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The alternative estimation of pion annihilation $\pi\pi \longrightarrow 2\gamma$ in vicinity of the chiral symmetry restoration

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- ✓ Confining separable DSE model
- ✓ U(3)$\otimes$ U(3) model with the 't Hooft interaction
- ✓ Conclusions

1 Confining separable Dyson-Schwinger equation model

Mesons can be described as $q\bar{q}$ bound states using the Bethe-Salpeter equation. In the ladder truncation, this equation reads

$$-\lambda(P^2)\Gamma(p,P) = \frac{4}{3}\int \frac{d^4q}{(2\pi)^4} D^{\rm eff}_{\mu\nu}(p-q)\gamma_\mu S(q_+)\Gamma(q,P)S(q_-)\gamma_\nu \ ,$$

where P is the total momentum, $q_\pm = q \pm P/2$,

$D^{\rm eff}_{\mu\nu}(k)$ an "effective gluon propagator".

The meson mass is identified from $\lambda(P^2 = -M^2) = 1$.

In conjunction with the rainbow truncation for the quark DSE

$$S(p)^{-1} \ = \ Z_2\, i\gamma\cdot p + Z_2\, m_0 + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2 D^{\rm eff}_{\mu\nu}(p-q)\gamma_\mu S(q)\gamma_\nu \, .$$

this equation forms the basis for the DSE approach to meson physics.

We consider here a simple separable interaction that has a finite range, accommodates quark confinement, and facilitates a decoupling of fermion Matsubara modes.

We base our approach on a confining separable model at $T = 0$ and defined by $D^{\rm eff}_{\mu\nu}(p-q) \rightarrow \delta_{\mu\nu}\, D(p^2, q^2, p\cdot q)$ with

$$D(p^2, q^2, p\cdot q) = D_0\ f_0(p^2)f_0(q^2) + D_1\ f_1(p^2)(p\cdot q)f_1(q^2)\ .$$

Here a Feynman-like gauge is chosen for phenomenological simplicity. This is a rank-2 interaction with two strength parameters D_0, D_1, and corresponding form factors $f_i(p^2)$.

The choice for these quantities is constrained by consideration of the resulting solution of the DSE for the quark propagator in the rainbow approximation. For the amplitudes defined by $S(p) = [i\not{p}A(p^2) + B(p^2) + m_0]^{-1}$ this produces

$$B(p^2) = \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{B(q^2) + m_0}{q^2A^2(q^2) + [B(q^2) + m_0]^2} ,$$

$$\left[A(p^2) - 1\right] p^2 = \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} D(p^2, q^2, p\cdot q) \frac{(p\cdot q)A(q^2)}{q^2A^2(q^2) + [B(q^2) + m_0]^2} .$$

The solution for $B(p^2)$ is determined only by the D_0 term, and the solution for $A(p^2) - 1$ is determined only by the D_1 term.

$$B(p^2) = b\ f_0(p^2) , \qquad A(p^2) = 1 + a\ f_1(p^2) ,$$

In the present separable model the strength $b = B(0)$, which is generated by solution of DSE and BSE, controls both confinement and dynamical chiral symmetry breaking. The propagator is confining if $m^2(p^2) \neq -p^2$ for real p^2 where the quark mass function is $m(p^2) = (B(p^2) + m_0)/A(p^2)$.

We use the exponential form factors (Model 1).

We also try to consider additional modification for form-factor $f_1(p^2)$. We represent it in the form (Model 2)

$$f_1(p^2) = \frac{N_1}{1 + \exp\left(\frac{p^2 - p_0^2}{\Lambda_1^2}\right)},$$

where N_1 is the normalization constant in order to have $f_1(0) = 1$ and equals

$$N_1 = 1 + \exp\left(-\frac{p_0^2}{\Lambda_1^2}\right).$$

	Experiment	model 1	model 2
- $\langle \bar{q}q \rangle^0$	$(0.236\,\mathrm{GeV})^3$	0.209	0.217
m_0	5 - 10 MeV	6.5	6.3
$m(p^2 = 0)$	$\sim$0.350 GeV	0.451	0.415
M_π	0.1385 GeV	0.140	0.140
f_π	0.093 GeV	0.92	0.92
M_σ	400-1200 MeV	722	667
$\Gamma_{\sigma \to \pi\pi}$	600-1000 MeV	418	345
Parameters			
a		0.622	0.554
Λ_0 , GeV		0.725	0.780
b , GeV		0.725	0.639
Λ_1 , GeV		1.7	1.69
p_0 , GeV			0.886

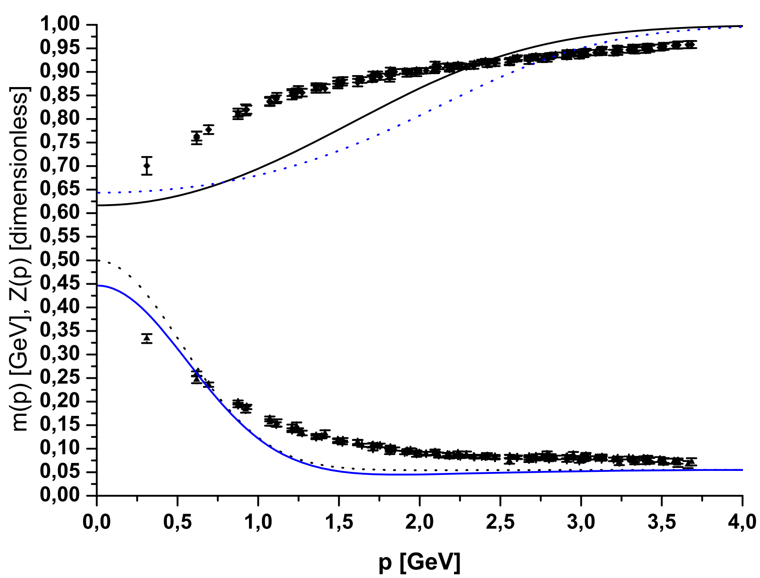


Figure 1: Momentum dependence of quark mass function $M(p)$ for the model 1 (doted lines) and the model 2 (solid lines) in comparison with lattice data. The value of current mass is taken 55 MeV.

The extension of the separable model studies to the finite temperature case, $T \neq 0$, is systematically accomplished by transcription of the Euclidean quark 4 - momentum via $q \to q_n = (\omega_n, \vec{q})$, where $\omega_n = (2n+1)\pi T$ are the discrete Matsubara frequencies. The effective $\bar{q}q$ interaction will automatically decrease with increasing T without the introduction of an explicit T-dependence which would require new parameters. We investigate the resulting behavior of the π and σ meson modes and decays in the presence of deconfinement and chiral restoration.

The result of the DSE solution for the dressed quark propagator now becomes

$$S^{-1}(p_n, T) = i\vec{\gamma}\cdot\vec{p}\, A(p_n^2, T) + i\gamma_4\omega_n\, C(p_n^2, T) + B(p_n^2, T) + m_0 \ ,$$

where $p_n^2 = \omega_n^2 + \vec{p}^2$ and there are now three amplitudes due to the loss of $O(4)$ symmetry. The solutions have the form $B = b(T) f_0(p_n^2)$, $A = 1 + a(T) f_1(p_n^2)$, and $C = 1 + c(T) f_1(p_n^2)$ and the DSE becomes a set of three non-linear equations for $b(T)$, $a(T)$ and $c(T)$. The explicit form is

$$\begin{aligned}
a(T) &= \frac{8D_1}{9}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_1(p_n^2)\, \vec{p}^2\, [1 + a(T) f_1(p_n^2)]\, d^{-1}(p_n^2, T) \ , \\
c(T) &= \frac{8D_1}{3}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_1(p_n^2)\, \omega_n^2\, [1 + c(T) f_1(p_n^2)]\, d^{-1}(p_n^2, T) \ , \\
b(T) &= \frac{16D_0}{3}\, T \sum_n \int \frac{d^3p}{(2\pi)^3}\, f_0(p_n^2)\, [m_0 + b(T) f_0(p_n^2)]\, d^{-1}(p_n^2, T) \ ,
\end{aligned}$$

where $d(p_n^2, T)$ is given by

$$d(p_n^2, T) = \vec{p}^2\, A^2(p_n^2, T) + \omega_n^2\, C^2(p_n^2, T) + [m_0 + B(p_n^2, T)]^2 \ .$$

Note that, at finite temperature, the strength $b(T)$ for the quark mass function will decrease with T so that this model can be expected to have a deconfinement transition at or before the chiral restoration transition associated with $b(T) \to 0$.

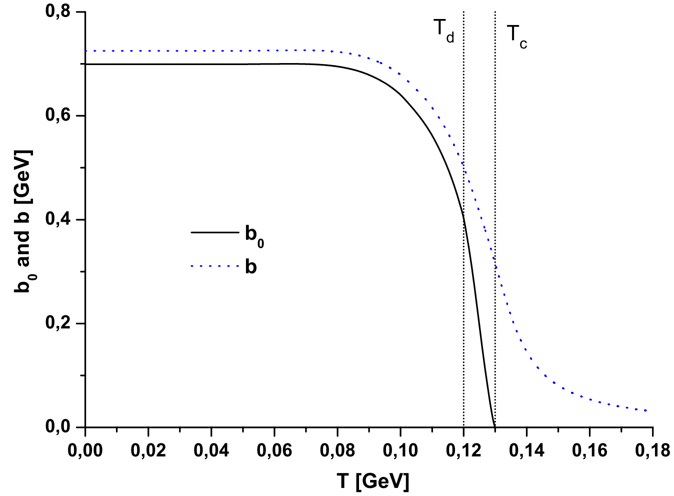


Figure 2: T-dependence of coefficients b_0, b.

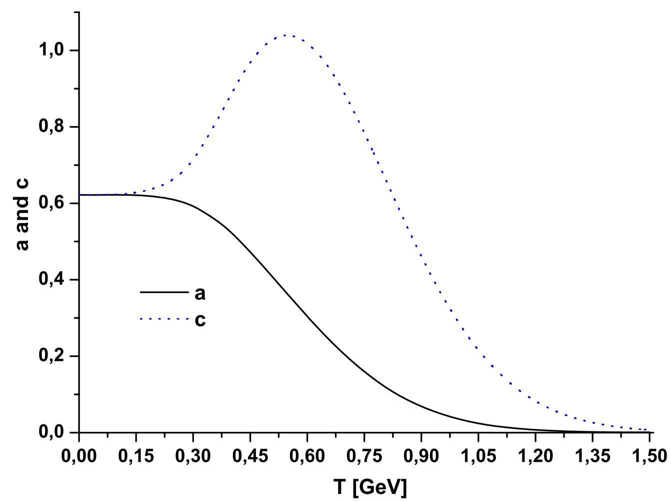


Figure 3: T-dependence of coefficients a, c.

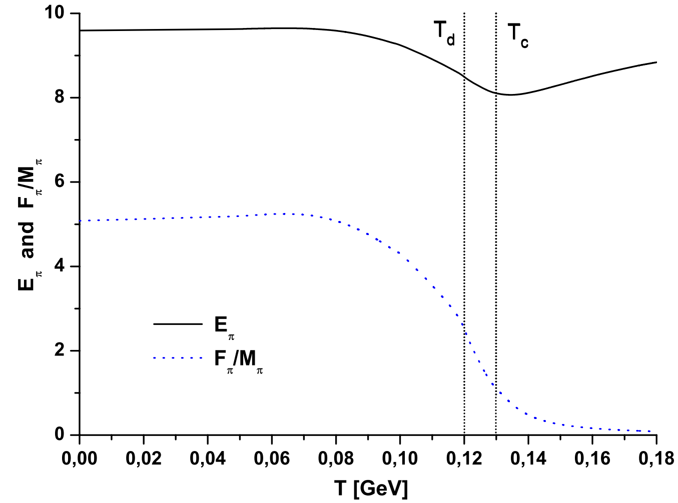


Figure 4: T-dependence of the π covariants .

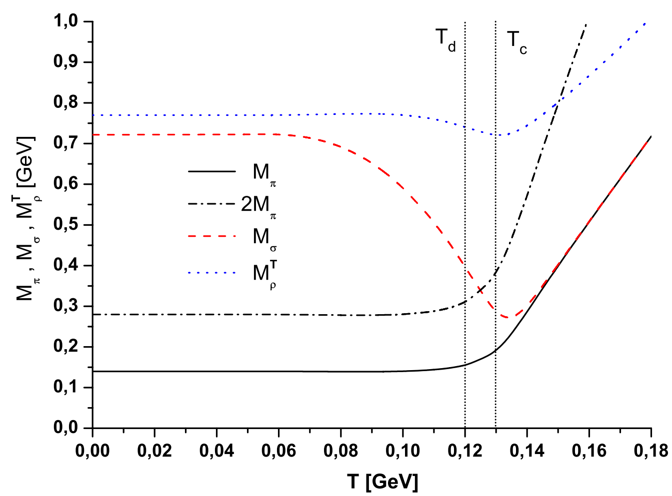


Figure 5: T-dependence of the π, σ masses.

2 U(3)⊗ U(3) model with the 't Hooft interaction

We use the three–flavor Nambu-Jona-Lasinio model, including the determinantal 't Hooft interaction that breaks the $U_A(1)$ symmetry, that has the following Lagrangian:

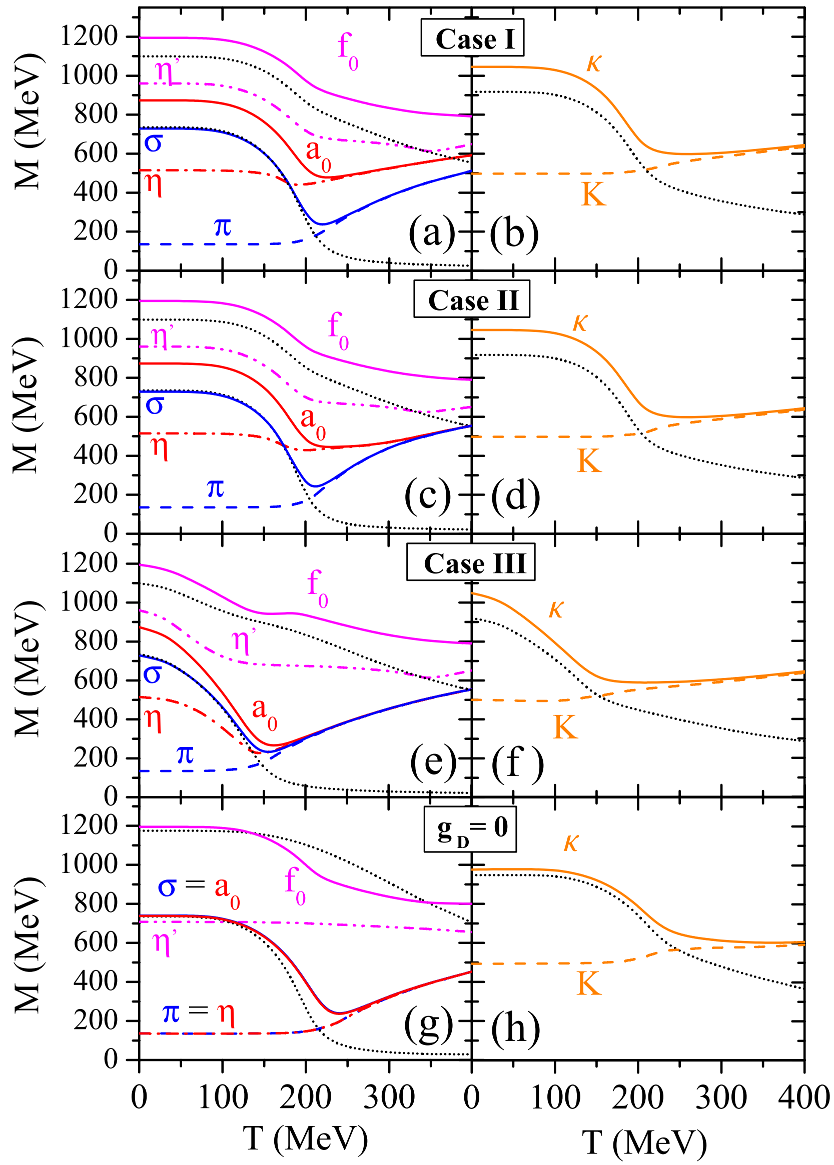
$$\begin{aligned}\mathcal{L} &= \bar{q}\,(\,i\,\gamma^{\mu}\,\partial_{\mu} - \hat{m})\,q + \frac{1}{2}\,g_S\,\sum_{a=0}^{8}[\,(\,\bar{q}\,\lambda^a\,q\,)^2 + (\,\bar{q}\,i\,\gamma_5\,\lambda^a\,q\,)^2\,] \\ &+ g_D\,\{\det[\bar{q}\,(1+\gamma_5)\,q] + \det[\bar{q}\,(1-\gamma_5)\,q]\}.\end{aligned}$$

Case I: the anomaly coefficient g_D is constant for all range of temperatures or densities.

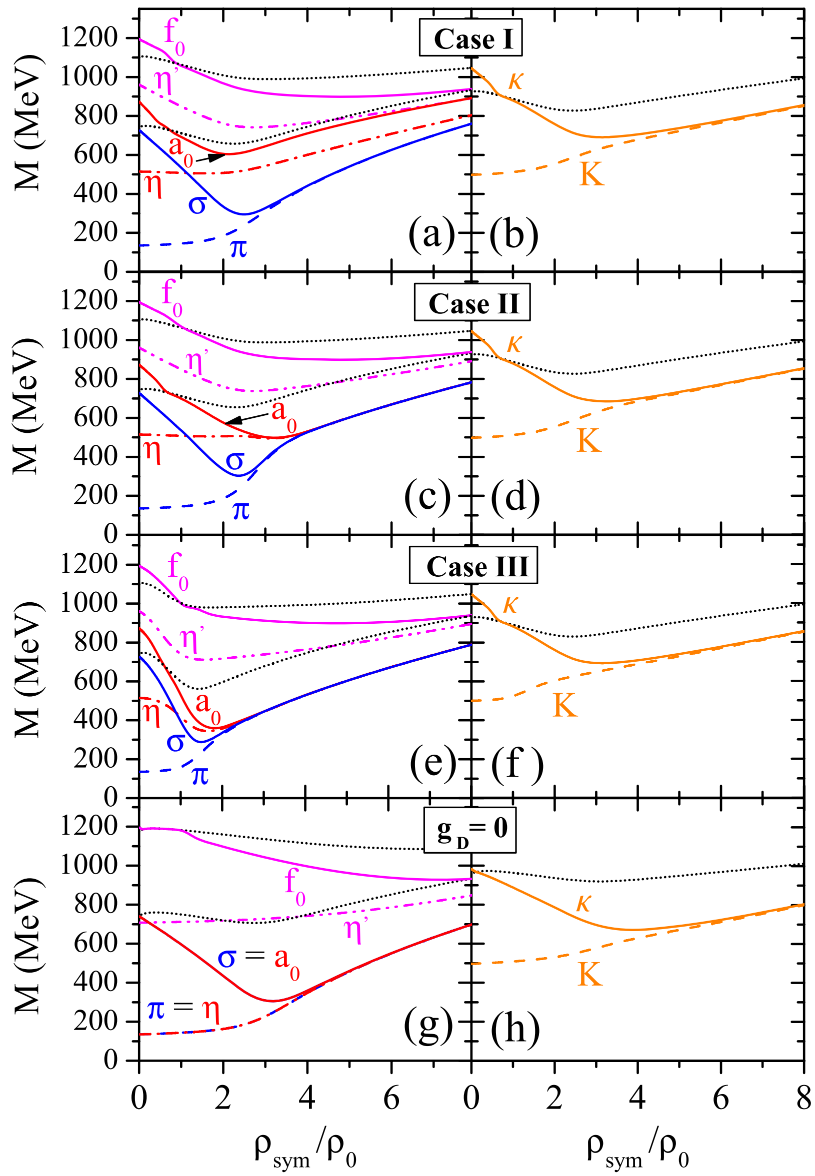
Case II: the anomaly coefficient g_D is a dropping function of temperature or density. The temperature dependence of g_D is extracted by making use of the lattice results for the topological susceptibility.

Case III: the anomaly coefficient has the form of a decreasing exponential ($g_D(T) = g_D(0)\exp[-(T/T_0)^2]$). This phenomenological pattern of restoration of the axial symmetry was proposed by Kunihiro in the framework of the present model. Here we consider a dependence of the anomalous coupling constant on density also inspired on the finite temperature scenario.

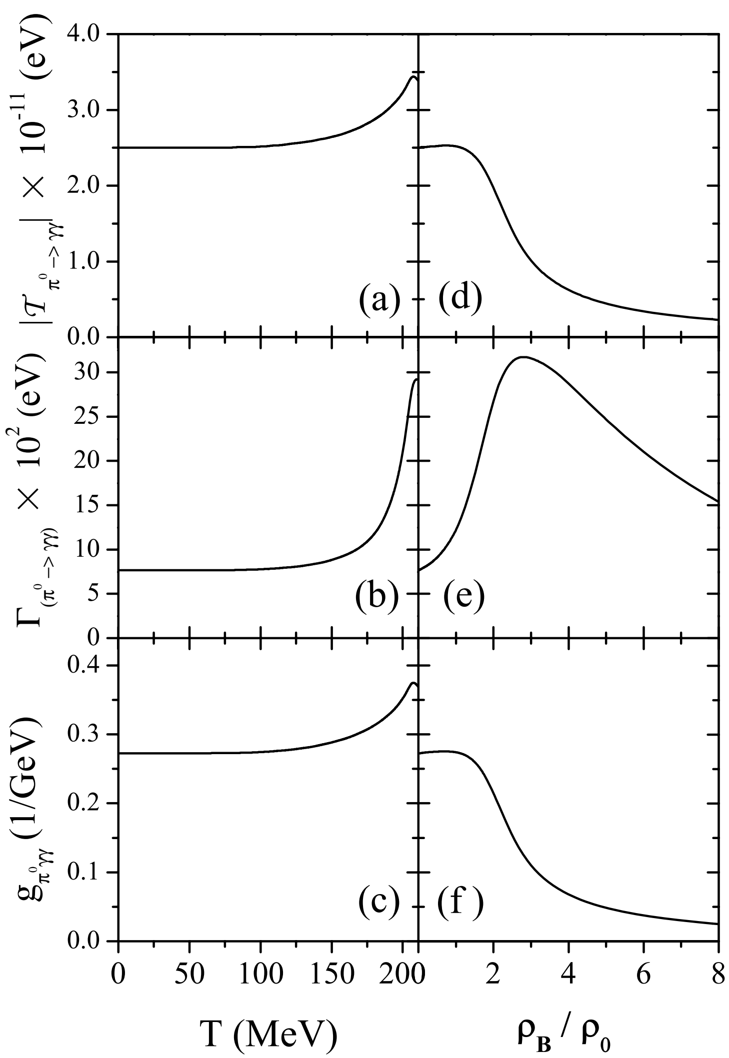
We also consider a simplistic scenario without $U_A(1)$ anomaly ($\mathbf{g_D = 0}$)



Case I
Case II
Case III
$g_D = 0$
M (MeV)
T (MeV)
f_0
η'
a_0
σ
η
π
κ
K
$\sigma = a_0$
$\pi = \eta$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)



Case I
Case II
Case III
$g_D = 0$
M (MeV)
ρ_{sym}/ρ_0
f_0
η'
a_0
η
σ
π
κ
K
$\sigma = a_0$
$\pi = \eta$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)



temperature (a), b) and c)), and as functions of the density (d), e) and f)).