# On external and internal structure of black holes in modified gravity

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# Ongoing collaboration with Italian colleagues

I. Investigation of early stages of inflation using non-perturbative approaches (e.g. stochastic inflation).

Bologna University and INFN

F. Finelli, G. Marozzi, A.A. Starobinsky, G.P. Vacca, G. Venturi, Stochastic growth of quantum fluctuations during inflation, arXiv:1102.0216.

II. Search for hidden features in CMB fluctuations.
Bologna University and INFN
F. Finelli, A. Gruppiso, F. Paci, A.A. Starobinsky, Searching for hidden mirror symmetries in CMB fluctuations from WMAP 7 year maps, arXiv:1111.5362.

Two different axes are found for which the CMB intensity pattern is anomalously symmetric (or anti-symmetric) with respect to reflection in planes orthogonal to them at the 99.86 (99.99)% confidence level. The directions of these axes are close to that of the CMB kinematic dipole and nearly orthogonal to the ecliptic plane, respectively.

III. Inflation with scalar fields non-minimally coupled to gravity, including the Higgs inflation.
Bologna University and INFN –
ongoing project with A.Yu. Kamenshchik et al.

Is the recently announced remaining range for the Higgs boson mass $m_H \sim (126 \pm \mathrm{few})$ Gev compatible with the extreme hypothesis that the Higgs boson is the inflaton, too, and there is no new particles beyond the Standard Model?
Still possible: while the lower bound from the one-loop result was $m_H \gtrsim 135$ Gev, the two-loop calculations of Bezrukov & Shaposhnikov and de Simone et al. (2009) lowered it to $m_H \gtrsim 126$ Gev. However, if $m_H$ is below 130 Gev, then the prediction for the slope of the power spectrum of matter density perturbations $n_s$ becomes very sensitive to $m_H$ and $m_t$ and may become too small (at present, $n_s \geq 0.948$ at the $2\sigma$ level from observations).

IV. Cosmology and compact objects with the generalized Chaplygin gas model of dark energy.
Bologna University, Universita dell'Insubria, Como and INFN, Milano – ongoing project with V. Gorini, A.Yu. Kamenshchik, U. Moschella et al.

# Modified gravity

A local, metric, macroscopic theory of gravity, possibly with additional scalar fields not directly interacting with non-gravitational matter, used in the non-perturbative classical or semi-classical regime (i.e. all possible solutions are considered) and without ghosts.
An example: scalar-tensor gravity and its particular case: $f(R)$ gravity.

## Why modified gravity?

Why going beyond the pure Einstein gravity (GR) interacting with dust (baryons + CDM) and radiation? – Existence of dark energy (DE).

Two cases where DE shows itself:
1) inflation in the early Universe – primordial DE,
2) present accelerated expansion of the Universe – present DE.

The whole known part of the history of our Universe in one line, according to the standard cosmological scenario:

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Remarkable qualitative similarity of $DS$ and $\overline{DS}$ makes possible (though not necessary) combined description of both DS stages (both types of DE) using one class of models.

# Possible forms of DE

1. Physical DE

New non-gravitational field of matter. DE proper place – in the rhs of gravity equations.

2. Geometrical DE

Modified gravity. DE proper place – in the lhs of gravity equations. Still gravitational field equations can be conventionally written in the Einstein form:

$$\frac{1}{8\pi G}\left(R_{\mu}^{\nu} - \frac{1}{2}\,\delta_{\mu}^{\nu}R\right) = -\left(T_{\mu\,(vis)}^{\nu} + T_{\mu\,(DM)}^{\nu} + T_{\mu\,(DE)}^{\nu}\right)\ ,$$

$G = G_0 = const$ - the Newton gravitational constant measured in laboratory. In the absence of direct interaction between DM and DE: $T_{\mu\,(DE);\nu}^{\nu} = 0$ .

Generically, DE can be both physical and geometrical, e.g. in the case of a non-minimally coupled scalar field or, more generically, in scalar-tensor gravity. So, there is no alternative "(either) dark energy or modified gravity".

# $f(R)$ gravity – the simplest form of geometrical DE

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^{\mu}_{\mu} \ .$$

The effective energy-momentum tensor of DE in $f(R)$ gravity:

$$8\pi G T^{\nu}_{\mu\,(DE)} = F'(R)\, R^{\nu}_{\mu} - \frac{1}{2} F(R)\delta^{\nu}_{\mu} + \left(\nabla_{\mu}\nabla^{\nu} - \delta^{\nu}_{\mu}\nabla_{\gamma}\nabla^{\gamma}\right) F'(R)\,.$$

Non-perturbative generalization of GR since it contains an additional scalar degree of freedom (a scalar particle dubbed scalaron).

# Examples of ghostless modified gravity

For almost all known models of modified gravity, in particular, for models with the action $S$ functionally depending on $R_{\mu\nu}R^{\mu\nu},\ C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma},\ R_{,\mu}R^{,\mu}$ etc. (where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor), ghosts appear if the theory is taken in full, in the non-perturbative regime.

Known exceptions (without tachyons in the Minkowski space-time, too):
1. Scalar-tensor gravity with
$F(\phi) > 0,\ \omega_{BD} > -3/2,\ V''(\phi) \geq 0$.
2. $f(R)$ gravity with $f'(R) > 0,\ f''(R) > 0$.
3. $f(R,G)$ gravity with $f_{RR}f_{GG} - f_{RG}^2 = 0$,
where $G = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R\mu\nu R^{\mu\nu} + R^2$ is the Gauss-Bonnet invariant. However, it has other problems (a bad dispersion law). For $f_{RR}f_{GG} - f_{RG}^2 \neq 0$, a ghost was found very recently (A. De Felice and T. Tanaka, Progr. Theor. Phys. **124**, 503 (2010)).

# Black hole solutions in ghostless modified gravity theories

No new static or stationary stable vacuum solutions apart from the Schwarzschild-Kerr-de Sitter one. In particular, no stable wormholes in ghostless scalar-tensor or $f(R)$ gravity (K. Bronnikov and A.S., 2007).

In $f(R)$ gravity, a small scalar curvature perturbation $\delta R \equiv R - R_{dS}$ (where $R_{dS}$ – the scalar curvature of a self-consistent de Sitter solution – is a root of the functional equation $R_{dS} f'(R_{dS}) = 2f(R_{dS})$), around these solutions satisfies the linear equation

$$\nabla_\mu \nabla^\mu \delta R + m_s^2 \delta R = 0, \;\; m_s^2 = \frac{1}{3}\left(\frac{f'(R_{dS})}{f''(R_{dS})} - R_{dS}\right).$$

So, it decays like a massive scalar field around a black hole (BH) if $m_s^2 > 0$.

# Inside BH

Generic spacelike singularity, or a null singularity instead of the Cauchy horizon (no complete agreement between researchers). For the Reissner-Nordstrom BH - a spacelike singularity certainly, if creation of electron-positron pairs is taken into account (I. Novikov and A.S., 1980). For the Kerr case – not clear which kind of singularity is realized. Anyway, no unambiguous continuation of a space-time metric through these singularities – loss of predictability, future evolution depends on an UV completion of gravity.
Inside a static BH: the Kantowski-Sachs type metric

$$ds^2 = d\tau^2 - a^2(\tau)dx^2 - b^2(\tau)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

where $x$ is the former $t$ and $b$ is the former $r$ outside the event horizon. Near the spacelike singularity $a(\tau) \propto |\tau|^{-1/3}$, $b(\tau) \propto |\tau|^{2/3}$.

# BH interior in a hypothetical UV-regular gravity

Conjectures: in such theory of gravity 1) curvature is bounded somehow; 2) as a result, $a(\tau)$ stops diverging and $b(\tau)$ bounces simultaneously; 3) after that an inflationary stage with some number of e-folds $N$ follows; 4) after the inflationary stage creation of matter and thermalization occurs and classical FRW-like expansion begins. This expansion finally changes to the Kasner vacuum anisotropic behaviour when $b(\tau)$ becomes of the order of $\tau$. Near the singularity, a similar bounce occurs once more and so on up to the future time infinity.

$a(\tau)$ grows monotonically – no Cauchy horizon. $b(\tau)$ oscillates but $b_{min}(\tau)$ and $b_{max}(\tau)$ in each cycle grow with the cycle number $n$. Under a weak assumption that $\sum_{m=1}^{n}\sum_{k=1}^{m} N_k$ grows faster than $\log n$ for $n \to \infty$, $\int d\tau/a(\tau)$ converges for $\tau \to \infty$ – the future event horizon forms.

# Conclusions

- No satisfactory alternative to the Schwarzschild-Kerr-de Sitter BH in known local metric macroscopic modified gravity theories without ghosts.
- Generic curvature singularity inside these BH.
- For a hypothetical UV-regular modified gravity with no dynamical singularities, an eternally oscillating universe without Cauchy horizons can exist inside the event horizon of a BH under some weak assumptions. Its conformal diagram is similar to that of the Schwarzschild BH but with the spacelike singularity $r = 0$ replaced by a surface of an infinite proper time with a bounded curvature but an irregular behaviour.