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Black Hole and Cosmological solutions in
GENERALIZED THEORIES OF GRAVITY
with symmetric affine connection

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Based on:

arXiv:1112.3023 (math-phys) (first part of a paper on D=3, D=4 mainly)

General properties and some spherical/cylindrical reductions, expansion near horizons, vecton – scalaron duality, topological portrait of Static (BH) - Cosmology solutions

arXiv:1011.2445 v1 (gr-qc) 1-dim. theory as a relativistic particle in a potential

arXiv:1008.2333 v1 (hep-th) attempt at a new general formulation of geom.

arXiv:1003.0782 v3 (hep-th) further generalizations, cosmological solutions.

arXiv: 0812.2616 v2 (gr-qc) the first paper on new interpretation of **Einstein** 3 papers of **1926**; simplified model, static solutions, existence of horizons, non-integrability, approximate solutions by various power series expansions.

Content of the talk

Brief summary of affine models based on WEE ideas

Dimensional reduction to spherical and cylindrical configurations

Vecton – Scalaron equivalence in Dilaton Gr. D = 2,1 (on E.O.M.)

Static/cosmological solutions near horizons (convergent series)

Simple and degenerate horizons (LC and Szekeres – Kruskal.)

Unified treatment of static and cosmological solutions

Integrability and nonintegrability (examples)

Topological portraits (idea and simplest examples)

Main principles (suggested by Einstein's approach)

1. **Geometry:** **dimensionless** `*action'* constructed of a ***scalar*** *density*; its variations give the geometry and main equations ***without complete specification of the analytic form of the Lagrangian .***
2. **Dynamics:** a concrete Lagrangian constructed of the ***geometric variables*** - homogeneous 4^{th} order function (e.g. , the square root of the determinant of the curvature) produces a physical **effective Lagrangian**.
3. **Duality** between the geometrical and physical variables and Lagrangians.

NB: This looks more artificial than the first two principles and works for rather special models (actually giving *exotic fields*, *tachyons* etc.) (Einstein. did not know this! He was looking for unified theory of EM and Gravity.)

GEOMETRY OF SYMMETRIC CONNECTIONS

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + a^i_{jk}$$

$$\Gamma^i_{jk}[g] = \frac{1}{2} g^{il}(g_{lj,k} + g_{lk,j} - g_{jk,l})$$

$$r^i_{jkl} = -\gamma^i_{jk,l} + \gamma^i_{mk}\gamma^m_{jl} + \gamma^i_{jl,k} - \gamma^i_{ml}\gamma^m_{jk}$$

NONSYMMETRIC RICCI CURVATURE

$$r_{jk} = -\gamma^i_{jk,i} + \gamma^i_{mk}\gamma^m_{ji} + \gamma^i_{ji,k} - \gamma^i_{mi}\gamma^m_{jk}$$

Symmetric part of the Ricci curvature

$$s_{ij} \equiv \frac{1}{2}(r_{ij} + r_{j\,i})$$

Anti-symmetric part of the Ricci curvature

$$a_{ij} \equiv \frac{1}{2}(r_{ij} - r_{j\,i}) = \frac{1}{2}(\gamma^m_{jm,i} - \gamma^m_{im,j})$$

$$a_{ij,\,k} + a_{jk,i} + a_{ki,j} \equiv 0$$

VECTON: $a_i \equiv a^m_{im}$

$$a_i \equiv \gamma^m_{mi} - \Gamma^m_{mi} \equiv \gamma_i - \partial_i \ln \sqrt{|g|}$$

$$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i}) \equiv -\frac{1}{2}(\gamma_{i,j} - \gamma_{j,i})$$

EDDINGTON'S SCALAR DENSITY

$$\mathcal{L} \equiv \sqrt{-\det(r_{ij})} \equiv \sqrt{-r}$$

$$s_{ij} = -\nabla^{\gamma}_{m}\gamma^{m}_{ij} + \frac{1}{2}(\nabla^{\gamma}_{i}\gamma_{j} + \nabla^{\gamma}_{j}\gamma_{i}) - \gamma^{m}_{ni}\gamma^{n}_{mj} + \gamma^{n}_{ij}\gamma_{n}$$

Expressing in terms of the `metric' and using notation $\nabla_i \equiv \nabla^{g}_{i}$

$$s_{ij} = R_{ij}[g] - \nabla_m a^{m}_{ij} + \frac{1}{2}(\nabla_i\, a_j + \nabla_j\, a_i) + a^{m}_{ni}a^{n}_{mj} - a^{m}_{ij}a_m$$

$a_{ij} \equiv -\frac{1}{2}(a_{i,j} - a_{j,i})$ depends only on the vecton

‘GEODESICS’ (PATHS)

$$\ddot{x}^{\,i} + \gamma^{i}_{jk}\,\dot{x}^{j}\,\dot{x}^{k} = 0$$

TRANSFORMATIONS PRESERVING PATHS

$$\hat{\gamma}^{i}_{jk} = \gamma^{i}_{jk} + \delta^{i}_{j}\,\hat{a}_{k} + \delta^{i}_{k}\,\hat{a}_{j}$$

GEO-RIEMANNIAN CONNECTIONS

$$\hat{\gamma}^{i}_{jk} = \Gamma^{i}_{jk}[g] + \delta^{i}_{j}\,\hat{a}_{k} + \delta^{i}_{k}\,\hat{a}_{j}$$

$\alpha\beta$ - CONNECTION

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha(\delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j) - (\alpha - 2\beta)g_{jk}\,\hat{a}^i$$

Weyl: $\beta = 0$

geo-Riemannian: $\alpha = 2\beta$.

Einstein $\alpha = -\beta = \frac{1}{6}$

LINEAR TERMS in $s_{ij} - R_{ij}(g)$

$$(\alpha + \beta)(\nabla_i \hat{a}_j + \nabla_j \hat{a}_i) + (\alpha - 2\beta)\, g_{ij} \nabla_m \hat{a}^m$$

QUADRATIC TERMS in $s_{ij} - R_{ij}(g)$

$$\hat{a}_i \hat{a}_j \left[(\alpha - 2\beta)^2 - 3\alpha^2\right] + 2\, g_{ij} \hat{a}^2 (\alpha - 2\beta)(\alpha + \beta)$$

In addition to this dependence on the vecton, the generalized Einstein equations will depend on it through dynamics specified by the chosen Lagrangian

FROM GEOMETRY TO DYNAMICS

REQUIREMENTS TO LAGRANGIAN DENSITIES

1. IT IS INDEPENDENT OF DIMENSIONAL CONSTANTS.

2. ITS INTEGRAL OVER SPACE-TIME IS DIMENSIONLESS.

3. IT CAN DEPEND ON TENSOR VARIABLES HAVING
a DIRECT GEOMETRIC MEANING and
a NATURAL PHYSICAL INTERPRETATION.

4. THE RESULTING GENERALIZED THEORY MUST AGREE WITH ALL ESTABLISHED EXPERIMENTAL CONSEQUENCES OF EINSTEIN'S THEORY.

r_{ij}, s_{ij}, a_{ij}, and $a_k \equiv a^i_{ik}$ satisfy requirement **3**.

Einstein's choice is $\mathcal{L} = \mathcal{L}(s_{ij}, a_{ij})$

A simple nontrivial choice of a geometric Lagrangian density generalizing the Eddington – Einstein Lagrangian ,

$$\mathcal{L} \equiv \sqrt{-\det(r_{ij})} \equiv \sqrt{-r} \ ,$$

is the following, depending on one dimensionless parameter:

$$\mathcal{L} = \mathcal{L}(s_{ij} + \nu a_{ij}) = \sqrt{-\det(s_{ij} + \nu a_{ij})}$$

$$\det(s_{ij}) < 0$$

When $\nu a_{ij} \longrightarrow 0$ it will give Einstein's gravity with the cosmological constant.

Now we define (following Einstein) the metric and field densities by a Legendre-like transformation

$$\frac{\partial \mathbf{L}}{\partial s_{ij}} \equiv \mathbf{g}^{ij}\,, \qquad \frac{\partial \mathbf{L}}{\partial a_{ij}} \equiv \mathbf{f}^{ij}$$

dual to

$$s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}\,, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$$

$$2\nabla_i^\gamma\, \mathbf{g}^{kl} = \delta_i^l\, \nabla_m^\gamma\, (\mathbf{g}^{km} + \mathbf{f}^{km}) + \delta_i^k\, \nabla_m^\gamma\, (\mathbf{g}^{lm} + \mathbf{f}^{lm})$$

$$\nabla_i^\gamma\, \mathbf{f}^{kl} = \partial_i\, \mathbf{f}^{kl} + \gamma_{im}^k\, \mathbf{f}^{ml} + \gamma_{im}^l\, \mathbf{f}^{km} - \gamma_{im}^m\, \mathbf{f}^{kl}$$

$$\nabla_i^\gamma\, \mathbf{f}^{ki} = \partial_i \mathbf{f}^{ki} \equiv \mathbf{a}^k. \qquad \nabla_i^\gamma\, \mathbf{g}^{ik} = -\frac{D+1}{D-1}\hat{\mathbf{a}}^k$$

The main equation $\nabla_i^\gamma\, \mathbf{g}^{jk} = -\frac{1}{D-1}(\delta_i^j \hat{\mathbf{a}}^k + \delta_i^k \hat{\mathbf{a}}^j)$

for any dimension D

Defining the Riemann metric tensor g_{ij} by the equations

$$g^{ij}\sqrt{-g} = \mathbf{g}^{ij}\,, \quad g_{ij}\,g^{jk} = \delta_i^k$$

$$\nabla_i\,g_{jk} = 0, \quad \nabla_i\,g^{jk} = 0 \qquad \hat{a}^k \equiv \hat{\mathbf{a}}^k/\sqrt{-g}$$

we can derive the expression for the connection coefficients

$$\gamma^i_{jk} = \Gamma^i_{jk}[g] + \alpha_D\,[\,\delta^i_j\,\hat{a}_k + \delta^i_k\,\hat{a}_j - (D-1)\,g_{jk}\,\hat{a}^i]$$

$$\alpha_D \equiv [(D-1)(D-2)]^{-1}\,, \qquad \beta_D \equiv -[2(D-1)]^{-1}$$

We thus have derived the connection using a rather general dynamics!
Not using any particular for of the geometric Lagrangian!

Using a simple dimensional reduction to the dimension 1+1 (similar to spherical or cylindrical reductions in the metric case) we easily derive the important relation between geom. and phys.

$$\mathcal{L} = -\frac{1}{2}\sqrt{|\det(s + \lambda^{-1}\, a)|} = -2\Lambda\sqrt{|\det(\mathbf{g} + \lambda\mathbf{f})|} = \mathcal{L}^*$$

Λ having the dimension L^{-2}

Using the above definitions, → $\quad s_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{g}^{ij}}\,, \qquad a_{ij} = \frac{\partial \mathbf{L}^*}{\partial \mathbf{f}^{ij}}$

we can then write the

generalized Einstein eqs.

In **dimension D** we can similarly derive the relation

$$\mathcal{L}^* \equiv \sqrt{-\det(s_{ij} + \nu a_{ij})} \sim \sqrt{-g}\,[\det(\delta_i^j + \lambda f_i^j)]^{1/(D-2)}$$

The generalized Einstein–Eddington-Weyl model in dimension D

$$\mathcal{L}_{eff} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^{1/(D-2)} + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Restoring the dimensions and expanding the root term up to the second order in the vector and scalar fields

$$\mathcal{L}_{eff} \cong \sqrt{-g}\left[R[g] - 2\Lambda - \kappa\left(\frac{1}{2}F_{ij}F^{ij} + \mu^2 A_i A^i + g^{ij}\partial_i\psi\,\partial_j\psi + m^2\psi^2\right)\right]$$

$$A_i \sim a_i,\; F_{ij} \sim f_{ij},\; \kappa \equiv G/c^4$$

NB: $\partial_i\psi$ Is proportional to F_{ij} for i < 4, j=4

Dimensional reductions of

$$\mathcal{L}_{\rm ph} = \sqrt{-g}\left[-2\Lambda\left[\det(\delta_i^j + \lambda f_i^j)\right]^\nu + R(g) + c_a\, g^{ij} a_i a_j\right]$$

Spherical reduction of the theory

$$ds^2_D = ds^2_2 + ds^2_{D-2} = g_{ij}\, dx^i\, dx^j \; + \; \varphi^{2\nu}\, d\Omega^2_{D-2}(k)$$

$$\mathcal{L}_D^{(2)} = \sqrt{-g}\left[\varphi R(g) + k_\nu\, \varphi^{1-2\nu} + \frac{1-\nu}{\varphi}(\nabla\varphi)^2 + X(\varphi,\mathbf{f}^2) - m^2\varphi\,\mathbf{a}^2\right]$$

$$X(\varphi,\mathbf{f}^2) \equiv -2\Lambda\varphi\left[1 + \frac{1}{2}\lambda^2\mathbf{f}^2\right]^\nu \qquad \mathbf{f}^2 \equiv f_{ij} f^{ij} \qquad \nu \equiv (D-2)^{-1}$$

Weyl
rescaling

$$g_{ij} = \hat{g}_{ij}\, w^{-1}(\varphi), \quad w(\varphi) = \varphi^{1-\nu} \qquad \mathbf{f}^2 = w^2\,\hat{\mathbf{f}}^2, \quad \mathbf{a}^2 = w\,\hat{\mathbf{a}}^2$$

Cylindrical reductions

$$ds_4^2 = (g_{ij} + \varphi\, \sigma_{mn}\, \varphi_i^m \varphi_j^n)\, dx^i dx^j + 2\varphi_{im}\, dx^i dy^m + \varphi\, \sigma_{mn}\, dy^m dy^n$$

$$\sigma_{22} = e^{\eta} \cosh \xi, \quad \sigma_{33} = e^{-\eta} \cosh \xi, \quad \sigma_{23} = \sigma_{32} = \sinh \xi$$

$$\varphi R(g) + \frac{1}{2\varphi} (\nabla \varphi)^2 + V_{\text{eff}}(\varphi, \xi, \eta) - \frac{\varphi}{2} [(\nabla \xi)^2 + (\cosh \xi)^2\, (\nabla \eta)^2]$$

$$V_{\text{eff}}(\varphi, \xi, \eta) = -\frac{\cosh \xi}{2\varphi^2} \left[Q_1^2 e^{-\eta} - 2 Q_1 Q_2 \tanh \xi + Q_2^2 e^{\eta} \right]$$

$$\mathcal{L}_W^{(2)} = \sqrt{-g} \left[\varphi R(g) - \frac{Q_1^2}{2\varphi^{5/2}}\, e^{-\eta} - \frac{\varphi}{2}\, (\nabla \eta)^2 \right]$$

$$\mathcal{L}^{(2)}_{DW} = \sqrt{-g}\left[\varphi R(g) + k_\nu\,\varphi^{-\nu} - 2\Lambda\varphi^{\nu}\left[1 + \frac{1}{2}\lambda^2\varphi^{2(1-\nu)}\mathbf{f}^2\right]^{\nu} - m^2\varphi\,\mathbf{a}^2\right]$$

3-dimensional theory

$$\mathcal{L}^{(2)}_{3} = \sqrt{-g}\left[\varphi R(g) - 2\Lambda\varphi - \lambda^2\Lambda\varphi\,\mathbf{f}^2 - m^2\varphi\,\mathbf{a}^2\right]$$

Vecton – Scalaron DUALITY

$$ds^2 = -4\,h(u,v)\,du\,dv\,, \qquad \sqrt{-g} = 2h \qquad f^n_{uv} \equiv a^n_{u,v} - a^n_{v,u}$$

L /2*h* = $\varphi R + V(\varphi,\psi) + X(\varphi,\psi;\mathbf{f}^2_n)$ $\qquad -2\mathbf{f}^2_n = (f^n_{uv}/h)^2$

L' /2*h* = $\varphi R + V(\varphi,\psi) + X_{\text{eff}}(\varphi,\psi;q_n)$ $\qquad$ & $\quad q_n(u,v) \equiv h^{-1}X_n\,f^n_{uv}$

Effective action on `mass shell'; *f* – from eq, & $\qquad X_n \equiv \dfrac{\partial X}{\partial\,\mathbf{f}^2_n}$

$$X_{\text{eff}}(\varphi, \psi; q_n) = X(\varphi, \psi; \bar{\mathbf{f}}_n^2) + \sum q_n(u,v)\sqrt{-2\bar{\mathbf{f}}_n^2}$$

where: $2\bar{\mathbf{f}}_n^2 = -(q_n/\bar{X}_n)^2$ $\quad\quad \bar{X}_n \equiv \frac{\partial}{\partial \bar{\mathbf{f}}_n^2} X(\varphi, \psi; \bar{\mathbf{f}}_n^2)$

$$\partial_u(h^{-1} X_n f_{uv}^n) = -Z_n a_u^n = \partial_u\, q_n(u,v)$$

This defines a_u^n in terms of $q_n(u,v)$ and (φ, ψ)

$$X_{\text{eff}} = -2\Lambda\sqrt{\varphi}\left[1 + q^2/\lambda^2\Lambda^2\varphi^2\right]^{\frac{1}{2}}$$ for ***D*** = 4

$V = 2k\varphi^{-\frac{1}{2}}, \quad \bar{Z} = -1/m^2\varphi$ **N.B**: normally, ***Z*** ~ to dilaton φ

$X_{\text{eff}}(\varphi;\, q(u,v)) = -q^2/\lambda^2\Lambda\varphi \quad V = -2\Lambda\varphi$ ***D*** = 3

The result: we can study **DSG** instead of **DVG**

General **dilaton gravity** coupled to massless vectors and eff. massive scalars

$$\mathcal{L}^{(2)} = \sqrt{-g}\,[\,U(\varphi)R(g) + V(\varphi) + W(\varphi)(\nabla\varphi)^2 + \\ + X(\varphi, \psi, F^2_{(1)}, ..., F^2_{(A)}) + Y(\varphi, \psi) + \sum_n Z_n(\varphi, \psi)(\nabla\psi_n)^2\,]\,.$$

Dilaton gravity **dual** to vecton gravity with **massless Abelian vector fields,** Weyl frame

$$\mathcal{L}^{(2)}_{\text{eff}} = \sqrt{-g}\left[\varphi R + V(\varphi, \psi) + X_{\text{eff}}(\varphi, \psi; q) + \sum Z(\varphi, \psi)(\nabla\psi)^2\right].$$

Dilaton – Scalar Gravity (DSG) **dual** to **massive vecton gravity** in Weyl frame

$$\mathcal{L}_{\text{dsg}} = \sqrt{-g}\left[\varphi R + U(\varphi, \psi, q) + \bar{Z}(\varphi)(\nabla q)^2 + \sum Z(\varphi, \psi)(\nabla\psi)^2\right]$$

A general theory of **HORIZONS** in DSG

L' /2h = $\varphi R + U(\varphi, \psi, q) + \bar{Z}(\varphi)(\nabla q)^2$ (omitting normal scalars)

Consider **STATIC** solutions that normally **have horizons** when there are **no scalars**

All the equations can be derived from the Hamiltonian (constraint)

H = $\dot{\varphi}\,\dot{h}/h + hU + \bar{Z}\,\dot{q}^2 + Z\,\dot{\psi}^2$ (**= 0** in the end)

Without the scalars the EXACT solutions is: $h = C_0^2\,[N_0 - N(\varphi)]$

where $N(\varphi) \equiv \int U(\varphi) d\varphi$ $\quad C_0 \tau = \int d\varphi\,[N_0 - N(\varphi)]^{-1}$

There is always a horizon, i.e. $h \longrightarrow 0 \text{ for } \varphi \longrightarrow \varphi_0$

Horizons are classified into:

:

regular **simple**, regular **degenerate**, **singular**

Differentiation w.r.t. dilaton

$$q' = P\bar{Z}^{-1}\,, \quad \psi' = EZ^{-1}\,, \quad (\chi P)' = -\frac{1}{2}HU_q\,, \quad (\chi E)' = -\frac{1}{2}HU_\psi\,,$$

$$\chi' = -HU\,, \qquad H' = -H(P^2\bar{Z}^{-1} + E^2 Z^{-1})\,.$$

$$\bar{Z}\dot{q} = p \qquad Z\dot{\psi} = \eta \qquad \dot{\varphi} = \chi$$

$$h/\chi \equiv H,\; p/\chi \equiv P,\; \eta/\chi \equiv E$$

$$(n+1)\,\chi_{n+1} = -(UH)^{(n)}\,,$$

$$(n+1)\,q_{n+1} = (\bar{Z}^{-1}P)^{(n)}\,,$$

Recurrence relations

$$X(\varphi,q)\,Y(\varphi,q) = \sum (XY)^{(n)}\,\tilde{\varphi}^n$$

$$(XY)^{(n)} \equiv \sum_{m=0}^{n} (X)^{(n-m)}(Y)^{(m)}$$

$$2(n+1)\,(\chi P)^{(n+1)} = -(U_q H)^{(n)}\,,$$

$$(n+1)\,H_{n+1} = -(\bar{Z}^{-1}P^2 H)^{(n)}\,.$$

We find a gen. sol. near horizon as locally convergent power series in: $\tilde{\varphi} \equiv \varphi - \varphi_0$

$$h = \sum h_n \, \tilde{\varphi}^n, \qquad \chi = \sum \chi_n \, \tilde{\varphi}^n, \qquad q = \sum q_n \, \tilde{\varphi}^n, \qquad \chi(\varphi) \equiv \dot{\varphi}$$

$$h_0 = \chi_0 = 0 \qquad q_0 \neq 0 \qquad \tilde{\varphi} \equiv \varphi - \varphi_0$$

The equations for these functions are **not integrable** and we do not know exact solutions of the recurrence relations

Practically the **same equations** are applicable to studies of the **cosmological models** with vecton. The best chance to **test the theory is in cosmology**

However, we can show that the global picture cannot be found without knowledge of horizons connecting static and cosmological solutions.

It is important to use local language

BUT!

The importance of being global

As distinct from the standard Einstein theory, the generalized one is **not integrable** even in dimension one (static states and cosmologies). Therefore, in addition to the above solutions we need a global information on the system, which we may attempt to present as

topological portrait.

We try to demonstrate that the portrait must include **both static and cosmological** solutions, and that the most important info is in the structure of horizons. Actually, it is not less important for cosmologies than for static states. We prefer to use the **local language** and do not use the term Black Hole which should be reserved for real physical objects

For the moment, the idea can be explained only on integrable systems and only on the plane. For nonintegrable systems we need **3D portraites**

$$V = u(\varphi)v(\psi) \qquad Z(\varphi) = (g_0/u(\varphi)) \int u(\varphi)d\varphi$$

Integral $\quad Zh^{-1}\dot{h} - g_1\partial\varphi = C_0 \quad$ Weyl frame

Rather general integrable models, ATF `96

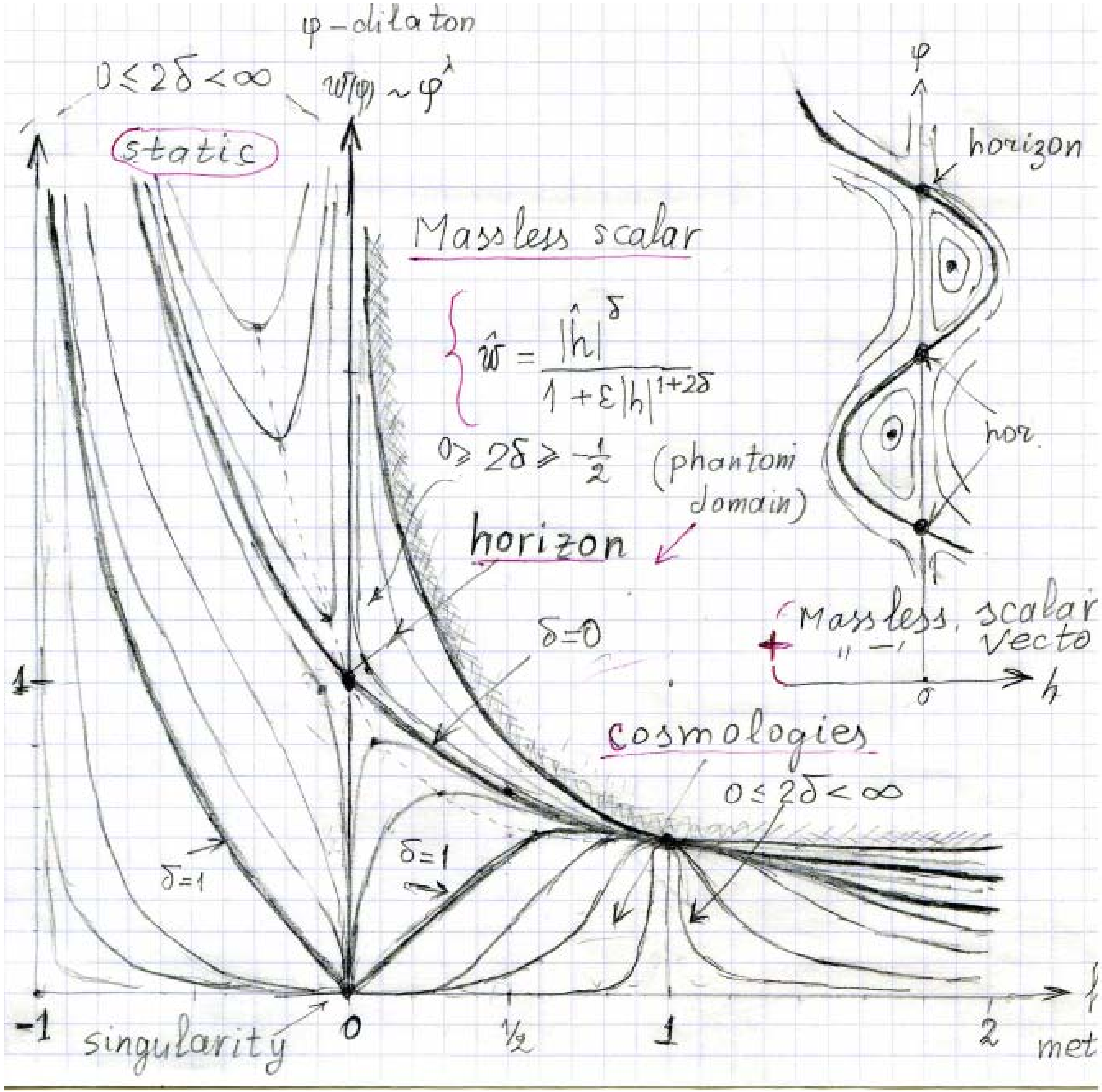
$$V = W(\bar{g}_4 w^2 - \bar{g}_1), \quad Z^{-1} = W(\bar{g}_3 + \bar{g}_2 \log w)$$

$$(F/W)^2 + 4\bar{g}_1 h + 2\bar{g}_2 C_0^2 \log h = \bar{C}_1$$

$$W = (1-\nu)/\phi, \; w = \phi^{1-\nu}, \; Z = -\gamma\phi$$

$$\bar{g}_1 = \bar{g}_2 = 0$$

$$w = \frac{|h|^\delta}{|1 + \varepsilon|h|^{1+2\delta}|} . \qquad w(\phi), \; w'/w \equiv W/U'$$



φ – dilaton
$0 \leq 2\delta < \infty$
$w(\varphi) \sim \varphi^\lambda$
static
Massless scalar
$\hat{w} = \frac{|\hat{h}|^\delta}{1 + \varepsilon |h|^{1+2\delta}}$
$0 \geq 2\delta \geq -\frac{1}{2}$
(phantom domain)
horizon
δ=0
δ=1
δ=1
cosmologies
$0 \leq 2\delta < \infty$
singularity
-1
0
½
1
2
met
φ
horizon
hor.
Massless scalar
" – vecto
h

Effects of *nonlinear Lagrangians*
must be studied (like in `B-I cosmology')

Crucial thing is to learn of how to find (partial) **portraits** in **non integrable** case (necessary **3D portraits!)**

Vecton dark matter can be produced in ***strong** gravitational fields* only.
Quantum gravity is necessary!

Inflation and ***dark matter*** are crucial things to study and test the theory in cosmological models

THE

END

Define the following densities of the weight two

$$d_0 \equiv 4!\,\det(s_{ij}) = \epsilon^{ijkl} s_{im} s_{jn} s_{kr} s_{ls} \epsilon^{mnrs} \equiv \epsilon \cdot s \cdot s \cdot s \cdot s \cdot \epsilon\,.$$

$$d_1 \equiv \epsilon \cdot s \cdot s \cdot s \cdot \bar{a} \cdot \epsilon\,, \qquad d_2 \equiv \epsilon \cdot s \cdot s \cdot a \cdot a \cdot \epsilon\,,$$

$$d_4 \equiv \epsilon \cdot a \cdot a \cdot a \cdot a \cdot \epsilon$$

where $\bar{a}$ denotes the matrix $a_i a_j$

$$\det(s_{ij} + \nu a_{ij}) = \frac{1}{4!}\,(d_0 + 6\nu^2 d_2 + \nu^4 d_4)$$

A more general Lagrangian

$$\mathcal{L} \equiv \alpha_0 \sqrt{|\,d_0 \;+\; \alpha_1 d_1 \;+\; \alpha_2 d_2 \;+\; \alpha_4 d_4|}$$