STRAIGHT-LINE PATHS APPROXIMATION FOR STUDYING HIGH-ENERGY ELASTIC AND INELASTIC HADRON COLLISIONS IN QUANTUM FIELD THEORY

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The problem of the asymptotic behaviour of high-energy elastic and inelastic amplitudes is studied by means of the functional methods of quantum field theory. The straight-line paths approximation (SLPA), making it possible to effectively calculate the functional integrals which arise, is formulated.

In a number of recent papers [1-11] the problem of the validity of the eikonal approximation for the two-particle elastic scattering amplitude at high energies was considered in the framework of various models of quantum field theory. In these papers the asymptotic behaviour of the sum of the Feynman diagrams of the ladder type (when all the possible crossings of "meson" lines between two "nucleons" are taken into account) was essentially investigated in the limit of high energies and fixed momentum transfers. One of the important results of these investigations is the fact that the principle logarithmic terms cancel in the sum of the ladder type diagrams in the asymptotic limit $S \to \infty$, t = fixed. Further, the sum of the ladder type diagrams tends asymptotically to the sum of quasipotential graphs for the two-particle scattering amplitude [12-13] and coincides with the Glauber-type eikonal expansion of the scattering amplitude at high energies and small angles.

In ref. [1] the functional integration methods in quantum field theory were used in studying this problem. As has been shown in these papers the functional integration methods present an effective tool for investigating the asymptotic behaviour of the scattering amplitudes. In subsequent papers [14,15] the functional integration methods have been used for studying the important problem of radiative corrections to the ladder type graphs for two-particle elastic scattering (see also papers [16,17]) and in investigating the inelastic scattering processes.

In this letter we present some results of investigation of the model of scalar nucleons interacting with neutral vector mesons [11]. We find closed analytic expressions for the two-nucleon elastic scattering amplitude and for the amplitudes of inelastic processes of meson production in nucleon collisions.

The asymptotic behaviour of these amplitudes in the high energy limit is studied in the framework of *the straight-line paths approximation* (SLPA) formulated below.

It is shown that the principal logarithmic terms in the scattering amplitudes cancel in the asymptotic limit $S \to \infty$, t - fixed when the diagrams with nucleon closed loops are neglected. It is shown then that the contributions of the radiative corrections to the ladder type graphs in the straight-line paths approximation are factorized and are determined by the quantity $H(t)$, which depends only upon the square of momentum transfers. In the region $|t| \ll m^2$ the quantity $H(t)$ has an exponential dependence on t and produces the diffraction peak in elastic scattering in accordance with the hypothesis of smoothness of the local quasipotential of two particles at high energies [18-20].

Such a behaviour of the elastic scattering amplitude was predicted recently in paper [21] and corresponds, in some sense, to the coherent interaction of virtual mesons which belong to the nucleon clouds.

Further the differential cross section of inelastic processes were obtained. Under the requirement of "softness" of secondary mesons the Poisson distributions in the number of particles emitted in collision is found.

We note that the total differential cross section summed over all the secondary mesons may have, generally speaking, no pronounced diffraction peak in the region $\mu^2 \lesssim |t| \ll m^2$.

Such a behaviour is due to the cancellation under certain conditions of two exponential factors, which correspond to the radiative correction contributions in the elastic scattering amplitude and to the total contributions from multi-meson production.

We would like to point out the analogy of such a regularity with the automodel behaviour [22] of deep inelastic processes of high energy hadron interaction. We choose an interaction Lagrangian of the following form

$$L_{\text{int}} = g{:}\psi^2(x)\mathrm{i}\overleftrightarrow{\partial}\psi(x)A_\alpha(x){:} + g^2{:}A_\alpha^2(x)\psi^*(x)\psi(x){:} \tag{1}$$

where g is some dimensionless coupling constant.

The one-particle Green function of the quantum field in the external field $A_\alpha(x)$ satisfies the equation

$$\{[\mathrm{i}\partial_\alpha + gA_\alpha(x)]^2 - m^2\}\, G(x,y|A) = -\delta^4(x-y) \tag{2}$$

The formal solution of eq. (2) for the Fourier transform of the Green function can be represented by means of the functional integral

$$G(p,q|A) = \mathrm{i}\int_0^\infty \mathrm{d}\tau \exp\{\mathrm{i}\tau(p^2-m^2)\} \int \frac{\mathrm{d}^4y}{(2\pi)^4} \exp\{\mathrm{i}(p-q)y\} \int [\delta^4\nu]_0^\tau \times \exp\{2\mathrm{i}g\int_0^\tau \mathrm{d}\xi\; [\nu_\alpha(\xi) + p_\alpha]\, A_\alpha[y + 2p\xi + 2\int_0^\xi \nu(\eta)\mathrm{d}\eta]\} \tag{3}$$

where

$$[\delta^4\nu]_{\tau_1}^{\tau_2} = \delta^4\nu \exp\{-\mathrm{i}\int_{\tau_1}^{\tau_2} \nu^2(\eta)\mathrm{d}\eta\} / \int \delta^4\nu \exp\{-\mathrm{i}\int_{\tau_1}^{\tau_2} \nu^2(\eta)\mathrm{d}\eta\} \tag{4}$$

is a normalized volume element of the functional space of the four-dimensional function $\nu_\alpha(\eta)$ determined on the interval $\tau_1 \leqslant \eta \leqslant \tau_2$.

The two-particle elastic scattering amplitude is defined as a vacuum average of the product of two Green functions [11]. Below, for simplicity, we shall neglect the vacuum polarization effects as well as the diagrams with closed nucleon loops. The propagator of the free vector field $\mathcal{D}_{\alpha\beta}$ is determined by the expression

$$\mathcal{D}_{\alpha\beta} = \delta_{\alpha\beta} - \kappa_\alpha\kappa_\beta/\mu^2/(\kappa^2-\mu^2) \tag{5}$$

Taking into account eq. (3) and eq. (5) we obtain the following closed expression for the two-particle scattering amplitude †

$$\begin{aligned} \mathrm{i}f(p_1,p_2;\, q_1q_2) &= g^2 \int \mathrm{d}^4y \exp\{y(p_1-q_1)\}\, \mathcal{D}_{\alpha\beta}(y) \int [\delta^4\nu_1]_{-\infty}^{\infty}[\delta^4\nu_2]_{-\infty}^{\infty}[2\nu_1(0) + p_1 + q_1]_\alpha \\ &\times [2\nu_2(0) + p_2 + q_2]_\beta \int_0^1 \mathrm{d}\lambda \exp\Big\{\tfrac{1}{2}\mathrm{i}g^2 \int \mathrm{d}^4\kappa\, \mathcal{D}_{\gamma\zeta}(\kappa) \Big[\sum_{i=1}^{2} j_\gamma^{(i)}(\kappa;\, p_i,q_i|\nu_i) j_\zeta^{(i)}(-\kappa;\, p_i,q_i|\nu_i) \\ &+ 2\lambda \exp\{\mathrm{i}\kappa y\} j_\gamma^{(1)}(\kappa;\, p_1,q_1|\nu_1) j_\zeta^{(2)}(-\kappa;\, p_2,q_2|\nu_2)\Big] - \mathrm{i}\int_{-\infty}^{\infty} \delta m^2 \mathrm{d}\xi\Big\} \end{aligned} \tag{6}$$

where

$$j_\gamma^{(i)}(\kappa;\, p_i,q_i|\nu_i) = 2\mathrm{i}\int \mathrm{d}\xi [\nu_{i\gamma}(\xi) + p_{i\gamma}\theta(\xi) + q_{i\gamma}\theta(-\xi)] \exp\{2\mathrm{i}\kappa_\gamma[p_{i\gamma}\xi\theta(\xi) + q_{i\gamma}\xi\theta(-\xi) + \int_0^\xi \nu_{i\gamma}(\eta)\mathrm{d}\eta]\} \tag{7}$$

is a conserving transition current. Obviously, an exact functional integration in expression (2.9) is not possible. Therefore, we use below the approximate method of calculating the integrals over ν_1 and ν_2 [23,24], called by us SLPA. The functional variables ν_1 and ν_2 formally introduced in eq. (3) for obtaining the solution for the Green function, describe the deviation of a particle trajectory from the straight-line paths. In fact, if we put $\nu = 0$ in formula (7) for the transition current, we would obtain

† In eq. (6) the mass renormalization $m_0^2 = m^2 + \delta m^2$ has been made. It removes divergences appearing in the integration over the variables ξ_1 and ξ_2 [14].

the classical current of the nucleon, moving with momentum p at $\xi > 0$ and with momentum q at $\xi < 0$.

We note, however, that the approximation $\nu = 0$ is known to be inapplicable at values of the proper time of the particle close to zero, when the particle classical trajectory changes its direction. In the language of Feynman graphs the approximation means that the quadratic κ-dependence in the nucleon propagator is neglected. It can lead, generally speaking, to the appearence of divergences of the integrals over $d^4\kappa$ at the upper limit.

A better approximation is given by averaging the nucleon current (7) over the functional variable ν, i.e.

$$\overline{j_\gamma(\kappa;\ p,q)} = -\left(\frac{2p_\gamma + \kappa_\gamma}{2\kappa p + \kappa^2 + i\epsilon} - \frac{2q_\gamma - \kappa_\gamma}{2\kappa q - \kappa^2 - i\epsilon}\right) \tag{8}$$

For this reason, in seeking the two-particle elastic scattering amplitude we shall use the SLPA, which consists in substituting in the exponential exponent in eq. (6) the current product averaged over the functional variables ν_1 and ν_2 [11].

Thus, in SLPA the expression for the elastic scattering amplitude takes the form

$$if(p_1,p_2;\ q_1,q_2) = g^2 H(t) \int d^4y \exp\{iy(p_1 - q_1)\}\ \Delta(y;\ p_1,q_1;\ p_2,q_2) \int_0^1 d\lambda \exp\{i\lambda\chi(y;\ p_1,p_2;\ q_1,q_2)\} \tag{9}$$

where

$$\Delta(y;\ p_1,q_1;\ p_2,q_2) = \int d^4\kappa\, \mathcal{D}_{\alpha\beta}(\kappa)\, [\kappa + p_1 + q_1]_\alpha\, [-\kappa + p_2 + q_2]_\beta \exp\{i\kappa y\} \tag{10}$$

$$\chi(y;\ p_1,p_2;\ q_1,q_2) = -\frac{g^2}{(2\pi)^4} \int d^4\kappa \exp\{i\kappa y\}\ \mathcal{D}_{\alpha\beta}(\kappa)\, \overline{j_\alpha^{(1)}(\kappa;\ p_1,q_1)\, j_\beta^{(2)}(-\kappa;\ p_2,q_2)} \tag{11}$$

$$H(t) = \exp \frac{ig^2}{2(2\pi)^2} \int d^4\kappa\, \mathcal{D}_{\alpha\beta}(\kappa) \sum_{i=1}^{2} \overline{j_\alpha^{(i)}(\kappa;\ p_i,q_i)\, j_\beta^{(i)}(-\kappa;\ p_i,q_i)} \tag{12}$$

It is interesting to note that the contribution of the radiative correction is factorized in a form of the factor $H(t)$ depending only on the square of the momentum transfer $t = (p_1 - q_1)^2$, as in the scalar field interaction model [14]. The analogous phenomena of the factorization of the radiative correction contribution in quantum electrodynamics was found in the articles [25-27].

In the high energy limit $S \to \infty$ at fixed momentum transfers t limited by the condition $|t| \ll m^2$ the expression for the elastic scattering amplitude has the form †

$$f(s,t) = i(s-u)\, v(t) \exp(at) \tag{13}$$

where

$$v(t) = \tfrac{1}{2} \int d^2 y_\perp \exp(i y_\perp \Delta_\perp) \left(\exp\left\{-i\frac{g^2}{2\pi} K_0(\mu|y_\perp|)\right\} - 1\right); \qquad t \approx -\Delta_\perp^2 \tag{14}$$

$$a = \{g^2/3(2\pi)^2 m^2\}\{\ln(m^2/\mu^2) + \tfrac{1}{2}\} \tag{15}$$

and K_0 is the Kelvin function of zero order.

The amplitudes of inelastic processes of meson production in two-nucleon collisions [11] at high energies can be determined by means of a generating function $f(p_1,p_2;\ q_1,q_2|A^{\text{ext}})$ having a meaning of the scattering amplitude of two nucleons in the presence of the external field A_α^{ext}.

In what follows we will consider the case in which the momenta of the secondary meson in the center of mass system satisfy the requirement of "softness" [15]:

† We note that taking into account the identity of nucleons leads on symmetrization of eq. (13) to terms vanishing in the limit $S \to \infty$ with fixed t.

$$\frac{1}{\sqrt{s}}\sum_{i=1}^{N}\kappa_{oi} \ll 1\ ; \qquad \left|\sum_{i=1}^{N}\varkappa_{i\perp}\right| \ll |\boldsymbol{p}_{1\perp}-\boldsymbol{q}_{1\perp}| \approx |\boldsymbol{p}_{2\perp}-\boldsymbol{q}_{2\perp}| \tag{16}$$

where the momenta of the initial nucleons are chosen along the Z-axis.

Under this requirement the amplitude of N-meson production is factorized and can be written in the following form:

$$f_{\text{inel}}(N) = f(p_1,p_2;\ q_1,q_2)\prod_{i=1}^{N} gE^{*}_{\alpha}(\kappa_i)\,[\overline{j^{(1)}_{\alpha}(\kappa_i;\ p_1,q_1)} + \overline{j^{(2)}_{\alpha}(\kappa_i;\ p_2,q_2)}] \tag{17}$$

where $E_{\alpha}(\kappa)$ is the polarization vector of a meson with the momentum κ.

We find also the asymptotic expression for differential cross sections of "soft" meson production, when the meson momenta satisfy eq. (16). It has the form:

$$\left(\frac{d\sigma}{dt}\right)_{n_1,n_2} \xrightarrow[t\text{-fixed}\ll m^2]{S\to\infty} \frac{1}{4\pi}v^2(t)\,w_{n_1}(s,t)\,w_{n_2}(s,t) \tag{18}$$

where

$$w_n(s,t) = \frac{1}{n!}\exp\{2at\}\int_{\Omega_p}\prod_{i=1}^{n}\frac{d\varkappa_i}{2\kappa_{oi}}\frac{-g^2}{(2\pi)^3}\,|\overline{j^{(l)}_{\alpha}(\kappa_i;\ p_l,q_l)}|^2 \tag{19}$$ v

The integration region Ω_p over secondary meson momenta is determined by the condition

$$-t \leqslant 2p\sum_{i=1}^{n}\kappa_i - \left(\Delta - \sum_{i=1}^{n}\kappa_i\right)^2 \leqslant S \tag{20}$$

where

$$t = \Delta^2\ ; \qquad \Delta = \left(q_1 - p_1 - \sum_{i=1}^{n_1}\kappa_i\right) = \left(q_2 - p_2 - \sum_{l=1}^{n_2}\kappa_l\right)$$

Consider now an approximation, in which the total momentum of the secondary mesons can be neglected in accordance with the requirement of "softness" (16). In this approximation eq. (19) takes the form of the Poisson distributions

$$w_n(s,t) = \frac{1}{n!}\exp\{2at\}\,[\bar{n}(s,t)]^n \tag{21}$$

where the quantity †

$$\bar{n}(s,t) = -\frac{g^2}{(2\pi)^3}\int\frac{d\varkappa}{2\kappa_o}\,|\overline{j^{(l)}(\kappa;\ p_l,q_l)}|^2 \tag{22}$$

is the average number of secondary particles produced in the two-nucleon collision at $S\to\infty$ and fixed t. Using eq. (8) for $\overline{j_{\alpha}}$, we find for $|t| \ll m^2$ that

$$\bar{n}(s,t) = -bt \tag{23}$$

The parameter b depends, in general, on a special form of cut-off of the integral in eq. (22) over the meson momentum. In the particular case when $R_{\perp}^2 \sim m^2$, $1 \gg \alpha^2 \gg \mu^2/m^2$, $\ln(m^2/\mu^2) \gg \ln(1/\alpha)^2$ where $\alpha = R_Z/p_0$ we get

$$b = \{2g^2/3(2\pi)^2m^2\}\{\ln(m^2/\mu^2) + \tfrac{1}{2}\} \tag{24}$$

which coincides with the double slope parameter of the diffraction exponential (15). Notice that the equality $2a = b$ is true also in the infrared asymptotic limit $\mu\to 0$.

For this case, after summing in eq. (19) over the number of secondary mesons, we find that the dependence on the variable t cancels and the diffraction peak in the total differential cross section disappears.

† The integration region in eq. (22) is effectively limited by $|\kappa_Z| \leqslant R_Z$, $|\varkappa_{\perp}| \leqslant R_{\perp}$.

This regularity was mentioned in paper [20] and is in analogy with the automodel behaviour of deep-inelastic processes of hadron interactions at high energy [22].

The straight-line paths approximation, used in this work corresponds to a physical picture in which colliding high energy nucleons at the act of interaction receive a small recoil connected with the emission of "soft" mesons and retain their individuality.

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