PARTICLE AND SPIN MOTION IN POLARIZED MEDIA

A.J. Silenko

Institute of Nuclear Problems, Belarusian State University, Minsk, Belarus
E-mail: silenko@inp.minsk.by

Abstract

Quantum mechanical equations of motion are obtained for particles and spin in media with polarized electrons in the presence of external fields. The motion of electrons and their spins is governed by the exchange interaction, while the motion of positrons and their spins is governed by the annihilation interaction. The equations obtained describe the motion of particles and spin in both magnetic and nonmagnetic media. The evolution of positronium spin in polarized media is investigated. Media with polarized electrons can be used for polarization of positronium beams.

1 Introduction

The quantum mechanical description of the motion of particles and spin in matter is a very important problem. The classical theory of motion of particles and spin has been developed in great detail (see [1, 2])). A quantum mechanical equation of motion of relativistic particles in an electromagnetic field was derived by Derbenev and Kondratenko [3]. The motion of the spin of relativistic particles in an electromagnetic field is described by the Bargmann–Michel–Telegdi (BMT) equation [4]. The Lagrangian with an allowance for terms quadratic in spin was obtained in [5, 6]. The corresponding equation of spin motion was presented in [7]. The interaction between polarized particles and polarized matter was analyzed in [8].

In the present paper, we find quantum mechanical equations of motion of particles and spin for relativistic particles with arbitrary spin that move in media with polarized electrons in the presence of external fields. The system of units $\hbar = c = 1$ is used.

2 Hamiltonian for particles in polarized media

For particles with arbitrary spin, the Hamiltonian can be derived with the use of the interaction Lagrangian $\mathcal{L}$, obtained in [5, 6]. This Lagrangian contains terms that are linear ($\mathcal{L}_1$) and quadratic ($\mathcal{L}_2$) in spin:

$$
\begin{gathered}
\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2, \quad \mathcal{L}_1 = \frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)(\mathbf{S}\cdot\mathbf{B}) - (g-2)\frac{\gamma}{\gamma+1}(\mathbf{S}\cdot\mathbf{v})(\mathbf{v}\cdot\mathbf{B}) + \right.\\
\left.\left(g-2+\frac{2}{\gamma+1}\right)(\mathbf{S}\cdot[\mathbf{E}\times\mathbf{v}])\right\},\\
\mathcal{L}_2 = \frac{Q}{2s(2s-1)}\left[(\mathbf{S}\cdot\nabla) - \frac{\gamma}{\gamma+1}(\mathbf{S}\cdot\mathbf{v})(\mathbf{v}\cdot\nabla)\right]\left[(\mathbf{S}\cdot\mathbf{E}) - \frac{\gamma}{\gamma+1}(\mathbf{S}\cdot\mathbf{v})(\mathbf{v}\cdot\mathbf{E}) + \right.\\
\left.(\mathbf{S}\cdot[\mathbf{v}\times\mathbf{B}])\right] + \frac{e}{2m^2}\frac{\gamma}{\gamma+1}(\mathbf{S}\cdot[\mathbf{v}\times\nabla])\left[\left(g-1+\frac{1}{\gamma}\right)(\mathbf{S}\cdot\mathbf{B}) - \right.\\
\left.(g-1)\frac{\gamma}{\gamma+1}(\mathbf{S}\cdot\mathbf{v})(\mathbf{v}\cdot\mathbf{B}) + \left(g-\frac{\gamma}{\gamma+1}\right)(\mathbf{S}\cdot[\mathbf{E}\times\mathbf{v}])\right], \quad \gamma = \frac{1}{\sqrt{1-\mathbf{v}^2}},
\end{gathered}
\tag{1}
$$

where $g = 2\mu m/(eS)$, $\mathbf{v}$ is the velocity operator, γ is the Lorentz factor, Q is the quadrupole moment, and $\mathbf{S}$ is the spin operator. The Hermitian form of formula (1) is obtained by the substitution $\mathcal{L} \to (\mathcal{L} + \mathcal{L}^\dagger)/2$. The total Hamiltonian is given by

$$\mathcal{H} = \sqrt{m^2 + \boldsymbol{\pi}^2} + e\Phi - (\mathcal{L}_1 + \mathcal{L}_2), \tag{2}$$

where $\boldsymbol{\pi} = \gamma m\mathbf{v}$ is the operator of kinetic momentum and Φ is the potential of the electromagnetic field. We neglect the commutators of the operators of dynamic variables.

The polarization of the electrons of the medium does not change the form of Hamiltonian (2) if a beam contains neither electrons nor positrons. This is attributed to the fact that the average field acting on particles in the medium is characterized by the electric field strength $\mathbf{E}$ and the magnetic induction $\mathbf{B}$. However, the form of the Hamiltonian is changed if the beam consists of electrons or positrons. There is an exchange interaction between electrons, which is very strong.[1)] In the nonrelativistic case ($v \ll c$), the magnetic field for electrons should be replaced in (2) by the effective quasimagnetic field [8]

$$\mathbf{B} \to \mathbf{G}_e = \mathbf{B} + \mathbf{H}^c_{eff} + \mathbf{H}^m_{eff}, \quad \mathbf{H}^c_{eff} = -\frac{4\pi|e|N}{mv^2}\mathbf{P}, \quad \mathbf{H}^m_{eff} = \frac{2\pi|e|N}{m}(\mathbf{P}\cdot\mathbf{n})\mathbf{n}, \tag{3}$$

where N and $\mathbf{P} =< \boldsymbol{\sigma}' >$ are the polarization density and vector (average spin), respectively, of polarized matter electrons and $\mathbf{n} = \mathbf{v}/v$. The main contribution to the effective quasimagnetic field, $\mathbf{H}^c_{eff}$, is made by the Coulomb exchange interaction, or the Coulomb scattering. The exchange magnetic scattering yields the lesser contribution, $\mathbf{H}^m_{eff}$.

For nonrelativistic positrons in polarized media, the effective field with an allowance for the annihilation interaction, $\mathbf{H}^a_{eff}$, is determined by

$$\mathbf{G}_p = \mathbf{B} + \mathbf{H}^a_{eff} = \mathbf{B} - \frac{\pi|e|N}{m}\mathbf{P}. \tag{4}$$

Formulas (3) and (4) can be represented in a more compact form by introducing the magnetization vector (magnetic moment of a unit volume) $\mathbf{M}$:

$$\mathbf{G}_e = \mathbf{B} + \frac{8\pi}{v^2}\mathbf{M} - 4\pi(\mathbf{M}\cdot\mathbf{n})\mathbf{n}, \quad \mathbf{G}_p = \mathbf{B} + 2\pi\mathbf{M}, \quad \mathbf{M} = -\frac{|e|N}{2m}\mathbf{P}. \tag{5}$$

For isotropic magnetic materials, one can introduce a magnetic permeability μ_m:

$$\mathbf{G}_e = \mathbf{B} + \frac{2(\mu_m - 1)}{\mu_m v^2}\mathbf{B} - \frac{\mu_m - 1}{\mu_m}(\mathbf{B}\cdot\mathbf{n})\mathbf{n}, \quad \mathbf{G}_p = \frac{3\mu_m - 1}{2\mu_m}\mathbf{B}. \tag{6}$$

An appropriate expression for the Hamiltonian is expressed as ($\mathbf{G} = \mathbf{G}_e, \mathbf{G}_p$)

$$\mathcal{H} = \sqrt{m^2 + \boldsymbol{\pi}^2} + e\Phi + \frac{e}{2m}\Bigl\{g(\mathbf{S}\cdot\mathbf{G}) + (g-1)(\mathbf{S}\cdot[\mathbf{E}\times\mathbf{v}])\Bigr\}. \tag{7}$$

3 Equations of motion of particles and spin

The equation of particle motion in both polarized and unpolarized media is the same if the Hamiltonian remains unchanged.

[1)] Recall that the exchange interaction is responsible for the ferromagnetism.

For electrons and positrons, the equation of particle motion is given by

$$\frac{d\boldsymbol{\pi}}{dt} = e\mathbf{E} + e[\mathbf{v}\times\mathbf{B}] - \frac{e}{2m}\nabla\Big\{g(\mathbf{S}\cdot\mathbf{G}) + (g-1)(\mathbf{S}\cdot[\mathbf{E}\times\mathbf{v}])\Big\}. \tag{8}$$

For particles with arbitrary spin, the equation of spin motion with regard to the terms quadratic in spin is given in [7]. For nonrelativistic electrons and positrons, the equation of the spin motion takes the form

$$\frac{d\mathbf{S}}{dt} = \frac{e}{2m}\Big\{g[\mathbf{S}\times\mathbf{G}] + (g-1)\,[\mathbf{S}\times[\mathbf{E}\times\mathbf{v}]]\Big\}. \tag{9}$$

The effect of the exchange interaction on the spin motion is stronger than that on the particle motion. The equations can be used for dia-, para-, and ferromagnetic media.

4 Polarization of positronium beams by polarized media

Positronium beams can be polarized under passing through the polarized medium. Such a possibility takes place due to a dependence of the ortho-para-conversion (spin-conversion) rate on the polarization of the positronium.

Positronium atoms have spin 0 (para-positronium, p-Ps) or 1 (ortho-positronium, o-Ps). Only o-Ps atoms pass through the medium because p-Ps atoms annihilate very quickly. As a rule, the positronium energy does not exceed several eV. In the matter, the o-Ps lifetime can be shortened by several processes, namely, the pick-off annihilation, the ortho-para-conversion that is the spin-conversion, and chemical reactions [9].

The spin-conversion takes place in para- and ferromagnetic media, whose molecules contain unpaired electrons. In these media, the spin-conversion rate is generally much more than the rates of the pick-off annihilation and other processes. As particular, in the oxygen gas the spin-conversion rate is dozens of times larger than the pick-off annihilation one [10]. The same conclusion can be made from an analysis of experimental data on the spin-conversion in solutions of HTMPO [11].

We consider the simplified description of the positronium polarization process. For a more detailed description, the method and results obtained in Ref. [12] can be used.

Let all the unpaired electrons of the matter be polarized along the z-axis (Fig. 1). The spin exchange interaction can cause changing the o-Ps spin or its projection. This process can result in both the ortho-para-conversion and the prompt annihilation of the o-Ps.

However, the simple analysis shows that the ortho-para-conversion is only possible when the o-Ps spin projection onto the z-axis equals either -1 or 0 (see Figs. 1a,b). When it equals 1, flipping the spin is forbidden (see Fig. 1c).

The operator of the spin exchange interaction has the form

$$P = -J(1 + 4\mathbf{s}\cdot\mathbf{s}')/2, \tag{10}$$

where $\mathbf{s}$ and $\mathbf{s}'$ are the spin operators of the o-Ps electron and the matter electron, respectively, and J is the exchange integral that determines splitting energy levels. The operator P mixes the states of the o-Ps and p-Ps.

The o-Ps with $S_z = -1$ interacting with a matter electron ($s_z = 1/2$) is described by the spin wave function $|1,-1;1/2,1/2\rangle$. As a result of simultaneous flipping the spins of the o-Ps electron and the matter one, the o-Ps can convert into the p-Ps with the spin wave funstion $|0,0;1/2,-1/2\rangle$. This process is characterized by the matrix element

$$\langle 0,0;1/2,-1/2|P|1,-1;1/2,1/2\rangle = -J/\sqrt{2}.$$

Moreover, flipping the electron spins without the o-Ps→p-Ps conversion can also occur. The o-Ps spin becomes zero. The matrix element characterizing this process equals

$$\langle 1,0;1/2,-1/2|P|1,-1;1/2,1/2\rangle = -J/\sqrt{2}.$$

Analogous processes take place for the o-Ps with $S_z = 0$. The spin exchange interaction between the o-Ps electron and the matter one with and without the ortho-para-conversion is described by the matrix elements

$$\langle 0,0;1/2,1/2|P|1,0;1/2,1/2\rangle = -J/2 \quad \text{and} \quad \langle 1,1;1/2,-1/2|P|1,0;1/2,1/2\rangle = -J/\sqrt{2},$$

respectively. Coupled electrons do not contribute to the spin-conversion because matrix elements characterizing the exchange interaction between the o-Ps and the pair of coupled electrons are zero.

For the o-Ps with $S_z = 1$, all the corresponding matrix elements are zero. Therefore, the majority of o-Ps atoms passes through the medium without the annihilation. The pick-off annihilation is possible for the o-Ps atoms with any spin projection.

The o-Ps lifetime is changed by a magnetic field into the medium. This field can be strong enough. As is known, such a field does not change the lifetime of the o-Ps with $S_z = \pm 1$ and shortens the lifetime of the o-Ps with $S_z = 0$ (see Ref. [9]). This circumstance accelerates the process of polarization of the o-Ps beam. However, the evaluation shows such an acceleration is not very significant. For example, the pick-off annihilation shortens the o-Ps lifetime more greatly than the magnetic field in ferromagnetic media.

5 Discussion and conclusions

Magnetic crystals can be effectively used for the rotation of the polarization vector of particles. Even for neutrons, whose magnetic moment is relatively small, for $B \sim 1$ T, the angle of rotation of the polarization vector per unit length is of the order of $\Delta\Phi/\Delta l \sim (c/v) \times 10^{-3}$ rad/cm.

The rotation of the polarization vector in magnetic crystals reaches especially large values for nonrelativistic electrons. It follows from (6) and (7) that the angular velocity of the spin precession of nonrelativistic electrons is increased by the factor of $(c/v)^2$ due to the exchange interaction. For $B \sim 1$ T, we have $\Delta\Phi/\Delta l \sim (c/v)^3 \times 1$ rad/cm in order of magnitude. In particular, when $v/c \sim 0.1$, we have $\Delta\Phi/\Delta l \sim 10^3$ rad/cm.

The use of magnetic crystals may also be sufficiently effective for the rotation of the polarization vector of relativistic electrons.

The Stern–Gerlach force, which splits beams according to the polarization of particles, is considerably increased. However, the use of polarized media for splitting electron beams according to the polarization is seriously hampered by the small value of the energy of interaction between the spin of electrons and a quasimagnetic field $\mathcal{W}^{(s)}$ (of the order of 1 eV or less) and a multiple scattering that increases the transverse energy of electrons. If the transverse energy of electrons is greater than $|\mathcal{W}^{(s)}|$, splitting the beam according to the polarization becomes impossible.

The formulas obtained in this work are also valid for beams of polarized nuclei.

Media with polarized electrons can be used for the polarization of positronium beams. The spin direction of positroniums coincides with the spin direction of polarized electrons.

The considered effect of the polarization of positronium beams is more important for para- and ferromagnetic media without conductivity electrons, e.g., ferrites. The spin exchange interaction of o-Ps atoms with conductivity electrons leads to the spin-conversion that does not depend on the o-Ps polarization. As a result, a presence of conductivity electrons can cause a strong background making the effect unobservable.

It is important that the intensity of the positronium beam passing through two samples magnetized in different directions, depends on the angle ϕ between the magnetization axes. This effect is similar to passing the unpolarized light through two polarizers when their axes do not coincide.

Polarized positronium beams can be used for an investigation of magnetic media. The investigation can include monitoring the process of magnetization and determining the magnetic structure of materials.

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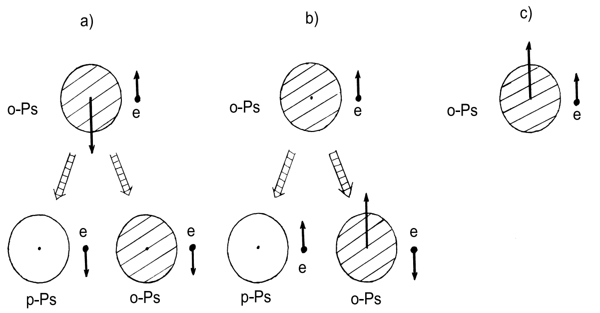

Figure 1. Spin-conversion of the o-Ps: a) the o-Ps (hatched) with the spin projection $S_z = -1$ can convert either into the p-Ps (white) or into the o-Ps with $S_z = 0$; b) the o-Ps with $S_z = 0$ can convert either into the p-Ps with $S_z = -1$ or into the o-Ps with $S_z = 1$; c) the o-Ps with $S_z = 1$ cannot convert.