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STRAIGHT-LINE PARTICLE PATHS APPROXIMATION FOR DESCRIPTION OF HIGH-ENERGY HADRON SCATTERING IN QUANTUM FIELD THEORY

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STRAIGHT-LINE PARTICLE PATHS APPROXIMATION FOR DESCRIPTION OF HIGH-ENERGY HADRON SCATTERING IN QUANTUM FIELD THEORY

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§1. Introduction

Much attention has recently been paid to the study of high energy particle scattering on the basis of ordinary methods of quantum field theory.

In a number of recent papers [1-11] the problem of the validity of the eikonal approximation for the two-particle elastic scattering amplitude at high energies was considered in the framework of various models of quantum field theory. In these papers the asymptotic behaviour of the sum of the Feynman diagrams of the ladder type (when all the possible crossings of "meson" lines between two "nucleons" are taken into account) was essentially investigated in the limit of high energies and fixed momentum transfers. One of the important results of these investigations is the fact that the principal logarithmic terms cancel in the sum of the latter type diagrams in the asymptotic limit $s \to \infty$, t - fixed. This result shows that the relativistic retardation effects probably vanish with increasing energy and that the scattering contributions of the virtual particle momenta off the mass shell decrease at high energy. Further, the sum of the ladder type diagrams tends asymptotically to the sum of quasipotential graphs for the two-particle scattering amplitude [12-15] and coincides with the eikonal expansion of the scattering amplitude at high energies and small angles[16-18].

In papers[1,2,5] the functional integration methods in quantum field theory were used in studying this problem[19]. As has been shown in these papers the functional integration methods present an effective tool for investigating the asymptotic behaviour of the scattering amplitudes. In subsequent papers[20,21] the functional integration methods have been used for studying the important problem of radiation corrections to the ladder type graphs for two-particle elastic scattering (see also papers[22,23]) and in investigating the inelastic scattering processes. Let us note that in these papers an approximate method of calculating the functional integrals in question was used, which had been developed in paper [24]. Since in the framework of the Feynman diagrams this approximation is equivalent to a definite modification of the propagators of particles with large momentum p when the terms of the type $k_i k_j$ (k_i and k_j are the momenta of different real or virtual mesons emitted by nucleons) are neglected, e.g.

$$\frac{1}{m^2 - (p - \sum_{i=1}^{n} k_i)^2} \to \frac{1}{2p \sum_{i=1}^{n} k_i - \sum_{i=1}^{n} k_i^2}$$

this approximation is sometimes called "the $k_i k_j = 0$ approximation"[x]. The validity of this approximation in the study of the problem of infrared asymptotics in quantum field theory was proved in papers[25-27].

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[x] We note that in papers[3-6] in fact a rougher approximation was used, when all the terms bilinear in the emitted meson momenta are neglected in nucleon propagators.

The possibility of applying this method to the study of high energy particle scattering is not a priori evident, but, nevertheless, is confirmed by calculations in the framework of the lowest orders of perturbation theory[28-30].

We note that this approximation has quite a simple physical meaning. Starting from the Feynman interpretation of the scattering amplitude as a sum over paths, the approximation in question is equivalent to taking into account paths which are closest to the classical trajectories of particles. When the small angle particle scattering is considered, the classical particle trajectories are represented by pairs of rays with the direction of the particle momenta before and after collision, respectively. For this reason the approximation which was used in papers[1,2,5,20,21] will be called <u>the straight-line particle paths approximation.</u>

In this paper a model of scalar nucleons interacting with vector mesons is investigated[x]. We find closed analytic expressions for the two-nucleons elastic scattering amplitude and for the amplitudes of inelastic processes of meson production in nucleon collisions.

The asymptotic behaviour of these amplitudes in the high energy limit is studied in the framework of the straight-line particle paths approximation formulated above.

It is shown that the principal logarithmic terms in the scattering amplitudes cancel in the asymptotic limit $s \to \infty$, t-fixed when the diagrams with nucleon closed loops are neglected. The elastic scattering amplitude in this limit takes the eikonal form. It is shown

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[x] The scalar nucleon - scalar meson interaction model was investigated in detail in the papers[1,2,20,21].

then that the contributions of the radiation corrections to the ladder type graphs in the straight-line particle paths approximation are factorized and are determined by the quantity $H(t)$, which depends only upon the square of momentum transfers. In the region $|t| \ll m^2$ the quantity $H(t)$ has an exponential dependence on t and produces the diffraction peak in elastic scattering in accordance with the hypothesis of smoothness of the local quasipotential of two particles at high energies /31-33/.

Such a behaviour of the elastic scattering amplitude was predicted recently in paper /34/ and corresponds, in some sence, to the coherenr interaction of virtual mesons belonging to the nucleon clouds.

Further the differential cross sections of inelastic processes were obtained. Under the requirement of "softness" of secondary mesons the Poisson distribution in the number of particles, emitted in collision, is found.

We note that the total differential cross section summed over all the secondary meson may have, generally speaking, no pronounced diffraction peak in the region $\mu^2 \lesssim |t| \ll m^2$.

Such a behaviour is due to the cancellation under certain conditions of two exponential factors, which correspond to the radiation correction contributions in the elastic scattering amplitude and to the total contributions from multi-meson production.

We would like to point out the analogy of such a regularity with the automodel behaviour of deep inelastic processes of high energy hadron interaction.

§2. Two-Particle Elastic Scattering Amplitude

We shall consider a model of scalar nucleons interacting with the vector field $A_\alpha(x)$, having a non-zero mass μ. We choose an interaction Lagrangian in the following form

$$L_{int} = g : \psi^*(x)\, i \overleftrightarrow{\partial}_\alpha\, \psi(x) A_\alpha(x) : + g^2 : A_\alpha^2(x) \psi^*(x)\psi(x) : , \qquad (2.1)$$

where g is some dimensionless coupling constant.

The one-particle Green function of the quantum field in the external field $A_\alpha(x)$ satisfies the equantion

$$\{[i\partial_\alpha + g A_\alpha(x)]^2 - m^2\} G(x,y|A) = -\delta^4(x-y). \qquad (2.2)$$

The formal solution of eq. (2.2) can be represented by means of the integral over proper time[37]

$$G(x,y|A) = i\int_0^\infty d\tau\, e^{-i\tau m^2} \exp\{ i\int_0^\tau d\xi\, [i\partial_\alpha(\xi) + gA_\alpha(x,\xi)]^2\}\delta^4(x-y). \qquad (2.3)$$

Using the "disentanglement" method of the differentiation operator in the exponential exponent[38] in eq. (2.3) the solution of eq.(2.2) may be written as the functional integral[24]

$$G(x,y|A) = i\int_0^\infty d\tau\, e^{-i\tau m^2} \int [\delta^4 \nu]_0^\tau \exp\{2ig \int_0^\tau d\xi\, \nu_\alpha(\xi) \cdot \qquad (2.4)$$

$$\cdot A_\alpha[x + 2\int_0^\xi \nu(\eta)d\eta]\}\delta^4[x-y-2\int_0^\tau \nu(\eta)d\eta],$$

where

$$[\delta^4\nu]_{\tau_1}^{\tau_2} = \frac{\delta^4\nu\, e^{-i\int_{\tau_1}^{\tau_2}\nu^2(\eta)d\eta}}{\int\delta^4\nu\, e^{-i\int_{\tau_1}^{\tau_2}\nu^2(\eta)d\eta}} \tag{2.5}$$

is a volume element of the functional space of the four-dimensional function $\nu_a(\eta)$ determined on the interval $\tau_1 \le \eta \le \tau_2$.

The expression for the Fourier transform of the Green function (2.4) takes the following form

$$G(p,q\,|\,A) = \frac{1}{(2\pi)^4}\int dx\, dy\, e^{ipx-iqy}\, G(x,y\,|\,A) =$$

$$= i\int_0^\infty d\tau\, e^{i\tau(p^2-m^2)} \int \frac{dy}{(2\pi)^4}\, e^{i(p-q)y} \int [\delta^4\nu]_0^\tau \,\cdot \tag{2.6}$$

$$\cdot \exp\{2ig\int_0^\tau d\xi\,[\nu_a(\xi)+p_a]\,A_a[y+2p\xi+2\int_0^\xi \nu(\eta)\,d\eta]\}.$$

We shall define the two-particle elastic scattering amplitude using the variational derivative method[x/]

$$i(2\pi)^4\delta(p_1+p_2-q_1-q_2)f(p_1,p_2;q_1,q_2) = \lim_{(p_1^2,p_2^2,q_1^2,q_2^2)\to m^2} (p_1^2-m^2)(p_2^2-m^2)(q_1^2-m^2)(q_2^2-m^2)\,\cdot$$

$$\cdot\{\exp[\frac{i}{2}\int d^4k\, D_{\alpha\beta}(k)\frac{\delta^2}{\delta A_\alpha(k)\,\delta A_\beta(-k)}]\,G(p_1,q_1\,|\,A)\,G(p_2,q_2\,|\,A)\,S_0(A)\}_{A=0}, \tag{2.7}$$

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x/ The particle identity is taken into account by symmetrization of eq. (2.7) over momenta of initial or final particles.

where $S_0(A)$ is a vacuum expectation value of the S-matrix in the presence of the external field A. Below, for simplicity, we shall neglect the vacuum polarization effects as well as the diagrams with closed nucleon loops. The propagator of the free vector field $D_{\alpha\beta}$ is determined by the expression

$$D_{\alpha\beta} = \frac{\delta_{\alpha\beta} - \frac{k_\alpha \cdot k_\beta}{\mu^2}}{k^2 - \mu^2} \quad . \tag{2.8}$$

Inserting (2.6) into (2.7), after a series of functional variable substitutions/1,2,5,23/ we obtain the following closed expression for the two-particle scattering amplitude

$$\begin{aligned} i f(p_1, p_2; q_1, q_2) = g^2 \int dy\, e^{iy(p_1 - q_1)} D_{\alpha\beta}(y) \int [\delta^4 \nu_1]_{-\infty}^{\infty} [\delta^4 \nu_2]_{-\infty}^{\infty} \cdot \\ \cdot [2\nu_1(0) + q_1 + q_1]_\alpha [2\nu_2(0) + q_2 + q_2]_\beta \int_0^1 d\lambda \exp\{ \frac{ig^2}{2} \int d^4k\, D_{\gamma\zeta}(k) \cdot \\ \cdot [\sum_{i=1}^{2} j_\gamma^{(i)}(k; p_i, q_i | \nu_i) j_\zeta^{(i)}(-k; p_i, q_i | \nu_i) + 2\lambda j_\gamma^{(1)}(k; p_1, q_1 | \nu_1) \cdot \\ \cdot e^{iky} \cdot j_\zeta^{(2)}(-k; p_2, q_2 | \nu_2)] - i \int_{-\infty}^{\infty} \delta m^2 d\xi \}, \end{aligned} \tag{2.9}$$

where

$$\begin{aligned} j_\gamma^{(i)}(k; p_i, q_i | \nu_i) = 2i \int d\xi\, [\nu_{i\gamma}(\xi) + p_{i\gamma}\theta(\xi) + q_{i\gamma}\theta(-\xi)] \cdot \\ \cdot \exp\{ 2ik_\gamma [p_{i\gamma}\xi\, \theta(\xi) + q_{i\gamma}\xi\, \theta(\xi) + \int_0^\xi \nu_{i\gamma}(\eta) d\eta]\} \end{aligned} \tag{2.10}$$

is a transition current satisfying the continuity equation

$$k_\gamma \, j_\gamma (k; p, q \,|\, \nu) = 0 . \tag{2.11}$$

In eq. (2.9) the mass renormalization $m_0^2 = m^2 + \delta m^2$ has been made. It removes divergences appearing in the integration over the variables ξ_1 and ξ_2 /20/.

We note that the terms $j_\gamma^{(i)}(k; p_i, q_i|\nu_i)$, $j_\zeta^{(i)}(-k; p_i, q_i|\nu_i)$, $(i=1,2)$ in eq. (2.9) describe the radiation corrections to each of the nucleon lines and the term $2 j_\gamma^{(1)}(k; p_1, q_1|\nu_1) j_\zeta^{(2)}(-k; p_2, q_2|\nu_2)$ describes the interaction between two nucleons. Obviously, that exact functional integration in expression (2.9) is not possible. Therefore, we give below an account of the approximate method of calculating the integrals over ν_1 and ν_2 /24/. As mentioned in the introduction, we call this method the straight-line particle paths approximation.

Let us consider in more detail the physical meaning of the functional variables ν_1 and ν_2. These variables, formally introduced in eq. (2.4) for obtaining the solution for the Green function, describe the deviation of a particle trajectory from the straight-line path. In fact, if we put $\nu = 0$ in formula (2.10) for the transition current, we would obtain

$$j_\gamma(k; p, q \,|\, 0) = -\left(\frac{2p_\gamma}{2kp + i\epsilon} - \frac{2q_\gamma}{2kq - i\epsilon} \right). \tag{2.12}$$

This corresponds to the classical current of the nucleon, moving with momentum p at $\xi > 0$ and with momentum q at $\xi < 0$.

We note, however, that the approximation $\nu = 0$ is known to be inapplicable at values of the proper time of the particle close to zero, when the particle classical trajectory changes its direction. In the language of Feynman graphs the approximation means that the quadratic k -dependence in the nucleon propagator is neglected

$$\frac{1}{m^2-(p+k)^2} \to -\frac{1}{2pk} .$$

It can lead, generally speaking, to the appearance of divergences of the integrals over $d^4 k$ at the upper limit.

A better approximation to the nucleon current is given by the average current value (2.10) over the functional variable ν ,i.e.

$$\overline{j_\gamma (k;p,q)} = \int [\delta^4 \nu]_{-\infty}^{\infty} j_\gamma (k;p,q|\nu) =$$

$$= i\int d\xi [k_\gamma \epsilon(\xi) + 2p_\gamma \theta(\xi) + 2q_\gamma \theta(-\xi)] \cdot$$

$$\cdot \exp\{2ik_\gamma [p_\gamma \xi\theta(\xi) + q_\gamma \xi\theta(-\xi) + ik^2|\xi|]\} = \qquad (2.13)$$

$$= -\left(\frac{2p_\gamma + k_\gamma}{2kp+k^2+i\epsilon} - \frac{2q_\gamma - k_\gamma}{2kq-k^2-i\epsilon}\right) .$$

For this reason in seeking the two-particle elastic scattering amplitude we shall use the straight-line particle paths approximation, which consists in substituting in the exponential exponent in eq. (2.9) the current products averaged over the functional variables ν_1 and ν_2 .

$$\overline{j_\gamma^{(1)} (k;p_1,q_1) j_\zeta^{(2)} (-k;p_2,q_2)} = \int [\delta^4 \nu_1]_{-\infty}^{\infty} [\delta^4 \nu_2]_{-\infty}^{\infty} j_\gamma^{(1)} (k;p_1,q_1|\nu_1) \cdot$$

$$\cdot j_\zeta^{(2)} (-k;p_2,q_2|\nu_2) = \left(\frac{2p_{1\gamma}+k_\gamma}{2kp_1+k^2+i\epsilon} - \frac{2q_{1\gamma}-k_\gamma}{2kq_1-k^2-i\epsilon}\right)\left(\frac{2p_{2\zeta}-k_\zeta}{-2kp_2+k^2+i\epsilon} + \frac{2q_{2\zeta}+k_\zeta}{2kq_2+k^2+i\epsilon}\right), \qquad (2.14)$$

$$\overline{j^{(i)}_{\gamma}(k;p_i,q_i)j^{(i)}_{\zeta}(-k;p_i,q_i)} = \tag{2.15}$$

$$=\left(\frac{2p_{i\gamma}+k_{\gamma}}{2kp_i+k^2+i\epsilon}-\frac{2q_{i\gamma}+k_{\gamma}}{2kq_i+k^2-i\epsilon}\right)\left(\frac{2p_{i\zeta}+k_{\zeta}}{2kp_i+k^2+i\epsilon}-\frac{2q_{i\zeta}+k_{\zeta}}{2kq_i+k^2-i\epsilon}\right).$$

Consequently, in the straight-line particle paths approximation the expression for the elastic scattering amplitude takes the form

$$iT(p_1,p_2;q_1,q_2)=g^2H(t)\int dy\, e^{iy(p_1-q_1)}\Delta(y;p_1,q_1;p_2,q_2)\int_0^1 d\lambda\, e^{i\lambda\chi(y;p_1,p_2;q_1,q_2)}, \tag{2.16}$$

where

$$\Delta(y;p_1,q_1;p_2,q_2)=\int d^4k\, D_{\alpha\beta}(k)[k+p_1+q_1]_{\alpha}[-k+p_2+q_2]_{\beta}e^{iky} \tag{2.17}$$

$$\chi(y;p_1,p_2;q_1,q_2)=-\frac{g^2}{(2\pi)^4}\int d^4k\, e^{iky}D_{\alpha\beta}(k)\overline{j_{\alpha}(k;p_1,q_1)j^{(2)}_{\beta}(-k;p_2,q_2)} \tag{2.18}$$

$$H(t)=\exp\left[\frac{g^2}{2(2\pi)^2}\int d^4k\, D_{\alpha\beta}(k)\sum_{i=1}^{2}\overline{j^{(i)}_{\alpha}(k;p_i,q_i)j^{(i)}_{\beta}(-k,p_i,q_i)}\right]. \tag{2.19}$$

It is interesting to note that the contribution of the radiation correction is factorized in a form of the factor $H(t)$, depending only on the square of the momentum transfer $t=(p_1-q_1)^2$, as in the scalar nucleon-scalar field interaction model/20/. The analogous phenomena of the factorization of the radiation correction contribution in quantum electrodynamics was found in the articles/39-41/.

In the high energy limit $s \to \infty$ at fixed momentum transfers t limited by the condition $|t| \ll m^2$ the quantities χ and H take the form

$$\chi(|\vec{y}_\perp|) = \frac{g^2}{2\pi} K_0(\mu|\vec{y}_\perp|) \tag{2.22}$$

$$H(t) = e^{at} , \tag{2.23}$$

where K_0 is the Kelwin function of zero order and

$$a = \frac{g^2}{3(2\pi)^2 m^2}\left(\ln\frac{m^2}{\mu^2} + \frac{1}{2}\right) . \tag{2.24}$$

Thus in this asymptotic limit the expression for the elastic scattering amplitude in the scalar nucleon-vector field interaction model has the form x/

$$f(s,t) = i(s-u)v(t)e^{at} , \tag{2.25}$$

where

$$v(t) = \frac{1}{2}\int d^2\vec{y}_\perp\, e^{i\vec{y}_\perp\vec{\Delta}_\perp}(e^{-i\chi}-1) \tag{2.26}$$

and

$$t = -\vec{\Delta}_\perp^2 . \tag{2.27}$$

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x/ We note that taking into account the identity of nucleons leads, on symmetrization of eq. (2.25), to terms vanishing in the limit $s \to \infty$ with fixed t.

As is clear from the formula (2.25) the contribution of the radiation effects leads to diffraction behaviour of the high energy small angle scattering amplitude which corresponds to the Gaussian form of the local quasipotential of the elastic scattering /14,15/ with the interaction radius of order $g\frac{h}{mc}$. The forces which arise from the exchange of mesons between the nucleons obviously have a radius $\frac{h}{\mu c}$, and it is supposed that $\frac{h}{\mu c} \gg g\frac{h}{mc}$.

Thus in the region of momentum transfer $\mu^2 \lesssim t \ll m^2$ it is very important to take into account the multiparticle meson exchange which leads to the eikonal structure of the quantity $v(t)$.

§3. Inelastic Scattering Amplitude

The amplitudes of inelastic processes of meson production in two-nucleon collisions at high energies can be determined by means a generating function $f(p_1, p_2; q_1, q_2 | A^{ext})$.

The quantity $f(p_1, p_2; q_1, q_2 | A^{ext})$ has a meaning of the scattering amplitude of two nucleons in the presence of the external field A_a^{ext} and is determined by (2.7), where after taking functional derivatives one should take $A_a(x) = A_a^{ext}(x)$.

In the straight-line particle paths approximation the quantity $f(p_1, p_2; q_1, q_2 | A^{ext})$ takes the form

$$i f(p_1, p_2; q_1, q_2 | A^{ext}) = g^2 \int dy\, e^{iy(p_1 - q_1)} dx\, e^{ix(p_2 - q_2)} \Delta(x-y, p_1, q_1; p_2, q_2) \cdot \tag{3.1}$$

$$\cdot \exp\{ ig \int d^4\ell\, A_a^{ext}(\ell) [\, \overline{j_a^{(1)}(\ell; p_1, q_1)}\, e^{i\ell x} + \overline{j_a^{(2)}(\ell; p_2, q_2)}\, e^{i\ell y}\,] \cdot$$

$$\cdot \int_0^1 d\lambda \exp\{ \frac{ig^2}{2} \int d^4k\, D_{\gamma\zeta}(k) [\, e^{ik(x-y)}\, \overline{2\lambda\, j_\gamma^{(1)}(k; p_1, q_1) j_\zeta^{(2)}(-k; p_2, q_2)} +$$

$$+ \sum_{i=1}^{2} \overline{j_\gamma^{(i)}(k; p_i, q_i) j_\zeta^{(i)}(-k; p_i, q_i)}\,)]\},$$

where the functional averaging values of the currents and their bilinear combination are defined by (2.13) and (2.14), (2.15), respectively.

The amplitude of N-vector quantum production can be found by functional derivatives

$$i(2\pi)^4\delta(p_1+p_2-q_1-q_2-\sum_{i=1}^{N}k_i)f(p_1,p_2;q_1,q_2;k_1,k_2,\dots,k_N)=$$

$$=\prod_{i=1}^{N}E^*_\alpha(k_i)\frac{\delta}{\delta A^{ext}(k_i)}if(p_1,p_2;q_1,q_2|A^{ext})|_{A^{ext}=0}=$$

$$=g^2\int dx\, e^{ix(p_1-q_1)}dy\, e^{iy(p_2-q_2)}\prod_{i=1}^{N}E^*_\alpha(k_i)[\overline{j^{(1)}_\alpha(k_i;p_1,q_1)}e^{ikx}+$$

$$+\overline{j^{(2)}_\alpha(k_i;p_2,q_2)}e^{iky}]\Delta(x-y;p_1,q_1;p_2,q_2)\exp[\frac{ig^2}{2}\int d^4k\, D_{\gamma\zeta}(k)\cdot \tag{3.2}$$

$$\cdot[e^{ik(x-y)}2\lambda\overline{j^{(1)}_\gamma(k;p_1,q_1)j^{(2)}_\zeta(-k;p_2,q_2)}+\sum_{i=1}^{2}\overline{j^{(i)}_\gamma(k;p_i,q_i)j^{(i)}_\zeta(-k;p_i,q_i)}]],$$

where $E_\alpha(k)$ is the polarization vector of a meson with the momentum k.

In what follows we will consider the case in which the momenta of the secondary meson in the center of mass system satisfy the requirement of "softness"/21/:

$$\frac{1}{\sqrt{s}}\sum_{i=1}^{N}k_{0i}\ll 1;\quad |\sum_{i=1}^{N}\vec{k}_{i\perp}|\ll|\vec{p}_{1\perp}-\vec{q}_{1\perp}|\approx|\vec{p}_{2\perp}-\vec{q}_{2\perp}|, \tag{3.3}$$

where the momenta of the initial nucleons are chosen along the z-axis.

Under the requirement the amplitude of N-meson production, defined by (3.2), is factorized and can be written in the following form:

$$f_{inel}(N) \equiv f(p_1, p_2; q_1, q_2; k_1, k_2, \ldots, k_N) =$$

$$= f(p_1, p_2; q_1, q_2) \prod_{i=1}^{N} g E^*_\alpha(k_i) [\overline{j^{(1)}_\alpha(k_i; p_1, q_1)} + \overline{j^{(2)}_\alpha(k_i; p_2, q_2)}], \tag{3.4}$$

where

$$\overline{j^{(\ell)}_\alpha(k; p_\ell, q_\ell)} = \left(\frac{2 p_{\ell\alpha} + k_\alpha}{2 p_\ell k - \mu^2} - \frac{2 q_{\ell\alpha} - k_\alpha}{2 q_\ell k + \mu^2} \right); \ (\ell = 1, 2). \tag{3.5}$$

§4. The Asymptotic Behaviour of Differential Cross Section of Multi-Meson Production

The differential cross section of N-meson production in two-nucleon collision is defined by

$$d\sigma_N = \frac{1}{2\sqrt{s(s-4m^2)}} |f_{inel}(N)|^2 (2\pi)^4 \delta(p_1 + p_2 - q_1 - q_2 - \sum_{i=1}^{N} k_i) \cdot$$

$$\cdot \frac{1}{(2\pi)^6} \frac{d\vec{q}_1 \, d\vec{q}_2}{2 q_{10} \cdot 2 q_{20}} \frac{1}{N!} \prod_{i=1}^{N} \frac{1}{(2\pi)^3} \frac{d\vec{k}_i}{2 k_{0i}}, \tag{4.1}$$

where $s = (p_1 + p_2)^2$.

Here we shall consider the asymptotic behaviour of differential cross sections of "soft" meson production, when the meson momenta satisfy eq. (3.3).

As we shall show below, for this case the interference terms in the expression (3.4) for the amplitude of inelastic processes can be neglected, i.e.

$$f_{\text{in}}(N) = f_{el}(s,t)\prod_{i=1}^{n_1} g E^*_\alpha(k_i)\, \overline{j^{(1)}_\alpha(k_i; p_1, q_1)} \prod_{\ell=1}^{n_2} g E^*_\beta(k'_\ell)\, \overline{j^{(2)}_\beta(k'_\ell, p_2, q_2)}, \quad (4.2)$$

where $t = \Delta^2$ and $\Delta = (q_1 - p_1 + \sum_{i=1}^{n_1} k_i) = (q_2 - p_2 - \sum_{\ell=1}^{n_2} k'_\ell)$.

Using (4.2) and the formula

$$\delta(p_1 + p_2 - q_1 - q_2 - \sum_{i=1}^{n_1} k_i - \sum_{\ell=1}^{n_2} k'_\ell) = \int d^4\Delta\, \delta(p_1 - q_1 - \sum_{i=1}^{n_1} k_i + \Delta)\delta(p_2 - q_2 - \sum_{\ell=1}^{n_2} k'_\ell - \Delta)$$

we obtain the following expression for the multi-meson production differential cross section

$$(d\sigma)_{n_1, n_2 \;\; s\to\infty,\; \Delta-\text{fixed}} \to \frac{1}{2s} \frac{d^4\Delta}{(2\pi)^4} |f_{el}(s,t)|^2 W_{n_1}(p_1, \Delta) W_{n_2}(p_2, -\Delta) \quad (4.4)$$

where

$$W_{n_1}(p_1, \Delta) = \frac{2\pi}{n_1!} \int \frac{d\vec{q}_1}{2q_{10}} \delta(p_1 - q_1 - \sum_{i=1}^{n_1} k_i + \Delta).$$

$$\prod_{i=1}^{n_1} \frac{d\vec{k}_i}{2k_{0i}} \frac{-g^2}{(2\pi)^3} \left| \overline{j^{(1)}(k_i; p_1, q_1)} \right|^2 \quad (4.5)$$

and an analogous equation for $W_{n_2}(p_2, -\Delta)$. The quantities $W_{n_1}(p, \Delta)$ and $W_{n_2}(p_2, -\Delta)$ depend on the variables

$$t = \Delta^2;\; r_1 = p_1\Delta \quad \text{and} \quad t = \Delta^2;\; r_2 = -p_2\Delta \quad (4.6)$$

respectively.

Using the variables (4.6) we rewrite the volume element $d^4\Delta$ in the form

$$d^4\Delta = \frac{4\pi}{\sqrt{s(s-4m^2)}}\, dt\, dr_1\, dr_2\, \frac{d\phi}{2\pi} \tag{4.7}$$

where ϕ is an azimuthal angle, and the physical region of integrations is determined by the inequalities

$$\begin{aligned} &-t \le 2r_1 \le s \\ &-t \le 2r_2 \le s, \quad s \gg m^2 \\ &-s \le t \le 0. \end{aligned} \tag{4.8}$$

Further we shall seek the asymptotic behaviour of the differential cross section $\left(\frac{d\sigma}{dt}\right)_{n_1,n_2}$ at $s \to \infty$ and fixed t .

Integrating eq.. (4.4) over dr_1 , dr_2 and using eq. (2.25) we get at $|t| \ll m^2$

$$\left(\frac{d\sigma}{dt}\right)_{n_1,n_2} \underset{t-\text{fixed}}{\xrightarrow{s\to\infty}} \frac{1}{4\pi} v^2(t)\, w_{n_1}(s,t)\, w_{n_2}(s,t), \tag{4.9}$$

where

$$\begin{aligned} w_n(s,t) &= \frac{e^{at}}{\pi} \int dr\, \Psi_n(t,r) = \\ &= \frac{1}{n!} e^{at} \int_{\Omega_p} \prod_{i=1}^{n} \frac{d\vec{k}_i}{2k_{0i}} \frac{-g^2}{(2\pi)^3} \overline{|j^{(\ell)}(k_i; p_\ell, q_\ell)|^2} . \end{aligned} \tag{4.10}$$

The integration region Ω_p over secondary meson momenta is determined by the condition

$$-t \leq 2p \sum_{i=1}^{n} k_i - (\Delta - \sum_{i=1}^{n} k_i)^2 \leq s \qquad (4.11)$$

or taking into account that in the case considered here by the condition

$$0 \leq 2p \sum_{i=1}^{n} k_i \leq s + t \,. \qquad (4.12)$$

Consider now an approximation, in which the total momentum of the secondary mesons can be neglected in accordance with the requirement of "softness" (3.3). In this approximation eq. (4.10) takes the form of the Poisson distributions

$$w_n(s,t) = \frac{1}{n!} e^{at} [\bar{n}(s,t)]^n , \qquad (4.13)$$

where the quantity[x/]

$$\bar{n}(s,t) = -\frac{g^2}{(2\pi)^3} \int \frac{d\vec{k}}{2k_0} \, \overline{|j^{(\ell)}(k;\, p_\ell + q_\ell)|^2} \qquad (4.14)$$

is the average number of secondary particles produced in the two-nucleon collisions as $s \to \infty$ with fixed t .

Using eq. (2.13) for $\overline{j_a}$, we find for $|t| \ll m^2$ that

$$\bar{n}(s,t) = -b\, t \,. \qquad (4.15)$$

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x/ The integration region eq. (4.14) is effectively limited by $|k_z| \leq R_z$, $|\vec{k}_\perp| \leq R_\perp$.

The parameter b depends, in general, on a special form of cut-off of the integral in eq. (4.14) over the meson momentum. In the particular case when

$$R_{\perp}^{2} \approx m^{2}$$

$$1 \gg a^{2} \gg \frac{\mu^{2}}{m^{2}} \qquad (4.16)$$

$$\ln \frac{m^{2}}{\mu^{2}} \gg \ln \left(\frac{1}{a}\right)^{2},$$

where $a = \frac{R_z}{p_0}$, we get

$$b = \frac{2 g^{2}}{3 (2\pi)^{2} m^{2}} \left(\ln \frac{m^{2}}{\mu^{2}} + \frac{1}{2} \right) \qquad (4,17)$$

which coincides with the double slope parameter of the diffraction exponential (2.24). Notice that the equality $2a = b$ is true also in the infrared asymptotic limit $\mu \to 0$.

For this case, after summing in eq. (4.10) over the number of secondary mesons, we find that the dependence on the variable t cancel and that the diffraction peak in the total differential cross section desappears.

This regularity was mentioned in paper/34/ and is in analogy with the automodel behaviour of deep-inelastic processes of hadron interactions at high energy/35,36/.

It was remarked above that in deriving (1.2) we have neglected interference terms in the inelastic amplitudes. Such terms, when taken into account in the expression form , give contributions of the type

$$\frac{g^2}{(2\pi)^3} \int \frac{d\vec{k}}{k_0} \overline{j^{(1)}(k;p_1,q_1)}\,\overline{j^{(2)}(k;p_2,q_2)} \qquad (4.18)$$

which under the condition (4.16) at asymptotically large energies $s \to \infty$ and fixed t are negligible compared with (4.14).

We note that in principle it is interesting to investigate in more detail the dependence of the quality $\bar{n}(s,t)$ on forms of the cut-off in the integral over meson momentum under various relations between the cut-off parameter and particle masses.

In conclusion we emphasize that in this paper we developed a method of summing the ladder and cross ladder Feynman diagrams for the amplitudes of inelastic and elastic processes, taking into account radiation corrections but not diagrams with closed nucleon loops.

The straight-line particle path approximation, used in this work corresponds to a physical picture in which scattering high energy nucleons at act of interaction receive a small recoil connected with the emission of "soft" mesons and retain their individually (leading particle in the terminology in cosmic ray physics).

The method can be used for studying the role of the vacuum polarization effects and also of the diagrams containing closed nucleon loops /42,43/.

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