac{\varepsilon_\perp}{p_0}$, $\frac{\mu^2}{m^2}\gg\frac{m^2}{s}$, $s\to\infty$	
$\bar{n}_{r.c.}$	$2\,\mathrm{Re}\left\{\frac{g^2}{(2\pi)^4}\frac{i}{2}\int\frac{d^4k}{k^2-\mu^2+i\varepsilon}\cdot\left[\frac{k_\mu+2p_{1\mu}}{2p_1k+k^2}-\frac{k_\mu+2q_{1\mu}}{2q_1k+k^2}\right]^2\right\}$	$t\,\frac{2g^2}{3(2\pi)^2m^2}\left[\ln\left(\frac{m^2}{\mu^2}\right)+\frac{1}{2}\right]+O\left(\frac{t^2}{m^4}\right)$	$t\,\frac{2g^2}{3(2\pi)^2m^2}\ln\left(\frac{m^2}{\mu^2}\right)$	$\bar{n}_{r.c.}=\bar{n}_{r.c.}(t)$	$\bar{n}_{r.c.}$
$\bar{n}_{m.c.}$	$-\frac{g^2}{(2\pi)^3}\int_{\lvert k_z\rvert\leqslant\alpha p_0}\frac{dk_z}{2k_0}\int_{\lvert k_\perp\rvert\leqslant\varepsilon_\perp}d^2k_\perp\cdot\left[\frac{p_{1\mu}}{p_1k}-\frac{q_{1\mu}}{q_1k}\right]^2$	$-t\,\frac{g^2}{6(2\pi)^2m^2}(4A+B)$	$t\,\frac{2g^2}{3m^2(2\pi)^2}\ln\left(\frac{\varepsilon_\perp^2}{\mu^2}\right)$	$t\,\frac{2\alpha^2g^2}{(2\pi)^2}\int_{\lvert k_\perp\rvert\leqslant\varepsilon_\perp}d^2k_\perp\frac{1}{(\mu^2+k_\perp^2)^3}+O(\alpha^4)$	$\bar{n}_{m.c.}$
		Vector « mesons »			

● RIASSUNTO (*)

Nel contesto dei modelli di teoria dei campi quantistici convenzionali si studia il problema della produzione multipla di mesoni « molli » nelle collisioni di due nucleoni di alta energia. Si usa l'approssimazione dei percorsi rettilinei per ottenere la distribuzione poissoniana del numero di particelle secondarie e per ricercare il numero medio di particelle in funzione della scelta dei parametri di taglio per gli impulsi delle particelle secondarie nelle varie regioni asintotiche.

(*) *Traduzione a cura della Redazione.*

Резюме автором не представлено.