Constraints on spin observables in $\bar{p}p \to \overline{\Lambda}\Lambda$

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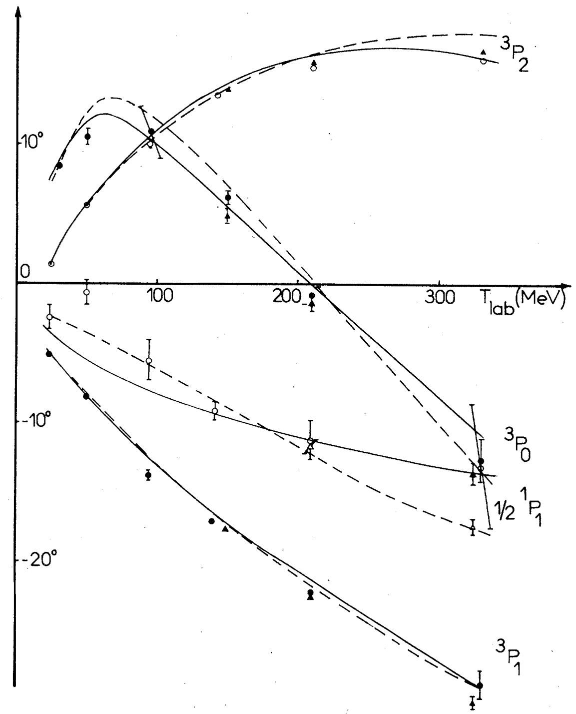
Outline

- Introduction
- Spin physics at LEAR
- Algebra of spin observables
- Checking PS185 data
- Outlook

Do we need spin observables?

Yes, with more support from community and committees.
Consider for instance NN scattering
Tensor ⇒ Spin-Orbit
⇒ Vector mesons (Breit)

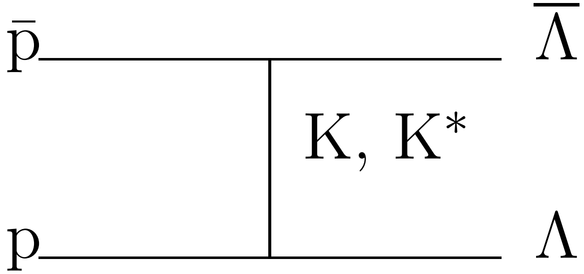


Spin physics at LEAR

- Antiproton–nucleus inelastic scattering $\bar{\mathrm{p}}\mathrm{A} \to \bar{\mathrm{p}}\mathrm{A}^*$ and antiprotonic atoms
- Antiproton–carbon analysing power,
- Antiproton–proton analysing power,
- Charge-exchange analysing power and D_{nn}
- Strangeness exchange $\bar{\mathrm{p}}\mathrm{p} \to \overline{\mathrm{Y}}\mathrm{Y}$

Aims of the $\bar{p}p \to \overline{Y}Y$ experiment (PS185)

- Study how strangeness is produced



- Kaon exchange
- Quark mechanisms

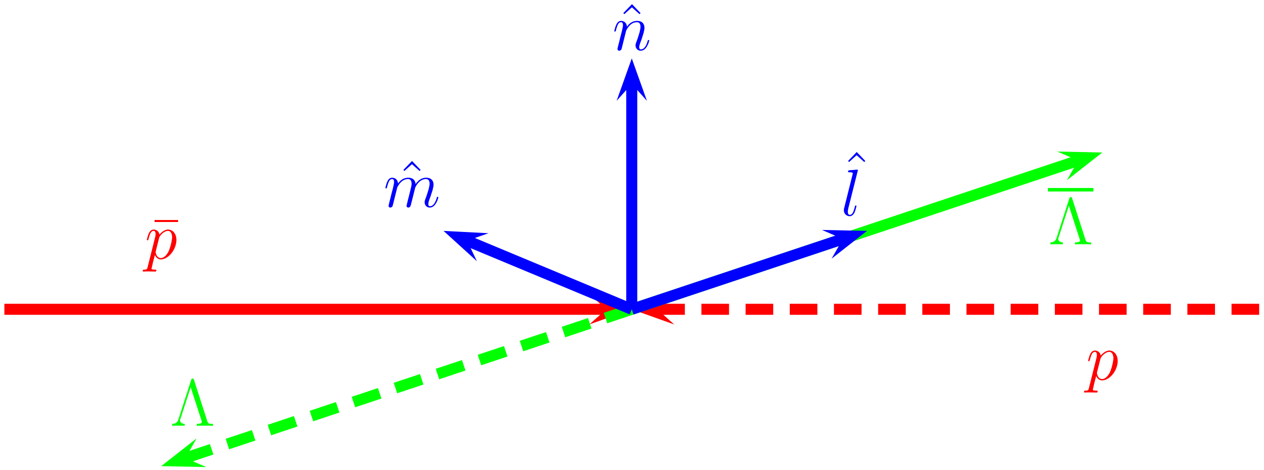
How many spin observables do we need?

- Not enough $\Rightarrow$ ambiguities
- Too many $\Rightarrow$ consistency problems
- πN example, $I = |a|^2 + |b|^2$,

  $$A = (|a|^2 - |b|^2)/I \; , B = 2\,\Re\mathrm{e}[ab^*]/I \; , C = 2\,\Im\mathrm{m}[ab^*]/I \; .$$

  A priori A and B and sign of C.
  But $A = 1 \;\Rightarrow\; B = C = 0$
- Anyhow, one should check $A^2 + B^2 + C^2 = 1$ for these possibly independent measurements.
- More involved for $\bar{\mathrm{p}}\mathrm{p} \to \overline{\Lambda}\Lambda$, with more amplitudes and many more observables.

Formalism: notations



$$\mathcal{M} = (a+b)I + (a-b)\,\sigma_1.\hat{n}\,\sigma_2.\hat{n} + (c+d)\,\sigma_1.\hat{m}\,\sigma_2.\hat{m}$$
$$+ (c-d)\,\sigma_1.\hat{l}\,\sigma_2.\hat{l} + e\,(\sigma_1+\sigma_2).\hat{n} + g\,(\sigma_1.\hat{l}\,\sigma_2.\hat{m} + \sigma_1.\hat{m}\,\sigma_2.\hat{l})\ ,$$

Formalism

- Amplitudes

$$\mathcal{M} = (a+b)I + (a-b)\,\sigma_1.\hat{n}\,\sigma_2.\hat{n} + (c+d)\,\sigma_1.\hat{m}\,\sigma_2.\hat{m}$$
$$+\,(c-d)\,\sigma_1.\hat{l}\,\sigma_2.\hat{l} + e\,(\sigma_1+\sigma_2).\hat{n} + g\,(\sigma_1.\hat{l}\,\sigma_2.\hat{m} + \sigma_1.\hat{m}\,\sigma_2.\hat{l})\ ,$$

- Observables

$$I_0 = \mathrm{Tr}[\mathcal{M}\mathcal{M}^\dagger]\ , \qquad P_n I_0 = \mathrm{Tr}[\sigma_1.\hat{n}\mathcal{M}\mathcal{M}^\dagger]\ ,$$
$$A_n I_0 = \mathrm{Tr}[\mathcal{M}\sigma_2.\hat{n}\mathcal{M}^\dagger]\ , \qquad C_{ij} I_0 = \mathrm{Tr}[\sigma_1.\hat{\imath}\,\sigma_2.\hat{\jmath}\mathcal{M}\mathcal{M}^\dagger]\ ,$$
$$D_{ij} I_0 = \mathrm{Tr}[\sigma_2.\hat{\imath}\mathcal{M}\sigma_2.\hat{\jmath}\mathcal{M}^\dagger]\ , \qquad K_{ij} I_0 = \mathrm{Tr}[\sigma_1.\hat{\imath}\mathcal{M}\sigma_2.\hat{\jmath}\mathcal{M}^\dagger]\ ,$$

$$C_{0\alpha ij} I_0 = \mathrm{Tr}[\sigma_1.\hat{\imath}\,\sigma_2.\hat{\jmath}\,\mathcal{M}\,\sigma_2.\hat{\alpha}\,\mathcal{M}^\dagger]\ .$$

Formalism (cont.)

$$I_0 = |a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |g|^2 \ ,$$

$$P_n I_0 = 2\,\Re\mathrm{e}(ae^*) + 2\,\Im\mathrm{m}(dg^*) \ ,$$

$$A_n I_0 = 2\,\Re\mathrm{e}(ae^*) - 2\,\Im\mathrm{m}(dg^*),$$

$$C_{nn} I_0 = |a|^2 - |b|^2 - |c|^2 + |d|^2 + |e|^2 + |g|^2 \ ,$$

$$C_{mm} I_0 = 2\,\Re\mathrm{e}(ad^* + bc^*) + 2\,\Im\mathrm{m}(ge^*) \ ,$$

$$C_{ll} I_0 = 2\,\Re\mathrm{e}(-ad^* + bc^*) - 2\,\Im\mathrm{m}(ge^*) \ ,$$

$$C_{ml} I_0 = 2\,\Re\mathrm{e}(ag^*) + 2\,\Im\mathrm{m}(ed^*) \ ,$$

$$D_{nn} I_0 = |a|^2 + |b|^2 - |c|^2 - |d|^2 + |e|^2 - |g|^2 \ ,$$

$$D_{mm} I_0 = 2\,\Re\mathrm{e}(ab^* + cd^*) \ ,$$

Formalism (end)

$$
\begin{aligned}
D_{ml} I_0 &= 2\,\Re\mathrm{e}(cg^*) - 2\,\Im\mathrm{m}(be^*)\ ,\\
K_{nn} I_0 &= |a|^2 - |b|^2 + |c|^2 - |d|^2 + |e|^2 - |g|^2\ ,\\
K_{mm} I_0 &= 2\,\Re\mathrm{e}(ac^* + bd^*)\ ,\\
K_{ml} I_0 &= 2\,\Re\mathrm{e}(bg^*) + 2\,\Im\mathrm{m}(ec^*)\ ,\\
C_{0nmm} I_0 &= \quad 2\,\Re\mathrm{e}(de^*) + 2\,\Im\mathrm{m}(ag^*)\ ,\\
C_{0nlm} I_0 &= \quad 2\,\Re\mathrm{e}(ge^*) - 2\,\Im\mathrm{m}(-a^*d + b^*c)\ ,\\
C_{0nml} I_0 &= \quad 2\,\Re\mathrm{e}(ge^*) - 2\,\Im\mathrm{m}(-a^*d - b^*c)\ ,
\end{aligned}
$$

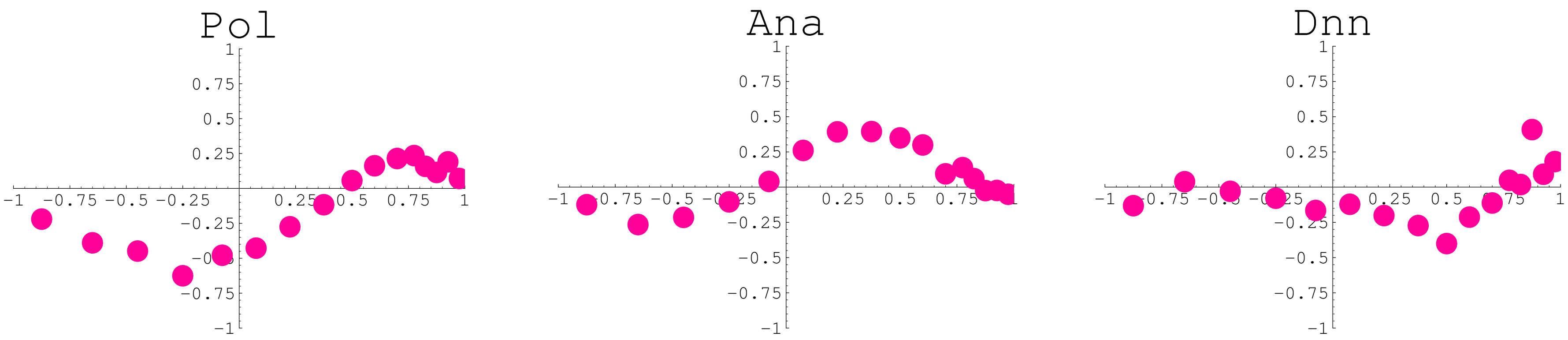
Deriving inequalities on observables

- From identities analogous to $A^2 + B^2 + C^2 = 1$ for πN, one derives $A^2 + B^2 < 1$, etc.
- Empirical but systematic study of pairs and triplets of observables.
- Generalised density matrix. Incoming and outgoing particles in a big density matrix, whose positivity gives a number of inequalities.

Results on single observables

By construction (e.g., fraction on polarisation along an axis) or from their expression such as $\mathcal{O}_i = (|a|^2 - |b|^2)/(|a|^2 + |b|^2)$, one has (trivially verified):

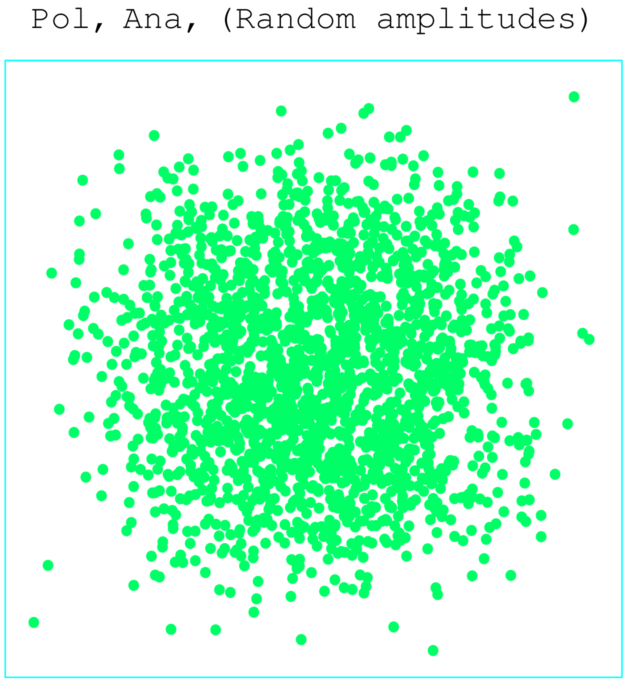
$$-1 \le \mathcal{O}_i \le +1 \, .$$

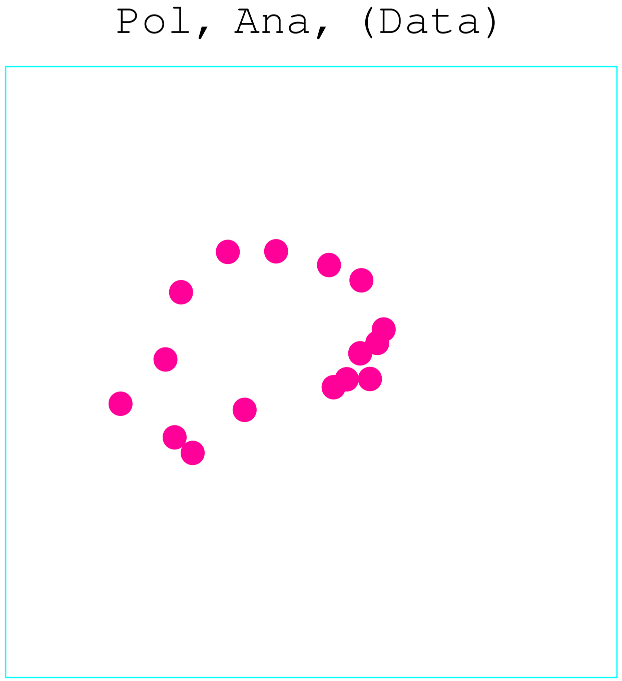


Results on pairs of observables-1

Three type of behaviour are identified:

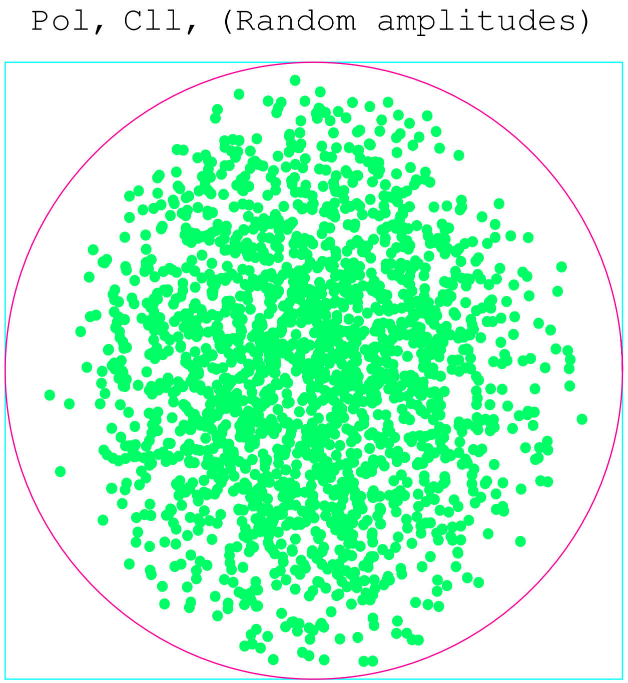
- Nothing: the entire square $\{-1 \leq x, y \leq +1\}$ is allowed

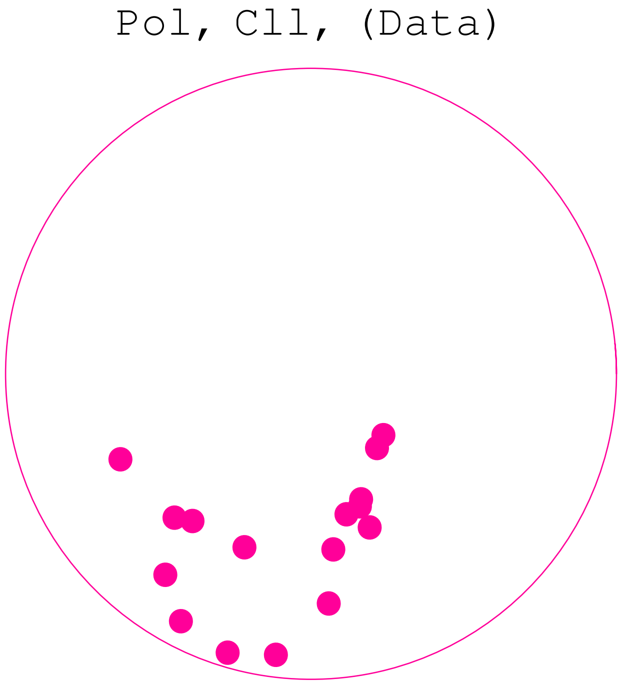




Results on pairs of observables-2

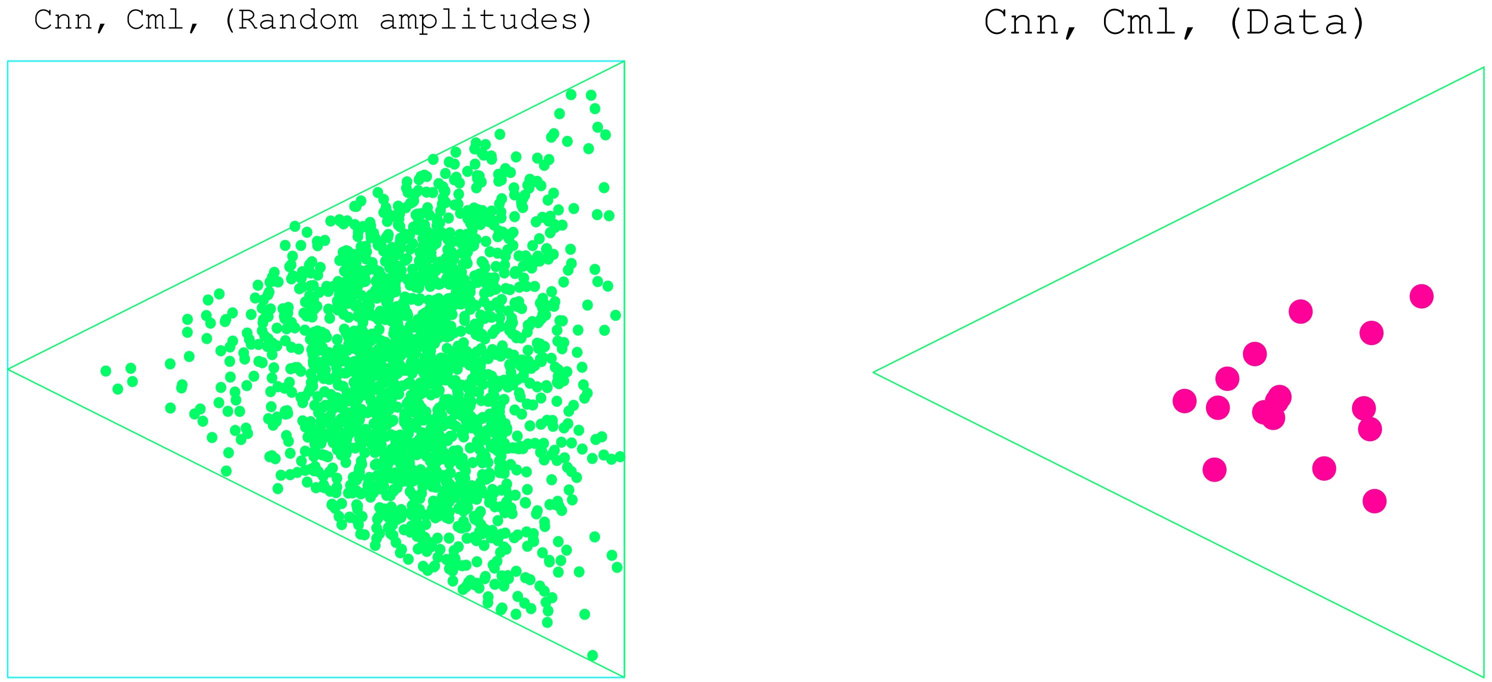
- Unit disk: $x^2 + y^2 \leq 1$



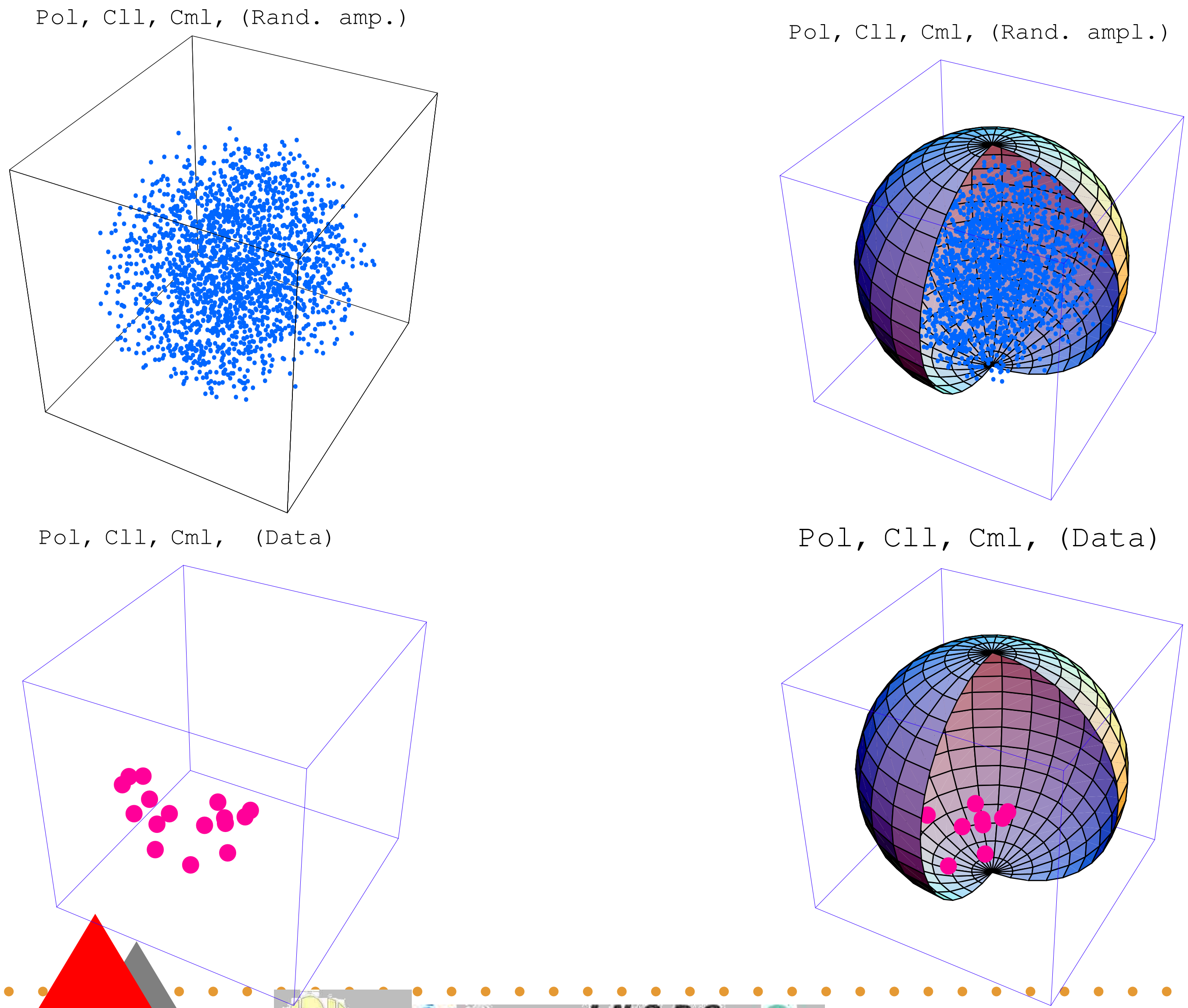


Results on pairs of observables-3

- Triangle: $x^2 - (\pm y + 1)^2 \leq 0$ or $x \leftrightarrow y$

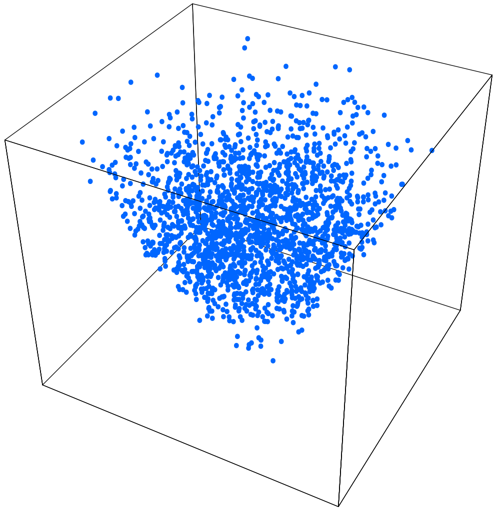


Triplets: 1. spherical constraint

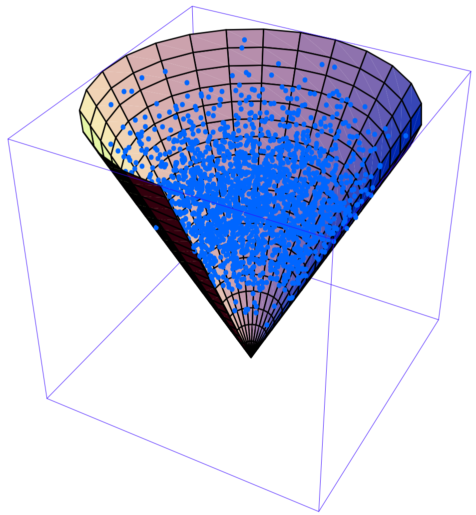


Triplets: 2. Cone

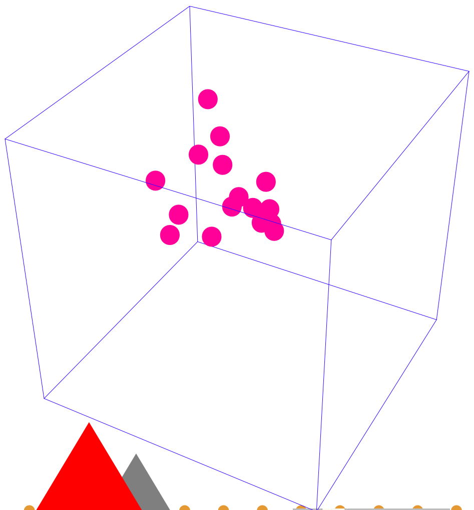
Pol, Cml, Cnn, (Rand. amp.)



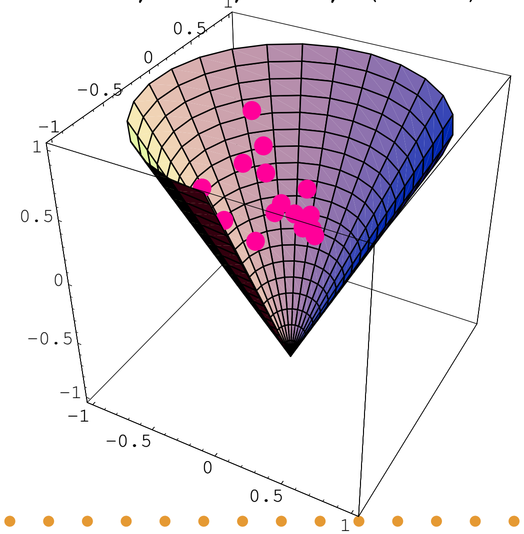
Pol, Cml, Cnn, (Rand. ampl.)



Pol, Cml, Cnn, (Data)

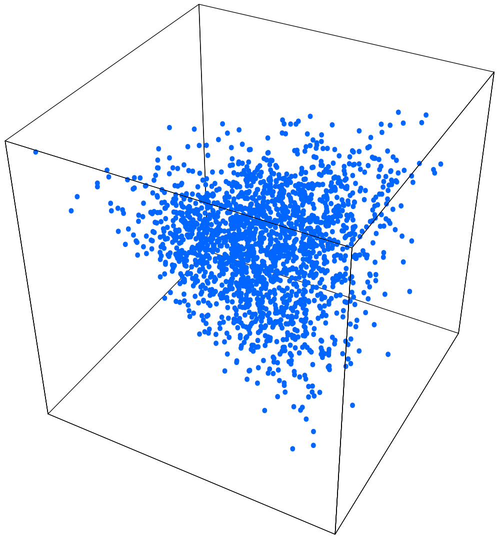


Pol, Cml, Cnn, (Data)

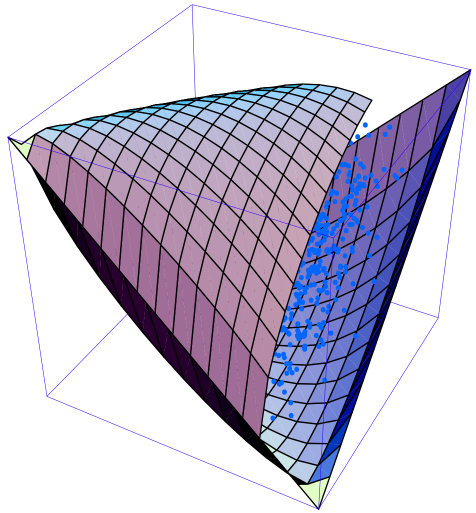


Triplets: 3. Cubic

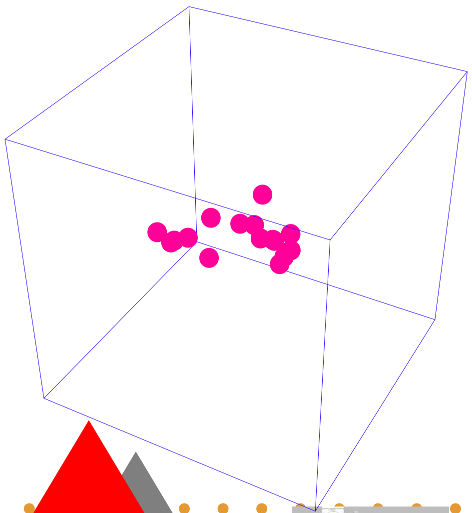
Pol, Ana, Dnn, (Rand. amp.)



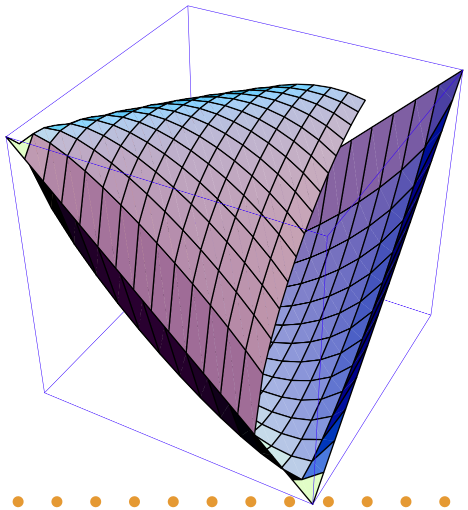
Pol, Ana, Dnn, (Rand. ampl.)



Pol, Ana, Dnn, (Data)



Pol, Ana, Dnn, (Data)



Cubic constraint: comments

This latter case is the most interesting. There is no restriction for any pair of observables, but a non-trivial constraint for the three of them.
It reads

$$x^2 + y^2 + z^2 \pm 2xyz \leq 1 \ .$$

Outlook

- Beautiful measurements at LEAR, in particular for strangeness exchange.
- Data look consistent so far. Model-independent tests successfully passed. And indeed, a reconstruction of the amplitudes (up to an overall phase) has been announced.
- Methods rediscovered and further developed here. Applicable to other reactions. Work in progress.
- Thanks for the organisers of this conference.