BIG-BANG, COLOR CONFINEMENT AND COMPUTER TECHNOLOGY

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BIG-BANG (STANDARD COSMOLOGICAL MODEL)

- At small times very high temperature, no composite particle, a plasma of elementary constituents : ν' s, leptons, quarks, vector bosons (γ, W, Z, Gluons), ..., expanding and cooling down. At time $t \approx 10^{-6} sec$ $T \approx 10 Gev$ quarks start binding into hadrons.
- When the reaction rate for this process equals the expansion rate of the universe $G_N^{\frac{1}{2}} T^2$ quarks should decouple and survive as relic particles.

$$
\begin{aligned}
n_q \sigma_0 &= G_N^{\frac{1}{2}} T^2; (\sigma^0 \equiv lim_{v \to 0}[v\sigma] \approx m_\pi^{-2}) \\
n_\gamma &\approx T^3; \frac{n_p}{n_\gamma} \approx 10^{-9}; T \approx 10 Gev \\
\frac{n_q}{n_\gamma} &= \frac{G_N^{\frac{1}{2}}}{T\sigma_0}; \frac{n_q}{n_p} \approx 10^{-12}
\end{aligned}
$$

BIG-BANG (STANDARD COSMOLOGICAL MODEL)2

If nothing special happens $n_q \approx 10^{-12} n_p$ is the expected abundance of quarks, or $\approx 10^{12}$ quarks / (gram of matter). Experiment: no quark seen in more than $1Kg$ of matter: $\frac{n_q}{n_p} \leq 10^{-27}$.

$\rightarrow$ Something happened in between. Inhibition factor $\approx 10^{-15}$

- Production of quarks in high energy collisions
  $\sigma_p \equiv \sigma(p + p \rightarrow q(\bar{q}) + X) \leq 10^{-40} cm^2$
  Expected $\sigma_p \approx 10^{-25} cm^2$. Again an inhibition by a factor 10^{-15}!
- Natural explanation : QUARK CONFINEMENT $\equiv$ $n_q = 0, \sigma_p = 0$ due to some symmetry. A phase transition occurred in the cosmic evolution at some T_c with a change of symmetry (ORDER-DISORDER TRANSITION).

DECONFINEMENT IN THE LAB.

- Heavy ion high energy collisions [CERN SpS, RHIC(BNL), ALICE (CERN LHC), GSI (DARMSTADT), NICA (DUBNA)]

- A hot spot is produced in the collision, at a temperature $T > T_c$, which then expands and cools down to hadrons.

- No smoking-gun signal of deconfinement found, but a number of interesting properties of hot hadron matter.

(DE)CONFINEMENT FROM FIRST PRINCIPLES: LATTICE

- Numerical simulations of QCD on a discretized Space-Time (LATTICE) : an approximant to the functional integral which defines the theory at zero and finite T. Approximations reasonably under control. An approach from first principles.
- Clear evidence found for a "deconfining" phase transition at $T_c \approx 170 Mev$, which confirms the cosmological argument. Can vary number of quarks, quark masses,... and study the nature of transition.
- Main question: is the transition ORDER-DISORDER ? If so what is the SYMMETRY involved, or what is the Mechanism of Confinement? In what follows I will focus on the first question.

FINITE TEMPERATURE QCD

- $T = 0$: Space-time N^4 lattice .
  Input parameters the quark mass m and the coupling constant $\beta = 2N_c/g^2$.
  Physical scale the correlation length $\frac{1}{\Lambda} \approx 1fm$. Lattice spacing $a(\beta) = \frac{1}{\Lambda}exp(-\frac{\beta}{b_0})$. Continuum limit $\beta \to \infty$ or $a(\beta) \ll \frac{1}{\Lambda}$.
  Lattice size $Na(\beta) \gg \frac{1}{\Lambda}$.
- $T \neq 0$ Lattice $N_t N_s^3$, $N_s \gg N_t$. Periodic B.C. in t for bosons, anti-periodic for fermions.[Feynmann 1949]
  $T = \frac{1}{N_t a(\beta)}$. T is controlled by varying β.
- The key quantity is the partition function
  $Z = \int \Pi dU_k(\vec{n}, n_t), d\psi(\vec{n}, n_t) d\bar{\psi}(\vec{n}, n_t) exp[-\beta A(U, \psi, \bar{\psi})]$
  an integral on $6N_s^3 N_t$ variables.

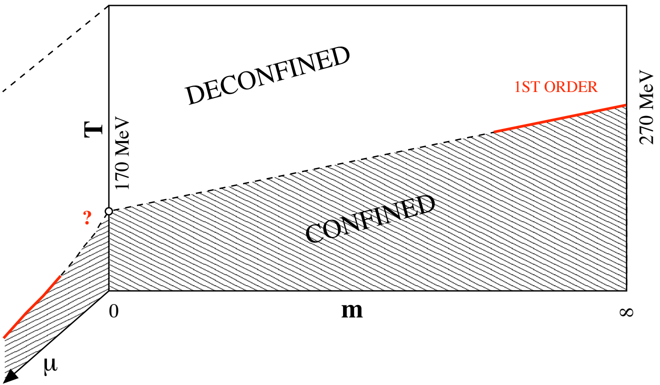
PHASE TRANSITIONS

- A phase transition is a discontinuity of a derivative of the free energy with respect to a thermodynamic variable, typically the temperature T.
  Order of the phase transition $\equiv$ order of the lowest diverging derivative. E.g. the specific heat for first order, corresponding to a discontinuity of heath content (a non zero latent heat).
  Cross-over $\equiv$ rapid, continuous change (∞ order)
  A change of symmetry can not be a cross-over.
- For a finite number of d.o.f. the partition function is analytic, and such is the free energy. Singularities only develop in the infinite volume limit [Lee-Yang 1952]. Dependence on the spacial size N_s identifies the order and the universality class of the transition (Finite-size scaling).

$N_f = 2$ QCD

- $N_f = 2$ QCD. Two quark flavors with equal mass m.
  At $T = 0$,$m \approx 0$ chiral symmetry and $U_A(1)$ broken.
  At some T_c chiral restoration expected, at some $T'_c \geq T_c$ $U_A(1)$ restoration.
- If chiral modes (π, σ) dominate, renormalization group and $(4 - \epsilon)$ arguments [Pisarski and Wilczek, 84] imply that
  1) either $T'_c = T_c$, and the transition is first-order, at $m = 0$ and also at small $m \neq 0$,
  2) or $T'_c > T_c$ and then the transition can be second order, in the universality class of $O(4)$. In this case at small $m \neq 0$ there is a cross-over and second-order end points at $m, \mu \neq 0$ [See figure 1]. No way to distinguish confined from deconfined.
  End point detectable in H.I.C. , but never observed. New experiments planned at RHIC and GSI.

FIG.1: $N_f = 2$ PHASE DIAGRAM



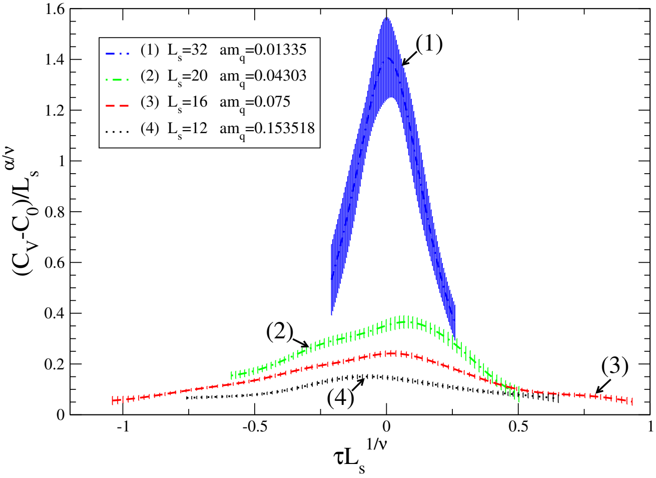
FINITE-SIZE SCALING(REN.GROUP)

- LOOKING FOR $O(4)$.
  Two scales , m and the correlation length, $\lambda \propto \tau^{-\nu}$, $\tau \equiv (1 - \frac{T}{T_c})$.

  $$C_V - C_V^0 \propto N_s^{\alpha} \Phi(\tau N_s^{\frac{1}{\nu}}, m N_s^{y_h})$$

  C_V^0 an UV subtraction constant, independent on m and N_s.
  α, ν, y_t critical indexes (anomalous dimensions) typical of the order and universality class of the transition.
  Second order $O(4)$: $\alpha = -.23, \frac{1}{\nu} = 1.34, y_h = 2.49$
  Weak first order : $\alpha = 1.00, \frac{1}{\nu} = 3.00, y_h = 3.00$
- Make simulations at different values of m and spatial sizes N_s, keeping the scaling variable $m N_s^{y_h}$ fixed. The peak of $C_V - C_V^0$ scales as N_s^{α}, the width as $N_s^{\frac{-1}{\nu}}$. If $O(4)$ is the universality class the curves at different $m's$ which should coincide appear as in Fig.2. $O(4)$ is definitely excluded, and with it the possibility that there is a cross-over at $m \neq 0$.

FIG.2 $O(4)$ SCALING:$\alpha = -.23, \frac{1}{\nu} = 1.34, mN_s^{2.49} = 74.7$



FINITE-SIZE SCALING(REN.GROUP)-2

- Fig.3 :same analysis with first order scaling.
- Keep $\tau N_s^{\frac{1}{\nu}}$ fixed. $O(4)$ at large volumes $C_V - C_V^0$ is analytic as $N_s \to \infty$

$$C_V - C_V^0 \propto m^{\frac{-\alpha}{\nu y_h}} \phi(\tau N_s^{\frac{1}{\nu}})$$

  For first order a singularity develops as $N_s \to \infty$

$$C_V - C_V^0 \propto m^{-1} \phi_1(\tau N_s^3) + N_s^3 \phi_2(\tau N_s^3)$$

  The second term is proportional to the latent heath and gives a peak diverging with the volume. The width of the peak shrinks as $\frac{1}{N_s^3}$
- Again $O(4)$ scaling is excluded (Fig.4). First order is consistent and must be very weak, since the second term is not clearly visible within errors : the width scales correctly.(Fig.5)

FIG.3 FIRST-ORDER SCALING: $\alpha = 1, \frac{1}{\nu} = 3, mN_s^3 = 437.45$

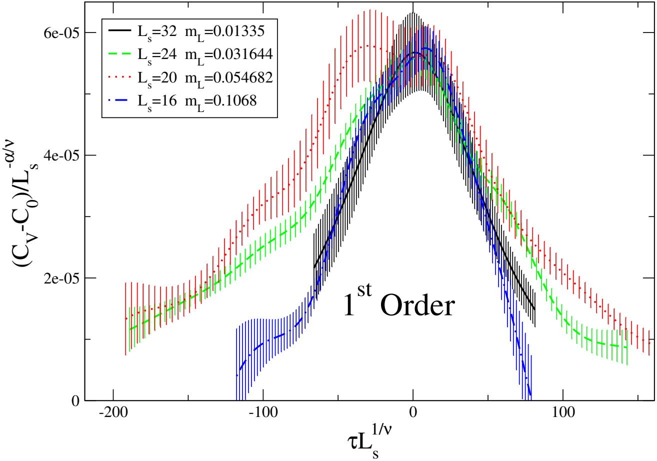


FIG.4 $O(4) : C_V - C_V^0 \propto m^{\frac{-\alpha}{\nu y_h}} \phi(\tau N_s^{\frac{1}{\nu}})$

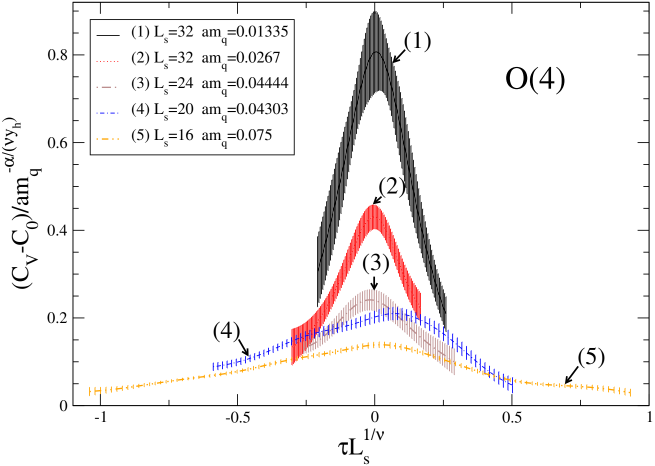
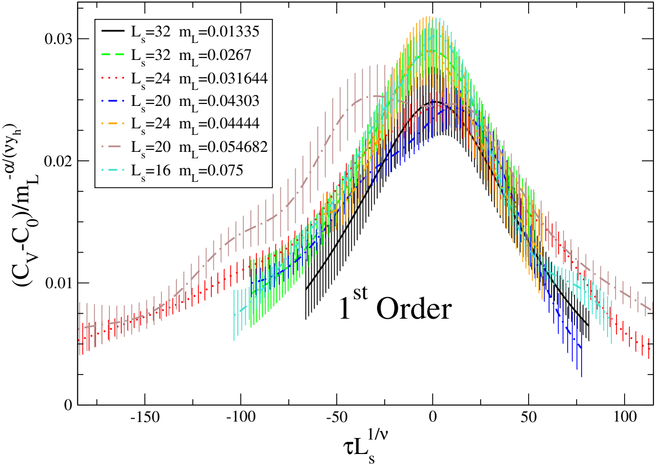


FIG.5 1st ORDER : $C_V - C_V^0 \propto m^{\frac{-\alpha}{\nu y_h}} \phi(\tau N_s^{\frac{1}{\nu}})$



STATE OF THE ART.

- No discontinuity observed within statistical errors.
- No way in principle to state by direct simulations on a lattice that a quantity is analytic (continuous with all derivatives) in the infinite volume limit. A weak first order and a crossover can be undistinguishable to all effects at volumes not big enough, except maybe for the scaling of the widths of the peaks. However they are very different in view of understanding confinement: with a crossover it is impossible in principle to define confined and deconfined.The usual statement that there is a crossover is meant in the sense that no jump is observed within errors.
- The only way would be to have a second order $O(4)$ scaling at the chiral point: then a general argument brings to a cross-over at $m \neq 0$ .But no evidence for $O(4)$.
- A numerical program to clarify the issue requires computer time of tens of teraflops$\times$ year.

COMPUTER TECHNOLOGY.

- All purpose computers comfortable but very expensive: developing to reach 1 petaflop (1000 teraflops)
- Dedicated computers much cheaper. INFN : APE, APE100, APE1000, APE-NEXT (≈ 1 teraflop). New developments on the way, available,maybe, in 2-3 years . Sizable investment needed in R and D.
- Big investment in graphical units(GPU) , for video-games, a big market. GPU a dedicated unit, very fast in parallel operations, like QCD simulations. USA Co. NVIDIA has developed a system consisting of a CPU and a GPU (TESLA), with the following strategy: the CPU runs the code and sends to the GPU the most demanding jobs, like the inversion of huge matrices, which run very fast. We have a set of such machines, 3 units for a total cost of ≈ 50KE. Some effort needed to adapt and optimize the codes. At present we can run at a total of 12 Teraflops. This helps us in waiting the next big APE and maybe will allow progress in our problem