SUPERFIELDS
European Research Council
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Black Hole Flow Equations and Duality

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ROUND TABLE 4 ITALY-RUSSIA@DUBNA
Black Holes in Mathematics and Physics
Dubna, December 17th, 2011

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Istituto Nazionale di Fisica Nucleare

Black Hole Flow Equations and Duality

P. Fre': hep-th/9812160 AC with P. Fre', R. D'Auria, M. Trigiante hep-th/9807136

AC, G.Dall'Agata, *JHEP* 03 (2007) 110, hep-th/0702088

S. Ferrara

recent review by G. Dall'Agata: arXiv.org/1106.2611

’70: Striking analogy:

	Thermodynamics	*Black Hole Mechanics*
Zeroth Law	The temperature T is uniform over a body in thermal equilibrium.	The surface gravity κ is is constant over the horizon.
First Law	$TdS = dE + PdV - \Omega dJ$	$\kappa dA = 8\pi(dM - \Omega dJ)$
Second law	$\Delta S \geq 0$	$\Delta A \geq 0$

1976 Black holes emit Hawking radiation

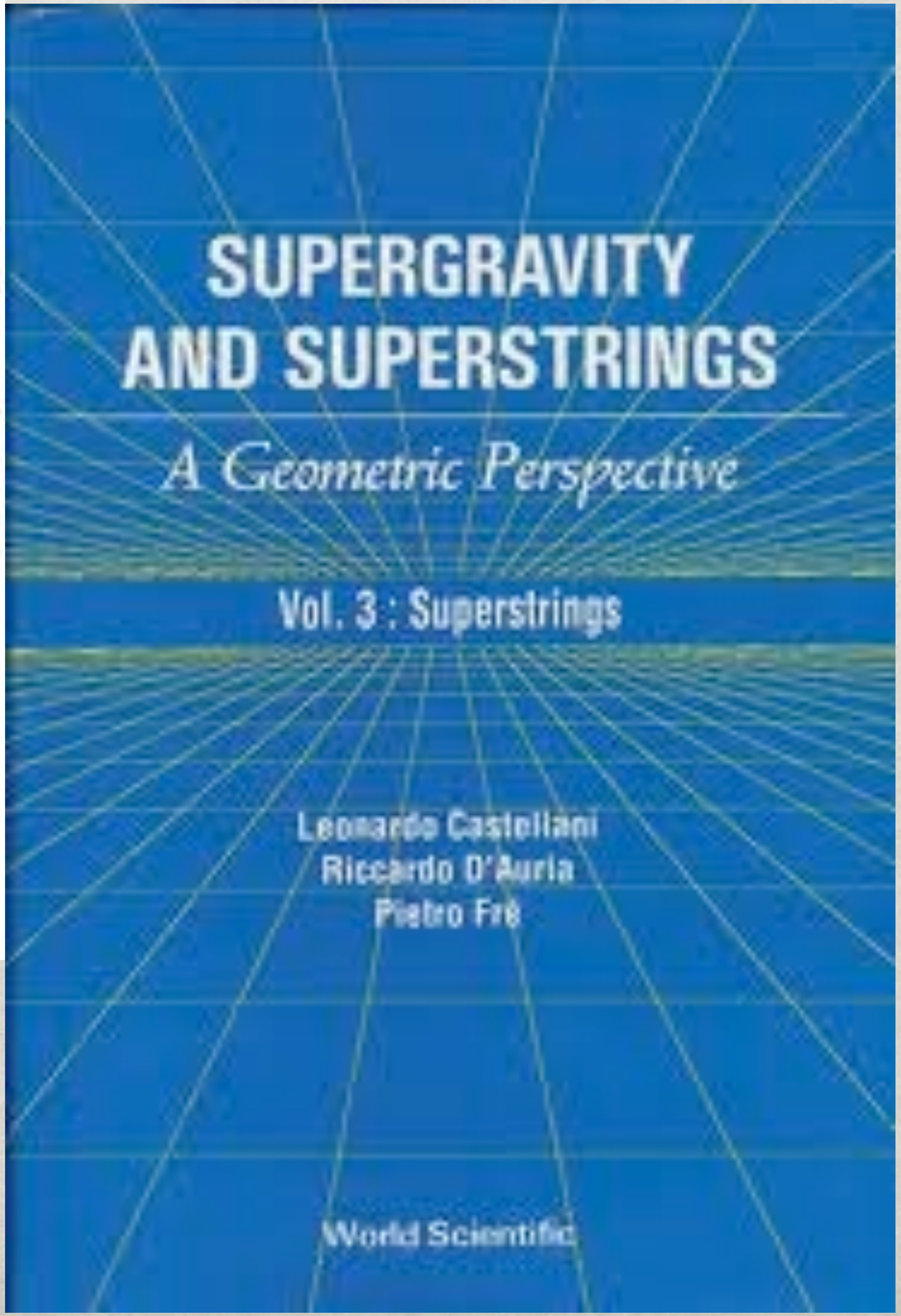
Black holes have an entropy proportional to the area of the horizon

BEKENSTEIN-HAWKING

$$S = \frac{k_B}{l_P^2}\frac{A}{4} \qquad l_P^2 = G\hbar/c^3$$

The microscopic degrees of freedom that give rise to the entropy are not visible in the classical theory.

L. Castellani
R. D'Auria
P. Fre'
80'



“The dark side of String Theory” (G. Horowitz, Trieste 1992)

“The Hydrogen Atom of Quantum Gravity”, (J. Maldacena 1996)

“BH’s are the Harmonic Oscillator of the 21st Century” (A. Strominger, 2009)

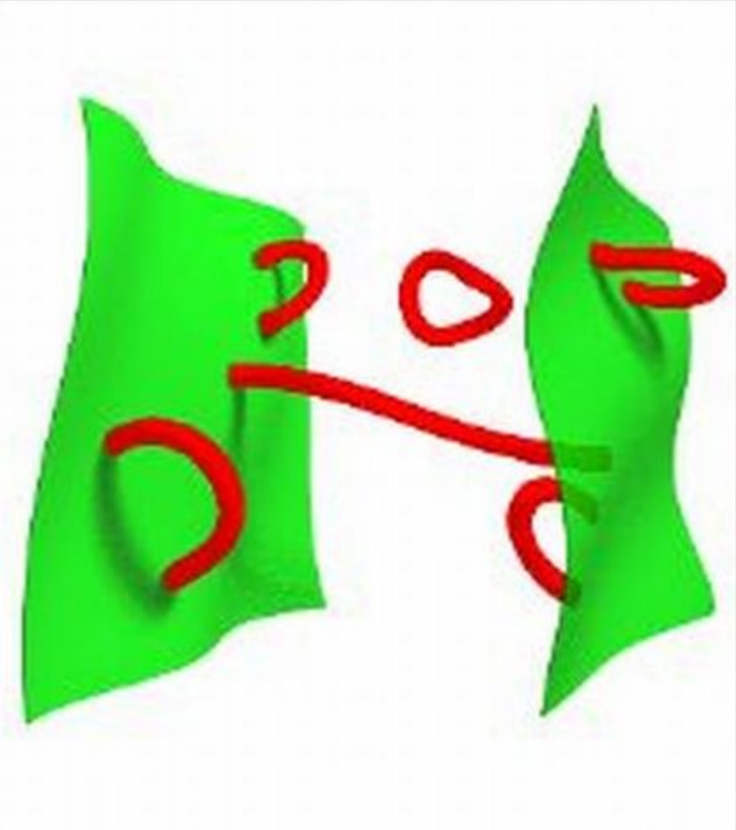
“Gedanken Black Holes” (B. Coppi, Dubna 2011)

Punchline:

1)Super-Gedanken Black Holes behave very similarly to Gedanken Black Holes

2)Both arise as solutions of first order flow equations

3)Their masses and entropies can be determined on the basis of symmetries alone



String theory, as a quantum theory of gravity, provides a microscopic quantum description of the thermodynamic properties of some extremal charged black holes

The description uses properties of some string theory solitons called

D-branes

(extended membranes of various spacetime dimensions when wrapped around the compact extra dimensions they look like charged particles)

$$S_{BH} = \log \Omega(M, Q, P)$$

goal: explain this formula, identify the microstates

Are they the fundamental degrees of freedom of quantum gravity?

In very simple situations String Theory has correctly given the microscopic description of the BH entropy

STROMINGER AND VAFA 1996

$$Sgr\ A^{\star} : r_{+} \sim 7 \cdot 10^{9}\ Km \quad S_{BH} \sim 10^{100}!!!$$

Black Holes in Gravity

•Schwarzschild	M	mass
•Kerr	J	angular momentum
•R.N.	Q= (p,q)	e-m charges

$$e^{-1}\mathcal{L} = R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

String/M-theory $\Longrightarrow$ Einstein-Maxwell Supergravity +**Scalar fields**

$$d, N \, : \{g_{\mu\nu}, A_\mu^\Lambda, \phi^i\} \qquad G/H$$

d spacetime dimensions, N supersymmetries

many scalars: sigma model on G/H
G: group of Type E7, H m.c.s.

Solutions in classical limit: p-branes, domain walls,...
p=0 : black holes

Symmetric Spaces G/H in Sugra

- Scalars live on G/H, charges are in fundamental representation of G

G global symmetry, H local symmetry: "classical" e-m duality, exchanges eqs of motion and Bianchi identities GAILLARD&ZUMINO

in full quantum theory charges are quantized and the duality is broken to discrete subgroup G(Z)=U-duality

HULL&TOWNSEND

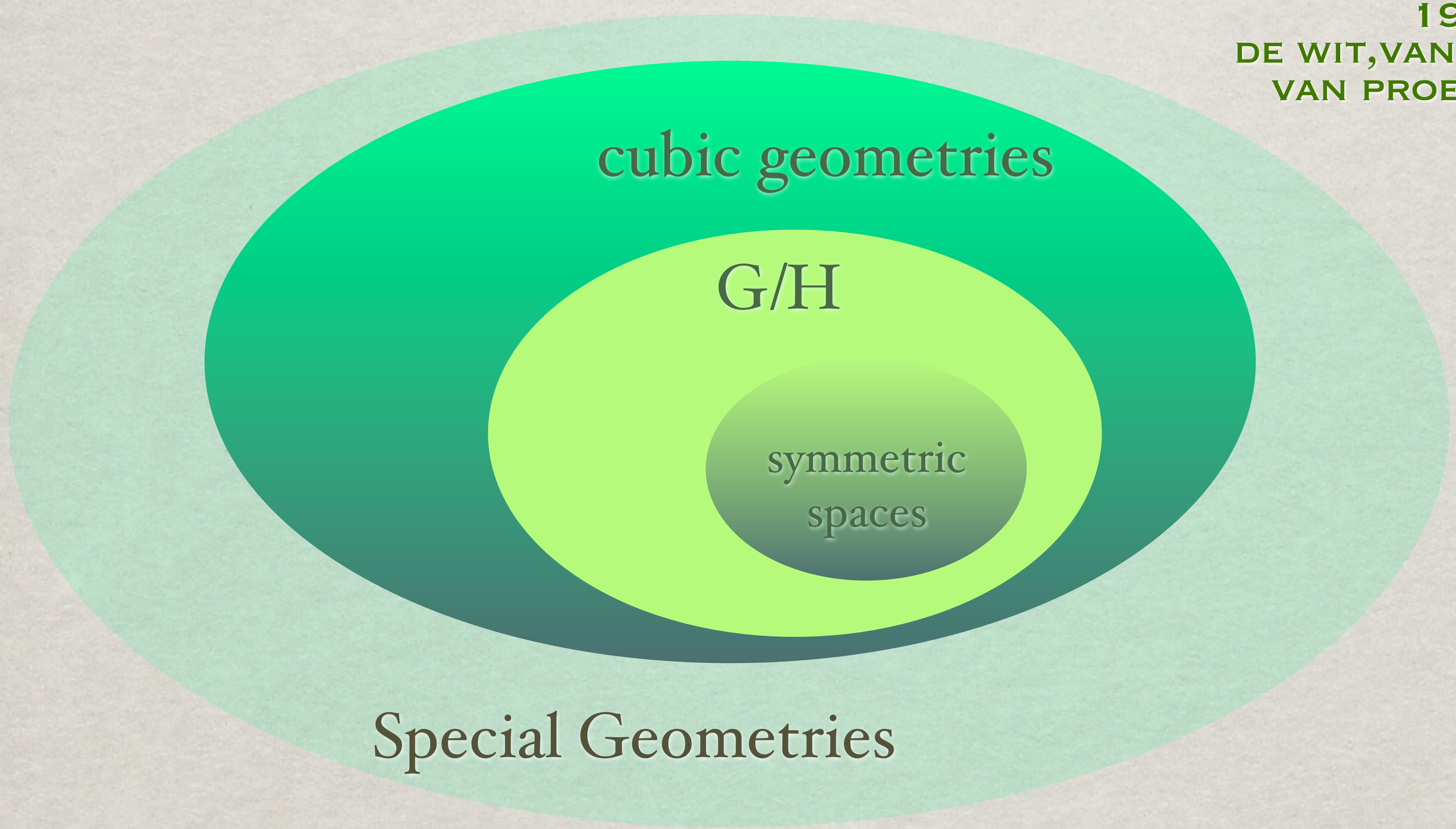
N=8: d=4 $\frac{E_{7(7)}}{SU(8)}$ d=5 $\frac{E_{6(6)}}{USp(8)}$

N=2: Special geometry or very special, defined by cubic F(X)

$$F(X) = \frac{1}{3!} d_{ijk} \frac{X^i X^j X^k}{X^0}$$

can be lifted to 5d

CREMMER, VAN PROEYEN 1985
DE WIT, VANDERSEYPEN, VAN PROEYEN 1993



- Reissner-Nordstrom

$$ds^2 = -(1-\frac{2M}{r}+\frac{Q^2}{r^2})dt^2 + (1-\frac{2M}{r}+\frac{Q^2}{r^2})^{-1}dr^2 + +r^2 d\Omega^2$$

$$r_\pm = M \pm (M^2 - Q^2)^{1/2}$$

(Extremal: $c = 2ST = \frac{1}{2}(r_+ - r_-) \to 0$)

T=0 but nonzero S,
stable, 1 horizon

- BPS (Bogomolny-Prasad-Sommerfeld) states:

preserve a certain fraction of N $\quad S^2 = S \quad S \cdot \mathcal{Q}|BPS\ state >= 0$

BPS bound $\quad M \geq |Q| \quad$ stable ground states

- BH's properties dictated by geometry: BEKENSTEIN-HAWKING

Thermodynamics: T, S

$$S_{BH} = \pi\, r_+^2 = \pi \left[M + \sqrt{M^2 - (P^2 + Q^2)} \right]^2$$

Dynamics: Attractor Mechanism FERRARA-KALLOSH 1995

$$S_{BH} = \frac{k_B}{l_P^2}\frac{1}{4}A_H = \pi V_{BH}(\phi^i_H; p, q)$$

Two strategies for BH:

A) Bottom Up:

start from string/M-theory or lower d compactification:
work with an effective supergravity theory

take specific geometry of spacetime, ansatz for various fields

solve equations of motion (by harmonic functions):
various degrees of susy preserved

interplay between 4d and 5d
extremal/non extremal BH, rings, nuts, bolts,
multicentre, rotating....

Two strategies for BH:

B) Top Down:

use symmetry of the theory (geometry, group theory) and extract general features of physically distinct classes of solutions

B1) U-duality charge orbits have been broadly classified FERRARA

B2) Nilpotent orbits (talk by P. Fre'):SG equations of motion become equivalent to lightlike geodesics motion on the pseudo-riemannian manifold of the 3d sigma model G_3/H_3 obtained by time reduction.

FRE' SORIN TRIGIANTE relate nilpotent orbits to Tits Satake Universality classes and Lax pair representations: integrability

Messages

FERRARA-KALLOSH 1995

i) Extremal BH solutions of extended SG have ``attractor behaviour" and they are associated to 1st order flow equations

{ BPS (susy)
non BPS (non susy) AC, G.Dall'Agata 2007

ii) U-Duality plays a fundamental role in determining

- BH effective potential $\mathbf{V_{BH}}$ (attractors, entropies)
- Fake superpotential **W** (flow equations, mass)
- Q orbits on **G/H** or nilpotent orbits (distinct classes of BH's)

iii) Singular BH's (S=0, NO attractors) have a **W** and are interesting,
Multi-centre BH's : use ``horizontal symmetry" SL(p,R)

Menu

- The Extremal Black Hole Attractor Flows and $\mathbf{V_{BH}}$
  Susy/Non-Susy $$V_{BH} = W^2 + g^{ij} D_i W D_j W$$
- The role of electric-magnetic U-duality: (N=2, N=8)
  - **W** "fake" superpotential
  - Orbits of charge vector **Q**
- Singular black holes, Multi-centre black holes
- Summary and Outlook

THE ATTRACTOR MECHANISM AND V_{BH}

FERRARA-KALLOSH-STROMINGER 1996

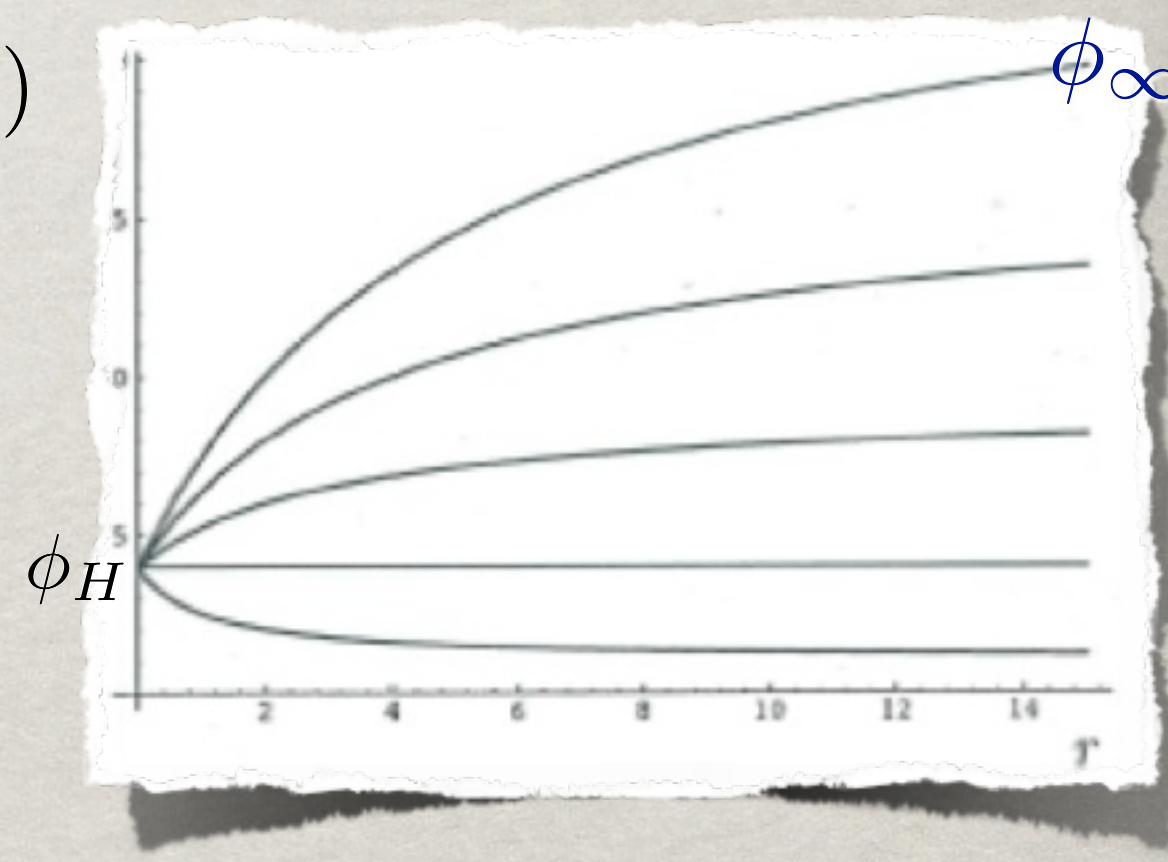
- Susy BH's with e-m charges (q,p) arise as solitonic solutions of a 1d quantum mechanical problem: radial evolution $\phi^i(r)$

$$r \to r_H \begin{cases} \phi^i(r) \to \phi^i_H(r_H) = \phi^i(p,q) \\ \dot{\phi}^i(r) \to 0 \end{cases}$$

scalars at the horizon do not depend on

$$\lim_{r\to\infty} \phi^i(r) = \phi^i_\infty$$

$\{\phi^i_\infty\}$ = moduli space



$\Longrightarrow$"NO SCALAR HAIR", no memory of boundary values

- Attractor fixed points are extrema of an effective potential

$$V_{BH}(p,q;\phi^i) = -\frac{1}{2}Q^T \mathcal{M} Q \qquad Q = (p^\Lambda, q_\Lambda) \quad Sp(2n,\mathbb{R})$$

$$\partial_\phi V_{BH} = 0 \qquad \mathcal{M}(\mathcal{N})$$

2n x 2n matrix

at the horizon:

$$S_{BH} = \frac{A}{4} = \pi V^*_{BH}(\phi_H(p,q);p,q)$$

Bekenstein-Hawking

- SUSY $\not\Leftarrow$ $\Longrightarrow$ EXTREMALITY

T. ORTIN 1996

- Extremal: c=2ST=0 minimal mass for a given charge config.

- BPS bound: $M \geq |Q|$

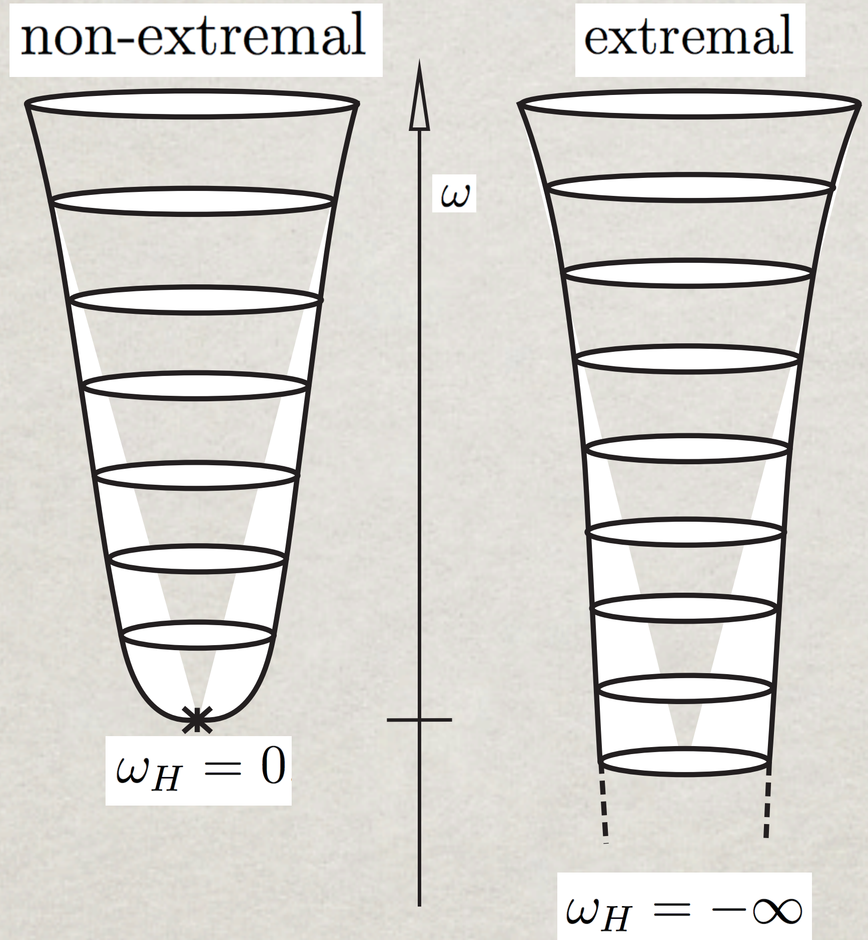


Figure 2: *Schematic representation of non-extremal and extremal black hole throats using proper-distance coordinates.*

- Consider static, spherically symmetric, asymptotically flat BH's in d=4

- Symmetries imply that $g_{\mu\nu}$ and ϕ^i depend only on r: $\phi^i = \phi^i(r)$

$$ds^2 = -\mathrm{e}^{2U(r)}dt^2 + \mathrm{e}^{-2U(r)}\left(c^4 \frac{dr^2}{\sinh^4(cr)} + \frac{c^2}{\sinh^2(cr)} d\Omega^2_{S^2}\right)$$

- Start from 4d N=2 supergravity with vector fields

$$\mathcal{L}_{4d} = -\frac{R}{2} + g_{i\bar{\jmath}}\partial_\mu\phi^i\partial_\nu\bar{\phi}^{\bar{\jmath}} + \mathcal{I}_{\Lambda\Sigma}(\phi)F^\Lambda_{\mu\nu}F^{\Sigma\,\mu\nu} + \mathcal{R}_{\Lambda\Sigma}(\phi)F^\Lambda_{\mu\nu}\widetilde{F}^{\Lambda\mu\nu}$$

e-m charges: $\int_{S^2} F^\Lambda = 4\pi p^\Lambda \qquad \int_{S^2} G_\Lambda = 4\pi q_\Lambda$

- Integrating over $\mathbb{R}_t \times S^2$ you get

$$\begin{cases} \mathcal{L} = (U'(r))^2 + g_{i\bar{\jmath}}\phi'^i\bar{\phi}'^{\bar{\jmath}} + \mathrm{e}^{2U}V_{BH}(\phi,q,p) - c^2 \\ H = (U'(r))^2 + g_{i\bar{\jmath}}\phi'^i\bar{\phi}'^{\bar{\jmath}} - \mathrm{e}^{2U}V_{BH}(\phi,q,p) - c^2 \end{cases}$$

Need H=0 to have that 1d eqs of motion are consistent with the 4d ones

- For N=2 supergravity the effective potential reads

$$V_{BH}(\phi,q,p) = |\mathcal{Z}|^2 + 4g^{i\bar{\jmath}}\partial_i|\mathcal{Z}|\partial_{\bar{\jmath}}|\mathcal{Z}|$$

$\mathcal{Z} = e^{K/2}(X^\Lambda q_\Lambda - \mathcal{F}_\Lambda p^\Lambda)$ N=2 central charge $\mathcal{F}_\Lambda = \partial_\Lambda F(X)$

$Q = (p^\Lambda, q_\Lambda); \quad \mathcal{V} = (X^\Lambda, \mathcal{F}_\Lambda) \ : \quad Sp(2n_v+2)$

Sections of Kahler-Hodge manifold (Special Geometry)

- For extremal solutions c = 2ST=0), the action takes Bogomolny form:

$$S = \int dr \left[\left(U' \pm \mathrm{e}^U |\mathcal{Z}| \right)^2 + \left| \phi^{i\prime} \pm 2\mathrm{e}^U g^{i\bar{\jmath}} \partial_{\bar{\jmath}} |\mathcal{Z}| \right|^2 \mp 2 \frac{d}{dr} \left(\mathrm{e}^U |\mathcal{Z}| \right) \right]$$

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Flow equations

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Flow equations

ADM mass

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Flow equations

ADM mass

- The flow stops at $\partial_i |\mathcal{Z}| = 0 \Rightarrow \partial_i V_{BH} = 0$

where $ds^2 = -\frac{r^2}{|\mathcal{Z}|_*^2} dt^2 + \frac{|\mathcal{Z}|_*^2}{r^2} \left(dr^2 + r^2 \Omega_{S^2}^2\right)$

Ferrara-Gibbons-Kallosh 1996

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$AdS_2 \times S^2$

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FERRARA-GIBBONS-KALLOSH 1996

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$$S_{BH} = \frac{A}{4} = \pi |\mathcal{Z}|_*^2 \left(\phi_*(p,q), p, q \right)$$

First Order Flow Equations

- eqs of motion: $$\begin{cases} U'' = e^{2U} V_{BH} \\ \phi^{i\prime\prime} + \Gamma^i_{jk}\phi^{j\prime}\phi^{k\prime} = e^{2U} g^{i\bar{\jmath}}\partial_{\bar{\jmath}} V_{BH} \end{cases}$$

- effective BH potential: $$V_{BH} = W^2 + 4 g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$$

- superpotential: $$W(\phi, \bar{\phi}) = |Z|$$

- ADM mass: $$e^U W|_\infty \sim M_{ADM}$$

- BPS attractor: $$\frac{\partial V_{BH}}{\partial \phi^i} = 0 \Longrightarrow D_i Z = 0 \quad Z \neq 0$$

stability: check Hessian

First Order Flow Equations

- eqs of motion:

$$\begin{cases} U' = \pm e^U W \\ \phi^{i\prime} = \pm 2 e^U g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W \end{cases}$$

- superpotential for BPS flows: $W(\phi, \bar{\phi}) = |Z|$

- effective BH potential: $V_{BH} = W^2 + 4 g^{i\bar{\jmath}} \partial_i W \partial_{\bar{\jmath}} W$

- ADM mass: $e^U W|_\infty \sim M_{ADM}$

- BPS attractor: $\dfrac{\partial V_{BH}}{\partial \phi^i} = 0 \Longrightarrow D_i Z = 0 \quad Z \neq 0$

stability: check Hessian, possible saddle points

Set up for N- Extended Supergravities

- N- extended susy algebra: $\{\mathcal{Q}_{\alpha A}, \mathcal{Q}_{\beta B}\} = \epsilon_{\alpha\beta} Z_{AB}(p, q; \phi)$

$$V_{BH} = -\frac{1}{2} Q^T \mathcal{M}(\mathcal{N}) Q = \frac{1}{2} Z_{AB} \overline{Z}^{AB} + Z_I Z^I$$

$$\partial_\phi V_{BH} = 0$$

$$\begin{cases} Z_{AB} = -Z_{BA} \\ Z_I \end{cases}$$

central charges

matter charges

A,B in SU(N)
I: fundam of matter group when present

$$(N = 2 : Z_{AB} = \epsilon_{AB} Z, \quad Z_I = D_i Z)$$

$$\begin{cases} Q = (p^\Lambda, q_\Lambda) \\ Z_{AB} = f^\Lambda{}_{AB} q_\Lambda - h_{AB\Lambda} p^\Lambda \end{cases}$$

$\mathcal{N}(\phi)$ kinetic matrix for vector fields

$(f^\Lambda{}_{AB}, h_{AB\Lambda})$ $Sp(2n, \mathbb{R})$

- BPS bound: $M_{ADM}(\phi, Q) \geq |z_1(\phi, Q)| \geq \ldots \geq |z_{[N/2]}(\phi, Q)|$

BPS states: M=highest eigenvalue of central charge

Fake Superpotential for non susy BH

Fake Supergravities

Freedman, nunez, Schnabel, skenderis, townsend, 2003
Celi, AC, Dall'Agata, Van Proeyen, Zagermann 2004

- Gravitational theories in d-dim that are susy only through linear order in fermion fields. Contain some "fake BPS equations" for the warp factor and scalar fields that are of first order and solve ordinary Einstein and scalar field equations

- The scalar potential can formally be written in terms of a superpotential (matrix) in the "stability form"

- Applications: curved domain walls in SUGRA, cosmological solutions; adding vectors, also BH's, superstars,...

- Caution when you have many (hyper)-scalars

townsend, skenderis pseudosupersymmetries, cosmological solutions

NON-BPS EXTREMAL BLACK HOLES

- Defining a real $W(\phi, \overline{\phi})$, extremal black holes are described by

$$\begin{cases} U' = -e^{U} W \\ \phi'^{i} = -2e^{U} g^{i\bar{\jmath}} \partial_{\bar{\jmath}} W \end{cases}$$

iff $V_{BH}(\phi, q, p) = W^2 + 4g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$

- BPS BH's are a special case with W = |Z|

- But **other possible solutions are the non-BPS BH's** !

- $\partial_i W(\phi, \bar{\phi}) = 0$ **gives non-BPS critical points!**

$W(\phi, \overline{\phi})$ "fake" superpotential

Non-BPS Extremal Black Holes

- Defining a real $W(\phi,\overline{\phi})$, extremal black holes are described by

$$\begin{cases} U' &= -\mathrm{e}^U W \\ \phi'^i &= -2\mathrm{e}^U g^{i\bar{\jmath}}\partial_{\bar{\jmath}} W \end{cases}$$

This is a PDE with b.c. the critical point of the superpotential

iff $V_{BH}(\phi,q,p) = W^2 + 4g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$

- BPS BH's are a special case with W = |Z|

- But **other possible solutions are the non-BPS BH's** !

- $\partial_i W(\phi,\bar{\phi}) = 0$ **gives non-BPS critical points!**

$W(\phi,\overline{\phi})$ "fake" superpotential

General Answer:

AC&G. Dall'Agata 2007

- look for a real "fake" superpotential $W(\phi, \bar{\phi}) \neq |Z|$

a) $V_{BH} = W^2 + 4g^{i\bar{\jmath}}\partial_i W \partial_{\bar{\jmath}} W$ same effective potential

b) drives first order flows $$\begin{cases} U' = \pm e^U W \\ \phi^{i\prime} = \pm 2e^U g^{i\bar{\jmath}}\partial_{\bar{\jmath}} W \end{cases}$$

c) $\partial_i W(\phi, \bar{\phi}) = 0$ gives non-BPS critical points

- Construct it using **duality invariance:** $W = W(\{i_n\})$

The example: N=8

$$G/H = \frac{E_{7(7)}}{SU(8)}$$

Susy algebra: $\{\mathcal{Q}_{\alpha A}, \mathcal{Q}_{\beta B}\} = \epsilon_{\alpha\beta} Z_{AB}(p,q;\phi)$

70 scalars, 56 charges

U-duality: $E_{7(7)}(\mathbf{Z})$

$$Z_{AB} \xrightarrow{\text{SU(8)}} \begin{pmatrix} z_1 & & & \\ & z_2 & & \\ & & z_3 & \\ & & & z_4 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

A,B=1,...8

normal frame

eigenvalues: $\{z_i = \rho_i e^{i\varphi/4}\} \quad i = 1,2,3,4$ 5 parameters

$$M \geq z_h$$

Cartan quartic invariant (Cremmer-Julia):

$$I_4 = Tr(Z\overline{Z})^2 - \frac{1}{4}(Tr\, Z\overline{Z})^2 + 4(Pf\, Z + Pf\, \overline{Z}) = T_{abcd}q^a q^b q^c q^d$$

$$\frac{\partial I_4}{\partial \phi^i} = 0$$

$A = Z\overline{Z}$ invariants: $\{\mathrm{Tr}A, \mathrm{Tr}A^2, \mathrm{Tr}A^3, \mathrm{Tr}A^4, \mathrm{Re\ Pf}\, Z\}$

- **Question:** What is a complete set of duality invariants for N=2?
- **Answer:** Sp(2n+2,R) invariants are

Cerchiai Marrani Ferrara Zumino 2009

$$i_1 = Z\overline{Z}$$

$$i_2 = g^{i\bar{\jmath}} Z_i \overline{Z}_{\bar{\jmath}} \qquad (Z_i = D_i Z \ , \ \overline{Z}_{\bar{\imath}} = \overline{D}_{\bar{\imath}} \ \overline{Z}) ,$$

$$i_3 = \frac{1}{6}\left[Z N_3(\overline{Z}) + \overline{Z}\overline{N}_3(Z_i)\right] , \qquad i_4 = \frac{i}{6}\left[Z N_3(\overline{Z}) - \overline{Z}\overline{N}_3(Z)\right] ,$$

$$i_5 = g^{i\bar{\imath}} C_{ijk} C_{\bar{\imath}\bar{\jmath}\bar{k}} \overline{Z}^j \overline{Z}^k \, Z^{\bar{\jmath}} Z^{\bar{k}} ,$$

cubic norms:

$$N_3(\overline{Z}) = C_{ijk} \overline{Z}^i \ \overline{Z}^j \ \overline{Z}^k , \qquad \overline{N}_3(Z) = C_{\bar{\imath}\bar{\jmath}\bar{k}} Z^{\bar{\imath}} \ Z^{\bar{\jmath}} \ Z^{\bar{k}} .$$

Ansatz:

$$W(\phi, \bar{\phi}) = W(i_1, i_2, i_3, i_4, i_5)$$

AC, Dall'Agata, Ferrara, Yeranyan 2009

W and Hamilton-Jacobi

Andrianopoli, D'Auria, Orazi, Trigiante 2009

- Interpret $U(r), \phi^a(r)$ as coordinates of an Hamiltonian system where the radial variable plays the role of time. Then first order description is equivalent to solving HJ problem for Hamilton's characteristic function

$$\mathcal{W}(U,\phi) = 2e^U W(\phi)$$

- Hamilton-Jacobi equation

$$W^2 + 2g^{ab}\frac{\partial W}{\partial \phi^a}\frac{\partial W}{\partial \phi^b} = V$$

- boundary conditions

$$U(r=\infty) = 0\,, \phi^a(r=\infty) = \phi^a_\infty$$

- implications on duality invariance and stability (a la Liapunov)
  - W= Hamilton's principal function for non -BPS flows

How to find W

AC, Dall'Agata, Ferrara, Yeranyan 2009

Find W for generic charge configuration by

1. Take W for STU model in S=T=U limit

Bellucci, Ferrara, Marrani, Yeranyan 2008

2. Compute it in simple charge configuration and then boost it to generic charges by a duality transformation

$$W^2 = \frac{i_1+i_2}{4} + \frac{3}{8}\left[\left(4\,i_3\sqrt{-I_4} - (i_1+i_2)\,I_4 + \left(i_1 - \frac{i_2}{3}\right)^3\right)^{1/3} + \right.$$
$$\left. + \left(-4\,i_3\sqrt{-I_4} - (i_1+i_2)\,I_4 + \left(i_1 - \frac{i_2}{3}\right)^3\right)^{1/3}\right].$$

$\Longrightarrow$ non polynomial expression, but at non-BPS attractor point:

$$i_2 = 3i_1 = \frac{3}{4}\sqrt{-I_4}\,,\, i_3 = 0 \Longrightarrow S_{BH} = W^2 = \sqrt{-|I_4|}$$

Full non-BPS BH solution:

Given W you can solve flow eqs by (universal) harmonic functions: $t = x - iy$

$$\left[\begin{array}{rcl} e^{-4U} &=& (\mathcal{H}_1)^3 \mathcal{H}_0 - b^2, \\ x &=& \dfrac{b\sqrt{-I_4}}{2(p^1)^2(\mathcal{H}_1)^2}, \\ y &=& \dfrac{e^{-2U}\sqrt{-I_4}}{2(p^1)^2\mathcal{H}_1^2}. \end{array}\right.$$

$$\begin{cases} \mathcal{H}_0 = \dfrac{(-I_4)^{1/4}}{\sqrt{2}q_0} H_0 \\ \mathcal{H}_1 = -\dfrac{(-I_4)^{1/4}}{\sqrt{2}q_0} H^1 \end{cases}$$

$$\begin{cases} H_0 = h_0 - \sqrt{2}q_0 r \\ H^1 = h^1 + \sqrt{2}p^1 r \end{cases}$$

AC, Dall'Agata, Ferrara, Yeranyan 2009

Results for T^3, ST^2, STU agree with time reduction approach

Bossard,Michel, Pioline arXiv:0908.1742

"non standard diagonalization problem", sextic polynomial in W^2 whose coefficients are SU(8) invariants

CHARGE ORBITS

Attractors & Duality

- Kallosh-Kol (1996): Area of horizon for N=8 extremal BH is proportional to $\sqrt{\pm I_4}$, where $I_4 = T_{abcd}Q^aQ^bQ^cQ^d$ of E_7 for 1/8 preserved susy. A=0 $(I_4 = 0)$ for 1/8, 1/4, 1/2 susy

sen; cvetic, Hull 1996

Andrianopoli, D'Auria, Ferrara 1997, 1998

- Ferrara-Maldacena (1998): different susy features are distinguished by U-invariant conditions on charges Q=(p,q)

Lu, Pope, Stelle 1998

- Ferrara-Gunaydin (1998): for fixed values of I_4 in d=4 and of I_3 in d=5, charge vectors Q for supergravities on symmetric spaces describe orbits whose nature is related to the susy properties of fixed points

Bellucci, Ferrara, gunaydin, marrani 2006

- Orbits of the fundamental representation of the U-duality groups in extended supergravities based on symmetric spaces classify in an invariant way the extremal BPS and non BPS regular and singular Black Hole solutions

- Each orbit correspond to an allowed entropy

Ferrara gunaydin 1998

- Classification of BPS states preserving different numbers of susy is in close parallel to the classification of the little groups and orbits of timelike, lightlike and spacelike vectors in Minkowski space:

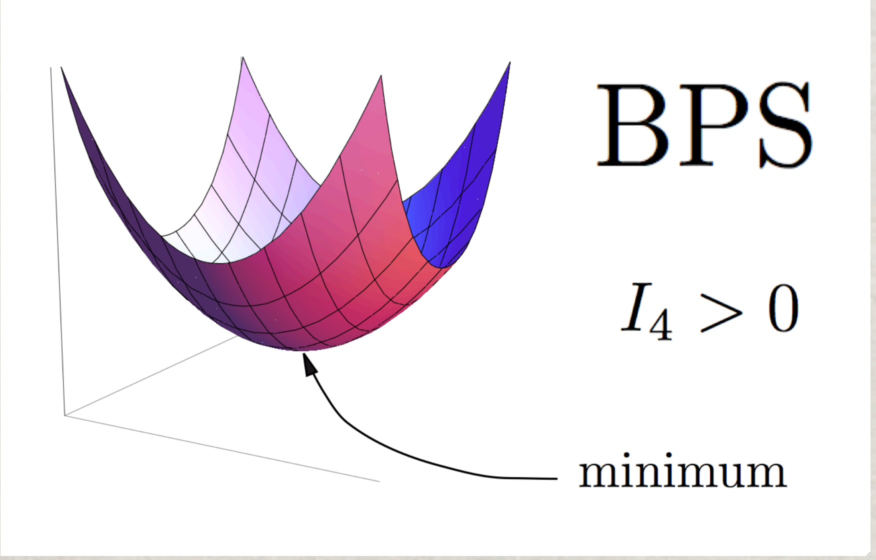
Lightlike: $I_4 = 0$

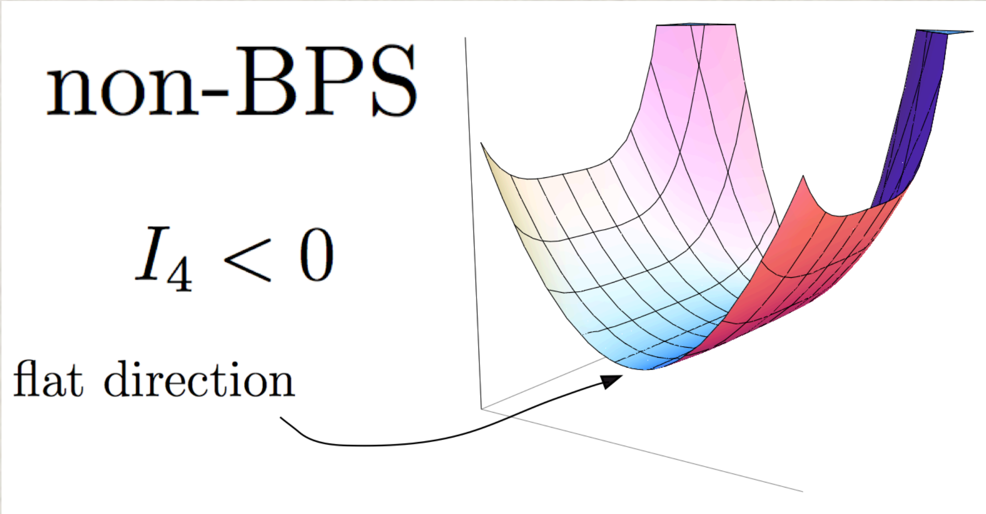
Spacelike: $I_4 > 0$

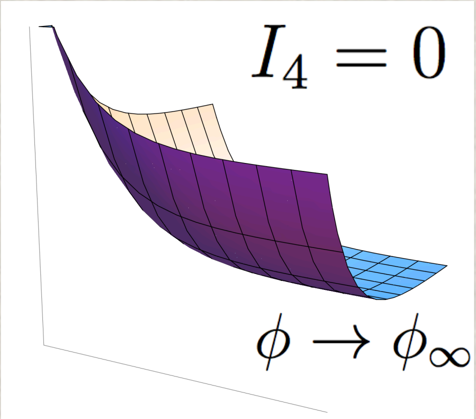
Timelike : $I_4 < 0$

BH potentials for different values of the quartic invariant I_4

(pictures by G. Dall'Agata)







small BH's

Charge Orbits for N=8

KALLOSH-KOL1996,
FERRARA MALDACENA 1996
FERRARA KALLOSH 2006,
CERCHIAI,FERRARA,
MARRANI, ZUMINO 2009

Large Orbits

$I_4 \neq 0$

1/8 BPS: $S_{BH} = \pi\sqrt{I_4} = \pi\rho^2 \qquad I_4 > 0$

$$\{z_1 = \rho e^{i\varphi}, z_2 = z_3 = z_4 = 0\};$$

non BPS: $S_{BH} = \pi\sqrt{-I_4} = 4\pi\rho^2 \qquad I_4 < 0$

$$\{z_1 = z_2 = z_3 = z_4 = \rho e^{i\pi/4}\};$$

Small Orbits

$I_4 = 0$

1/8 BPS: $\frac{\partial I_4}{\partial q^a} \neq 0 \qquad \{\rho_1 \geq \rho_2 \geq \rho_3 \geq \rho_4\,, \varphi\}$

1/4 BPS: $\frac{\partial I_4}{\partial q^a} = 0\,, \quad \frac{\partial^2 I_4}{\partial q^a \partial q^b}|_{Adj} \neq 0 \qquad \{\rho_1 = \rho_2\,, \rho_3 = \rho_4\,, \varphi\}$

1/2 BPS: $\frac{\partial^2 I_4}{\partial q^a \partial q^b}|_{Adj} \neq 0 \qquad \{\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho\,, \varphi = 2k\pi\}$

SINGULAR BLACK HOLES

Singular Black Holes

- S=0, $I_4 = 0$ vanishing classical entropy
- NO Attractor behaviour: $\partial_\varphi W|_H \neq 0$; W has a runaway solution W=0 at boundary of moduli space
- In principle can still compute $W(I_4 \to 0)$ by a suitable limit of large BH's

  ANDRIANOPOLI, FERRARA, D'AURIA, TRIGIANTE 2010
- Can compute W as a function of the invariants $W(\{i_n\})$
- In N=2 "small" BH's can be BPS or non-BPS, differently from N=8

  AC, FERRARA, MARRANI 2010
- Small BH's may play a role in strings/finiteness of N=8 SG

  BIANCHI, KALLOSH, FERRARA 2010

Large Orbits:

$$I_4 > 0 \quad : \begin{cases} \text{BPS} & i_1 > \lambda_1, \lambda_2, \lambda_3 & W_{BPS} = \sqrt{i_1} \\ \text{non BPS} & \lambda_1 > i_1, \lambda_2, \lambda_3 & W_{nonBPS} = \sqrt{\lambda_1} \end{cases}$$

$$I_4 < 0 \quad \text{non BPS} \qquad \lambda_1 \neq \lambda_2 \neq \lambda_3 \qquad W_{nonBPS} = \ldots$$

Small Orbits: $I_4 = 0$

•Lightlike

$$I_4 = 0 \quad r = 3 : \qquad \begin{cases} \text{BPS} \quad i_1 > \sqrt{\lambda_1}; & W_{\text{BPS}} = \sqrt{i_1} \\ \text{nonBPS} \quad i_1 < \sqrt{\lambda_1}; & W_{\text{non BPS}} = \sqrt{\lambda_1} \end{cases} .$$

•Critical

$$\partial I_4 = 0\,, \; r = 2 : \qquad \begin{cases} \text{BPS} \quad i_2 > i_1 > \frac{i_2}{3} & W_{\text{BPS}} = \sqrt{i_1} \\ & i_1 = \lambda_1; \lambda_2 = \lambda_3 = \frac{i_2 - i_1}{2} \\ \text{nonBPS} \quad i_1 < \frac{i_2}{3} & W_{\text{non BPS}} = \sqrt{\frac{i_2 - i_1}{2}} \end{cases}$$

•Doubly critical

$$\partial^2_{Adj} I_4 = 0\,, \quad r = 1 : \quad \text{BPS} \quad i_1 = \lambda_1 = \lambda_2 = \lambda_3; \quad W_{\text{BPS}} = \sqrt{i_1}$$

AND...

ONE MORE THING

BH Technology Transfer: from one to many centres by U-duality

U-duality can take a long way in classifying physically distinct (extremal) 1-centre black holes, their orbits and attractors.

What can we infer for multi-centre?

- Need to give up spherical symmetry
- Stationary solutions
- Many charge vectors $\mathcal{Q}_a^M \equiv \left(p_a^\Lambda , q_{a\,\Lambda}\right) \qquad a = 1, \ldots, p, \quad , \quad M = 1, \ldots, \mathbf{f}$
- mutual non locality: $\mathcal{W} \equiv \langle \mathcal{Q}_1, \mathcal{Q}_2 \rangle = \frac{1}{2} \mathcal{Q}_a^M \, \mathcal{Q}_b^N \mathbb{C}_{MN} \epsilon^{ab}$
- Horizontal Symmetry $SL_h\left(p, \mathbb{R}\right)$
- More Invariants for groups of type E7 (and E6 in 5d) $I_{abcd} \qquad \mathbf{I}_6$
- example of Bossard+Ruef

BOSSARD + RUEF:

EXTREMAL SOLUTIONS OF STU IN $N=2$ $d=4$ (W FLAT 3d BASE)

	Q_a	Q_b	$\langle Q_a, Q_b \rangle$
BPS (Denef)	BPS $I_4 > 0$	BPS $I_4 > 0$	$\langle Q_a, Q_b \rangle = 0$
			$\langle Q_a, Q_b \rangle \neq 0$
ALMOST BPS (Goldstein + Katmadas)	BPS	non BPS	$\langle Q_a, Q_b \rangle \neq 0$
$\bar{D}6\,D4\,D2\,D0$	non BPS		$\langle Q_a, Q_b \rangle = 0$
INTERACTING COMPOSITE non BPS	non BPS $I_4 < 0$	non BPS $I_4 < 0$	$\langle Q_a, Q_b \rangle \neq 0$

3 NILPOTENT ORBITS (3 SETS OF EQUATIONS)

many centres SKG identities:

$$< Q_a, Q_b >= -2Im(Z_a \bar{Z}_b - g^{i\bar{\jmath}} D_i Z_a \bar{D}_{\bar{\jmath}} \bar{Z}_b)$$

$$Q_a^T \mathcal{M} Q_b = -2Re(Z_a Z_b + g^{i\bar{\jmath}} D_i Z_a \bar{D}_{\bar{\jmath}} \bar{Z}_b)$$

$$-\frac{1}{2} Q_a^T \mathcal{M} Q_b - \frac{i}{2} < Q_a, Q_b >= Z_a Z_b + g^{i\bar{\jmath}} D_i Z_a \bar{D}_{\bar{\jmath}} \bar{Z}_b$$

Summary

"Gedanken Black Holes" (B. Coppi, Dubna 2011)

Punchline:

1)Super-Gedanken Black Holes behave very similarly to Gedanken Black Holes

2)Both arise as solutions of first order flow equations

3)Their masses and entropies can be determined on the basis of symmetries alone

Results

i) Extremal BH solutions of extended SG have attractor behaviour even without supersymmetry

ii) They are associated to 1st order flow equations {easier to find full solutions by harmonic functions} driven by **W** which gives entropy at horizon and ADM mass at infinity

iii) U-Duality constrains **W** and allows for a classification of orbits of charge vector Q in terms of invariants, characterizing different physical features

iv) Singular BH's (S=0) have a **W** and are interesting

N=8: 1/8,1/4,1/2 ; N=2: 1/2,0, W known

v) Multicentre solutions and non extremality are to be explored

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- 4d/5d connection can give many clues

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- Integrability: next seminar by P. Fre'

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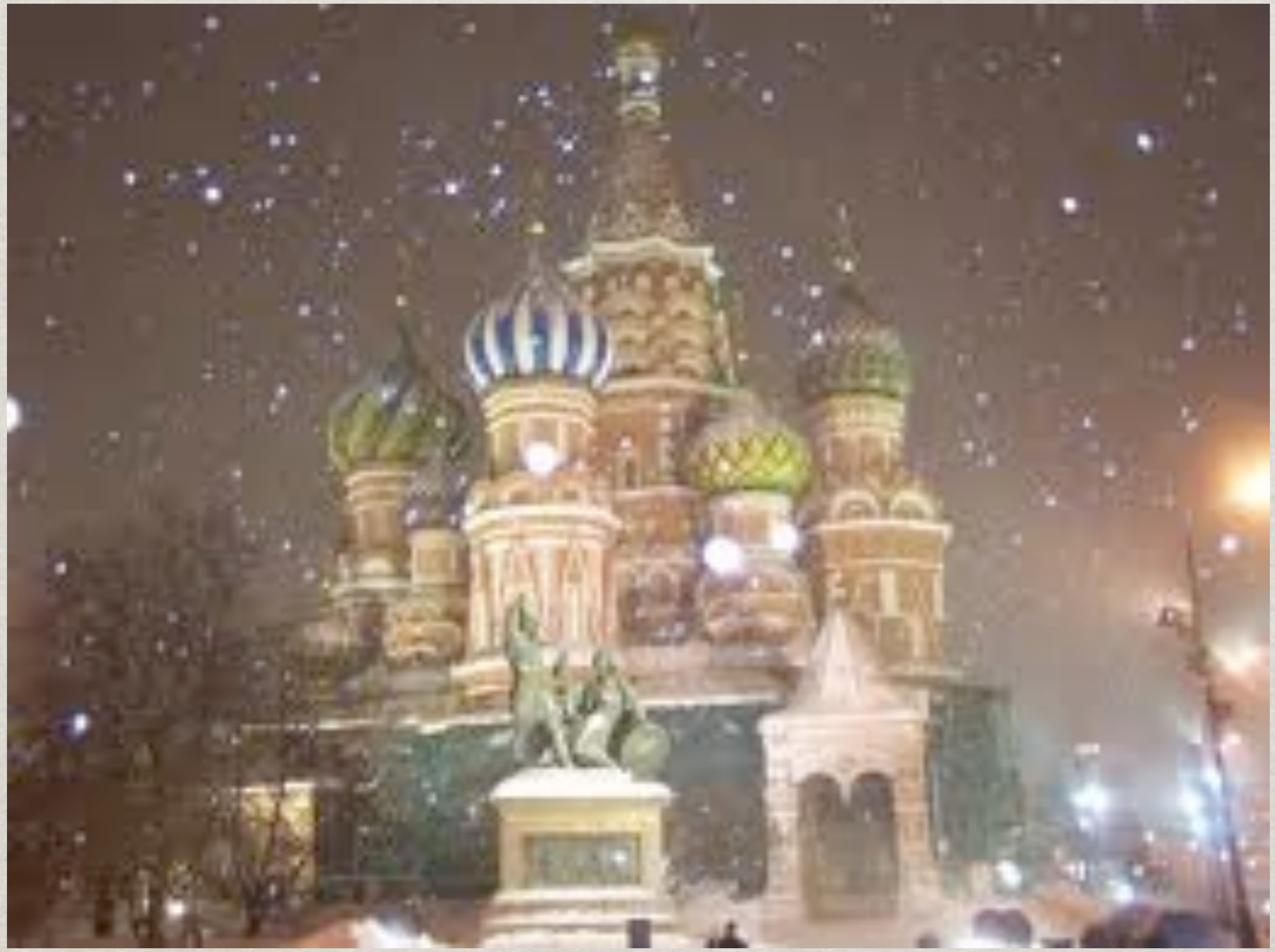

Thank you!!!