Some observable effects from primordial black holes

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CONTENT. Potpourri (or Irish rague, according to Jerome K. Jerome):

 Primordial black holes and the observed Galactic 511-keV line. C. Bambi, A.D. Dolgov, A.A. Petrov Phys. Lett. B670 (2008) 174, err. 681 (2009) 504.
 Black holes as antimatter factories.
 C. Bambi, A.D. Dolgov, A.A. Petrov, JCAP 0909 (2009) 013. Implications of primordial black holes on the first stars and the origin of the super-massive black holes.
 Bambi, D. Spolyar, A.D. Dolgov,
 K. Freese, M. Volonteri, Mon. Not.
 Roy. Astron. Soc. 399 (2009) 1347.
 Relic gravitational waves from light primordial black holes. A.D. Dolgov,
 D. Ejlli, Phys.Rev. D84 (2011) 024028. All these works are united by the hypothesis of abundant cosmological population of PBHs either in the early universe, $M_{BH} < 10^8$ g, efficiently creating GWs, or in the contemporary universe for an explanation of 0.511 MeV line, $M = 10^{16} - 10^{21}$ g, or at the first star format epoch, $M > 10^{22}$ g, and creation of heavy and superheavy BH.

Proton to positron transformation (NB: non conservation of B and L) by the Schwinger process at the Schwarzshild radius, i.e. production of e^+e^- -pairs by static electric field. The production probability per unit time and volume is

 $W = rac{m_e^4}{\pi^2} \left(rac{E}{E_c}
ight)^2 \sum_{n=1}^{\infty} rac{e^{-n\sqrt{\pi}E_c/(2E)}}{n^2},$ where $E_c = m_e^2/e.$ $p + BH \rightarrow e^+ + BH$ Rough estimate of E by equality of Coulomb repulsion and gravitational attraction:

$\frac{\alpha Q}{2}$	$m_p M_{BH}$
$\overline{r_g^2}$ ($\frac{1}{m_{Pl}^2 r_g^2}$

Hence $E \sim \alpha Q/r_g^2 \sim m_p m_{Pl}^2/M_{BH}$. Demanding $E \geq E_c$ we find

 $M_{BH} \le 10^{20} \,\mathrm{g}$.

The electric field can be considered as homogenous: $dE/(drEp_e) \ll 1$ or, what is the same, $\lambda/r_g \ll 1$. Process of PBH charging.

Different mobilities of protons and electrons in interstellar gas result in positive charging of stellar bodies. Electrons are much stronger coupled to the surrounding medium. (The usual mechanism of star charging, V. Schwarzman, 1970?) Equations of motion for p and e fluids:





The equilibrium electric charge is

$$lpha Q pprox rac{r_g m_p}{4K^{1/4}} \left(T_e/T_p
ight)^{3/4} \sqrt{rac{\ln(\lambda_p/r_g)}{\ln(\lambda_e/r_g)}},$$

where $K = m_p/m_e \approx 2000$. For $T_p = T_e$, $\alpha Q \approx 0.15 m_p r_g$ and:

$$rac{E_c}{E} = 0.6 \left(rac{M}{10^{20} \ {
m g}}
ight) \, .$$

A larger T_e/T_p (is it possible?) leads to larger Q and larger electric field. Thus heavier PBH are allowed. **Positron energy**

$$E_{e^+} = 140\,MeVrac{10^{20}\,g}{M}.$$

Production rate:

 $\Gamma_{e^+} \sim (10^{20} \, g/M)^2 e^{(-0.528M/10^{20}g)} \, .$

If we take $M \sim 10^{21}$ g, the suppression of the production rate would be by $5 \cdot 10^{-5}$, still not bad. The process of discharge is much faster than the process of charging.

Could this be the source of the intense annihilation line from the galactic center?

Observed 511 keV gamma ray line flux

 $\Phi \approx 1.0 \cdot 10^{-3} \text{ photons cm}^{-2} \text{ s}^{-1},$

energy width of about 3 keV, the angular full width at half maximum is about 9° . Great annihilator.

Problem: too large positron energy for $E = E_c$ but might be OK for smaller E i.e. larger M. Another possible mechanism could be PBH evaporation, similar to other works but the mass spectrum of PBHs is different:

 $rac{dN}{dM} = C \exp\left[-\gamma \ln^2(M/M_0)
ight],$ (AD and J. Silk, 1994).



 γ ray spectra from PBHs in the Buldge (blue) and of the background (red) in $\gamma/\text{cm}^2/\text{s}$. PBHs are assumed to have log-normal distribution, $M_0 = 6 \cdot 10^{16}$ g and $\gamma = 1$. Their number is fixed by the condition that their total mass in the innermost 0.6 kpc is $5 \cdot 10^9 M_{\odot}$, to explain the 511 keV line.



Cosmic isotropic gamma ray spectra from PBHs (blue) and of the measured background (red) It is assumed that PBHs make all cosmological dark matter and have log-normal distribution with $\gamma = 1$ and $M_0 = 6 \cdot 10^{16}$ g.



Diffuse cosmic neutrino spectrum from primordial BHs



Gamma ray spectra from PBHs (blue) and of the measured background (red) from the Bulge. The BHs are assumed to have the same mass: $M = 10^{15}$ g and produce the right amount of e^+ to explain the observed 511 keV line.



Gamma ray spectra from PBHs (blue) and of the measured background (red) from the Bulge The BHs are assumed to have the same mass: $M = 10^{16}$ g and produce the right amount of e^+ to explain the observed 511 keV line.



Gamma ray spectra from PBHs (blue) and of the measured background (red) from the Bulge The BHs are assumed to have the same mass: $M = 10^{17}$ g and produce the right amount of e^+ to explain the observed 511 keV line.

Other mechanisms:

 Production by type Ia supernovae,
 P.A. Milne, L.S. The, M.D. Leising,
 Astrophys. J. Suppl. 124, 503 (1999).
 By low mass X-ray binary systems,
 J. Knodlseder et al., Astron. Astrophys. 441, 513 (2005).

3. Energetic electrons and photons produced at accretion on the supermassive black hole at the Galactic Center, T. Totani, Publ. Astron. Soc. Jap. 58, 965 (2006). 4. Positron production by collision of energetic photons produced in accretion to super-massive central black hole and to surrounding primordial black holes with mass about 10¹⁷ g, L. Titarchuk and P. Chardonnet, Astrophys. J. 641, 293 (2006).

5. Annihilating light dark matter particles. C. Boehm, D. Hooper, J. Silk, M. Casse and J. Paul, Phys. Rev. Lett. 92, 101301 (2004); D. Hooper, F. Ferrer, C. Boehm, J. Silk, J. Paul, N.W. Evans and M. Casse, Phys. Rev. Lett. 93, 161302 (2004); C. Boehm and P. Fayet, Nucl. Phys. B 683, 219 (2004). 7. Decaying unstable relics and, in particular, sterile neutrinos. C. Picciotto and M. Pospelov, Phys. Lett. B 605, 15 (2005).

 MeV right-handed neutrino interacting with baryonic matter. J.M.
 Frere, F.S. Ling, L. Lopez Honorez, E. Nezri, Q. Swillens and G. Vertongen, Phys. Rev. D 75, 085017 (2007).
 Strangelets. D.H. Oaknin and A.R.
 Zhitnitsky, Phys. Rev. Lett. 94, 101301 (2005). 10. Positrons originating from primordial antimatter (antistars) by capture of protons. C. Bambi and A.D. Dolgov, Nucl. Phys. B 784, 132 (2007).
11. Decays of milli-charged particles.
J.H. Huh, J.E. Kim, J.C. Park and S.C. Park, arXiv:0711.3528 [astro-ph].

However, type Ia supernovae have a different galactic distribution and their rate is probably an order of magnitude smaller than the one necessary to explain the observed flux. Low mass X-ray binaries also do not fit the enhanced Bulge component. Lastly, light dark matter particles are theoretically not well motivated, might be inconsistent with observations and their mass should be very close to m_e , because from the comparison of the Galactic gamma ray emission above and below 511 keV, one can conclude that the injected energy of the positrons cannot be larger than about 3 MeV.

Contemporary gravitational waves from PBHs.

Based on:

A.D., P.D. Naselsky, I.D. Novikov

e-Print: astro-ph/0009407; A.D., D. Ejlli, Phys.Rev. D84 (2011) 024028. Modified cosmological thermal history: matter dominance (by PBH) in the early universe, all previous relics are diluted, including inflationary GWs. Early period of structure formation at very small scales.

Universe heating by PBH evaporation, 2nd RD stage, "return to normality". Possibly observable GW induced by PBH scattering and binaries in high density PBH clusters. To avoid conflict with BBN we need:

 $\tau_{BH} < 0.01 sec < t_{BBN} \sim 1 \, {\rm s}, \label{eq:tbb}$ where

$$au_{BH} pprox rac{5\cdot 2^{11}\pi}{N_{eff}} rac{M^3}{m_{Pl}^4},$$

(grey factor is neglected). Here $N_{eff} \sim 100$ is the number of species with $m < T_{BH} = m_{Pl}^2/(8\pi M)$. Correspondingly $M_{BH} < 2 \cdot 10^8$ g. I. Cosmological story of PBH.

PBHs are formed if the density contrast at horizon scale is of the order of unity, $\delta \rho / \rho \sim 1$. Hence PBHs formed at cosmological time t_p , have masses:

 $M=t_pm_{Pl}^2\,,\quad t_p=r_g/2\,,$

where $r_g = 2M/m_{Pl}^2$ and

 $m_{Pl} = 1.22 \times 10^{19} GeV \approx 2.18 \times 10^{-5} g.$

Mass spectrum of PBHs.

1. Flat, inflationary perturbations lead to a power law spectrum.

2. Modified Afleck-Dine baryogenesis, leads to log-normal spectrum (AD and J. Silk):

 $rac{dN}{dM} = C \exp{[(M-M_0)^2)/M_1^2]}\,.$

and possibly to a large cosmological mass fraction of PBH. Relative cosmological energy density of BHs at production is

 $\Omega_{BH}(t_p)\equiv\Omega_p,$

model dependent parameter. Normally $\Omega_p \ll \Omega_{tot} \approx \Omega_R \approx 1$, thus the universe was at RD stage before and after production of BH with $\rho = 3m_{Pl}^2/(32\pi t^2)$, till BH started to dominate, if they lived long enough. At RD stage $\Omega_{BH} \sim a(t) \sim t^{1/2}$, until Ω_{BH} rises up to unity at $t = t_{eq} = M/(m_{Pl}^2 \Omega_p^2)$. t_{eq} is the onset of BH dominance. Condition of PBH dominance, $\tau_{BH} > t_{eq}$, demands:

$$M > 5.6 \cdot 10^{-2} \left(\frac{N_{eff}}{100}\right)^{1/2} \frac{m_{Pl}}{\Omega_p}.$$

After evaporation $\Omega_{BH} \rightarrow 0$, while $\Omega_{tot} = 1$ remains and the 2nd RD stage begins. Rise of density perturbations.

At MD stage primordial density perturbations rise as $\Delta \equiv \delta \rho / \rho \sim a(t)$. For sufficiently long MD stage, Δ would reach unity and after that quickly rises to $\Delta \gg 1$.

High density clusters of PBHs would be formed and GW emission could be strongly amplified. The regions with high n_{BH} would emit GW much more efficiently than in the homogeneous case. The emission of GW is proportional to vn_{BH}^2 and, both the BH velocity in dense regions and n_{BH} would be by several orders of magnitude larger than those in the homogeneous universe. Perturbations could become large if $\tau_{BH} > t_1$, where t_1 is the moment when $\delta \rho / \rho \sim 1$. To this end PBM mass should be bounded from below:

$$M > 10^{3} \mathrm{g} \frac{10^{-6}}{\Omega_{p}} \left(\frac{10^{-4}}{\Delta_{in}}\right)^{3/4} \left(\frac{N_{eff}}{100}\right)^{1/2}$$

After Δ reached unity, rapid structure formation would take place: violent relaxation with non-dissipating dark matter. Maximum velocity in the cluster is limited by the condition of sufficiently large M to reach $\Delta \equiv \delta \rho / \rho \geq 1$ and reads:

$$v_{max} pprox 0.01 \Delta_{cl}^{1/6} \left(rac{\Delta_{in}}{10^{-4}}
ight)^{-1/3}$$

and with Δ_{cl} , as large as 10^6 , BHs can be moderately relativistic.

The density contrast $\Delta_{cl} \sim 10^6$ is assumed to be similar to that of the contemporary galaxies.

There is another effect (absent for galaxies) of increase of Δ_{cl} by several orders of magnitude due to decrease of $\rho_{cosm} \sim 1/t^2$:

 $\Delta_{cl} \sim (\tau_{BH}/t_1)^2$.

GW from PBH binaries.

Gravitationally bound systems of PBH pairs captured by dynamical friction. Luminosity of GW radiation from a single binary:

 $L = rac{32 M_1^2 M_2^2 (M_1 + M_2)}{5 r^5} pprox rac{64}{5} rac{M^5}{r^5 m_{_{I\!\!Pl}}^8}.$

Radius is expressed through ω_{orb} :

$$\omega_{orb}^2 = rac{M_1 + M_2}{m_{Pl}^2 R^3}.$$

Stationary or inspiral regimes? Stationary is more probable. PBHs evaporated before they coalescenced. Average distance between BHs in the high density cluster:

$$d_b = 0.1 r_g \Omega_p^{rac{2}{3}} \left(rac{M}{m_{Pl}}
ight)^{rac{4}{3}} \left(rac{100}{N}
ight)^{rac{2}{3}} \left(rac{10^6}{\Delta}
ight)^{rac{1}{3}}$$

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The present day energy density of GWs from binaries in stationary regime:

$$\Omega_{GW}^{(stat)}(f;t_0) = 4.88 \cdot 10^{-10} \epsilon$$
.

Frequency range from a few Hz:

$$f \geq 5 \mathrm{Hz} \left(rac{10^5 \mathrm{g}}{M}
ight)^{1/2}$$



Density parameter today $h_0^2 \Omega_{GW}$ as a function of frequency for PBH binaries in stationary approximation for $\epsilon \sim 10^{-5}$ and $M \sim 10^7$ g (solid line) and $M \sim 1$ g (dashed line).

Gravitons from BH evaporation. Average graviton energy:

$$\omega_{av}=3T_{BH}=rac{3m_{Pl}^2}{8\pi M}.$$

Gravitons carry about 1% of the total evaporated energy and thus their contribution into cosmological energy density would be about 10^{-6} .

For $\omega < \omega_{av}$ the graviton density fraction drops down to $10^{-6} (\omega/\omega_{av})^4$. Non-thermal spectrum because of redshift.



Frequency [Hz]

Density parameter per logarithmic frequency, $h_0^2 \Omega_{GW}(f;t_0)$, as a function of frequency today for M = 1 g (solid line) and $M = 10^5$ g (dashed line).

The existing and near-future detectors are not sensitive to such GW but Ultimate DECIGO (2035), which will be sensitive to $\Omega = 10^{-20}$ at f = 1 Hz may put the limit:

$$M > 10^{3.6} m_{Pl}$$

or discover them.

However, electromagnetic detectors based on resonance graviton to photon transformation are promising. The principle of such detector was proposed by Braginsky and Mensky. There is a renewed interest on these new detectors and prototype has been constructed at Birmingham University with sensitivity of the order $h_{rms} \sim 10^{-14}$ Hz^{-1/2} at $f \sim 10^8$ Hz.



 $\log [h_0^2 \Omega_{GW}(f)$ vs. $\log (f[Hz])$ for different models of production of stochastic background of GWs

Conclusion

Cosmological scenario with dominance of PBH is plausible.

This early MD-stage may be observable through high frequency GW. Heavy relics from after-inflationary heating would be forgotten.

Baryogenesis might successfully proceed in the course of BH evaporation. If DM and baryon asymmetry are produced in BH evaporation, it is natural to expect that $\Omega_{DM} \sim \Omega_b$. BBN is safe, though some distortion is possible.

Impact on CMB is weak or high frequency GW could distort it (?).

DM of PBHs and first stars.

PBHs would be captured by the first stars. Due to dynamical friction they sink to the center and form a single BH. For PBHs heavier than $\sim 10^{22} g$, the timescale for dynamical friction turns out to be shorter than the typical stellar lifetime. The central BH would accrete very rapidly and swallow the whole star.

As a result a large BH with mass $10 - 10^3 M_{\odot}$ would be created. So for $M_{PBH} \gtrsim 10^{22}$ g, the lifetimes of Pop. III stars may be shortened with implications for reionization of the Universe and for the first supernovae. In addition, since the stars are inside much larger haloes, they can in principle accrete even more matter (depending on the accretion mechanism).

Thus, the end-products of the scenario are BHs of masses $(10-10^5) M_{\odot}$. These may be the seeds which produced the super-massive BHs seen at high redshifts; the Intermediate Mass Black Holes; as well as the black holes at the center of every normal galaxy today and whose origin is as yet uncertain. The PopIII stars: $M \sim (1 - 100) M_{\odot}$. Heat source: the ordinary fusion. According to Heger and Woosley: for $M = 100 M_{\odot}$: central temperature is 1.2×10^8 K, central density 31 g/cm³, $R = 7 R_{\odot}$; for $10 M_{\odot}$, $T_{centr} = 9.6 \times 10^7$ K, $\rho_{centr} = 226 g/cm^3$, $R = 1.2 R_{\odot}$, and the stellar fusion luminosity is:

 $L_* = 6.5 \times 10^{39} \, erg/s \, \text{ for } 100 M_{\odot},$ $L_* = 4.2 \times 10^{37} \, erg/s \, \text{ for } 10 \, M_{\odot}).$ In addition to the ordinary stellar fusion there could be heat sources from PBHs, which include accretion onto the BHs, Hawking radiation, and the Schwinger mechanism.

We have found that the ordinary stellar fusion luminosity dominates over the heat sources due to PBHs. Total Mass in PBHs inside the star. First stars form at the centers of $10^6 M_{\odot}$ DM haloes. We assume an initial Navarro Frenk–White profile for both DM and baryons. As the gas collapses to form a star, its gravity pulls DM (i.e. PBHs) with it. Adiabatic contraction (Sellwood and McGaugh (2005)) leads to the DM density inside the star:

 $\rho_{DM} \approx 5 \, (n_b \ \mathrm{cm}^3)^{0.8} \ \mathrm{GeV/cm}^3.$

This result is independent of the nature of DM. For $n_b \approx 10^{24}$ /cm³, the DM to baryon matter ratio of a typical Pop. III star is about 10^{-4} . The number of PBHs inside the star:

$$N_{BH} \sim 10^7 \left(rac{10^{24} \ g}{M_{BH}}
ight) \left(rac{M_*}{100 \ M_\odot}
ight),$$

where M_* is the mass of the star. More precisely the total mass in PBHs is

 $M_{PBH}^{tot} \sim 6.3 \times 10^{30} \text{ g} \text{ for } 100 M_{\odot},$ $M_{PBH}^{tot} \sim 4.1 \times 10^{29} \text{ g} \text{ for } 10 M_{\odot}.$ Accretion of stellar material on PBH. The maximum accretion luminosity for a single PBH cannot exceed the Eddington limit

$$L_E = 6.5 \cdot 10^{28} \left(\frac{M_{BH}}{10^{24} \text{ g}} \right) \text{ erg/s.}$$

The extra heat produced by accretion onto the PBHs inside the star has negligible impact on the star physics. We estimate the accretion rate according to Bondi (1952). The accretion time scale is about:

$$au_a \sim 10^5 \left(rac{10^{24}~{
m g}}{M_{BH}}
ight) \left(rac{T}{{
m keV}}
ight)^{rac{3}{2}} \left(rac{{
m g/cm^3}}{
ho_b}
ight)~{
m yr}$$

So even a single PBH with mass greater than 10^{24} g inside the star could eat the entire star. The PBHs are thought to comprise at least some measurable fraction of the DM in the universe. If the PBHs do not comprise the entire DM, then the PBH mass could be larger than we have discussed heretofore, though contributing only a small fraction of the critical density. Formation of a larger BH at the center of the star via Dynamical Friction. According to Chandrasekhar the deceleration of BH moving at a velocity v_{BH} with respect to the fluid of light particles with Maxwell velocity distribution with dispersion σ , is

$$\begin{split} \frac{d}{dt} \vec{v}_{BH} &= -4\pi \, G_N^2 \, M_{BH} \, \rho_b \, \ln \\ \frac{\vec{v}_{BH}}{v_{BH}^3} \left[\mathrm{erf}(X) - \frac{2X \exp(-X^2)}{\sqrt{\pi}} \right] \,, \end{split}$$

where $X \equiv v_{BH}/(\sqrt{2}\sigma)$, ρ_b is the density of particles in the star, and $\ln \approx \ln (M_*/M_{BH})$.

Since the characteristic gravitational time scale

$$au_g = \sqrt{rac{r^3}{M_*(r)G_N}} \sim \left(rac{3}{4\pi
ho_b G_N}
ight)^{1/2} \ pprox 1900 \left(rac{1 \ {
m g/cm}^3}{
ho_b}
ight)^{1/2} \ {
m s}$$

is much shorter than the lower limit on the characteristic dynamical friction time scale

$$\begin{split} \tau_{DF} &= \frac{\sigma^3}{4\pi \, G_N^2 M_{BH} \rho_b \ln \Lambda} \approx \\ 5 \cdot 10^{10} \, \left(\frac{10^{24} \mathrm{g}}{M_{BH}} \right) \left(\frac{\sigma}{3 \cdot 10^7 \mathrm{~cm/s}} \right)^3 \\ & \left(\frac{1 \mathrm{~g/cm^3}}{\rho_b} \right) \left(\frac{10}{\ln \Lambda} \right) \mathrm{~s} \,, \end{split}$$

the equation of motion can be quite accurately solved.

For the time of BH formation we find: (10^{24} c)

$$\begin{split} \tau_f \approx 1.4 \cdot 10^4 \, \left(\frac{10 - \mathrm{g}}{M_{BH}}\right) \\ \left(\frac{\sigma}{3 \cdot 10^7 \mathrm{~cm/s}}\right)^3 \left(\frac{1 \mathrm{~g/cm^3}}{\rho_b}\right) \left(\frac{10}{\ln\Lambda}\right) \mathrm{~yr}\,, \end{split}$$

where $v_{BH}^{in} \approx \sigma$ is the initial PBH velocity, so while v_{BH}^{f} is the final PBH velocity, when $R_{f} = 4 \cdot 10^{2}$ cm, that is, when the orbit of the PBH is equal to the Schwarzschild radius of the final BH.

Summary and conclusions

Primordial black holes in the mass range $M_{PBH} \sim 10^{17} - 10^{26}$ g are viable dark matter candidates. If they make part of the cosmological dark matter, they could make up a significant mass fraction of the first stars. PBHs with $M < 10^{22}$ g do not have a significant effect on the evolution of primordial stars, because their timescales for Bondi accretion and for dynamical friction are larger than the lifetime of a Main Sequence star of $1 - 100 M_{\odot}$.

PBHs with $M > 10^{22}$ g might sink quickly to the center of the star by dynamical friction and form a larger black hole, which could swallow the whole star in a short time.

So, Pop. III stars would likely have lived for a short time, with implications for the reionization of the Universe after the cosmic dark ages and the nature of the first supernovae; in fact they may preclude any supernovae from the first stars. Although the BH swallowing the star shortens the star's lifetime and its contribution to reionization, the newly formed hole can become a new, alternative source of ionizing photons. The 10–100 M_{\odot} BHs that form by swallowing Pop. III stars may grow even larger: they reside in 1000 M_{\odot} of gas that are in excess of the Jeans mass and may fall into the BH. Black holes of mass 1-1000 M_{\odot} may result. The accretion could continue in the $10^6 M_{\odot}$ minihaloes of dark matter which $10^5 M_{\odot}$ of baryonic matter of low density gaseous material, $\rho \sim 10^{-24}$ g/cm³. The end-products are $10 - 10^5 M_{\odot}$ black holes, these objects may serve as the progenitors of the super-massive black holes which are in the center of every normal galaxy today.

PBHs with $M_{BH} > 10^{26}$ g, are observationally constrained to be only a fraction of the total DM, yet could be important in the first stars.

It would only take one such black hole to be pulled into the star via dynamical friction (timescale ~ 10^7 yr for a 1 M_{\odot} black hole to get from 1 pc out into the center of the star and to quickly eat up the whole star. If the effects described above do not take place, one could place bounds on the black hole abundances of various masses. E.g., if PBHs swallowed primordial stars too quickly, the cosmological metal enrichment would be problematic and in absence of viable alternatives, the current allowed mass range $M_{PBH} \sim 10^{16} - 10^{26}$ g could be further reduced to $\sim 10^{16} - 10^{22}$ g.