Shining Black Holes: Coherent Field and Global Plasma Structures Associated with Them. Relevant Collective Modes

Dubna, December 2011

Black Holes Categories

1. Shining Black Holes
2. Dark Black Holes
3. Gedanken Black Holes
4. etc.


Irresistible Oblivion
currentsofspace.blogspot
2 0 1 1

Shining Black Holes

- Characterized by intense radiation emission with different spectra and different plasma configurations associated with them.
- An approach, that I prefer, is that of looking for coherent processes and global, fields and plasma configurations.
- The computational physics approach that has been taken with increasing frequency lately is that of generating and investigating plasma turbulent states emerging from the development of the Magneto Rotational Instability with relatively short wavelengths.

For the first approach analytic methods have been adopted, considering the intrinsic limitations that these involve.

Note that

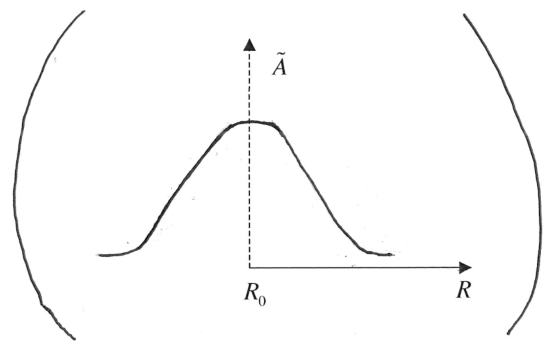
- Issues to be dealt with are numerous. Probably the most important is that of considering non-Maxwellian distributions in momentum space. In particular, the effects of these cannot by represented by a scalar pressure in the relevant total momentum conservation equations.
- Other issues such as the need or not-need of a "seed" magnetic field from which the considered configuration may grow will need special attention.
- General Relativity effects are taken into account through effective gravitational potentials that are valid at the considered distances from the black hole.

Overview of Obtained Results

1. Axisymmetric "standard" disk configurations imbedded in a vertical ("seed") magnetic field have been shown to be subjected to a spectrum of modes leading to axisymmetric or to non-axisymmetric spiral configurations. For this the linearized approximation has been used and structures with radial scale distances smaller than the height of the disk have been found.

   The driving factors of the relevant modes are the radial gradient of the rotation frequency $(d\Omega/dR)$ or the vertical gradient of the plasma temperature, i.e. $\frac{d\ln T}{dz} \bigg/ \frac{d\ln n}{dz} > \frac{2}{3}$.

An important feature of spiral modes is that they are radially localized



unlike axisymmetric modes that are purely oscillatory and not localized radially. These spiral modes are expected to be more robust, than axisymmetric modes having equal growth rates initially.

2. Local axisymmetric stationary configurations are found as non linear solution of a set of two characteristic equations (the "Master Equation" and the Vertical Equilibrium Equation). These can be

- A periodic sequence of plasma rings
- Solitary ring pairs

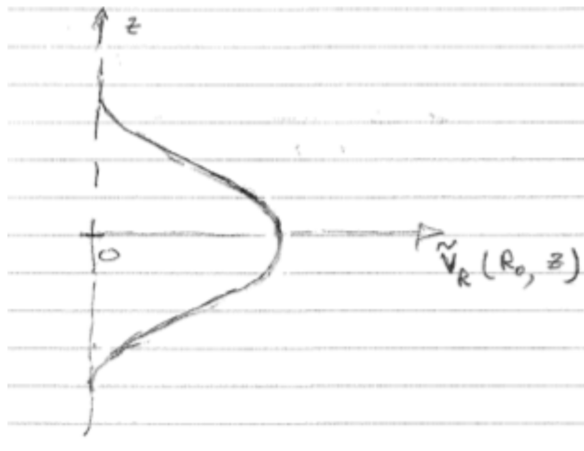
Pairs of current channels carrying oppositely directed currents are found in correspondence of these rings. A model for the emission of jets as sequences of "smoke-rings" is proposed.

3. Locally co-rotating tridimensional structures are found with out seed fields (adopting a local "rigid-rotor" approximation).

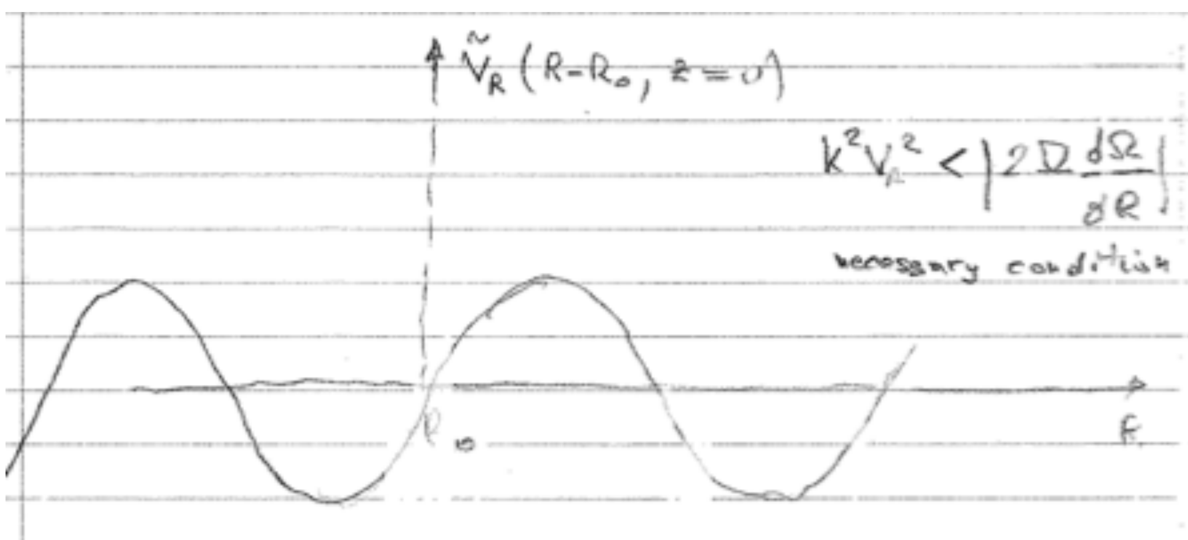
Emerging from a "Standard" Disk Configuration Imbedded in a (seed) Vertical Field

I. Axisymmetric Mode Profile

i) Vertical ("ballooning")



ii) Horizontal (oscillatory)



3-D Spiral Modes

i) Vertical profile: ballooning like that of axisymmetric modes

ii) Radial Profile

The effort to identify the plasma configurations that can exist around collapsed objects goes back a long way…

THE ASTROPHYSICAL JOURNAL, 179:269–275, 1973 January 1

MAGNETIC CONFIGURATION IN THE NEIGHBORHOOD OF A COLLAPSED STAR

R. H. COHEN, B. COPPI, AND A. TREVES*
Massachusetts Institute of Technology
Received 1972 January 31; revised 1972 May 11

ABSTRACT

It is shown that the magnetic configuration in the neighborhood of a collapsed star with parameters appropriate for models of X-ray stars or pulsars is nearly force-free, with $(\nabla \times \boldsymbol{B})/B$ nonconstant. In the case where the magnetic axis coincides with the rotation axis, a differential equation for the magnetic surfaces is derived. A proper double-expansion technique is used to obtain a significant asymptotic solution of this equation and to derive explicit expressions for the relevant magnetic-field components.

Subject headings: collapsed stars — magnetic fields — pulsars — X-ray sources

I. INTRODUCTION

We consider a rotating collapsed (neutron) star with a magnetic-field configuration that is symmetric about its axis of rotation. We point out that, given the expected high value of the magnetic field, in the vicinity of the polar caps the current flow is nearly parallel to the magnetic field, $\boldsymbol{J} \simeq \alpha \boldsymbol{B}$; hence the field is approximately force-free. In addition, by considering the nature of the electromotive force driving this current and the resultant current flow, one must conclude that α is not constant. For this reason the treatment of force-free field configurations which are found in the literature (Lüst and Schlüter 1954; Chandrasekhar 1956; Chandrasekhar and Kendall 1957; Woltjer 1958; Morikawa 1969) cannot be utilized.

We therefore resort to an asymptotic solution of the general force-free field equations, valid in the vicinity of the rotation axis. In particular we refer, as in the analysis of the equilibrium of laboratory plasmas (Solovev and Shafranov 1970), to the magnetic surfaces of the considered configuration. These surfaces are labeled by the streaming function Ψ which satisfies the equation $\boldsymbol{B}\cdot\nabla\Psi = 0$. Our solution enables us to give explicit expressions for the magnetic surfaces and field lines, and to formulate a precise criterion to establish limits for the current which flows through the star surface. Here we summarize our analysis, while a more detailed treatment of the same problem will be published elsewhere (Cohen, Coppi, and Treves 1972).

We assume that the collapsed star under consideration is surrounded by a plasmasphere and distinguish two regions: an active region near the symmetry axis, where poloidal currents can flow, and an inactive equatorial region where the plasma corotates with the star. The existence of poloidal currents in the active region results from slippage of the plasma with respect to the magnetic field; this slippage can be associated with the finite plasma resistivity and with relativistic effects and produces an electromotive force. The two regions are separated by the magnetic surface which intersects the star at colatitude θ_c; if we assume that the poloidal magnetic field is dipolar, we find that θ_c is of order $(\omega_0 a/c)^{1/2}$ where ω_0 is the angular velocity of the star and a its radius.

We consider a region well inside the speed-of-light cylinder and write the equation of

* Permanent address: Istituto di Fisica dell'Università, Milano, Italy.

8

Brief Comments on Pulsar Models

In dealing with axisymmetric pulsar magnetospheres we have to take

$$\mathbf{B} \simeq \frac{1}{R}\left[\nabla\psi \times \mathbf{e}_\phi + I(\psi,\, z)\mathbf{e}_\phi\right]$$

as poloidal currents producing slowing down $\left[\omega_0 = \omega_0(t)\right]$ have to be present. That is, I is not a function of ψ only and is an odd function. The relevant magnetic configuration equation was derived originally in 1971 (published in Ap. J., 1973).

In this case the magnetic force $\mathbf{F}_M$ is given by

$$\mathbf{F}_M = \frac{1}{c}\mathbf{J}\times\mathbf{B} = -\frac{1}{4\pi R^2}\left\{(\Delta_*\psi)\nabla\psi + I\nabla I - (\nabla I \times \nabla\psi)\right\}$$

and has a **toroidal** component.

Two-dimensional Plasma and Field Configuration Around Black Holes

General Relativity corrections are neglected at first. The plasma is rotating around a central object with a velocity

$$\mathrm{V}_\phi = R\Omega(R,\, z)$$

where

$$\Omega(R,\, z) \simeq \Omega_k(R) + \delta\Omega(R,\, z)\ ,$$

$\Omega_k \equiv \left(GM_*/R^3\right)^{1/2}$ is the Keplerian frequency for the central object of mass M_* and whose gravity is prevalent (that is, the plasma self gravity can be neglected) and $|\delta\Omega|/\Omega_k < 1$. We assume, for simplicity that $I = I(\psi)$. Then

$$\mathbf{F}_{Mp} == -\frac{1}{4\pi R^2}\left\{(\Delta_*\psi) + I\frac{dI}{d\psi}\right\}\nabla\psi,$$

as in the case considered earlier of magnetically confined plasmas.

In the case that we consider, the total momentum conservation equation, that includes both the toroidal rotation velocity and the effect of the gravitational field of the a central object, is

$$-\rho\left(\Omega^2 R\mathbf{e}_R + \nabla\Phi_G\right) = -\nabla p + \frac{1}{c}\mathbf{J}\times\mathbf{B} \qquad (\mathrm{I})$$

where

$$\Phi_G = \frac{GM_*}{\sqrt{R^2+z^2}}, \qquad \nabla\Phi_G \simeq -\frac{V_k^2}{R}\left(\mathbf{e}_R + \frac{z}{R}\mathbf{e}_z\right), \qquad V_k^2 \equiv \frac{GM_*}{R} \equiv \Omega_k^2 R^2 .$$

Then we have

$$\mathbf{B}\cdot\nabla p = \rho R\left(\Omega^2 - \Omega_k^2\right)B_R - z\rho\Omega_k^2 B_z \neq 0$$

and if we apply the $\nabla\times$ operator on Eq. (I) we obtain

$$\nabla \times \left(\rho \nabla \Phi_G + \rho \Omega^2 R \mathbf{e}_R \right)$$

$$= \mathbf{e}_\phi \left\{ \frac{\partial \rho}{\partial z} \left(R\Omega^2 + \frac{\partial \Phi_G}{\partial R} \right) + \rho R 2\Omega \frac{\partial \Omega}{\partial z} - \frac{\partial \rho}{\partial R} \frac{\partial \Phi_G}{\partial z} \right\}$$

$$= \frac{1}{4\pi R^2} \left[-\frac{2}{R} \left(\Delta_* \psi + I \frac{dI}{d\psi} \right) \mathbf{e}_R + \nabla \left(\Delta_* \psi \right) \right] \times \nabla \psi \, .$$

This can be rewritten as

$$2\Omega_k R \frac{\partial}{\partial z} \left(\rho \delta \Omega \right) + z \Omega_k^2 \left(\frac{\partial \rho}{\partial R} + \frac{3}{2} \frac{z}{R} \frac{\partial \rho}{\partial z} \right)$$

$$= \frac{1}{4\pi R^2} \left\{ \left[\frac{2}{R} \left(\Delta_* \psi + I \frac{dI}{d\psi} \right) - \frac{\partial}{\partial R} \left(\Delta_* \psi \right) \right] \frac{\partial \psi}{\partial z} + \left[\frac{\partial}{\partial z} \left(\Delta_* \psi \right) \right] \frac{\partial \psi}{\partial R} \right\} \qquad (*)$$

and we call it the "**Master Equation**".

In order to proceed further we consider a radial interval $|R - R_0| < R_0$ around a given radius R_0. Then

$$\Omega \simeq \Omega_k(R_0) + (R - R_0)\left.\frac{d\Omega_k}{dR}\right|_{R=R_0} + \delta\Omega$$

and we comply with the isorotation condition $\Omega = \Omega(\psi)$ defining ψ_z / ψ_{0k} by

$$(R - R_0)\left.\frac{d\Omega_k}{dR}\right|_{R=R_0} = \Omega_k^0 \frac{\psi_z}{\psi_{0k}}$$

and ψ_1 / B_0 by

$$2\Omega R\delta\Omega \simeq 2\Omega_k^0 \left.\frac{d\Omega_k}{dR}\right|_{R=R_0} \frac{\psi_1}{B_0} = -\Omega_D^2 \frac{\psi_1}{B_0 R_0},$$

where $\Omega_D^2 \equiv -R d\,\Omega^2 / dR^2 = 3\Omega_k^2$ is considered to be the "driving factor" for the onset of the magnetic configurations that are analyzed and $\left|\psi_1 / \left(B_0 R_0^2\right)^2\right| < 1$.

We note that the vertical momentum conservation equation is, considering the expression for F_{Mz} given earlier,

$$0 = -\frac{\partial p}{\partial z} - z\rho\Omega_k^2 - \frac{1}{4\pi R^2}\frac{\partial \psi}{\partial z}\left(\Delta_* \psi + I\frac{dI}{d\psi}\right). \qquad (**)$$

Clearly, we have two equations, (*) and (**), which give $\psi(R, z)$ and $p(R, z)$ for reasonable choices of the density $\rho(R, z)$, the poloidal current function $I(\psi)$, and $\delta\Omega(\psi_1)$.

THE MASTER EQUATION AND COMPOSITE DISK STRUCTURES

The analysis of the disk structures, that can be formed in the vicinity of compact objects such as black holes leads to conclude that these are composite structures. These are characterized by a "core" of highly ordered magnetic field configurations with relatively strong fields and a thermal "envelope" where the magnetic field does not play an important role. In fact, there is an increasing body of experimental observations that supports the existence of composite structures around a broad variety of objects.

We observe that for a "conventional" thin disk configuration

$$\left|\frac{z}{R}\frac{\partial\rho}{\partial z}\right| \sim \left|\frac{\partial\rho}{\partial R}\right| \text{ and } \frac{\partial}{\partial z} \gg \frac{\partial}{\partial R}.$$

On the other hand, for the configurations we shall consider

$$\frac{\partial}{\partial R} \gtrsim \frac{\partial}{\partial z} \gg \frac{1}{R} \quad \text{and} \quad I\frac{dI}{d\psi} \sim \Delta_* \psi \,.$$

In this case $\nabla^2 \simeq \partial^2/\partial R^2 + \partial^2/\partial z^2$ and the Master Equation reduces to

$$\frac{\partial}{\partial z}\left[R\rho\left(\Omega^2 - \Omega_k^2\right)\right] + \Omega_k^2 z \frac{\partial}{\partial R}\rho + \frac{1}{4\pi}\left(B_z \nabla^2 B_R - B_R \nabla^2 B_z\right) \simeq 0 \,,$$

that is independent of the toroidal field component.

In this connection we note that the derivation of the Master Equation is compatible with a pressure tensor of the form

$$\underline{\underline{P}} = p_{th}\underline{\underline{\mathbf{I}}} + p^F \mathbf{e}_\phi \mathbf{e}_\phi$$

where p^F indicates the anisotropic pressure of a fast particle population that may be present, $p_{th} = p_e + p_i$ and p_e and p_i are the electron and the ion thermal population pressure. In the theoretical model for the three plasma regimes considered later we shall argue that a highly non-thermal distribution may prevent an axisymmetric configuration to develop allowing instead the formation of tridimensional spirals whose excitation can be associated with gradients of the plasma mean energy.

Ring Sequence Solutions

$$\psi_1 \simeq \psi_*(R_*)\exp\left(-\frac{z^2}{2\Delta_z^2}\right)$$

$$\rho \simeq \rho_*(R_*)\exp\left(-\frac{z^2}{2\Delta_z^2}\right)$$

$$R_* \equiv (R - R_0)/\delta_R$$

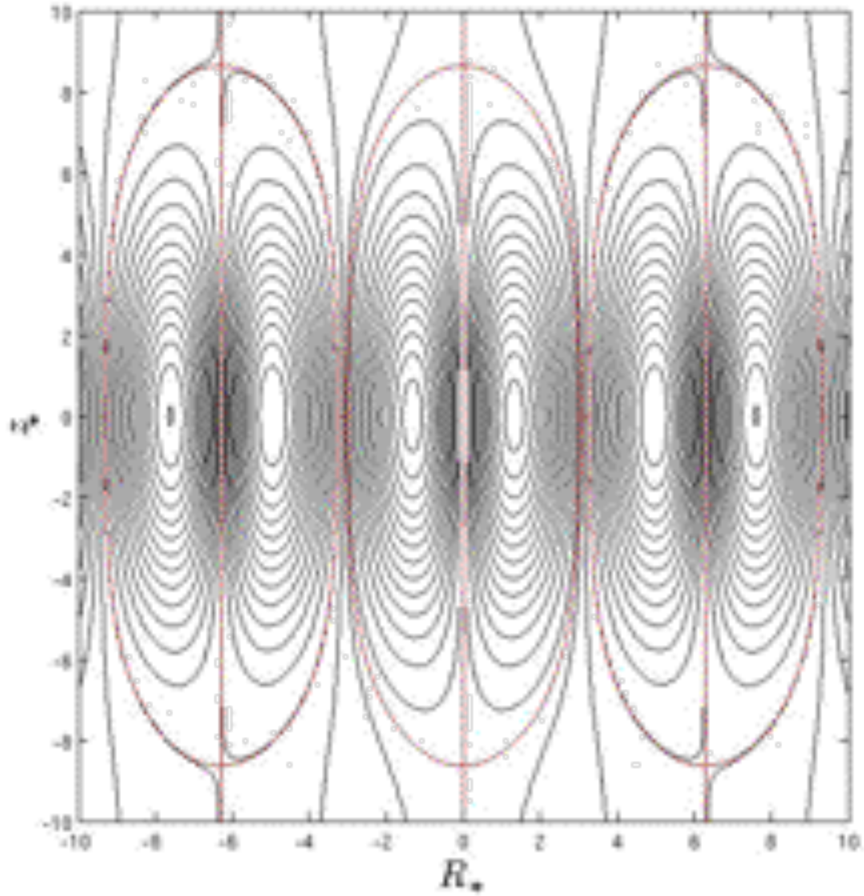
$$\delta_R \simeq \left(\frac{\psi_*^{02}}{4\pi\rho_N\Omega_0^2}\right)^{1/3}\frac{1}{R_0} \qquad \psi_* \sim \psi^0$$

$$\rho_*(R_*) \simeq \rho_N D_N \frac{\sin^2 R_*}{1+\varepsilon_* \cos R_*}$$ where $\varepsilon_* \leq \frac{1}{4}$ is a free parameter

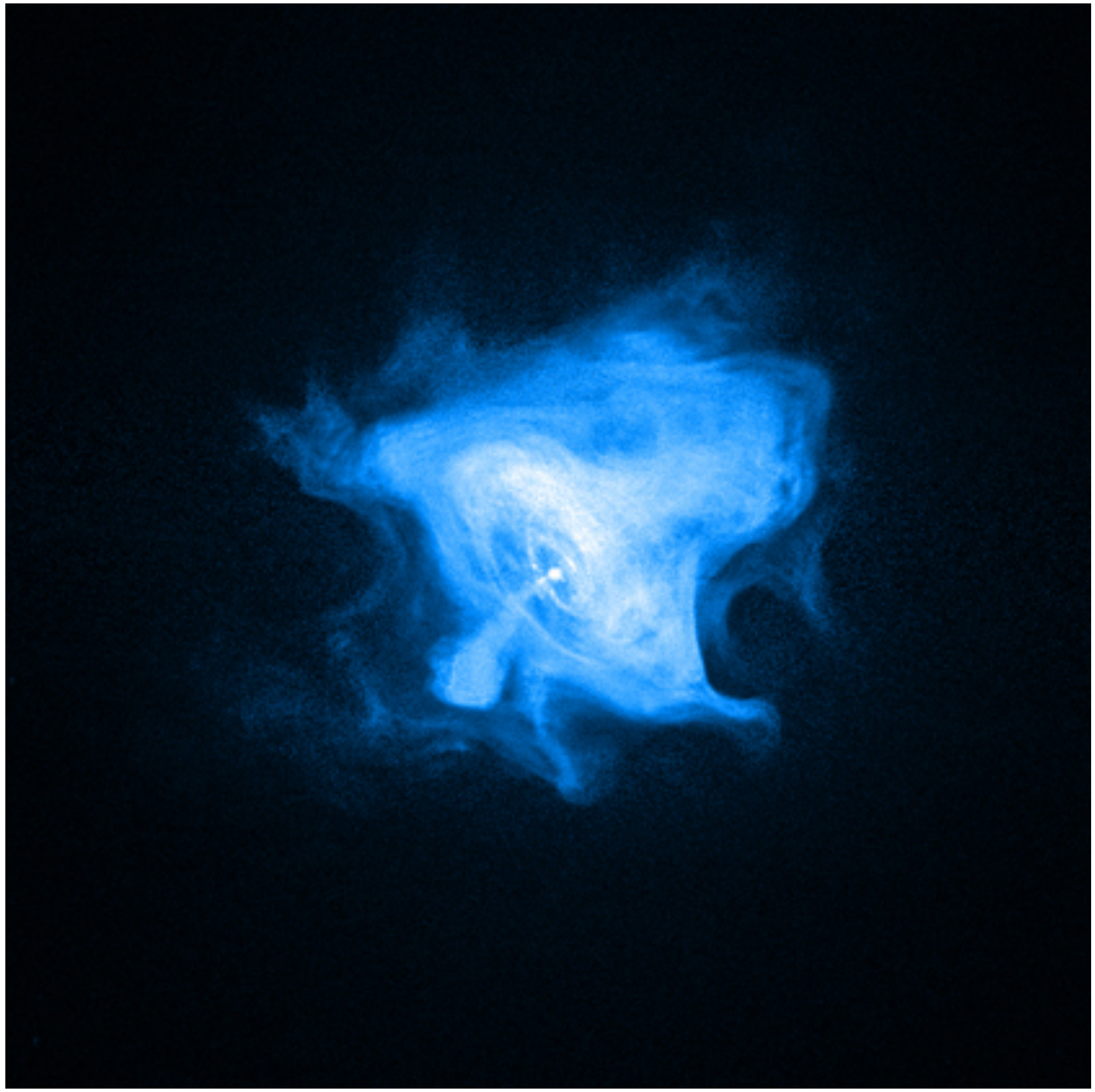
$$\psi_*(R_*) = \psi_*^0 \frac{2}{3\varepsilon_*} D_N \left[\sin R_* + \frac{\varepsilon_*}{2}\sin(2R_*)\right]$$

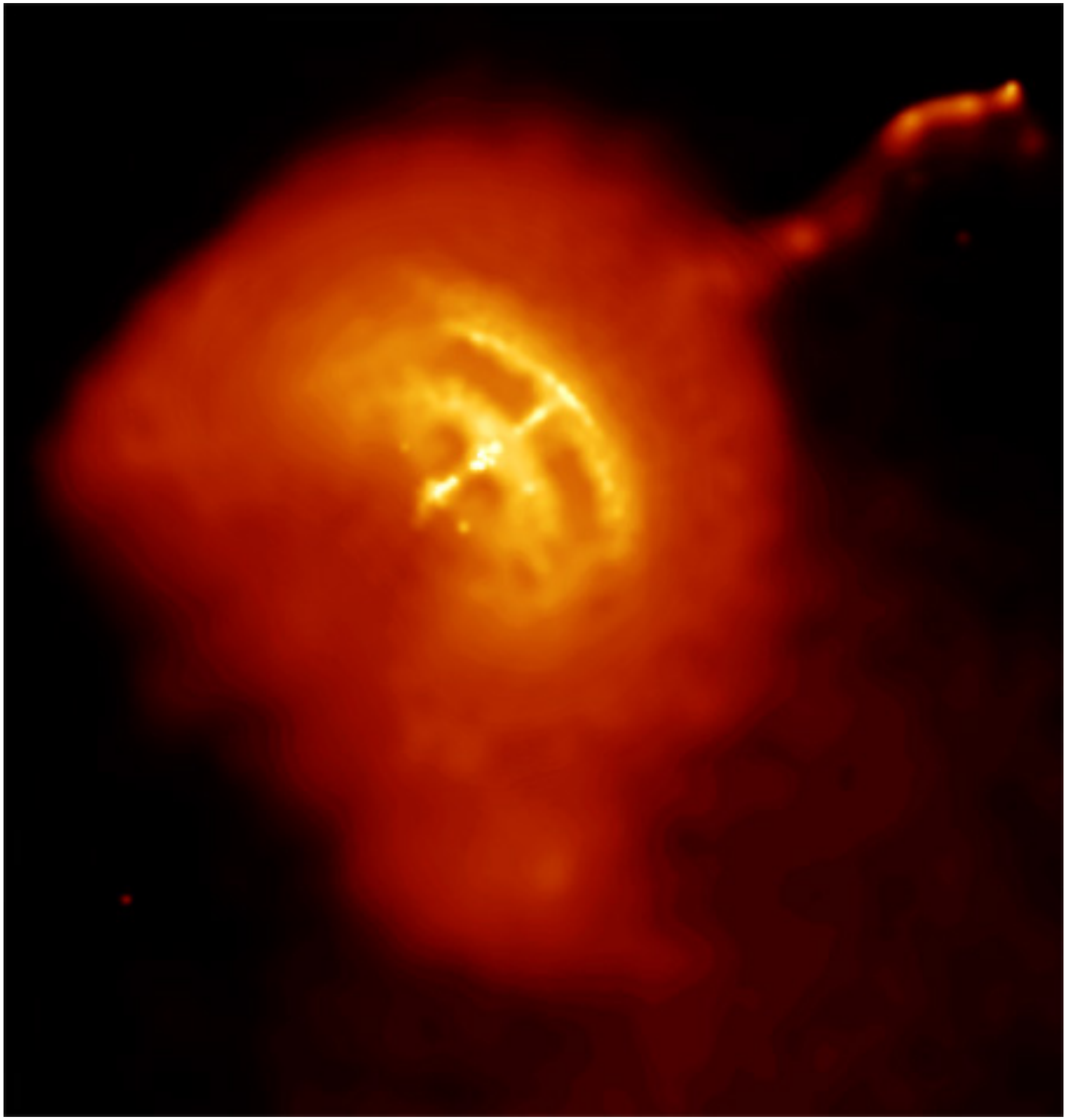
and we may take $D_N = \frac{3}{2}\varepsilon_*$

Note that $J_\phi \simeq -\frac{c}{4\pi R_0 \delta_R^2}\frac{d^2}{dR_*^2}\psi_*(R_*)\exp\left(-\frac{z^2}{2\Delta_z^2}\right)$



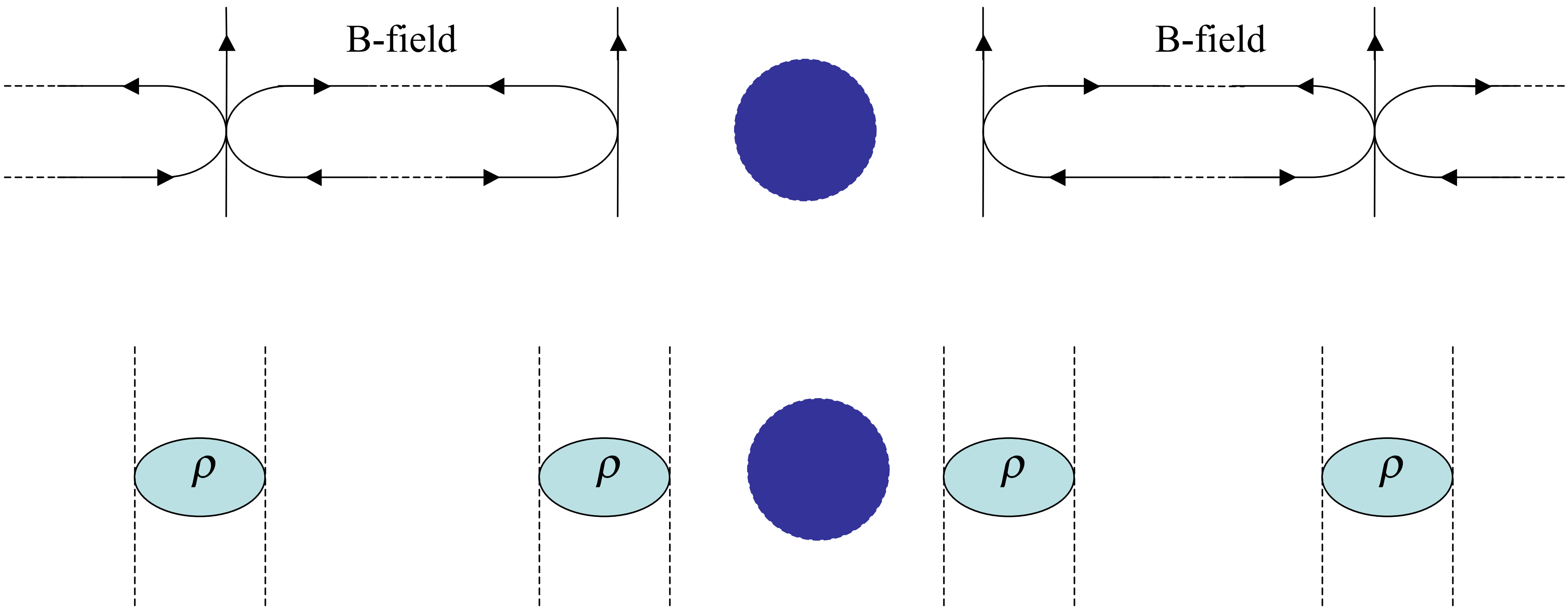
Closed and open magnetic surfaces in the core of a composite disk structure. Here $R_* = (R - R_0)/\delta_R$.

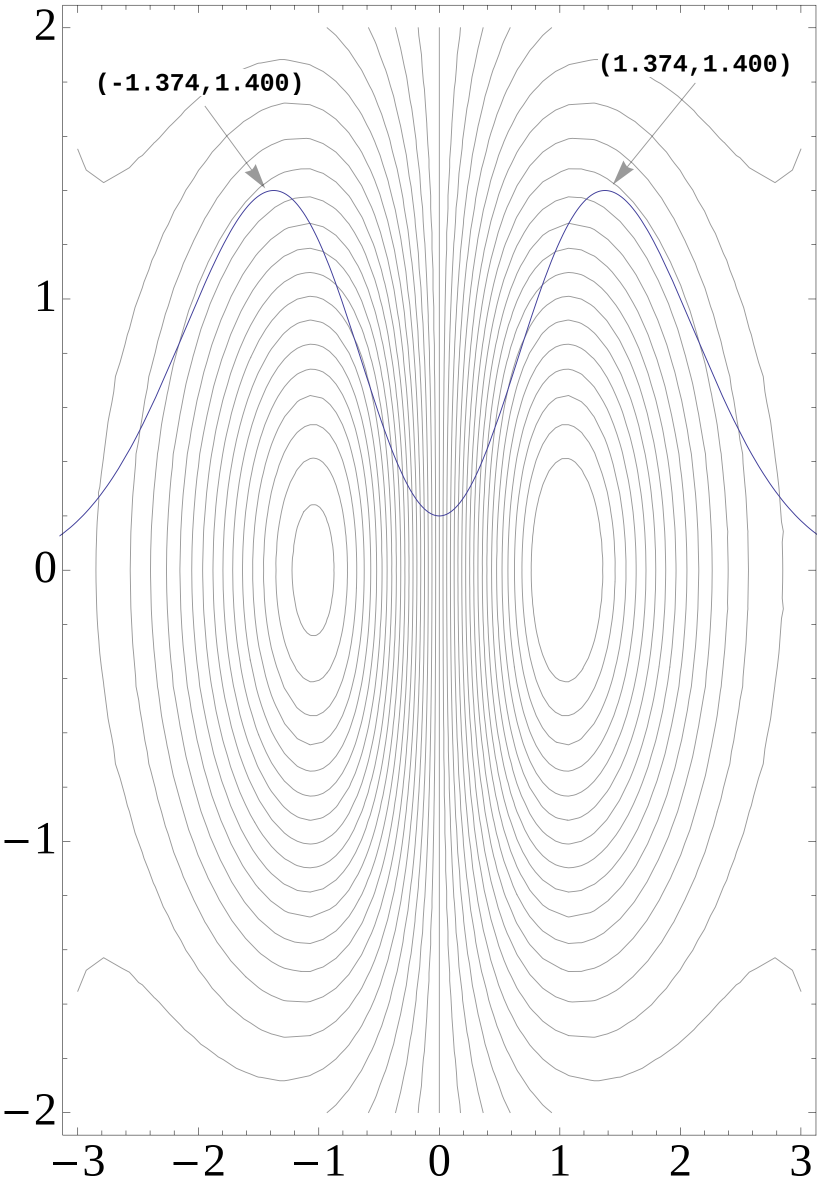




Solitary Rings (Thin Structure Ordering)

$$\frac{\partial^2}{\partial z^2} \gg \frac{\partial^2}{\partial R^2} \qquad |R - R_0| < R_0$$





(-1.374,1.400)
(1.374,1.400)
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

Local Description

$$\left(R-R_0\right)\sim\Delta_R^2\ll R_0^2$$

$$\psi\simeq\psi_0\left(R\right)+\psi_1\left(R-R_0,\ \mathrm{z}\right),\qquad\left|\psi_1\right|<\left|\psi_0\right|$$

$$\psi_1\simeq\psi_1^0\exp\left(-\frac{z^2}{2\Delta_z^2}\right)\int_0^{R_*}dR_*\exp\left(-R_*^2\right)$$

$$R_*=\left(R-R_0\right)/\Delta_R$$

$$B_z\sim B_z^1\equiv\frac{\psi_1^0}{\Delta_R R_0}>B^0\equiv\frac{\psi_0}{R_0^2}\Rightarrow\Delta_R<\frac{\psi_1^0}{\psi_0}<R_0$$

$$\Delta_z^2=\frac{1}{\left(4\pi\rho_0\right)}\frac{B^0B_z^1}{\Omega_D^2}$$

$$\Omega_D^2\equiv-2\Omega\frac{d\Omega}{dR}$$

$$n \simeq n_0 \exp\left[-\frac{z^2}{2\Delta_z^2} - \frac{(R-R_0)^2}{\Delta_R^2}\right] \Rightarrow \text{ density profile}$$

$$B_z \simeq \frac{\psi_1^0}{R_0 \Delta_z} \exp\left[-\frac{z^2}{2\Delta_z^2} - \frac{(R-R_0)^2}{\Delta_R^2}\right] \Rightarrow \text{ vertical field component}$$

$$B_R \simeq -\frac{\psi_1^0}{R_0 \Delta_z}\left[\int_0^{R_*} dR_* \exp\left(-R_*^2\right)\right]\frac{z}{\Delta_z} \exp\left(-\frac{z^2}{2\Delta_z^2}\right) \Rightarrow \text{ radial component}$$

$$R_* \equiv (R-R_0)/\Delta_R$$

$$J_\phi \simeq \frac{c}{4\pi}\frac{\psi_1^0}{R_0 \Delta_z^2}\left(1-\frac{z^2}{\Delta_z^2}\right)\exp\left(-\frac{z^2}{2\Delta_z^2}\right)\int_0^{R_*} \exp\left(-R_*^2\right) dR_* \quad \text{toroidal current density}$$

$$\int_{-\infty}^{+\infty} J_\phi \, dz = 0 \quad \Rightarrow \quad \text{Important feature.}$$

3-D Co-rotating Configurations

The tridimensional co-rotating configurations (TCR) that we have analyzed are **localized** around $R = R_0$ and represented by

$$\mathbf{B}_p \simeq \frac{1}{R_0} \nabla \psi \times \mathbf{e}_\phi \quad \text{and} \quad \mathbf{B}_\varphi = 0$$

where

$$\psi = \bar{\bar{\psi}}\left(R_*, \bar{z}^2\right) \sin\left[m_\varphi\left(\varphi - \Omega_0 t\right)\right],$$

$$R_* \equiv \frac{R - R_0}{\Delta_R} \qquad \Delta_R^2 \ll R_0^2$$

$$\bar{z} \equiv \frac{z}{\Delta_z} \qquad \Delta_z^2 \ll R_0^2$$

$\Delta_R^2 \ll \Delta_z^2$ (for the configurations analyzed until now).

In particular, ψ is an

odd function of R_*,

and an

even function of $\bar{z}$.

A specific (important case) is

$$\bar{\bar{\psi}} \propto \exp\left(-\frac{\bar{z}^2}{2}\right).$$

The relevant rotation velocity $\mathrm{v}_\varphi \mathbf{e}_\varphi$ is

$$\mathrm{v}_\varphi = \Omega_0 R \qquad \text{(locally rigid rotor)}$$

where

$$\Omega_0 = \Omega_k\left(R_0\right) = \frac{GM_*}{R_0^3}.$$

Therefore,

$$\mathrm{v}_\varphi = \mathrm{v}_k\left(R_0\right) + \left(R - R_0\right)\Omega_0.$$

The corresponding plasma density structures are of the form

$$\rho = \bar{\bar{\rho}}\left(R_*^2, \bar{z}^2\right),$$

with, for instance

$$\bar{\bar{\rho}} \propto \exp\left(-\bar{z}^2\right),$$

and $\bar{\bar{\rho}}$ can represent a periodic Sequence of Rings, as a function of R_*, e.g.

$$\bar{\bar{\rho}} \simeq \rho_N \left[\frac{2}{3} - \cos R_* + \frac{1}{3} \cos^3 R_* \right] \exp\left(- \bar{z}^2 \right),$$

for

$$\bar{\bar{\psi}} = \psi_N \left[\sin R_* + \frac{\varepsilon_*}{4} \sin\left(2R_* \right) \right] \exp\left(-\frac{\bar{z}^2}{2} \right),$$

or Ring Pairs with $\bar{\bar{\rho}} \to 0$ for $R_*^2 \to \infty$.

3-D Spiral Configurations (Tightly Wound)

Represented for instance by

$$\psi = \psi_N \exp\left(-\frac{\bar{z}^2}{2}\right)\left\{\sin\left[m_\varphi\left(\varphi - \Omega_0 t\right) + R_*\right] + \frac{\varepsilon_*}{4}\sin\left[2m_\varphi\left(\varphi - \Omega_0 t\right) + 2R_*\right]\right\}.$$

Master Equation in the Presence of Anisotropic Pressure

B. Basu
Boston College, Chestnut Hill, MA 02167

Let us consider that the pressure tensor $\underline{\underline{\mathbf{P}}}$ is of the form

$$\underline{\underline{\mathbf{P}}} = p_\perp \underline{\underline{\mathbf{I}}} + (p_\parallel - p_\perp)\frac{\mathbf{BB}}{B^2},$$

where $\mathbf{B} = B_R \mathbf{e}_R + B_z \mathbf{e}_z = (\nabla\psi \times \mathbf{e}_\varphi)/R$. Next, we refer to the momentum conservation equation,

$$\tilde{n}\left(\nabla\Phi_G - \Omega^2 R \mathbf{e}_R\right) = -\nabla \cdot \underline{\underline{\mathbf{P}}} + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B},$$

where

$$\Phi_G = -\frac{GM_*}{\sqrt{R^2+z^2}}.$$

Master Equation in the Presence of Anisotropic Pressure

Applying the $\mathbf{e}_\varphi \cdot \nabla \times$ operator to the momentum balance equation, expanding Φ_G in z^2/R^2, and taking $\Omega = \Omega_K \equiv (GM_*/R^3)^{1/2}$, we find the Master Equation

$$\Omega_K^2 z\left(\frac{\partial\rho}{\partial R} + \frac{3}{2}\frac{z}{R}\frac{\partial\rho}{\partial z}\right)$$

$$= \left(\Delta_A + \frac{1}{R^2}\right)\left[(p_\parallel - p_\perp)\frac{B_R B_z}{B^2}\right] + \frac{\partial^2}{\partial R \partial z}\left[(p_\parallel - p_\perp)\frac{B_R^2 - B_z^2}{B^2}\right] + \frac{1}{R}\frac{\partial}{\partial z}\left[(p_\parallel - p_\perp)\frac{B_R^2}{B^2}\right]$$

$$+ \frac{1}{4\pi}\left[\frac{\partial}{\partial R}\left(B_R \frac{\partial B_z}{\partial R}\right) - \frac{\partial}{\partial z}\left(B_z \frac{\partial B_R}{\partial z}\right)\right] - \frac{1}{8\pi}\frac{\partial^2}{\partial R \partial z}\left(B_R^2 - B_z^2\right),$$

where

$$\Delta_A \equiv \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R}.$$

Vertical Equilibrium Equation in the Presence of Anisotropic Pressure

$$\frac{\partial}{\partial z}\left[p_\perp + (p_\parallel - p_\perp)\frac{B_z^2}{B^2}\right] + \left(\frac{\partial}{\partial R} + \frac{1}{R}\right)\left[(p_\parallel - p_\perp)\frac{B_R B_z}{B^2}\right]$$

$$= -\rho\Omega_K^2 z + \frac{B_R}{4\pi}\left(\frac{\partial B_z}{\partial R} - \frac{\partial B_R}{\partial z}\right)$$

The simplest configuration from which the modes that we shall analyze can emerge is a thin currentless disk that is threaded by a relatively weak vertical magnetic field B_z and where the only component of the plasma flow velocity is toroidal. In particular, we assume that the central plasma pressure p_0 exceeds the magnetic pressure $B^2/8\pi$ The particle density profile for an up-down symmetric disk is represented by $n \simeq n_0\left(1 - z^2/H_0^2\right)$ near the equatorial plane at the reference distance $R = R_0$ from the axis of symmetry. The thickness of the (density) disk, of the order $2H_0$, is considered to be small relative to R_0. Different vertical temperature profiles, corresponding different heating processes, are represented near the equatorial plane by different values of the parameter

$$\eta_T \equiv -\left.\frac{\mathrm{d}\ln T}{\mathrm{d}z^2}\right)_{z=0} H_0^2, \tag{1}$$

where $2T \equiv p/n = T_\mathrm{e}+T_i$ and p is the total plasma pressure. The radial equilibrium equation, to lowest order in the ratio H_0^2/R_0^2, reduces to

$$\Omega^2\left(R_0\right) = \frac{GM_*}{R_0^3} \equiv \Omega_k^2\left(R_0\right), \tag{2}$$

where $\Omega\left(R\right)$ is the rotation frequency, $\Omega_k\left(R_0\right)$ is the Keplerian frequency, M_* is the mass of the central object and $v_\phi = \Omega R$ is the toroidal velocity. The relevant vertical equilibrium equation is

$$0 = -\frac{\partial p}{\partial z} - z\Omega_k^2\rho, \tag{3}$$

where ρ is the mass density. Then Eq. (2) gives $4T_0\left(1+\eta_T\right) = H_0^2\Omega_k^2 m_i$ for $z^2/H_0^2 \ll 1$, where $m_i = \rho/n$. We shall consider a variety of temperature profiles including the flat profile corresponding to $\mathrm{d}T/\mathrm{d}z^2 = 0$ over the height of the disk $n = n_0\exp\left(-z^2/H_0^2\right)$ describes the entire density profile. The scale distance for the pressure gradient is defined by $H_\mathrm{p}^2 \equiv H_0^2/\left(1+\eta_T\right)$ for $z^2 \ll H_0^2$.

that gives

$$E_R = -\Omega_k (R) \frac{R}{c} B_z \tag{4}$$

and

$$E_z = -\frac{1}{ne} \frac{\partial p_e}{\partial z} \tag{5}$$

where p_e is the electron pressure, $p = p_e + p_i$ and $p_e \simeq p_i$. Realistically, $E_z << E_R$.

Normal mode equations

We consider normal mode perturbations from the indicated initial state represented by

$$\hat{v}_\phi = \tilde{\tilde{v}}_\phi (R - R_0, z) \exp\left(\gamma_0 t - \mathrm{i}\omega_0 t + \mathrm{i} m_\phi \phi\right) \tag{6}$$

in an interval $|R - R_0|$ around R_0, such that $|R - R_0| \ll R_0$, where γ_0 is the mode growth rate, ω_0 the frequency, m_ϕ the toroidal mode number and $\hat{v}_\phi$ the perturbed toroidal velocity. The basic linearized equations that describe the departure from the initial state include

$$\hat{\boldsymbol{E}} + \frac{1}{c}\left(\hat{\boldsymbol{v}} \times \boldsymbol{B} + \boldsymbol{v} \times \hat{\boldsymbol{B}}\right) = 0, \tag{7}$$

$$-\frac{1}{c}\frac{\partial \hat{\boldsymbol{B}}}{\partial t} = \nabla \times \hat{\boldsymbol{E}}, \tag{8}$$

and the total momentum conservation equation

$$\boldsymbol{A}_m \equiv \rho\left(\frac{\partial}{\partial t}\hat{\boldsymbol{v}} + \hat{\boldsymbol{v}} \cdot \nabla \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \hat{\boldsymbol{v}}\right)$$

$$+ \nabla\left(\hat{p} + \frac{\hat{\boldsymbol{B}} \cdot \boldsymbol{B}}{4\pi}\right) - \frac{1}{4\pi}\boldsymbol{B} \cdot \nabla \hat{\boldsymbol{B}} + z\hat{\rho}\Omega_k^2 \boldsymbol{e}_z = 0. \tag{9}$$

Here, we have taken into account that the initial magnetic field B_z is considered to be varying over scale distances of the order of R_0 and have used standard symbols.

Furthermore, it is reasonable to assume that the collisional mean free path is short relative to the distance Δ_z over which the mode is localized vertically and to the mode radial wavelengths. Thus, the thermal conductivity can be neglected and the adiabatic equation of state can be adopted, that is,

$$\frac{\partial}{\partial t}\hat{p} + \boldsymbol{v} \cdot \nabla \hat{p} + \hat{\boldsymbol{v}} \cdot \nabla p + \Gamma p \nabla \cdot \hat{\boldsymbol{v}} = 0 \tag{10}$$

where $\Gamma = 5/3$ is the adiabatic index. Then, given Eq. (6) we obtain

$$\left[\gamma_0 - \mathrm{i}\omega_0 + \mathrm{i}m_\phi \Omega_k (R)\right] \hat{p} + \hat{v}_z \frac{\partial p}{\partial z} + \Gamma p \nabla \cdot \hat{\boldsymbol{v}} = 0 \tag{11}$$

and we choose to consider modes that co-rotate with the plasma at $R = R_0$.

Therefore, we take

$$\omega_0 = m_\phi \Omega_k (R_0) + \delta\omega_0, \tag{12}$$

where $|\delta\omega_0| \ll m_\phi \Omega_k (R_0)$ is not linear in m_ϕ, and Eq. (6) reduces to

$$\hat{v}_\phi \simeq \tilde{\tilde{v}}_\phi (R - R_0, z) \exp\{\gamma_0 t - \mathrm{i}\delta\omega_0 t + \mathrm{i}m_\phi[\phi - \Omega_k (R_0)\, t]\}. \tag{13}$$

We define

$$\gamma_t \equiv \gamma_0 + \mathrm{i}\Omega_k' (R - R_0) - \mathrm{i}\delta\omega_0, \qquad \hat{\boldsymbol{v}}_\mathrm{p} = \gamma_t \hat{\boldsymbol{\xi}}_\mathrm{p} \tag{14}$$

for $\hat{\boldsymbol{v}}_\mathrm{p} = \gamma_t \hat{\boldsymbol{\xi}}_\mathrm{p}$ where the subscript p indicates the relevant poloidal component.

Axisymmetric modes

At first we consider, for simplicity, axisymmetric modes (Coppi 2008a) with $m_\phi = 0$, that are purely growing. Thus, $\gamma_t = \gamma_0$ and we look for normal modes of the form

$$\hat{\mathrm{v}}_\phi \simeq \tilde{\mathrm{v}}_\phi^0 G_0(z) \exp\left[\gamma_0 t + \mathrm{i} k_R (R - R_0)\right]$$

that are a special case of those represented by Eq. (6) and where $k_R^2 R_0^2 \gg 1$, $k_R \sim k_0 \equiv (-2\Omega' R\Omega)^{1/2}/v_\mathrm{A} \equiv 1/L_c$, $\Omega' = \mathrm{d}\Omega/\mathrm{d}R$, $\Omega_k' = -3\Omega_k/(2R_0)$ and $v_\mathrm{A} \equiv B_z/(4\pi\rho)^{1/2}$ is the Alfvén velocity. In addition $G_0(z)$ is an even or odd function of z that is localized over a distance $L_c < \Delta_z \lesssim H_\mathrm{p}$ represented, for instance, by $G_0 = \exp\left[-z^2/\left(2\Delta_z^2\right)\right]$. The unstable modes that are found have

$$\Omega_D^2 \equiv -2\Omega_k' R\Omega_k = 3\Omega_k^2 > k_R^2 v_\mathrm{A}^2$$

corresponding to the fact that the radial gradient of the rotation frequency is a key driving factor for the relevant instability. Clearly, $k_R^2 v_\mathrm{A}^2$ is the representative bending factor of the magnetic field lines.

Here we consider $\gamma_0^2 \sim \left(\Delta_z^2/H_0^2\right)\Omega_k^2 < \Omega_k^2$ and therefore $k_R^2 v_A^2 \simeq \Omega_0^2\,(1-\epsilon_k)$ with $\epsilon_k < 1$, that is $k_R^2 \simeq k_0^2\,(1-\epsilon_k)$. Then since $\rho \simeq \rho_0\left(1 - z^2/H_0^2\right)$

$$\gamma_0^2\tilde{\xi}_\phi + 2\Omega_k\gamma_0\tilde{\xi}_R = v_A^2\frac{d^2}{dz^2}\tilde{\xi}_\phi, \qquad (15)$$

$$\frac{d^2}{dz^2}\left\{\Omega_D^2\left(1-\frac{k_R^2}{k_0^2}\right)\tilde{\xi}_R - \Omega_D^2\frac{z^2}{H_0^2}\tilde{\xi}_R + v_A^2\frac{d^2}{dz^2}\tilde{\xi}_R - \gamma_0^2\tilde{\xi}_R\right\} \simeq$$

$$\frac{4}{3}k_0^2\gamma_0^2\left[\left(1+\frac{3}{4}\frac{k_R^2}{k_0^2}\right)\tilde{\xi}_R + \frac{\gamma_0}{2\Omega_k}\tilde{\xi}_\phi\right] - \mathrm{i}k_R C_0\Omega_k^2\frac{d}{dz}\left[\frac{z^2}{H_0^2}\tilde{\xi}_z\right], \qquad (16)$$

where $\tilde{\xi}_R \simeq (i/k_R)\,d\tilde{\xi}_z/dz$. Since we look for localized solution soltuion in z it is convenient to take the Fournier transform of Eqs. (15) and (16) that gives

$$\left\{\left(\frac{\gamma_0 H_0}{v_\mathrm{A}}\right)^2\left(1+\frac{4}{3}\frac{k^2 v_\mathrm{A}^2}{k^2 v_\mathrm{A}^2+\gamma_0^2}\right)-k^2\left(\epsilon_k-\frac{k^2}{k_0^2}\right)H_0^2\right\}\tilde{\xi}_{zk}$$

$$\simeq k\frac{\mathrm{d}^2}{\mathrm{d}k^2}\left(k\tilde{\xi}_{zk}\right)-\frac{C_0}{3}k_0^2\frac{\mathrm{d}^2}{\mathrm{d}k^2}\tilde{\xi}_{zk}, \tag{17}$$

and the problem is reduced to solving a second order differential equation.

We note that at marginal stability $\left(\gamma_0^2=0\right)\tilde{\xi}_{\phi k}$ and Eq. (17) reduces to

$$0\simeq (kH_0)^2\left(\epsilon_k-\frac{k^2}{k_0^2}\right)\tilde{\xi}_{zk}+k\frac{\mathrm{d}^2}{\mathrm{d}k^2}\left(k\tilde{\xi}_{zk}\right)-\frac{C_0}{\mathrm{v}_A^2}\Omega_k^2\frac{\mathrm{d}^2}{\mathrm{d}k^2}\tilde{\xi}_{zk}.$$

Thus we obtain the known ballooning solution

$$\tilde{\xi}_{zk}=\tilde{\xi}_{zk}^{\partial}\exp\left(-\frac{k^2}{2\sigma}\right)$$

where

$$\sigma^2=\frac{k_0^2}{H_0^2}\equiv\frac{1}{\Delta_0^4}$$

The next eigensolution is odd with

$$\tilde{\xi}_{zk} = \tilde{\xi}_{zk}^{0} \bar{k} \exp\left(-\frac{\bar{k}^2}{2}\right) \cdot$$

In this case

$$C_0^0 = -\frac{2}{3}$$

and

$$E_k^0 = \frac{13}{3} \cdot$$

Vertical fluxes of particles and thermal energy

The considered modes can produce particle density transport, in the vertical direction, that is of contrary sign to that of the temperature transport and modify the density and temperature profiles in such a way as to lead η_T toward 2/3, corresponding to a polytropic. Thus, if $\eta_T > 2/3$ a particle inflow toward the equatorial plane is induced. When $\eta_T < 2/3$, including the case where $\eta_T = 0$ or where the surface of the disk can be hotter than the interior, the particle transport is away, from the equatorial plane. These arguments are based on the quasilinear analysis that gives the vertical particle flux produced by unstable modes as

$$\Gamma_{pz} = \langle\langle \hat{n}\hat{v}_z \rangle\rangle \simeq -\frac{4}{5}\gamma_0 \left\langle\left\langle \left|\hat{\xi}_z\right|^2 \right\rangle\right\rangle \times \left[\frac{\partial}{\partial z}n - \frac{3}{2}\frac{n}{T}\frac{\partial}{\partial z}T\right], \tag{18}$$

where $\langle\langle\rangle\rangle$ indicates an average over a radial distance ΔR such that $1/k_R < \Delta R < R_0$. The corresponding temperature flux is $\left\langle\left\langle \hat{T}\hat{v}_z \right\rangle\right\rangle \simeq -\langle\langle \hat{n}\hat{v}_z \rangle\rangle T/n$.

The outflows produced by these modes when $\eta_T < 2/3$ can be considered as candidates to explain the origin of the particle fluxes (winds) that have been observed to emanate from disk structures such as those at the core of AGN's.

The transport process described by Eq. (18) is similar to that proposed for the theoretical explanation of the experimentally observed particle inflow in magnetically confined toroidal plasmas that is associated (Coppi & Spight 1978) with the outflow of electron thermal energy related to the ratio of the gradients of the radial electron temperature and the particle density.

Mode growth rates

$$\gamma_0^2 = \left(\frac{\sqrt{3}6}{35} \right) \frac{v_A}{H_0} \Omega_k \left(\eta_T - \frac{2}{3} \right), \quad \epsilon_k = \varepsilon_0 + \frac{2}{5} \left(\eta_T - \frac{2}{3} \right)$$

where $\varepsilon_0 \equiv 1/\left(k_0 H_0\right) < 1$.

In the case where $C_0 \sim 1$ and $\epsilon_k > \varepsilon_0^{2/3}$ we find

$$\Delta_z \simeq \Delta_0 \left(\frac{3\epsilon_k}{|C_0|} \right)^{1/4}$$

and

$$\gamma_0 \simeq \sqrt{\frac{3}{7}} \left(\frac{v_A}{H_0} \Omega_k \right)^{1/2} \left(\epsilon_k \, |C_0| \right)^{1/4}$$

showing that the dependence of Δ_z and γ_0 on ϵ_k is weak, in this case.

The next (odd) eigenfunction, corresponds to the ballooning mode investigated already by Coppi & Keyes (2003). In this case the dispersion relation is $\Gamma_0^2 = 2 + 3C_0^0$ while $E_k^0 = 5 + C_0^0$. Clearly the mode can be unstable even if $C_0^0 < 0$ that is for $\eta_T < 2/3$ and

$$\gamma_0^2 \simeq \frac{6}{7} \frac{v_A}{H_0} \left[\frac{\sqrt{3}3}{5} \left(\eta_T - \frac{2}{3} \right) \Omega_k + \frac{v_A}{H_0} \right].$$

We observe that, relative to the case investigated by Coppi & Keyes (2003) the relevant growth rate is increased by the term $k_0 H_0 \left(\eta_T - 2/3\right) 3/5$.

Tridimensional, tightly wound spirals

Referring to Eq. (13), we consider modes that are represented by

$$\hat{v}_\phi = \tilde{v}_\phi \left(R - R_0, z\right)$$

$$\exp\left\{\gamma_0 t + \mathrm{i}\left[k_R \left(R - R_0\right) + m_\phi \left(\phi - \Omega_o t\right) - \delta\omega_0 t\right]\right\} \qquad (19)$$

where $\Omega_0 \equiv \Omega_k(R_0)$, $|k_R| \simeq k_0$, $\tilde{\mathrm{v}}_\phi(R - R_0, z)$ varies over a radial scale distance that is considerably larger than $L_c \equiv 1/k_0$ and we take m_ϕ to be relatively low. These modes are of the spiral type and we discuss at first the case where $\delta\omega_0 = 0$ and γ_0 is given approximately by the dispersion relations for axisymmetric modes analyzed earlier for

$$\gamma_0^2 > m_\phi^2 \left(\frac{\mathrm{d}\Omega}{\mathrm{d}R}\right)^2 \Delta_R^2 \sim m_\phi^2 \Omega^2 \left(\frac{\Delta_R^2}{R^2}\right),$$

where Δ_R is the radial width over which the mode is localized. In fact, this is a rather easy condition to satisfy as $\gamma_0 \gtrsim v_\mathrm{A}/H_0$, and $v_\mathrm{A}^2/H_0^2 > m_\phi^2 \Omega_k^2 (\Delta_R/R)^2$ for $\beta > m_\phi^2 (\Delta_R/R)^2$ where $\beta \sim c_s^2/v_\mathrm{A}^2$, c_s is the sound velocity and $H_0\Omega_k \sim c_\mathrm{s}$.

We observe that the low-m_ϕ spiral modes considered here are directly connected to the axisymmetric modes discussed earlier. Referring to the radial displacement $\hat{\xi}_R$ we note that its expression, consistent with that of $\hat{v}_\phi$ given by Eq. (19), can be expanded as follows

$$\hat{\xi}_R \simeq \exp\left[\mathrm{i}k_R(R - R_0) - \mathrm{i}m_\phi(\Omega_k t - \phi) + \gamma_0 t\right]$$
$$\times \left[F_0(R - R_0)\tilde{\xi}_{R0}(z) + \tilde{\xi}_{R1}(z, R - R_0) + \tilde{\xi}_R^1(z, R - R_0)\right]$$
$$\equiv \hat{\xi}_{R0} + \hat{\xi}_{R1} + \hat{\xi}_R^1, \qquad (20)$$

where $k_R = k_0 + \delta k_R$, $-1 < \delta k_R/k_0 < 0$ and

$$\frac{\mathrm{d}F_0}{\mathrm{d}R} \sim \frac{F_0}{\Delta_R}, \left|\frac{\hat{\xi}_{R1}}{\hat{\xi}_{R0}}\right| \sim \frac{1}{\Delta_R|\delta k_R|} < 1, \left|\frac{\hat{\xi}_R^1}{\hat{\xi}_{R0}}\right| \sim \frac{1}{\Delta_R k_0} < 1.$$

$$F_0 \simeq \exp\left[-\frac{7}{6} m_\phi k_R \frac{\Omega_k'}{\gamma_0}(R-R_0)^2\right].$$

It is important to point out that, in order that F_0 be a localized function of $(R-R_0)$ the sign of k_R and m_ϕ have to be related by the requirement that

$$m_\phi k_R \Omega' < 0,$$

and, since $\Omega' < 0$, $m_\phi k_R > 0$, this corresponds to trailing spirals.

Then we define

$$\Delta_R \equiv \left|\frac{6}{7}\frac{\gamma_0}{\Omega' m_\phi k_R}\right|^{1/2} \sim \left(\frac{\gamma_0}{\Omega_k}\frac{R_0}{k_0 m_\phi}\right)^{1/2}$$

and condition (66) becomes, approximately,

$$\frac{m_\phi}{k_0 R_0} < \frac{\gamma_0}{\Omega_k} < 1$$

with $\Delta_R k_0 \sim (\gamma_0 k_0 R_0)^{1/2} / \left(\Omega_k m_\phi\right)^{1/2} \gg 1$. The condition $|\delta k_R / k_0| > 1/|\Delta_R k_0|^{1/2}$ can be easily satisfied as well for $\gamma_0 \sim \mathrm{v}_A / H_0$, $\Delta_R \sim (1/k_0)\left[R_0 / \left(m_\phi H_0\right)\right]^{1/2}$. In particular, taking into account that the vertical profiles of the modes represented by Eq. (20) are given by the theory of the corresponding axisymmetric $\left(m_\phi = 0\right)$ modes, the spiral modes we consider become the form

$$\hat{\xi}_z \simeq \tilde{\xi}_z^0 \exp\left[-\frac{(R-R)^2}{\Delta_R^2} - \frac{z^2}{2\Delta_z^2}\right]$$

$$\times G_0^0(z) \sin\left\{k_R(R-R_0) - m_\phi\left[\Omega(R_0)t - \phi\right]\right\} \exp(\gamma_0 t)$$

Convective spirals

Another kind of spirals is of special interest in view of the role they can have in the outward transport of angular momentum as needed for mass accretion by the central object. These spiral are oscillatory, in the frame where they corotate with the plasma at a given radius $R = R_0$, and are of the convective type, radially, over scale distances that can be considerably larger than the "microscopic" scale distances such as Δ_0 that have been introduced. In fact, they may characterize the "equilibrium" state of a large class of accretion disks providing a means for angular momentum flow away from the central object.

The complete form of this mode can be represented as

$$\hat{\xi}_z \simeq \tilde{\xi}_z^0 \exp\left[-\frac{z^2}{2\Delta_z^2}\right]$$
$$\times \exp\left\{\mathrm{i}\left[m_\phi\left(\phi - \Omega_k t\right) + k_R\left(R - R_0\right)\right]\right.$$
$$\left.-\mathrm{i}\left[\left(\delta\omega_0\right)t - \frac{\sigma_R}{2}\left(R - R_0\right)^2\right]\right\}$$

over scale distances such that

$$|\delta\omega_0| > \left|\left(R - R_0\right)\frac{\mathrm{d}\Omega_k}{\mathrm{d}R}\right| \qquad (*)$$

corresponding to

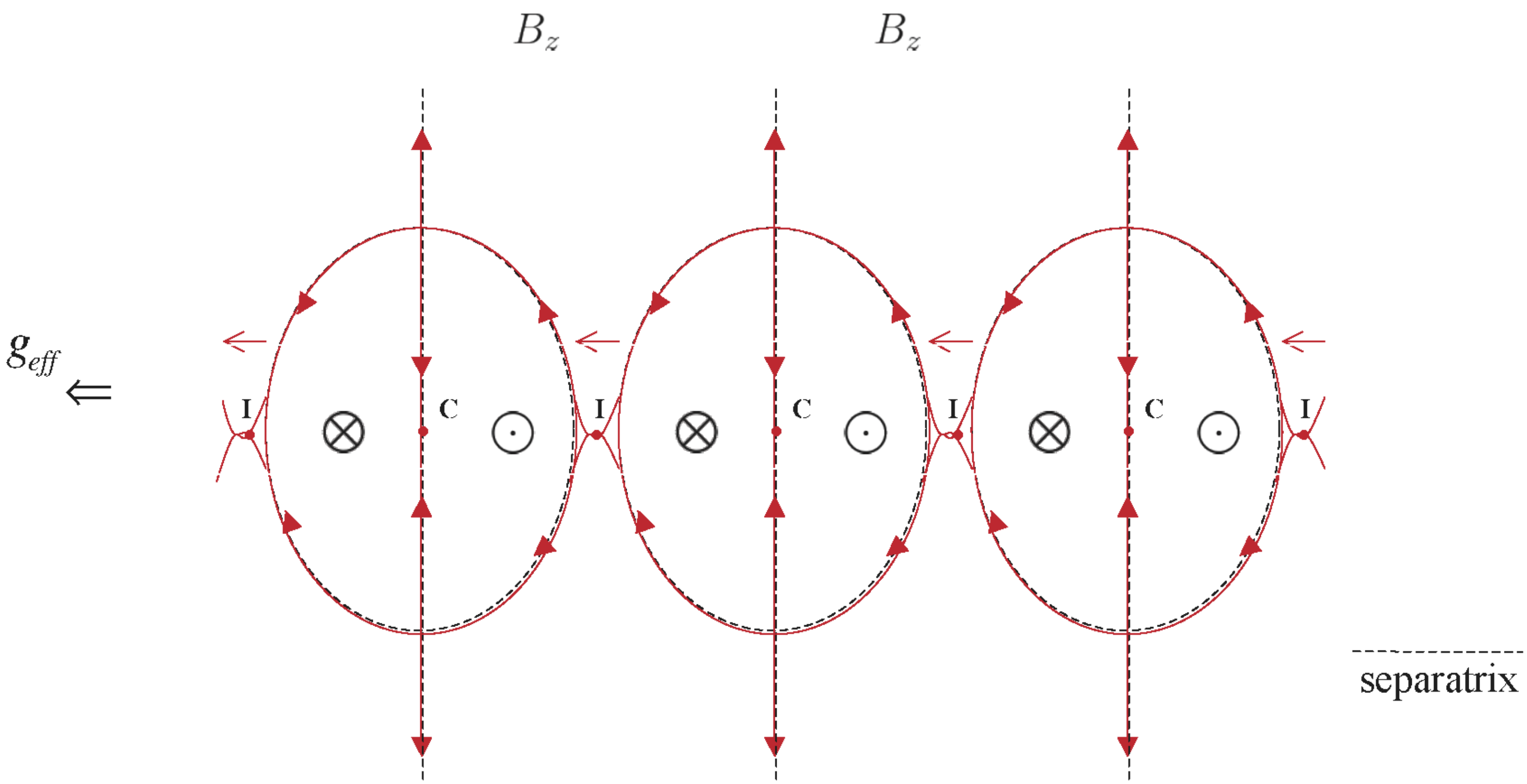
$$|R - R_0| < R_0\frac{1}{\beta^{1/6}},$$

where $\beta = 8\pi p_0 \big/ B^2$, $k_R^2 \simeq k_0^2$ and $\delta\omega_0$ can be derived from an earlier Eq. after replacing ω_0 by $\delta\omega_0$.

Clearly, this mode is oscillatory (in R) over shorter scale distances than that required by condition (*) as

$$\sigma_R \simeq \frac{7}{3}\frac{m_\phi k_R}{\delta\omega_0}\frac{\mathrm{d}\Omega}{\mathrm{d}R}\,.$$

Convective modes localized over shorter scale distances can be constructed out of this class of modes providing a means to transport energy and angular momentum associated with the mode away from R_0, provided $m_\phi \Omega' k_R < 0$. Clearly, this indicates a trailing spiral configuration as in the case of localized modes.



Plasma flow patterns according to the Bursty Accretion scenario.

Plasma Regimes And Regions

Now, taking into account the characteristics of the observed radiation emission from black hole candidates, we may envision a sequence of three plasma regions developing in the vicinity of a rotating and “active” black hole. These regions differ by the kinds of plasma and magnetic field geometry that are present in them. In particular, we consider

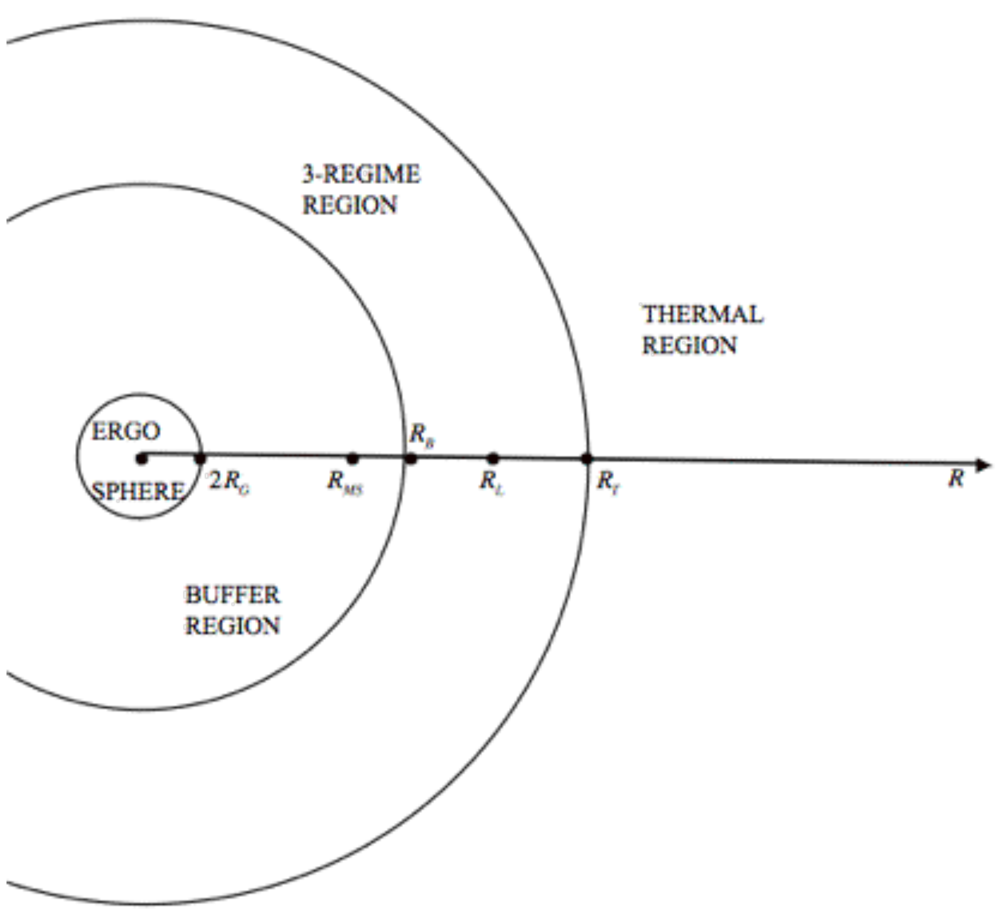
i) a “Buffer Region”

ii) a “Three-regime Region”

iii) a “Structured Low Temperature Region”

The Buffer Region is assumed to be bounded by the Ergosphere and to extend to a distance close to the radius of the marginally stable (e.g. $R_{MS} \simeq 9R_G$) retrograde orbit. This region is assumed to be strongly turbulent. Thus coherent structures originating from external regions should remain excluded from it. The source of energy for this region is considered to be the rotational energy of the black hole [27].

In the region surrounding to the Buffer Region three plasma regimes can emerge (see Fig. 4). Each regime is characterized both by different particle distributions in velocity space and by different coherent plasma structures. In particular, we may identify



Sketch in the equatorial plane of the plasma regions surrounding a rotating black hole. Here $R_{MS} = 9R_G$ and R_L is the distance at which the maximum amplitude of the spiral modes is localized.

a) an “Extreme” (highly non thermal) Regime in which spiral structures are excited.

b) an “Intermediate Non-thermal” Regime in which plasma ring structures are present and rings are ejected vertically at the inner edge of the region.

c) a “Dissipative Thermal” regime where the ring structure is gradually dissipated within the Region before reaching the Buffer Region.

In fact, it is well established experimentally, on the basis of the characteristics of the radiation emitted from Binary Black Holes [2] that these can be attributed to 3 states:

i) a “Steep Power Law” (SPL) State,

ii) a “Hard” State,

iii) a Thermal State.

Transitions between states have been observed for the same object.

Tri-dimensional Structures

Referring to the "Extreme Regime" the assumption made in the derivation of the Master Equation that the electron distribution is represented by a scalar pressure p_e can no longer be made. In particular, if the pressure tensor has an anisotropy of the type the Master Equation is no longer valid and we may argue that a two dimensional configuration of a disk structure may not be established. Then dual spiral structures with the same basic characteristics as those described in *A&A* **504** (2009) are envisioned to become dominant. These consist of two spiral channels, one with a relatively high plasma density and one with a low density. The existence of the low density region characterized by relatively low runaway critical fields is consistent with the onset of spiral structures represented by the following density profiles

$$\hat{n} \simeq \tilde{n}_0^0 \frac{z}{\Delta_L^0} \exp\left[-\frac{\left(R - R_2\right)^2}{\Delta_R^2} - \frac{z^2}{\left(\Delta_L^0\right)^2} \right] \sin\left\{ k_R \left(R - R_L\right) - m_\phi \left[\Omega\left(R_L\right) - \phi \right] \right\}.$$

Here R_L is the radial distance around which the mode is localized, Δ_R and Δ_L^0 are the radial and vertical localization distances, respectively, $\Omega(R_L)$ is the frequency of the plasma rotation around the black hole, and m_ϕ/R_L and k_R are the toroidal and radial mode numbers, respectively. Moreover, $\mathrm{sgn}\left(k_R m_\phi\, d\Omega/dR\right) < 0$ corresponding to trailing spirals.

We note that the expressions for k_R, Δ_R, and Δ_L^0 found from the linearized theory are $k_R \simeq k_0 = \Omega_D/\mathrm{v}_A$, $\mathrm{v}_A^2 = B_0^2/(4\pi\rho_0)$, where B_0 is the vertical "seed" magnetic field from which the considered perturbation can emerge, ρ_0 is the plasma density on the equatorial plane, $\Delta_L^0 \simeq (H_0/k_0)^{1/2}$,

$$\Delta_R \simeq \left\{ \frac{\gamma_0}{\left|\frac{d\Omega}{dR}\right| m_\phi k_R} \right\}^{1/2} \sim \left(\frac{\gamma_0 R_L}{\Omega_k k_0 m_\phi} \right)^{1/2},$$

γ_0 is the linear growth rate of the unstable mode, $\gamma_0 < \Omega$, $H_0 \equiv c_s / \Omega_k \left(R_L \right)$ and c_s is the local velocity of sound. We observe that accretion should be allowed to proceed at relatively fast rates along the considered spiral structures.

Then we may estimate the spiral co-rotational radius to be at the distance $R_L \simeq \alpha_{MS} R_{MS} + \Delta_R^0$ where $R_{MS} \simeq 9 R_G$, and α_{MS} is an appropriate uncertainty parameter. In addition, we may estimate Δ_R^0 as $\Delta_R^0 \simeq \varepsilon_R \left(R_L H_0 \right)^{1/2}$ where $\varepsilon_R < 1$ is a second uncertainty parameter.

A&A 504, 321–329 (2009)
DOI: 10.1051/0004-6361/200811236

Astronomy & Astrophysics

Spectrum of plasma modes and relevant transport processes in astrophysical disks

B. Coppi

Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
e-mail: coppi@mit.edu

Received 27 October 2008 / Accepted 29 June 2009

ABSTRACT

A simple plasma disk structure imbedded in a magnetic field and in the (prevalent) gravity of a central object is shown to be subject to the excitation of significant axisymmetric and tridimensional modes. The key factors involved in the relevant instability are the plasma pressure vertical gradient and the rotation frequency (around the central object) radial gradient. A modestly peaked vertical profile of the plasma temperature is shown to drive a "thermo-rotational instability" with considerable growth rates. Unstable modes are found as well for "flat" temperature profiles. The tri-dimensional tightly wound spirals that are found have properties that, unlike the familiar galactic spirals, depend on the vertical profiles of their amplitudes. Both radially standing and convective (quasi-modes) spirals are identified. Within the considered spectrum, unstable modes are shown to produce opposing fluxes of particles and thermal energies in the vertical direction. Thus disks with relatively flat vertical temperature profiles can expel particles (winds) from the equatorial plane while transporting thermal energy inward. An effective "diffusion" coefficient, for energy and angular momentum, is derived from the structure of radially convective spiral modes and shown to be consistent with significant radial transport rates.

Key words. accretion, accretion disks – black hole physics – magnetohydrodynamics (MHD) – instabilities – magnetic fields – gravitation

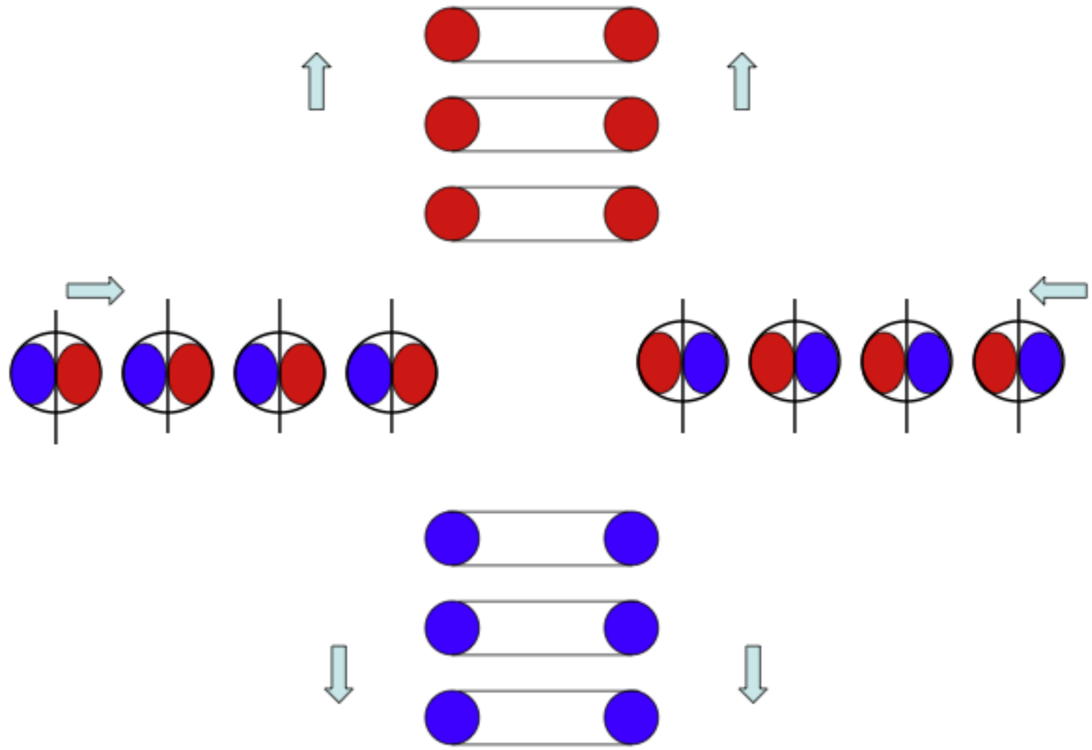
1. Introduction

Identifying the plasma collective modes that can be excited in plasma disk structures (Pringle 1981; Blandford 1976; Lovelace 1976; Coppi & Rousseau 2006) surrounding compact objects such as black holes can be important to explain experimental observations associated with objects of this kind. The geometry of these disk structures (Coppi & Coppi 2001) and their physical parameters, that include in particular the radial gradient of the rotation frequency, the vertical gradients of the particle density and temperature and the effects of the magnetic field in which they are imbedded (Coppi 2008a), determine the characteristics and the spectrum of the modes that can be excited. Novel transport processes of particles and thermal energy in the vertical direction as well as radial transport of angular momentum can be produced by the identified modes. In fact, these processes and the tri-dimensional structure of the spiral modes (Coppi 2008b) that are found can be correlated with important observations connected with black holes (Coppi & Rebusco 2008a). We point out that the properties of the tightly wound spirals that are introduced depend on the vertical profiles of their amplitudes, a feature that is not shared with the better known theory of galactic spirals (Bertin 2000). We note also that the theory presented here for both axisymmetric and tri-dimensional modes include, among others, the physical ingredients of basic modes such as the so called MRI instability (Velikhov 1959; Chandrasekhar 1960; Balbus & Hawley 1991), that is appropriate for different geometries such as cylinders, or the modes that can be excited in well confined plasmas (Coppi & Spight 1978) and can explain the experimentally observed inward transport of particles in connection with the outward transport of plasma thermal energy.

In Sect. 2 the characteristic parameters of the simplest plasma disk structure, imbedded in a vertical magnetic field and surrounding a compact object, are defined in view of the analyses presented in the next sections. In Sect. 3 the basic equations are given for the spectrum of axisymmetric and tri-dimensional normal modes that can be excited in a thin plasma disk. In Sect. 4 the realistic conditions on the density and temperature vertical profiles are examined under which nearly isobaric, weakly compressible modes are found. In Sect. 5 the theory of axisymmetric modes is given and the conditions for their marginal stability are derived. The radial gradient of the rotation frequency and the vertical gradient of the plasma pressure are recognized as the key factors for the excitation of these modes. In particular, when the ratio of the relative gradient of the plasma temperature exceeds the relative gradient of the density by 2/3 a new form of instability (the "thermo-rotational instability") emerges (Coppi 2008a). This is in addition to that of the ballooning modes analyzed by Coppi & Keyes (2003) that are found when the ratio of the gradients mentioned earlier is 2/3 (polytropic profile). In Sect. 6 the transport produced in the vertical direction by the unstable modes described in Sect. 5 is evaluated by the relevant quasi-linear theory. Then the suggestion is made that the outward particle transport (away from the equatorial plane) occurring in the presence of relatively flat temperature profiles be the seed for the observed winds emanating from disks structures surrounding black holes (Elvis 2000). In this case an inward (toward the equatorial plane) flux of thermal energy is produced. The mode growth rates of the unstable modes on which the relevant transport coefficients depend are evaluated by completing the analysis given in Sect. 5. In Sect. 7 the characteristics of the weakly damped oscillatory modes, can be found when the

Article published by EDP Sciences

When the particle distributions in momentum space have a non-thermal component such as that represented by which allows the formation of a composite axisymmetric disk structure in the Three-regime Region, the excitation of spiral modes can be prevented. Then the associated HFQPOs disappear. In addition we may argue that as a result of the interaction between the composite disk structure and the strong turbulence at the edge of the Buffer Region the last couple of plasma rings, carrying oppositely directed toroidal plasma currents that repel each other, could be ejected vertically. Following the arguments given in Section VI the plasma rings can be expected to "arrive" intermittently with a period related to the onset of the modes that transfer particles from one separatrix to the next. In particular, we may envision that jets results from the ejection of toroids ("smoke-rings") carrying currents in the same (toroidal) directions launched in opposite vertical directions. We also note that experimental observations indicate that jets emerge from evolving disk structures.



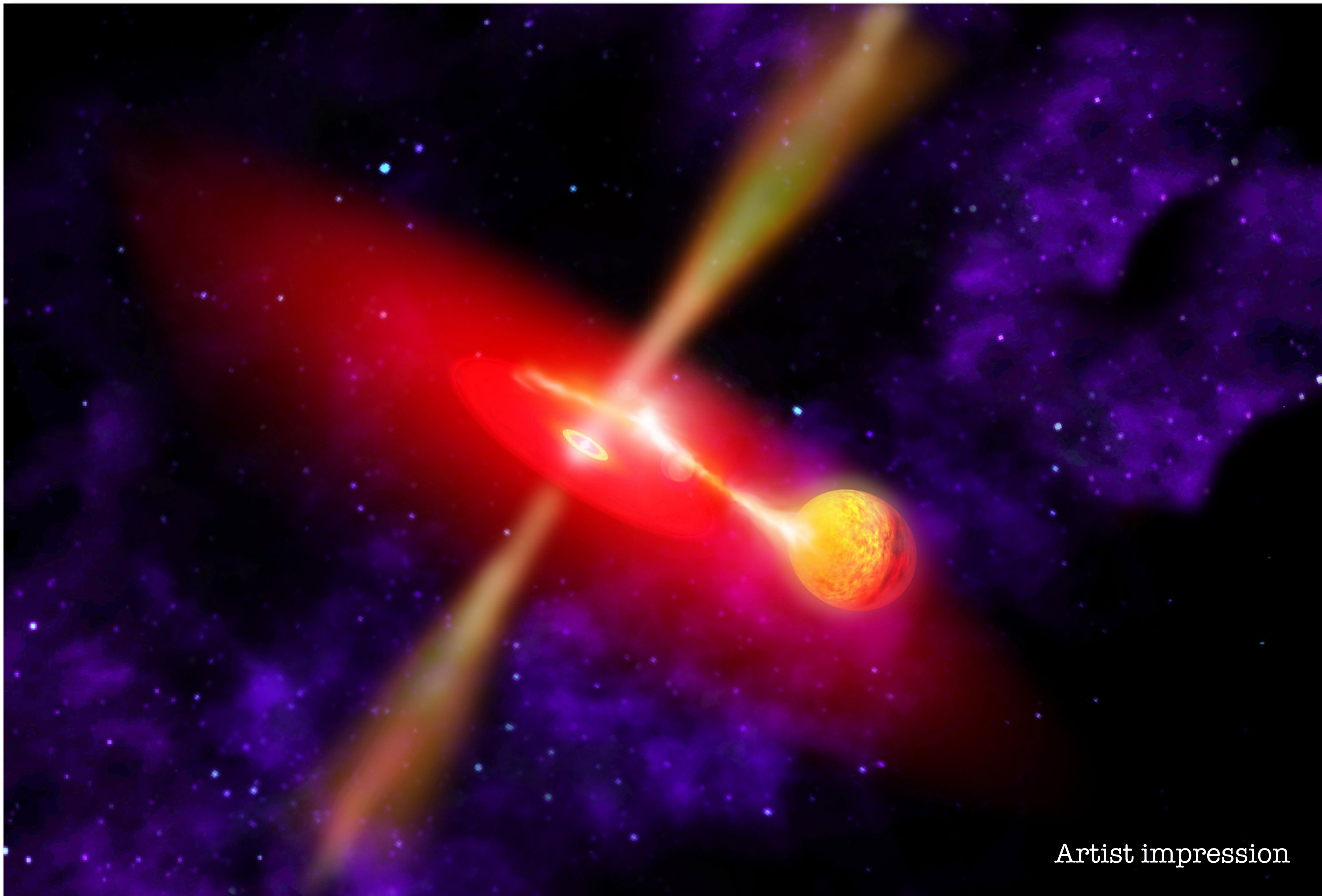
Ring ejection scenario.

In this connection we point out that a recent paper [28] suggests that the power associated with jets [29] is independent of the estimated angular momentum of the black holes with which they are connected. On the other hand it is reasonable to assume that the properties of the Buffer Region and of the plasmas contained in it depend on the black hole rotation. We point out also that the formation and ejection of jets with a purely toroidal magnetic field was proposed and analyzed in Ref. [30].

c) In the Dissipative Thermal regime the plasma can reach a relatively high temperature and maintain a thermal distribution as the coherent ring structure is dissipated before reaching the Buffer Region.

Finally, in the outermost region the plasma is considered to be relatively cold and in a well thermalized state. In this region a composite disk structure such as that described in Section III is assumed to be well established allowing the (accreting) plasma to flow along successive magnetic separatrices as proposed in Section VI.

Low mass X-ray Binaries



X-ray Observations: Active Spectral States

(Remillard&Mc Clintock 2006)

- **Thermal state** (High/Soft)

  high thermal disc fraction

- **Hard state** (Low/Hard)

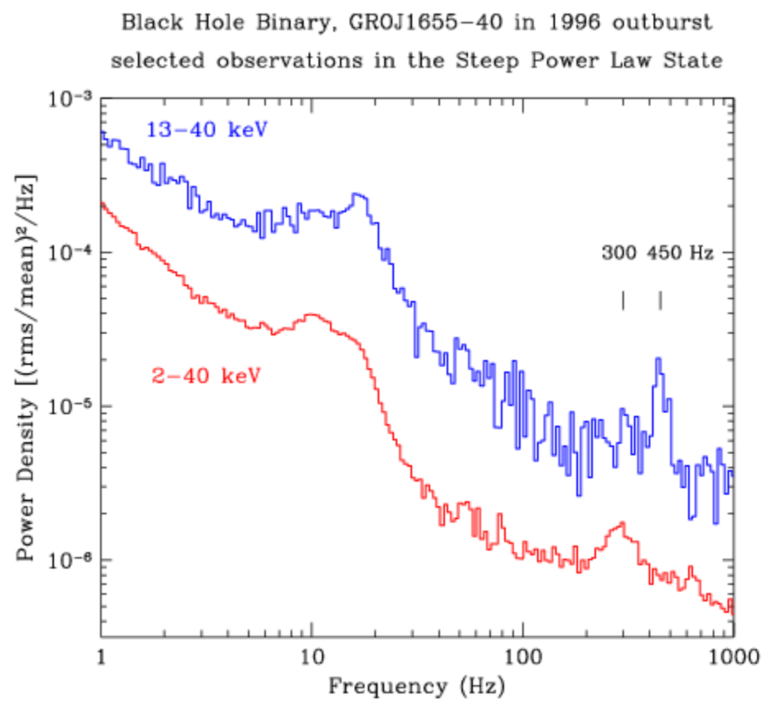
  power law (non-thermal emission) - Sometimes jets -

- **Steep power law** (Very High)

  highly non-thermal. Sometimes HFQPOs

High Frequency Quasi Periodic Oscillations (HFQPOs)

- Highly Coherent Peaks
  in the X-ray power spectra
- 0.1-1200 Hz
  HF-> **few hundred Hz**
- Show up alone OR in pairs OR more
- In **Black Holes**:
  stable **3:2**
- HFQPOs show up in the
  highly **non-thermal** (steep power law) state
- jets and HFQPOs exclude each other



Important Features

•High frequency QPOs lie in the range of ORBITAL FREQUENCIES of free particle orbits just few Schwarzschild radii outside the central source

•The frequencies scale with 1/M

(e.g, Mc Clintock&Remillard 2004)

Normal Modes in Plasma Accretion Structures (Coppi 2008)

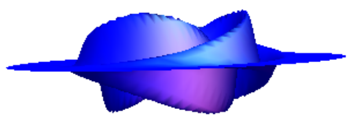
Tridimensional tightly wound spirals excited from a disc embedded in a "seed" vertical magnetic field.

$$\hat{v}_\phi = \tilde{\tilde{v}}_\phi (R - R_0, z) \exp(\gamma_0 t - i\omega_0 t + i m_\phi \phi)$$ They corotate at $\mathbf{R_0}$

$$\omega_0 = m_\phi \Omega(R_0)$$

Excitation mechanism: differential rotation and vertical gradients of plasma density and temperature.

3D plasma spirals (trailing)



$$\hat{\xi}_z \simeq \tilde{\xi}_z^0 \exp\left[-\frac{(R-R)^2}{\Delta_R^2} - \frac{z^2}{2\,\Delta_z^2}\right]\times$$

$$G_0^0(z)\sin\{k_R(R-R_0)-m_\phi[\Omega(R_0)t-\phi]\}\exp(\gamma_0 t)$$

$$\frac{\hat{\rho}}{\rho_0} \simeq -\frac{C_T}{H_0^2} z\tilde{\xi}_z\,,$$

Δ_R and Δ_z are the radial and vertical localizations

Where are they localized?

(B.Coppi, P.Rebusco and M.Bursa 2011,

in preparation)

We make 2 Assumptions

1) $$H_0 \simeq R_{MS} = \alpha_{MS} R_G$$

$$H_0 = \text{half height of the disk}$$

$$R_G = \frac{GM_*}{c^2}$$

2) $$\varepsilon_{th}^e \simeq \varepsilon_{th}^i = \alpha_T m_e c^2$$

Hence...

$$R_0 \simeq R_G \left(\frac{\alpha_{MS}^2}{\alpha_T}\frac{m_i}{m_e}\right)^{1/3} \simeq 12.3 \times R_G \times (\frac{\alpha_{MS}^2}{\alpha_T})^{1/3}$$

$$R_G = \frac{GM_*}{c^2} \qquad R_s = 2R_G$$

$$\Omega_K(R_0) = \left[\frac{c^2\ R_G}{R_0^3}\right]^{1/2} \simeq \frac{c}{R_G}\left[\frac{\alpha_T m_e}{\alpha_{MS}^2 m_i}\right]^{1/2}.$$

$$\alpha_T = \frac{\varepsilon_{th}^i}{m_e c^2} \qquad \alpha_{MS} = \frac{R_{MS}}{R_s}$$

R_{MS} = radius of marginally stable orbit

M_* = black hole mass

$$\nu = m_\phi \frac{\Omega_K}{2\pi} \simeq \frac{m_\phi}{3} \times \frac{10M_\odot}{M_*} \times \frac{1}{\alpha_{MS}}\left(\alpha_T \frac{m_p}{m_i}\right)^{1/2} \times 2.2 \times 10^2\ Hz.$$

√

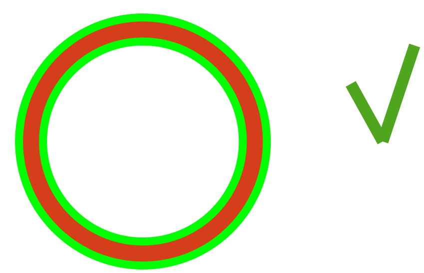
3:2?

Higher toroidal number m_ϕ modes decay into m_ϕ =2 and m_ϕ =3 modes, consistently with the observed twin peak QPOs spectra with the 3:2 ratio.

Ω_{lower} =2 Ω_K and Ω_{upper} =3 Ω_K

Right Ratio!

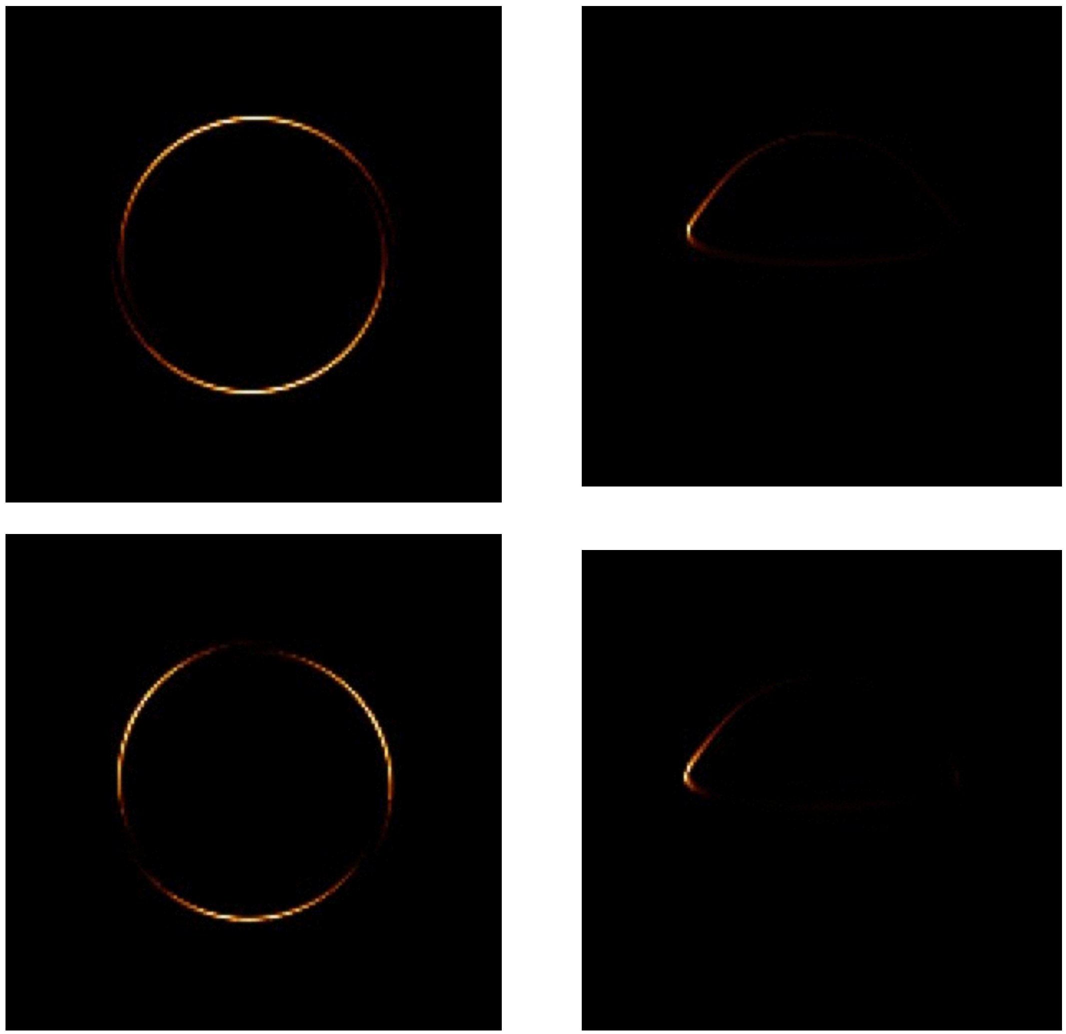
$$\frac{1}{\Delta_R^2} = -m_\phi k_R \frac{d\Omega}{dR} \frac{1}{\gamma_0}$$



Ray-tracing (courtesy of Michal Bursa)

m=2

m=3



Remarks:

- We propose that the excitation of tri-dimensional spiral modes be considered as the explanation for the emergence of QPO's
- The frequencies of the modes are tied to those of the local rotation frequencies of plasmas around black holes
- A specific physical process for the excitation of the relevant plasma collective modes is given, factor not covered by other proposed theories.
- It is essential to advance the presented theory by dealing, with non-thermal particle distributions in phase space.

GENERAL RELATIVITY CORRECTIONS

The phenomena we consider to guide the presented theory, such as High Frequency Quasi Periodic Oscillations (HFQPOs) are estimated to be related to processes taking place at distances $R \gtrsim 10 R_G$, where $R_G \equiv GM_* / c^2$. Therefore, we can extend the theory given in earlier sections by adopting effective gravitational potentials that include General Relativity effects and can be justified for these distances. In particular, when considering a non-rotating black hole we use the Paczynsky-Wiita gravitational potential

$$\phi_G \simeq -\frac{GM_*}{R - 2R_G}.$$

It is easy to verify that this gives the correct radius (also known as ISCO) for the marginally stable orbit ($R_{MS} = 6R_G$), that the rotation frequency is

$$\Omega_G \simeq \frac{1}{R - 2R_G}\left(\frac{GM_*}{R}\right)^{1/2},$$

and

$$\Omega_D^2 = -\frac{Rd\Omega_G^2}{dR} = \frac{3R - 2R_G}{R - 2R_G}\Omega_G^2.$$

As pointed out earlier Ω_D^2 has a prominent role in Eqs. (*) and (***) and is one of the driving factors of the spectrum of modes that can lead to the formation of the considered configurations. As we can see, Ω_G is increased by a factor $3/2$ and Ω_D^2 by a factor 3 for $R = 6R_G$ relative to the Newtonian values.

We observe that, numerically, $R_G \simeq 14.8\left[M_*/(10M_\odot)\right]$ km and $R_{MS} \simeq 89$ $\left[M/(10M_\odot)\right]$ km. Considering a disk structure whose height is $2H$, at a given radius $R \gg R_G$, and a mass accretion rate $\dot{M}$ about $10^{-9}\,M_\odot/\text{yr}$, a rudimentary estimate of the plasma density may be made by an average mass conservation equation like $\simeq 2\left[\bar{\bar{\dot{M}}}/\bar{\bar{H}}\bar{\bar{R}}\right]\times\left[5kms^{-1}/\bar{V}_R\right]\times 10^{17}\,\text{cm}^{-3}$ where $\bar{\bar{\dot{M}}} = \dot{M}/\left(10^{-9}\,M_\odot/\text{yr}\right)$, $\bar{\bar{H}} = H/\left(10^3\ \text{km}\right)$, $\bar{\bar{R}} = R/\left(10^4\ \text{km}\right)$. The corresponding Keplerian velocity is $\text{V}_\phi = c\left(R_G/R\right)^{1/2} \simeq 1.2\times 10^4\left(\bar{\bar{M}}/\bar{\bar{R}}\right)^{1/2}$ km s^{-1} where $\bar{\bar{M}} = M_*/\left(10M_\odot\right)$.

We note that the radius R_{MS} depends in a significant way on the value of the angular momentum $\mathbf{J} = J\mathbf{e}_z$ that a black hole can have. This is characterized by the dimensionless parameter

$$a_* = \frac{J}{M_* c R_G}$$

with $0 < a_* < 1$, $a_* \to 1$ being the so-called "extreme Kerr" limit. When $a_* \to 1$, $R_{MS} = R_G$ (for a direct orbit), $R_{MS} = 9R_G$ (for a retrograde orbit) while $R_{MS} = 6R_G$ for $a_* = 0$, as indicated earlier. Another important radius associated with the Kerr metric to consider is that of the Ergosphere on the equatorial plane $R_E^0 = 2R_G \equiv R_S$. As is well known, the Kerr metric is

$$ds^2 = -\left(1 - \frac{2R_G r}{r_a^2}\right)(cdt)^2 - (2F_K)(ad\phi)(cdt)$$

$$= \left(r^2 + a^2 + a^2 F_k\right)\sin^2\theta (d\phi)^2 + \frac{r_a^2}{\Delta_a^2} dr^2 + r_a^2 (d\phi)^2,$$

where Boyer-Lindquist coordinates are used, $r_a^2 \equiv r^2 + a^2 \cos\theta$, $a \equiv a_* R_G = J/(M_* c)$, $\Delta_a^2 = r^2(1 - 2R_G/r) + a^2$ and $F_K = \left(2rR_G/r_a^2\right)\sin^2\theta$.

In this case we may consider the effective potential for parti cles orbits in the plane $z = 0$, whose radial velocity is given by $\dot{R}^2/\left(2c^2\right) + \mathrm{V}_{eff}(R, E_N, L) = E_N/c^2 = \mathcal{E}$, where

$$V_{eff} = -\frac{R_G}{R} + \frac{L^2/c^2 - 2a^2\mathcal{E}}{2R^2} - \frac{R_G}{R^3}\left(\frac{L}{c} - a\sqrt{\mathcal{E}+1}\right)^2 \qquad (***)$$

and L is the particle specific angular momentum. For circular orbits $V_{eff} = \mathcal{E}$ and $dV_{eff}/dR = 0$, give $\mathcal{E}$ and L as functions of R. Then the radius R_{MS} is obtained from $d^2V_{eff}/dR^2 = 0$. In particular, we may adopt Eq. (***) to add General Relativity corrections to the relevant theory developed in the Newtonian limit.

[1]M. van der Klis and P. Murdin, *Encyclopedia of Astronomy and Astrophysics* (IOP, London, 2000), p. 2380.
[2]R. A. Remillard and J. E. McClintock, Annu. Rev. Astron. Astrophys. **44**, 49 (2006).
[3]D. Psaltis, e-print arXiv:cond-mat/0806, 1531 (2008).
[4]E. F. Taylor and J. A. Wheeler, *Exploring Black Holes* (Addison-Wesley, Boston, 2000), p. F-1.
[5]B. Coppi and F. Rousseau, Astrophys. J. **641**, 458 (2006).
[6]B. Coppi, Astron. Astrophys. **504**, 321 (2009).
[7]B. Coppi, Plasma Phys. Controlled Fusion **51**, 124007 (2009).
[8]B. Coppi, in *Plasmas in the Laboratory and in the Universe*, edited by G. Bertin *et al.* (AIP, New York, 2010), p. 45.
[9]D. Lynden-Bell, Nature (London) **223**, 690 (1969).
[10]J. E. Pringle and M. J. Rees, A&A **24**, 337 (1972).
[11]J. E. Pringle, Annu. Rev. Astron. Astrophys. **19**, 137 (1981).
[12]B. Coppi and M. N. Rosenbluth, Proceedings of the 1965 International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Vienna, 1965.
[13]B. Basu and B. Coppi, Geophys. Res. Lett. **10**, 900, doi:10.1029/GL010i009p00900 (1983).
[14]S. Kato, J. Fukue, and S. Mineshige, *Black-Hole Accretion* (Kyoto Univ. Press, Kyoto, 1998), p. 50.
[15]J. B. Hartle, *Gravity* (Addison-Wesley, San Francisco, 2003), p. 318.
[16]G. I. Ogilvie, Mon. Not. R. Astron. Soc. **888**, 63 (1997).
[17]V. C. A. Ferraro, Mon. Not. R. Astron. Soc. **97**, 288 (1987).
[18]R. H. Cohen, B. Coppi, and A. Treves, Astrophys. J. **179**, 269 (1973).
[19]Chandra X-ray Observatory Center NASA/CXC/SAO. ACIA/HETG Image (2002).
[20]B. Coppi and E. A. Keyes, Astrophys. J. **595**, 1000 (2003).
[21]R. V. E. Lovelace, Nature (London) **262**, 649 (1976).
[22]R. D. Blandford, Mon. Not. R. Astron. Soc. **176**, 465 (1976).
[23]T. E. Strohmayer, W. Zhang, J. H. Swank, I. Lapidus, and J. C. Lochner, Astrophys. J. **469**, L9 (1996).
[24]M. van der Klis, R. Wijnands, W. Chen, F. K. Lamb, D. Psaltis, E. Kuulkers, W. H. G. Lewin, B. Vaughan, J. van Paradijs, S. Dieters, and K. Horne, IAU Circ. **6424**, 2 (1996).
[25]B. Coppi and P. Rebusco, E.P.S. Conference on Plasma Physics, Crete, Greece, 2008.
[26]B. Pazcynsky and P. J. Wiita, Astron. Astrophys. **88**, 23 (1980).
[27]R. A. Blandford and R. L. Znajek, Mon. Not. R. Astron. Soc. **179**, 433 (1977).
[28]R. P. Fender, E. Gallo, and D. Russel, Mon. Not. R. Astron. Soc. **406**, 1425 (2010).
[29]R. A. Blandford and D. G. Payne, Mon. Not. R. Astron. Soc. **199**, 883 (1982).
[30]J. Contopoulos, Astrophys. J. **450**, 616 (1995).
[31]N. I. Shakura and R. A. Sunyaev, Astron. Astrophys. **24**, 337 (1973).

which is independent of r. The condition that the first, second, and fourth terms of equation (13) be dominated by $(4\chi/\hat{r}^3\Psi_c)\Psi_{\chi\chi}^{(1)}$ gives, respectively,

$$\sin^2\theta \ll 8\,, \qquad \sin^2\theta \ll 2\,, \qquad \sin^2\theta \ll \tfrac{4}{3}\,,$$

which together with expression (23) may be written

$$\sin^2\theta \ll \min\left[(\Psi_c/\hat{\alpha}_M)^{2/3}, \tfrac{4}{3}\right]. \tag{24}$$

Our solution is valid, then, as long as both the above condition and the force-free field approximation hold.

IV. MAGNETIC SURFACES AND FIELD LINES

A point (r, θ, ϕ) is related to a point (a, θ_0, ϕ_0) at the surface of the star on the same magnetic surface by the equation $\Psi(r, \theta) = \Psi(a, \theta_0)$, or equivalently, by $\Psi(\hat{r}, \chi) = \Psi(\hat{r} = 1, \chi_0)$ where $\chi_0 = (\sin^2\theta_0)/\Psi_c$. One may use the latter form of this equation to obtain an expression for the magnetic surfaces in polar coordinates. Writing $\Psi(\hat{r}, \chi) = \Psi^{(0)} + \Psi^{(1)} \equiv \Psi_c\chi + \Psi^{(1)}(\hat{r}, \chi)$, the preceding equation gives

$$(\chi - \chi_0) = [\Psi^{(1)}(\hat{r} = 1, \chi_0) - \Psi^{(1)}(\hat{r}, \chi)]/\Psi_c$$

implying that $\chi - \chi_0$ is of order $(\Psi^{(1)}/\Psi_c) \lesssim (\Psi^{(1)}/\Psi^{(0)})$. We may obtain an expression for $(\chi - \chi_0)$ correct to first order in $(\Psi^{(1)}/\Psi_c)$ by simply replacing $\Psi^{(1)}(\hat{r}, \chi)$ by $\Psi^{(1)}(\hat{r}, \chi_0)$ in the above equation. Assuming that the separated form (16) is applicable, so that $\Psi^{(1)} = -(\hat{r}^3\Psi_c/4)\psi(\chi)$, the result may be expressed in the form

$$\frac{\sin^2\theta}{\hat{r}} = \sin^2\theta_0 + \tfrac{1}{4}\Psi_c\psi(\chi_0)(\hat{r}^3 - 1)\,. \tag{25}$$

For a current distribution corresponding to $\hat{\alpha}$ chosen as in equation (11), the coefficient of $(\hat{r}^3 - 1)$ is positive for all magnetic surfaces in the range of our approximations; θ increases with r along a magnetic surface faster than for a dipole field. Since field

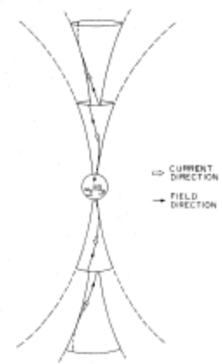


FIG. 1.—Sketch of magnetic surfaces and field lines

ANNALS OF PHYSICS **119**, 97–116 (1979)

Magnetic Equation for a Rotating Neutron Star

B. COPPI AND F. PEGORARO*

Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received July 19, 1978

The magnetic configuration in the plasma-sphere surrounding a neutron star is described in terms of a model equation that is constructed to be valid from the surface of the star to distances of the order of the light speed cylinder and beyond. Significant asymptotic solutions of this equation, that are valid in limited regions around the star, are presented.

1. INTRODUCTION

One of the problems that have to be dealt with when considering possible models for pulsars concerns the macroscopic properties of the plasma that can surround a rotating neutron star. The high magnetic field that is associated with this type of star and the relatively high frequency of rotation that is appropriate for pulsar models are the most important parameters determining the nature of the plasma-sphere surrounding the star.

In the following sections we give an analytical procedure, for a fluid-like description of this plasma-sphere, that leads to solve an equation in the scalar labeling the (magnetic) surfaces of the relevant magnetic configuration. A number of important effects, that it is necessary to consider when analyzing the possible types of plasma flow and magnetic field configurations at distances of the order of or larger than the radius of the "light-speed-cylinder," are discussed and taken into account in the above mentioned magnetic equation. These include the fact that the plasma does not strictly corotate with the star and that its motion is not "frozen-in" everywhere with the magnetic field, the influence of the gravitational and of the centrifugal forces, and the effects of a "braking" force on the plasma resulting from the emission of radiation and particles. Moreover, the braking mechanism has to satisfy the condition that the star rate of energy loss be equal to the rate of angular momentum loss times the frequency of rotation. For the sake of simplicity, a two dimensional model is analyzed assuming that the axis of rotation coincides with that of the magnetic field configuration (Goldreich and Julian, 1969). This magnetic configuration depends upon two expansion parameters which are related to the portion of star surface from which poloidal currents are drawn, and to the ratio between the toroidal and the poloidal magnetic field at the star surface. An estimate of these parameters can be obtained

* Permanent address: Scuola Normale Superiore, Pisa, Italy.

0003-4916/79/050097–20$5.00/0

respectively, where $\theta_c \simeq \psi_c^{1/2}$ as follows from Eq. (3.1). Therefore,

$$\Phi \neq 0 \text{ if } 0 < \psi < \psi_c \quad \text{and } \Phi = 0 \text{ if } \psi_c \leqslant \psi. \tag{3.4}$$

In addition, on each of the polar caps Φ is assumed to be a function of ψ only, as there is no braking force close to the star. Two regions around the star are thus postulated, an active and an inactive one (see Fig. 1). The active region extends from the polar caps ($r = a$, $\psi < \psi_c$) to a "critical surface" beyond which the motion of the plasma is no longer tied to the star magnetic field. Current is drawn from the polar caps along constant Φ-surfaces which, in the neighborhood of the star, coincide with magnetic surfaces. The inactive region is assumed to be bounded by the magnetic surface $\psi = \psi_c$ that is tangent to the critical surface. In addition, it is reasonable to assume that inside the inactive region there is no poloidal current so that the magnetic configuration, in a region relatively close to the star, is that of a dipole corotating with it. The braking of the plasma, which is connected with the rate of energy and

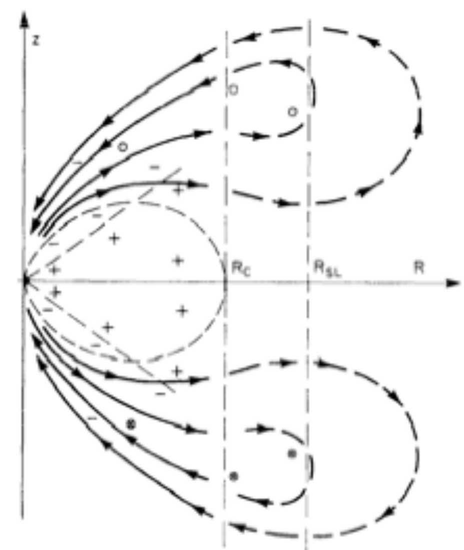


FIG. 1a. Current distribution in the rotating star magnetosphere. The star is indicated by the small circle around the origin. The light broken curve shows the boundary between the active and the inactive region while the straight broken line separates the region of positive charge (+) from the region of negative charge (−) close to the star. The current flow is indicated by heavy lines, solid for $R < R_c$ and broken for $R > R_c$ where the circuit closes. The orientation of the toroidal magnetic field in the active region is symbolized by crossed ⊗ and dotted ⊙ small circles. The dipole magnetic moment and the angular velocity of the star have been assumed to point in the same direction.