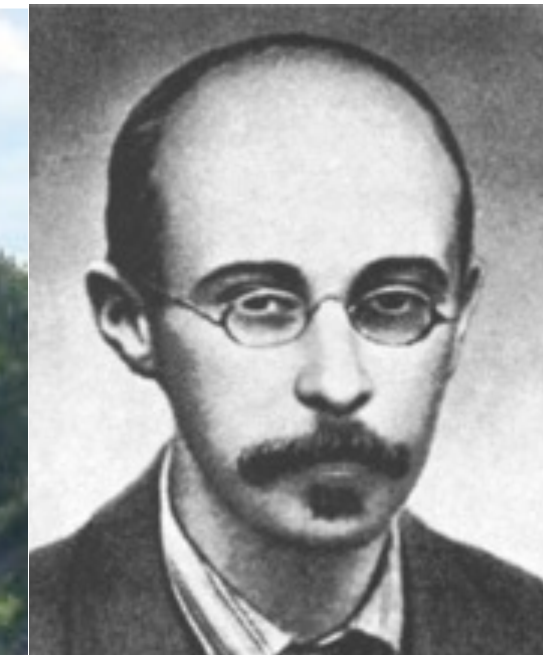

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Black Holes in Supergravity

Sergio Ferrara

(CERN - LNF-INFN - UCLA)

Round-Table Italia-Russia @ Dubna

Black-Holes in Mathematics and Physics

BLACK HOLES IN THE SUPERWORLD

M-THEORY : MYSTERY AND MAGIC

Black Holes are perhaps the most mysterious and fascinating outcome of Einstein's (1879-1955) theory of General Relativity.

Mathematically discovered by accident whilst trying to merge Newton's Law of gravitation with general (relativistic) covariance.

Nowadays they are predicted by fundamental (candidate) theories of QUANTUM GRAVITY (Superstring? M-theory?) and observed in the sky as relics of collapsing stars.

They seem to encompass many of the mysteries of the evolution of our Universe from its creation to its final destiny (the big crunch?) or its eternal existence (endless expansion?).

Astrophysical Black Holes have huge masses (solar mass scale ~ 2×10^{30} kg) while Quantum Gravity black holes have tiny masses (Planck mass scale ~ 2×10^{-8} kg) although much bigger than particle masses (proton mass scale ~ 1.6×10^{-27} kg)

SUPERGRAVITY BLACK HOLES are the black holes of the SUPERWORLD. Supersymmetry requires that they are EXTREMAL, i.e. have vanishing temperature, are marginally stable, but carry ENTROPY

The black hole Entropy makes a bridge between classical gravity and Quantum gravity

Its macroscopic definition (Bekenstein-Hawking) connects its value to the BH HORIZON AREA

$$S^{MA}_{BH} = \frac{1}{4} A_H$$

Its microscopic definition relates its value to the microstate counting

$$S^{MI}_{BH} = \log N_{MICROSTATES}$$

Remarkably these formulae (in certain approximations) give the same result in Superstring Theory!

(Strominger - Vafa)

WHAT IS THE SUPERWORLD ?

It is a hypothetical physical reality whose environment is not ordinary space but superspace.

SUPERSPACE (Salam, Strathdee; Ferrara, Wess, Zumino) is a mathematical entity which extends the notion of Riemann (1826-1866) manifold to a SUPERMANIFOLD.

Othe than usual coordinate points: x_μ ($\mu = 1 -- D$) in a D-dimensional spce M_D with Lorentz (1853-1928) signature, superspace includes GRASSMANN (1809-1877) anticommuting coordinates θ_α ($\alpha = 1 -- 2^{[D/2]}$) with two basic properties (

1) $\theta_\alpha \theta_\beta = - \theta_\beta \theta_\alpha \Rightarrow \theta_\alpha^2 = 0$ (nilpotency)

2) They transform as "Spinors" [Cartan (1869-1951), WEYL (1885, 1955)] under the Lorentz group.
SPINORS ARE RELATED to modules of CLIFFORD (1845-1879) algebras and to the universal covering group of the Lorentz group (spin group)

The group of motion in SUPERSPACE is SUPERSYMMETRY as much as the group of motion in ordinary space-time is the POINCARÉ (1854-1912) group

$$x_\mu \rightarrow x_\mu + i\bar{\epsilon}^\alpha (\gamma_\mu)_\alpha^{\ \beta} \theta_\beta$$

$$\theta_\alpha \rightarrow \theta_\alpha + \epsilon_\alpha$$

so that

$$[\delta_1, \delta_2]\, x_\mu = 2i\,\bar{\epsilon}_2^{\ \alpha} (\gamma_\mu)_\alpha^{\ \beta} \epsilon_{1\beta}$$

and the SUPERSYMMETRY ALGEBRA is a GRADED LIE ALGEBRA with basic anticommutator

$$\{Q_\alpha, Q_\beta\} = 2\,(\gamma_\mu C^{-1})_{\alpha\beta}\, P^\mu$$

(Wess, Zumino)
(Gol'fand, Likhtman)
(Volkov, Akulov)

with Q_α Majorana (1906-1938) spinors

The supermanifold where the SUPERGROUP acts is denoted by $\mathcal{M}_{D,\,2^{[D/2]}} = \mathcal{M}_{b,f}$ where (b, f) denote bosonic and fermionic coordinates.

Its total (graded) dimension is $b+f$. $b_{MAX}=11$, $f_{MAX}=32$

By replacing a single (spinor) coordinate θ_α by N of them θ^I_α ($I=1 \ldots N$) we get

" Extended Superspace" and the corresponding extended supersymmetry algebra

$$\{Q^I_\alpha, Q^J_\beta\} = 2(\gamma_\mu C)_{\alpha\beta} P^\mu \delta^{IJ} \quad (+\text{ central terms})$$

By writing the 4D extended algebra in a Weyl basis and using Van der Waerden spinors

$$\{Q^I_\alpha, \bar{Q}_{\dot\alpha J}\} = 2(\sigma_\mu)_{\alpha\dot\alpha} P^\mu \delta^I{}_J$$

$$\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} Z^{IJ} \quad (\text{central term})$$

It is precisely the presence of the central charge Z^{IJ} which make possible the existence of supersymmetric BLACK HOLES

SUPERTHEORIES:

Supertheories describe interactions in the SUPERWORLD. It is remarkable that such theories may encompass gauge interactions (SUPER YANG-MILLS THEORIES: Ferrara, Zumino; Salam, Strathdee) as well as gravitational interactions (SUPERGRAVITY: FERRARA, FREEDMAN, VAN NIEUWENHUIZEN; DESER, ZUMINO)

HOWEVER THESE THEORIES EXIST ONLY FOR FEW VALUES OF N AND OF THE SPACE-TIME DIMENSION D (Gell-Mann; Nahm).

SUPERYANG-MILLS at $D=4$ require $1 \leq N \leq 4$ and at most live at $D=10$ (Brink, Scherk, Schwarz)

SUPERGRAVITY at $D=4$ require $1 \leq N \leq 8$ and at most live at $D=11$ (Cremmer, Julia, Scherk)

FROM SCHWARZSCHILD TO REISSNER-NORSTRÖM
THE CASE OF EXTREMAL BLACK HOLES

The celebrated B-H solution of pure Einstein theory looks (in a chosen spherical coordinates)

$$ds^2_{Schw} = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

The naked singularity at $r=0$ is covered by the event horizon at $r = 2M$ (which is only a coordinate singularity)

Its generalization to a "charged" black hole, in the Einstein-Maxwell theory is the R-N black hole with metric

$$ds^2_{RN} = -\left(1-\frac{2M}{r}+\frac{q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{q^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega^2$$

Such metric exhibit two horizons: $r_\pm = M \pm \sqrt{M^2 - q^2} = M \pm r_0$

$r_+ =$ event horizon $r_- =$ Cauchy horizon

The Cosmic Censorship Principle requires $M \geq |q|$ otherwise there is no horizon and the singularity is no longer unphysical (i.e. covered by the horizon)

The thermodynamical properties of the B-H relate the area of the event horizon to the Entropy through the Bekenstein-Hawking formula:

$$S_{BH} = \frac{1}{4} A_H = \pi R_+^2$$

when R_+ is the event horizon radius for R-N while it becomes an "effective radius" in presence of other B-H attributes such as angular momentum J and/or scalar charges Σ. For instance, in presence of the latter $R_+^2 = r_+^2 - \Sigma^2 \leq r_+^2$.

Another thermodynamical quantity is the TEMPERATURE which is related to the so called SURFACE GRAVITY κ through the formula

$$T_{BH} = \frac{c}{2 S_{BH}} , \quad c = \frac{1}{2}(r_+ - r_-)$$

A black hole is extremal if $c = 0$ i.e. $r_+ = r_-$ which, for RN, happens when $M = |q|$.

A supergravity black-hole is "SUPERSYMMETRIC" (BPS-saturated) if its (ADM) mass equals the "highest eigenvalue" of the central charge matrix $Z^{IJ} = -Z^{JI}$ evaluated at asymptotic infinity.

This makes a difference if "scalar charges" Σ, as it happens in $N \geq 2$ SUPERGRAVITY, are present

When angular momentum is added (as well as magnetic charge) the horizon radii become (Kerr, Kerr-Newman)

$$r_\pm = M \pm \sqrt{M^2 - q^2 - p^2 - J^2/M^2}$$

So that, even for a neutral spinning BH (Kerr) we reach extremality when $M^2 = J$ (in Planck unit)

Nearly extremal Kerr-Black Holes have been observed in the sky in our galaxy GRS 1915+105.
It was discovered on 15 August 1992 — ($M_{BH} = 10\, M_\odot$)
Its extremality parameter $a^* = \dfrac{J}{G M_{BH}^2} \gtrsim 0.98$

(Milky Way)

(its spin is $J = 10^{78} \hbar$). $M_{BH} = 10\, M_\odot$

It has been argued that such BH has an exact CFT dual (Guica, Hartman, Song, Strominger)

BLACK HOLES AND SUPERSYMMETRY

One of the main properties of SUPERGRAVITY is the presence of scalar fields not minimally coupled to vector fields

$$\mathcal{L} \propto g_{\Lambda\Sigma}(\phi) F^\Lambda F^\Sigma + \theta_{\Lambda\Sigma}(\phi) F^\Lambda \tilde{F}^\Sigma \qquad (F^\Lambda = dA^\Lambda)$$

with the implication that the Maxwell-Einstein black hole gets a nontrivial modification.

In particular the B-H flow toward the horizon is accompanied by trajectories of scalar field evolutions from asymptotic infinity to the horizon.

$$\phi(r)_{r\to\infty} = \phi_0 \in \mathcal{M} \longrightarrow \phi(r)_{r\to r_H} = \phi_{crit}$$

The resulting analysis exploits the ATTRACTOR MECHANISM

(Ferrara, Kallosh, Strominger)

Scalar fields behave as dynamical systems.

In their evolution toward the B-H horizon of an extremal black hole they loose memory of their initial conditions (of ϕ_∞) and approach a critical point with zero velocity

$$\phi(r) \underset{r \to r_H}{\longrightarrow} \phi_{crit}(Q) \quad \text{as} \quad \dot{\phi}(r) \underset{r \to r_H}{\longrightarrow} 0$$

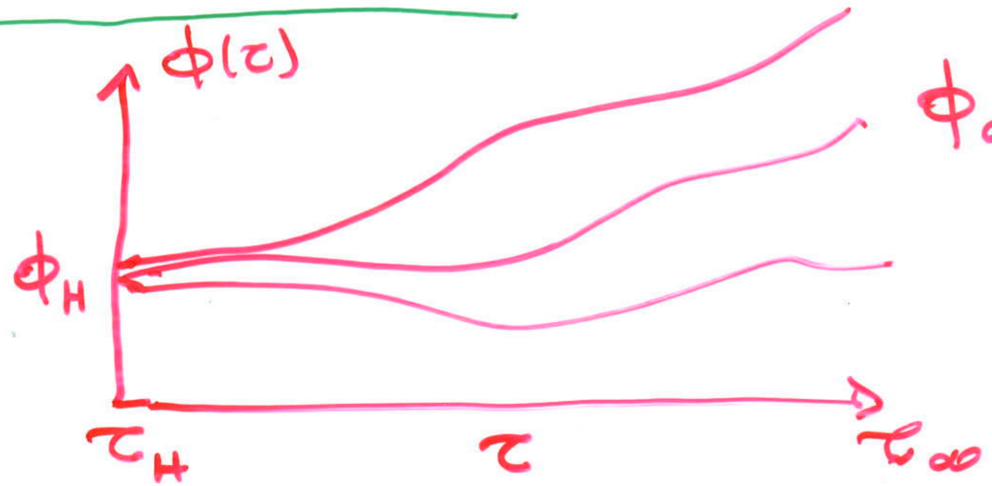
and consistency of the solution implies that ϕ_{crit} is a critical point of an "effective" black hole potential $V_{BH}(\phi, Q)$ extremized at $\partial V = 0 \Big|_{\phi = \phi_{crit}}$

Single centered black holes:

Attractor flow:

$\dot{\phi}_H \to 0$

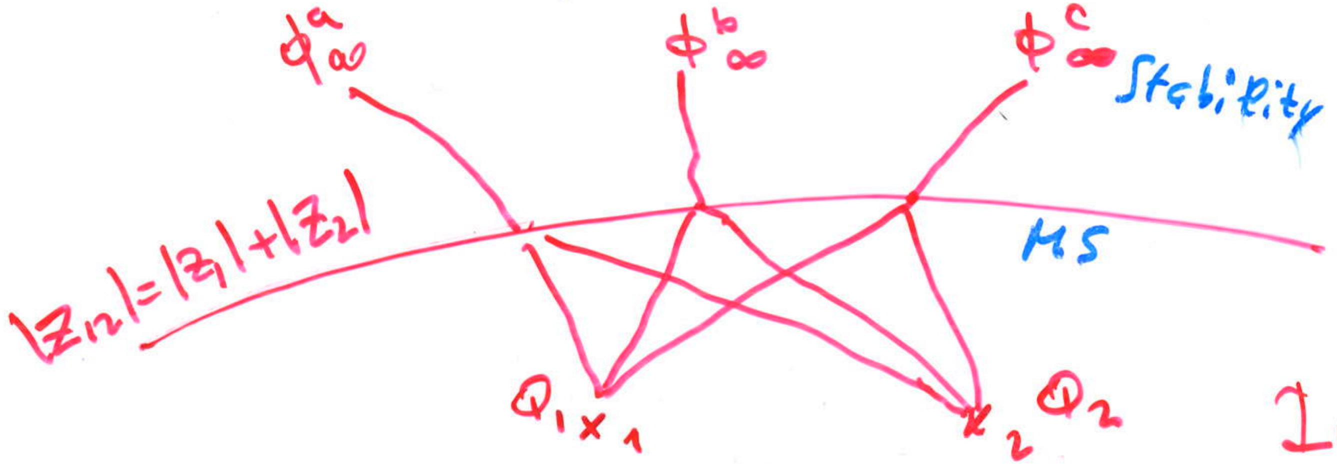
(Kallosh, Strominger, S.F.)



Multicentered black holes (split attractor flow)(p=2)

Existence of a marginal stability wall (Denef et al.)

Two center BH is stable until it crosses the MS line and then it decays into constituents



$S(Q_1,Q_2)= S(Q_1) + S(Q_2)$

$(Q_1, Q_2) \neq 0$

MS

$|x_1-x_2| \to \infty$

$\mathrm{Im}\, Z_1 Z_2^* \to 0$

Two centered black-holes: $(Q_1, Q_2) \neq 0$ (Denef...)

BPS ADM mass $|Z(Q_1) + Z(Q_2)|$

Marginal decay $\rightarrow$ $\mathrm{Im}\, \cancel{Z}_1 Z_2^* = 0$ $(\mathrm{Re}\, Z_1 Z_2^* > 0)$

Stability region $\rightarrow$ $(Q_1, Q_2)\, \mathrm{Im}\, Z_1 Z_2^* > 0$

Equilibrium distance $\rightarrow$ $|x_1 - x_2| = \frac{1}{2} (Q_1, Q_2) \frac{|Z_1 + Z_2|}{\mathrm{Im}\, Z_1 Z_2^*}$

Angular momentum $\rightarrow$ $J = \frac{1}{2} (Q_1, Q_2) \frac{x_1 - x_2}{|x_1 - x_2|}$

Entropy $\rightarrow$ $|I_4(Q_1)|^{1/2} + |I_4(Q_2)|^{1/2}$

e.m. duality invariants: $SL(2, R) \times G_4$

(FM, O, S, Y) $\downarrow$ horizontal symmetry

$\exists$ e.m. duality invariant for all $N=2$ rank 3 cosets

and for $N = 4, 5, 6, 8$ —

The "Attractor Mechanism" has a series of consequences

1) It explains why the Bekenstein-Hawking entropy, for extremal BH, is independent of scalar charges

2) it allows to classify BH solutions, i.e. critical points of the B-H potential, through the electric-magnetic DUALITY SYMMETRY of the theory

3) It allows to reduce the dynamical B-H flow to a "first order" evolution both for supersymmetric and non-supersymmetric black holes

4) It makes possible to have "extremal" solutions which are not supersymmetric

The Bekenstein-Hawking Entropy Area formula becomes:

$$S_{BH} = \frac{1}{4} A = \pi V_{crit}(\varphi, \phi_{crit})$$

(W = (fake) superpotential)

$$\left(V_{crit} = W^2_{crit} \text{ at } \partial W = 0 \right)$$

$$V_{BH} = W^2_{DO} + 4\, \partial_i W_{DO} \partial^i W_{DO}$$

Duality orbits classify the critical points of V.

For each duality orbit W_{DO} has a different expression, in the supersymmetric case $W = |Z|$ when $|Z|$ is the highest value of the central charge matrix. The duality orbits are modulus of groups of type E_7 as requested from the GAILLARD-ZUMINO analysis combined with the ATTRACTOR MECHANISM.

SUPERGRAVITY SEQUENCE (N=2 SYM. SPACES)

	G	R MODULE	PRIMITIVE SYM. INV.
J_3^{O}	$E_{7(-25)}$	56	I_4
J_3^{H}	$SO^*(12)$	32	I_4
J_3^{C}	$SU(3,3)$	20	I_4
J_3^{R}	$Sp(6,R)$	14'	I_4
T^3	$SL(2,R)$	4 (spin 3/2)	I_4
$J_{2,n}$	$SL(2,R)\times SO(2,n)$	$(2,2+n)$	I_4
CP^n	$U(1,n)$	$(1+n)_c$	I_2

THE SUPERGRAVITY SEQUENCE $(N \geq 3)$

N	G	R MODULE	PRIMITIVE SYMMETRIC INVARIANT
3	$U(3,n)$	$(3+n)_c$	I_2
4	$SL(2,R) \times SO(6,n)$	$(2,6+n)$	I_4
5	$SU(5,1)$	20	I_4
6	$SO^*(12)$	32	I_4
8	$E_{7(7)}$	56	I_4

The role of the exceptional group E_7 and its 56 dim. module.

1) It is the electric-magnetic duality symmetry of N=8 supergravity in four dimensions, since it relates 28 electric to 28 magnetic charges

2) The orbits of the 56 module classify the black holes with different fraction of SUPERSYMMETRY

3) It controls the ultraviolet divergences of perturbation theory since it is Anomaly free

4) Its Arithmetic subgroups may encode the nonperturbative quantum corrections

FUTURE DIRECTIONS:

1) Extension of black-hole solution to multi-center solutions and their classification (Dynamics: splitting and microstate counting)
(MULTI-CENTER ORBITS)

2) Quantum Corrections and the final role of E_7

3) Inclusion of the Attractor Mechanism in presence of higher derivative modification of GRAVITY as suggested by SUPERSTRING THEORY

4) ROLE OF $N=8$ BLACK HOLES IN A PERTURBATIVELY FINITE THEORY OF $N=8$ SUPERGRAVITY