# BOSE-EINSTEIN CORRELATIONS

as

# CORRELATIONS OF FLUCTUATIONS


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- From Poisson to Bose-Einstein distributions

  $\Rightarrow$ how to model (the presence of) bosonic particles ?

- Our proposition of numerical modelling of BEC - $\Rightarrow$ its ups and downs ...


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Acta Phys. Polon. B33 (2002) 2681

# NOTICE:

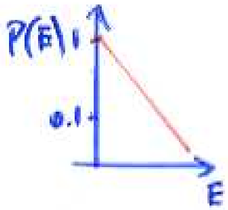
whatever one is doing it always means the following:

- One is trying *(consciously or not...)* to replace the original Boltzmann character of the particles provided by a given MCEG by their Bose-Einstein couterpart
- How?
- By:
  - artificially correlating the like-particles
    $\Longrightarrow$ mimicking their bunching in the momentum cells in phase space
  - enhancing events which (by chance!) are already showing some traces of BE statistics (i.e., bunching effect)
- Can it be done already in the MCEG itself?
- Yes, it can ! [7]

$\Longrightarrow$ $\Longrightarrow$ *example* $\Longrightarrow$ $\Longrightarrow$

[7] T.Osada, M.Maruyama and F.Takagi, *Phys. Rev.* D59, 014024 (1999); M.Biyajima, N.Suzuki, G.Wilk and Z.Włodarczyk, *Phys. Lett.* B386, 297 (1996).

- To model Boltzmann statistics: (*)




  - ⋆ choose particles with energies $E_i$ according to

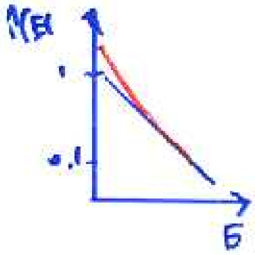
$$P(E_i) = \exp(-E_i/T)$$

  - ⋆ $\Longrightarrow$ Poissonian distribution of particles:

$$P(N) = \frac{<N>^N}{N!} e^{-<N>}$$

- To model Bose-Einstein statistics:

  - ⋆ out of some initial mass $M$ choose first particle with energy $E_1$ according to $P(E)$
  - ⋆ add to it with some probability $P_{BE}$ ([redacted]) other particles with energies $E_1$ - untill the first failure, when it happens
  - ⋆ choose second particle with $E_2$ etc ....
  - ⋆ repeat the whole procedure untill the whole initial mass $M$ is used
  - ⋆ $\Longrightarrow$ one gets a number $k$ of cells: $E_1, \dots, E_k$, with $N_i$ particles in each cell distributed according to geometrical distribution:




$$P(N_j) = \frac{1}{1+<N_j>}\left(\frac{<N_j>}{1+<N_j>}\right)^{N_j}$$

($<N_{j=1,\dots,k}>$ is mean multiplicity in the $j^{th}$ cell)



(*) Analytical discussion: K. Zalewski, Acta Phys. Polon. B33 (2002) 2643
and foundation and ref. therein

⋆ Notice that $\Longrightarrow$

- for the total $N = \Sigma_{j=1}^{k} N_j$ particles obtained in this way from mass $M$ and located in $k$ cells one gets Negative Binomial (NB) distribution

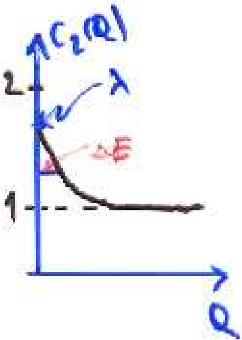
$$P(N;k) = \binom{N+k-1}{N} \frac{\left(\frac{<N>}{k}\right)^N}{\left(1+\frac{<N>}{k}\right)^{N+k}}$$

$$\stackrel{k\to\infty}{\Longrightarrow} \qquad P(N) = \frac{<N>^N}{N!} e^{<N>}$$

($one\ particle\ per\ cell = Boltzmann\ statistics$)

However!

- To get characteristic shape of $C_2(Q = |p_1 - p_2|)$ one has to allow for *smearing of the values of $E_i$ accepted in a given elementary cell* (by $\Delta E$) $\Longrightarrow$




⋆ the value of smearing translates to the width of $C_2(Q)$ (i.e., to the "radius" parameter $R$ in the fitting formulae [8] $C_2(Q) = 1 + \lambda \cdot \exp(-R \cdot Q)$)

⋆ the *number* of such cells influences $\lambda$ [9].

[8] The full-fledged MCEG based on this approach has been proposed and discussed in: T.Osada, M.Maruyama and F.Takagi, *Phys. Rev.* D59, 014024 (1999).

[9] This was first noted in paper by M.Biyajima, N.Suzuki, G.Wilk and Z.Włodarczyk, *Phys. Lett.* B386, 297 (1996).

For recent analysis see papers by K,G.Sarkisyan et al.: *Phys. Lett.* B487 (2000) 215 and B523 (2001) 35 (OPAL Coll. data); *Nucl. Phys.* B *(Proc. Suppl.)* 92 (2001) 75 and 211.

K. Zalewski: LNP 539 (2000) 291
(hep-ph/9810431)

(*) independent production (Boltzmann particles)
$\Rightarrow$ Poisson distribution

$$P(N) = \frac{\langle N\rangle^N}{N!} e^{-\langle N\rangle}$$

(*) symmetrization $\Rightarrow$ sum over the all permutations
$\Rightarrow$ it is equivalent to multiply the one particle density operator by $N!$

$\Rightarrow$ Poisson distribution goes over into a geometrical distribution

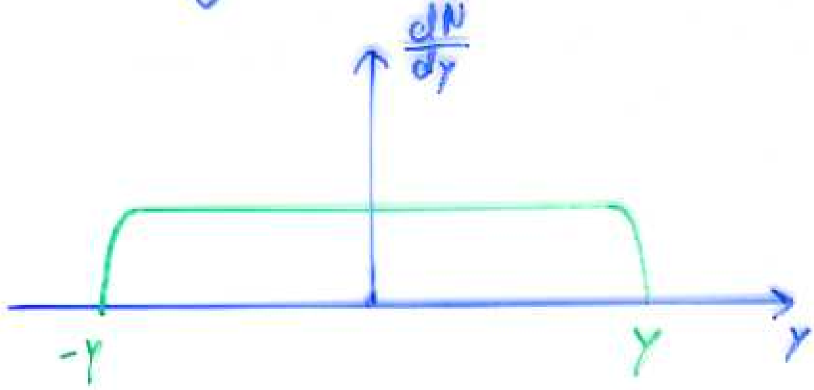
$$P(N) = \frac{1}{1+\langle N\rangle}\left[\frac{\langle N\rangle}{1+\langle N\rangle}\right]^N$$

This is the corner point of the method(s) proposed/discussed here and in OTM PR D59 (1999) 014024 ...

$\hookrightarrow$ complicated and tedious
only approximate conservation laws etc.

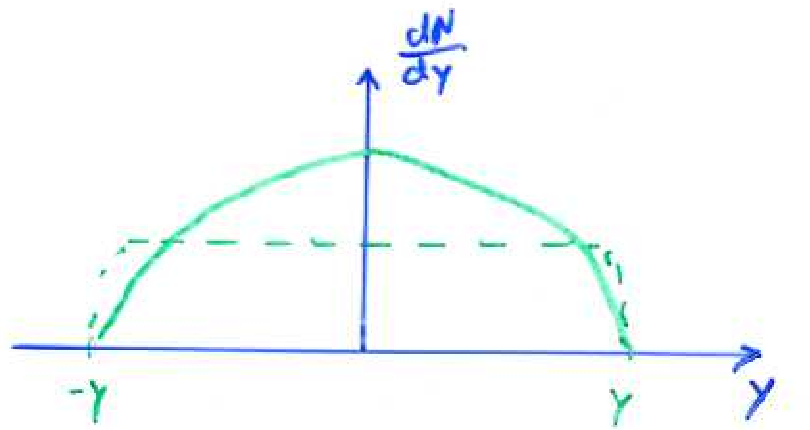
How "bosonization" acts?

⇓




← uncorrelated jet model for Boltzmann particles

⇓ (deGroot, Satz '70)




← the same model for Bose particles

⇒ the fact that produced particles are BOSONS apparently influences very much physical observables,

⇒ in a way sometimes "unexpected" at first sight...

Our proposition [14] for a general MCEG case (*Figure*):

- Resign *in each event!* from the original charge assignement provided by MCEG preserving, however, its:
  - number of particles with (+/-/0) charges
  - the energy-momentum and space-time structure

⟹ *in this way, by resigning from a part of information provided by MCEG, one introduces element of uncertainty into otherwise a purely classical structure of MCEG making it more "quantum-like"*

- Perform new assignement of charges along the proposed procedure (with some assumed form of probability $P$)

⟹ *in this way one one produces* - **in every single event!** - *EEC's* [15] *and changes the initial Boltzmann like statistics of the produced secondaries into the Bose-Einstein like one*

[14]See: O.V.Utyuzh, G.Wilk and Z.Włodarczyk, *Phys. Lett.* B522 (2001) 273 and hep-ph/0205087, to be published in *Acta Phys. Polon.* B33(2002). Cf. also: O.V.Utyuzh, *Fluctuations, correlations and non-extensivity in high-energy collisions*, PhD Thesis, available at http://www.fuw.edu.pl/ smolan/p8phd.html.

[15]T.Osada, M.Maruyama and F.Takagi, *Phys. Rev.* D59, 014024 (1999); M.Biyajima, N.Suzuki, G.Wilk and Z.Włodarczyk, *Phys. Lett.* B386, 297 (1996).

Proposition of "bosonization procedure" for a general algorithm: [1]

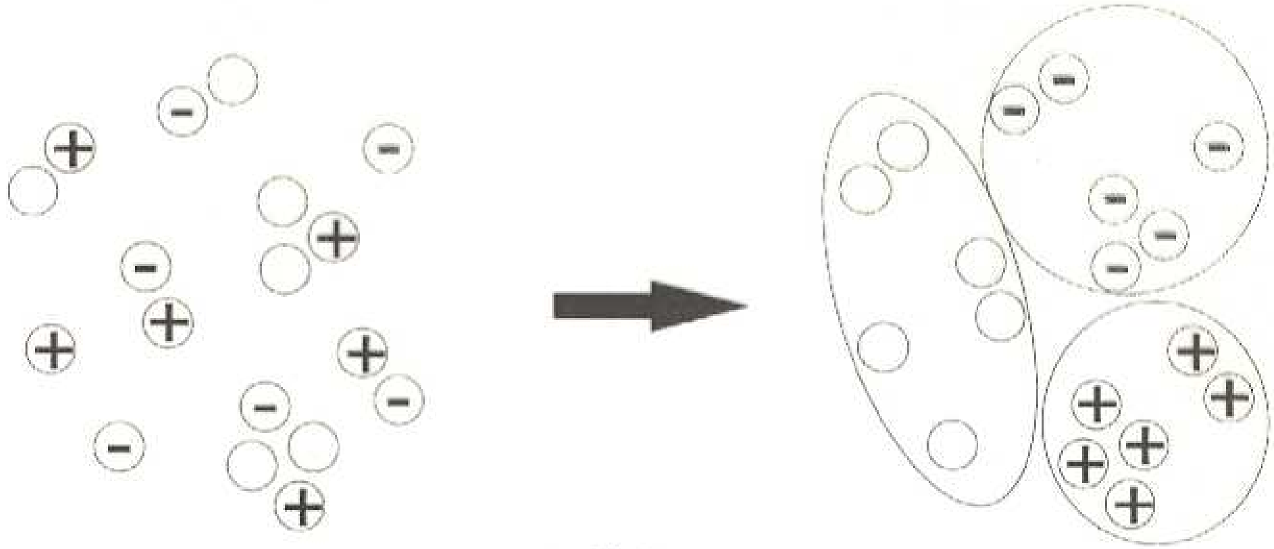


Fig. 1.

$$\{N = N(+) + N(-) + N(0)\} \text{ with } \{x_i, p_i\};\ i = 1, \ldots, N$$

$$\Longrightarrow$$

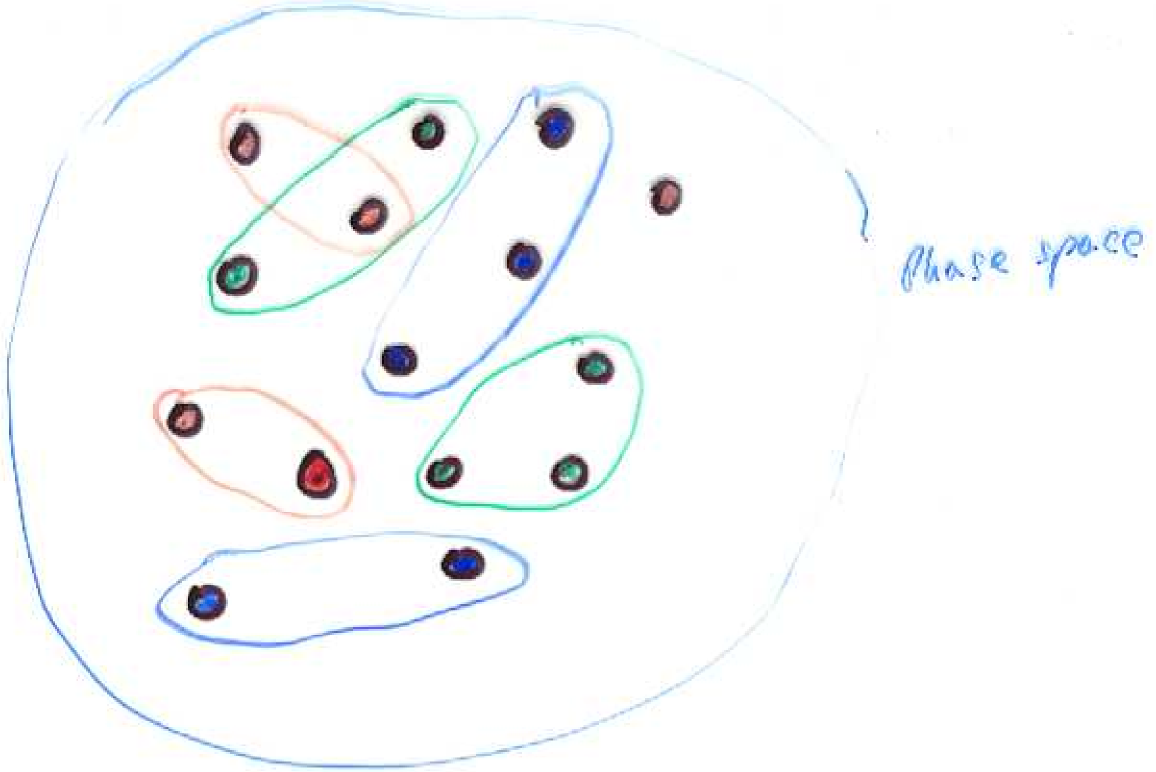
$$\{N = N(+) + N(-) + N(0)\} \text{ with } \{x_i, p_i\};\ i = 1, \ldots, N$$

**The same $N(+,-,0)$, $N$ and spatio-temporal and energy-momentum patterns BUT allocation of $(+,-,0)$ is now different[2]:**

*identical particles are maximally bunched in phase-space*

[1] See: W.A.Zajc, *A pedestrian's guide to interferometry*, w "Particle Production in Highly Excited Matter", eds. H.H.Gutbrod and J.Rafelski, Plenum Press, New York 1993, p. 435. **This concept is known and widely used when discussing the quantum optical analog of BEC known as HBT effect**, see R.Loudon, *The quantum theory of light* (IInd ed.) Clarendon Press - Oxford, 1983 or J.W.Goodman, *Statistical Optics*, John Wiley & Sons, 1985. Cf. also: K.Fiałkowski, in Proc. of the XXX ISMD, Tihany, Hungary, 3 October 2000, Eds. T.Csörgő et al., World Scientific 2001, p. 357; M.Stephanov, *Thermal fluctuations in the interacting pion gas*, hep-ph/0110077.

[2] O.V.Utyuzh, G.Wilk and Z.Włodarczyk, *Phys. Lett.* **B522** (2001) 273; cf. also: O.V.Utyuzh, *Fluctuations, correlations and non-extensivity in high-energy collisions*, PhD Thesis, available at http://www.fuw.edu.pl/ smolan/p8phd.html

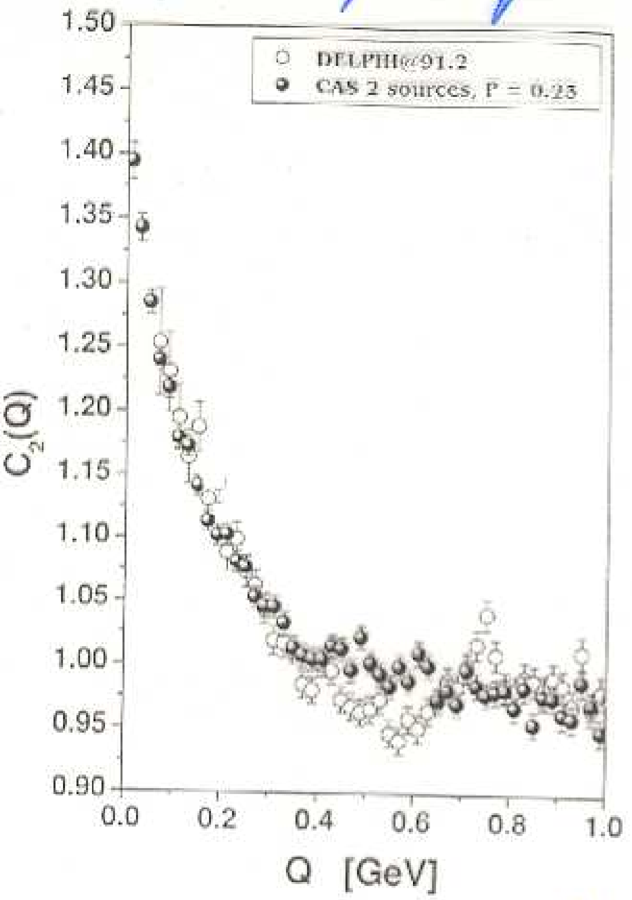


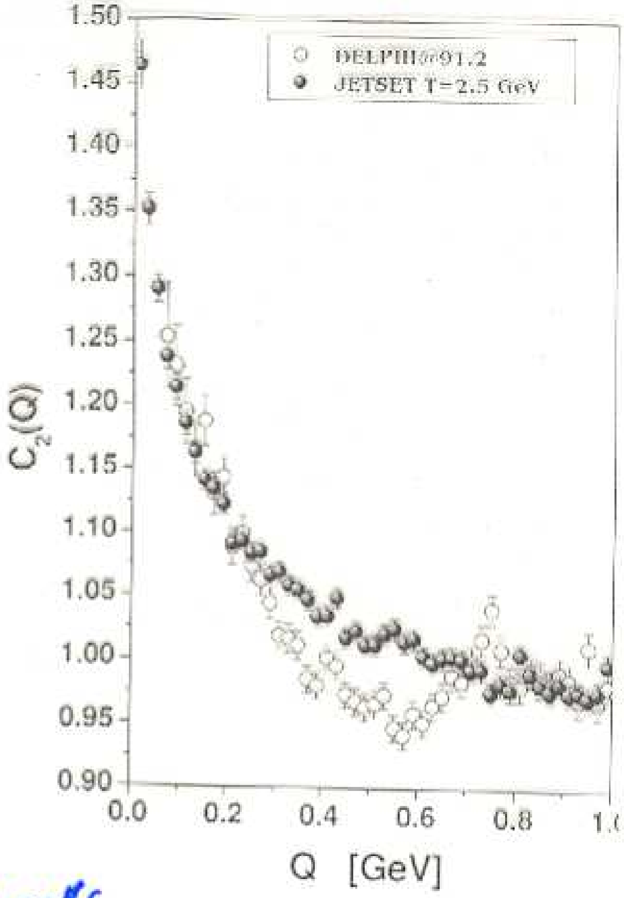

Parameter: P = probability that new particle "wants" to be assigned to the already present in a chosen cell

=>

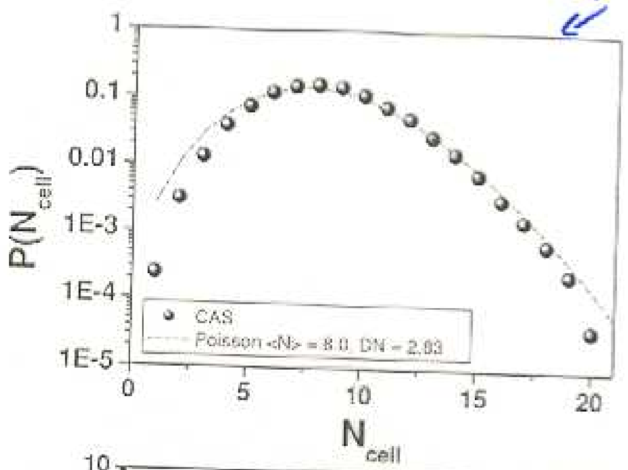
- a number of "cells" is formed with particles of like sign
- they can overlap in phase space (in OMT model they were adjacent)
- none of them is empty
- no "a priori" limit of particles in a cell

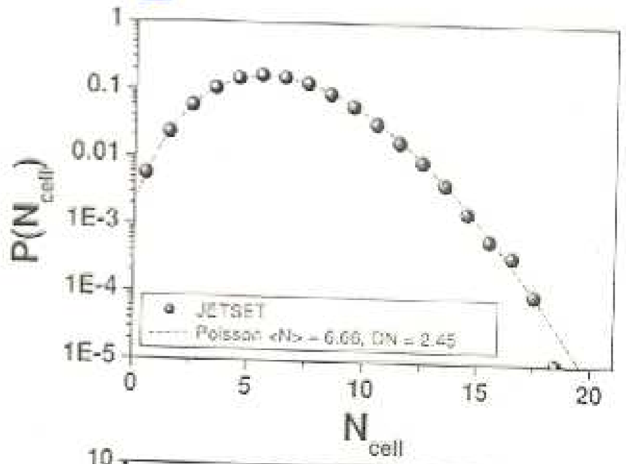
Preliminary comparison with (some...) data:

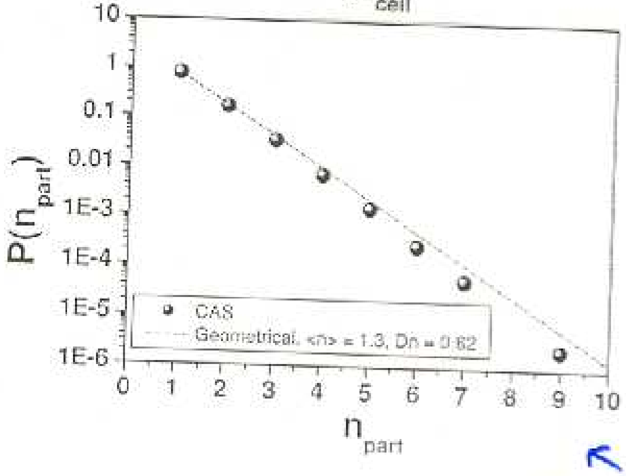


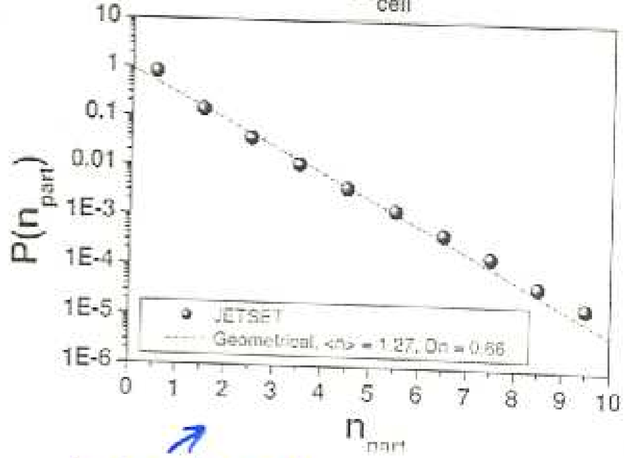




# of cells




# of particles in cell
(cell's occupancy)

The possible physical meaning ⟹

Scheme of charge flow in the cascade model used here:

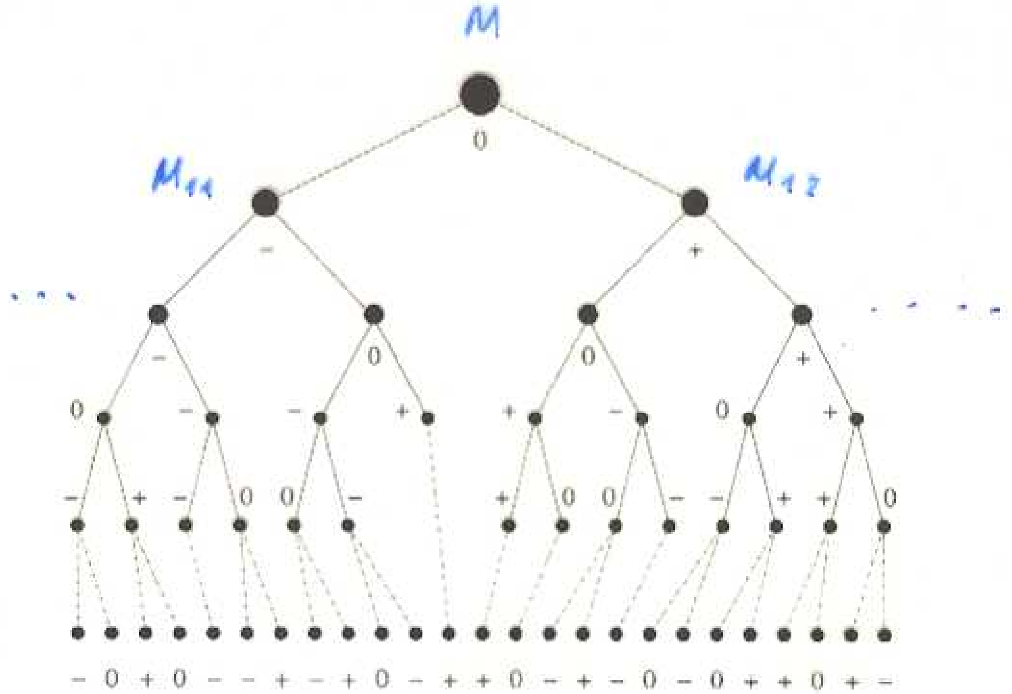



Figure 1: Example of the charge flow obtained with the simplest charge conservation pattern in vertices: $(0) \to (+)(-)$, $(+) \to (+)(0)$ and $(-) \to (-)(0)$. Notice that charge in each vertex does not exceed unity.

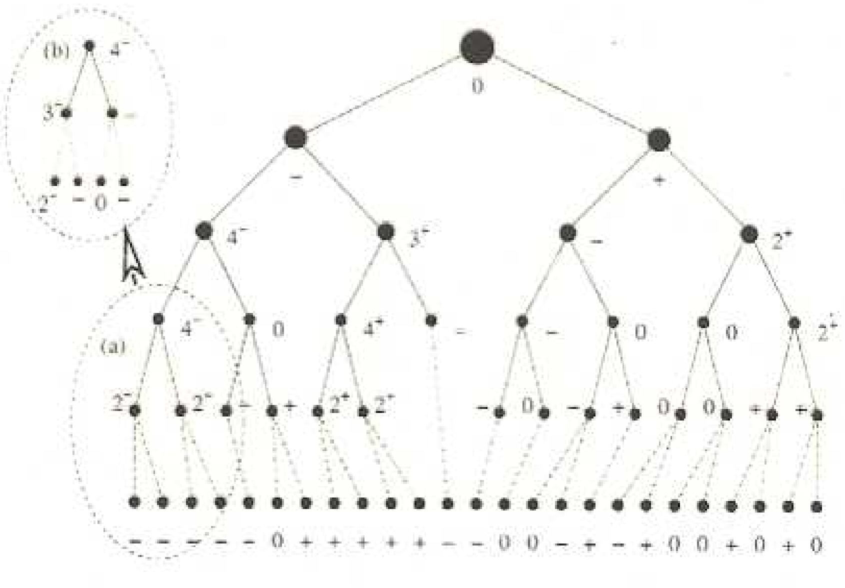



Figure 2: The same cascade as in Figure 1 in what concerns phase-space topology but with different assignment of final charges (preserving, however, the total numbers of neutral, positive and negative charges presented in Figure 1). To obtain such final assignment one has to allow for multiparticle charged vertices. Inlet shows different possible assignment with $2^-$ being either a resonance or state, which will further decay into two $1^-$ particles.

How it could be in string model?

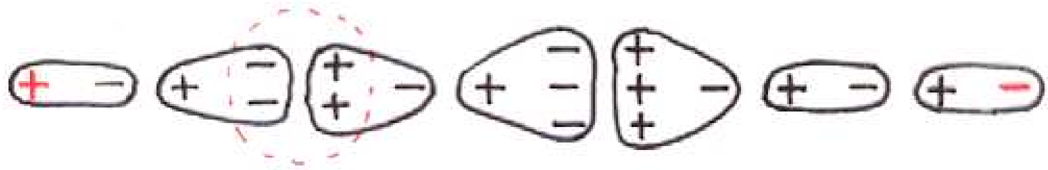
instead of neutral dipoles
one can have

SOME of them
could be CHARGED

⇒ Our algorithm is based
on the physical possibility
of occurence during the hadronization
process of multi-like-charged objects!

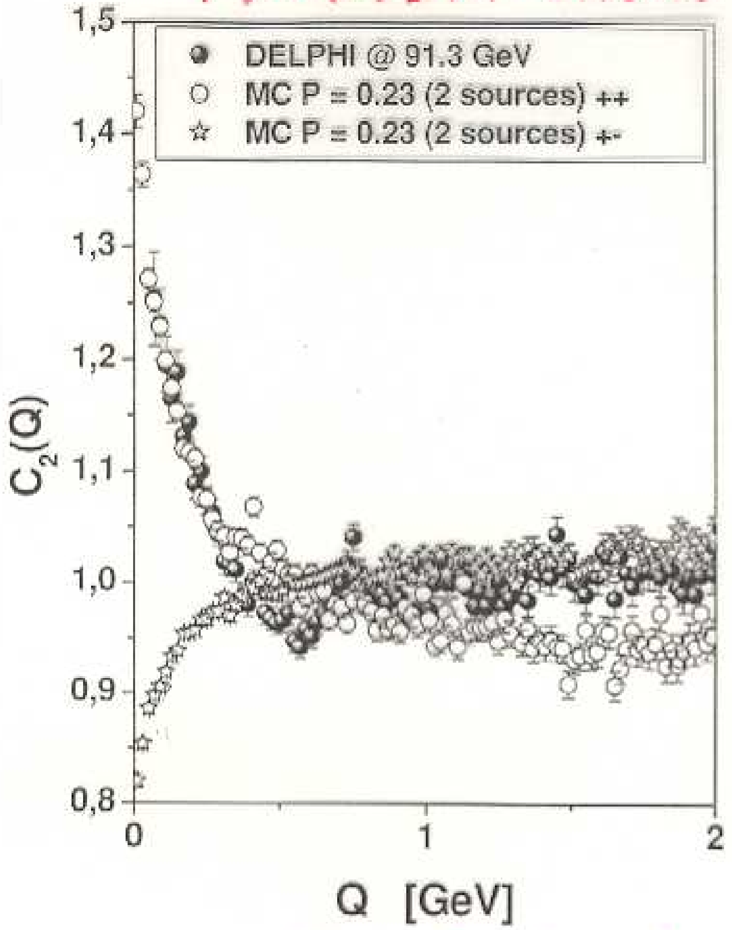
(*) Not yet attempted ....

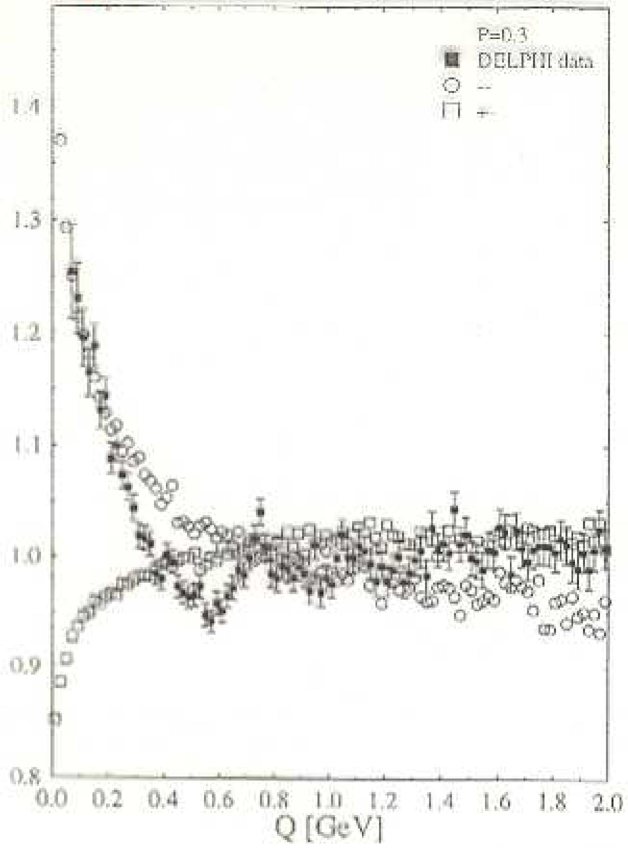
$$\frac{dn}{d^2p_t} \sim \exp\left(-\frac{\pi\left[\sqrt{p_t^2+\mu^2}\right]^2}{\kappa^2}\right)$$



To get such fluctuations one would need larger (from time to time, because of fluctuations ?[2] ⇐ A. Białas PLB (1999 ?)) value of $\kappa$ ⇐ connection with the properties of the QCD vacuum ?

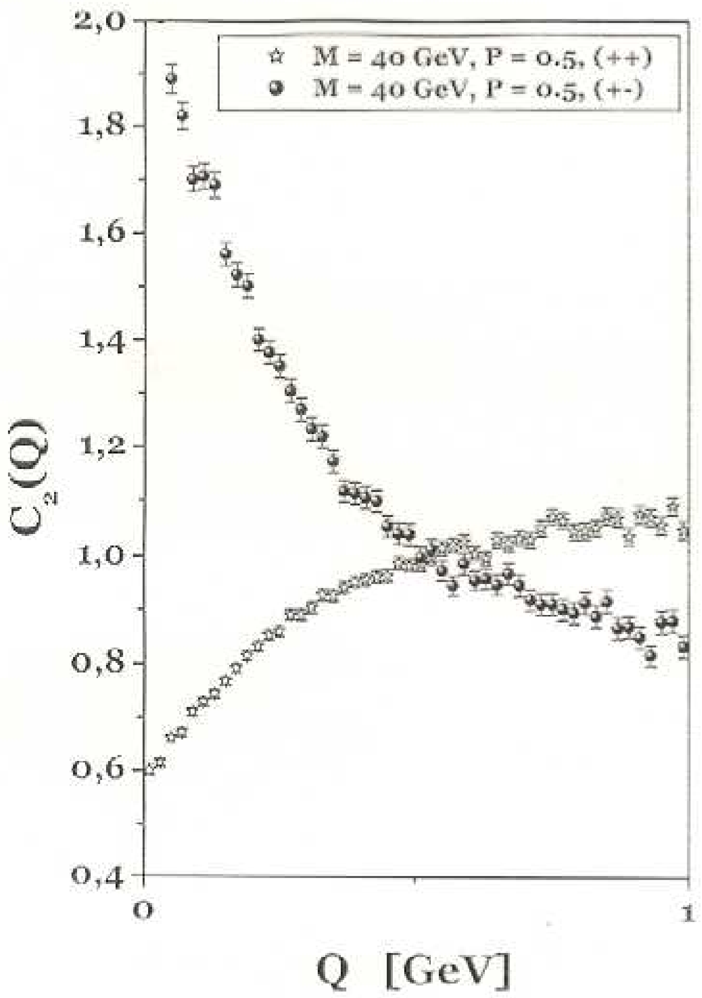
PROBLEM SHOWS UP!

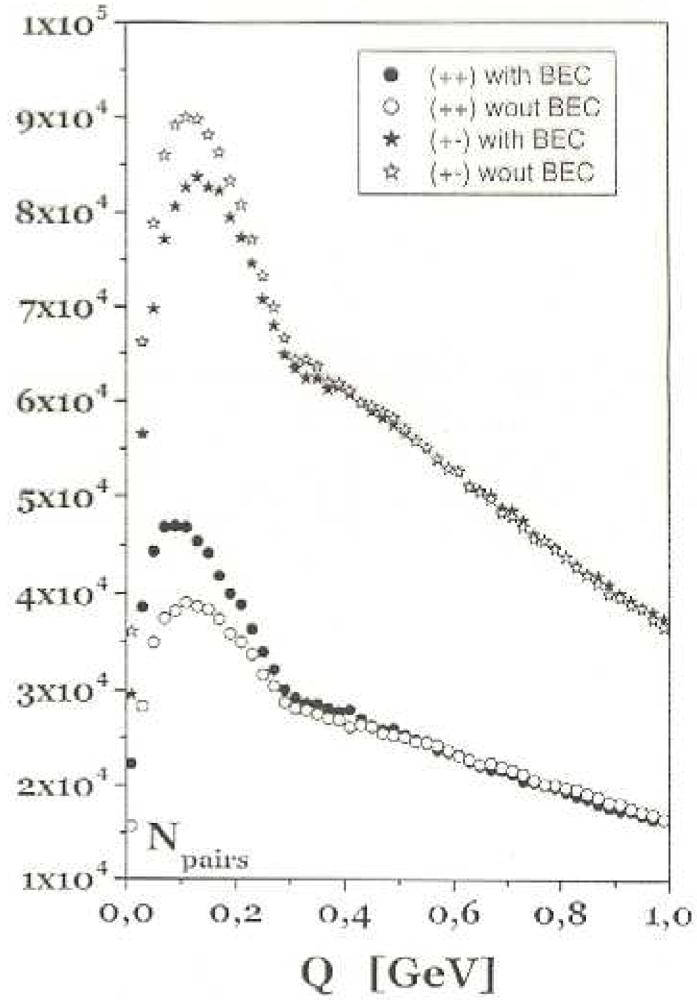



Correlations (++) arise because of (weaker) anticorrelations (+-)

(W. Kittel)



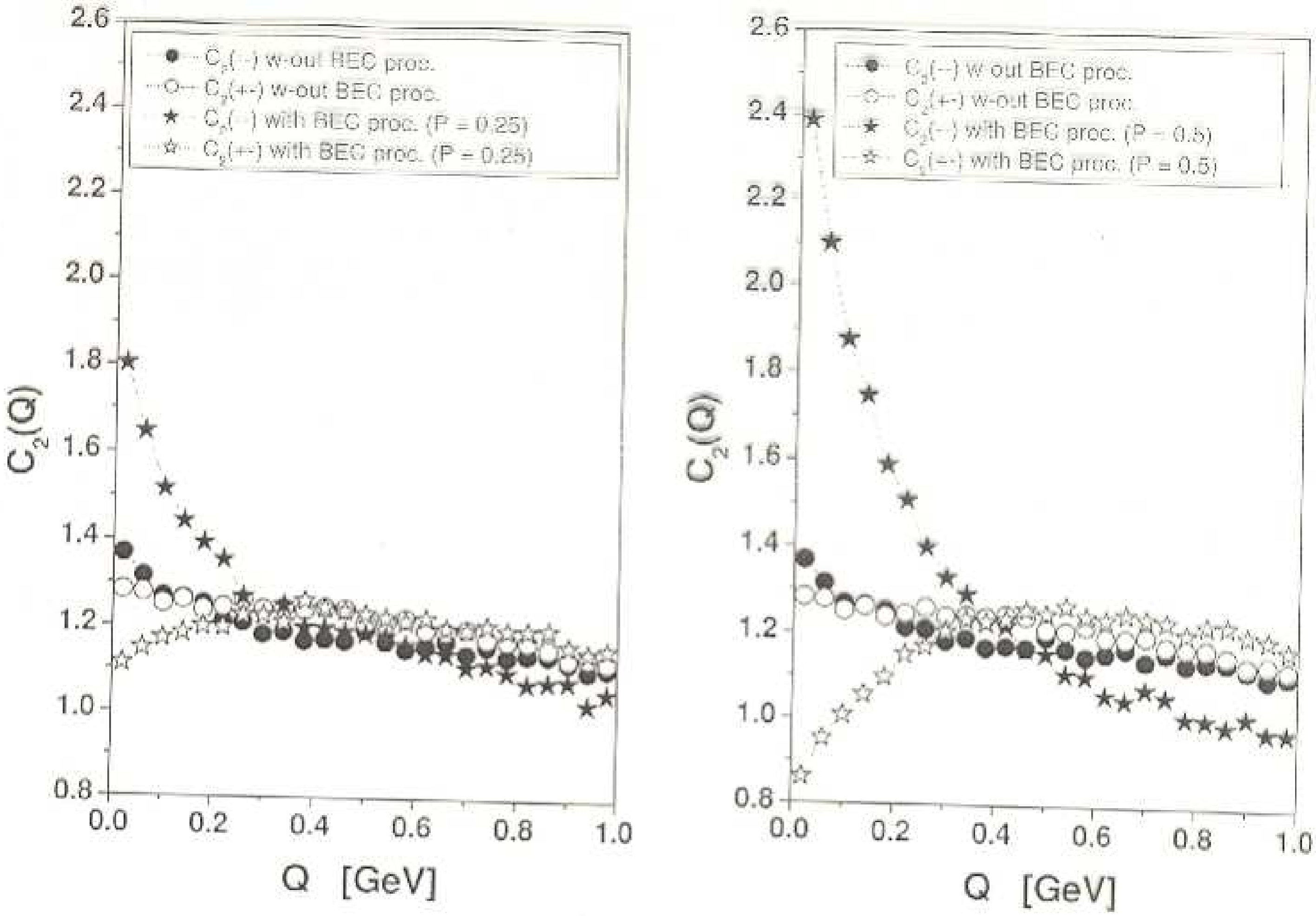



Illustration how to get BEC without unwanted unlike charge anticorrelations:

(i) start with parameters of MCEG which lead to slight positive correlations between (+-) together with some (++) etc

(ii) choose P in our algorithm in such a way as to kill positive (+-) correlation

? it is not clear whether (+-) can be made totally flat....

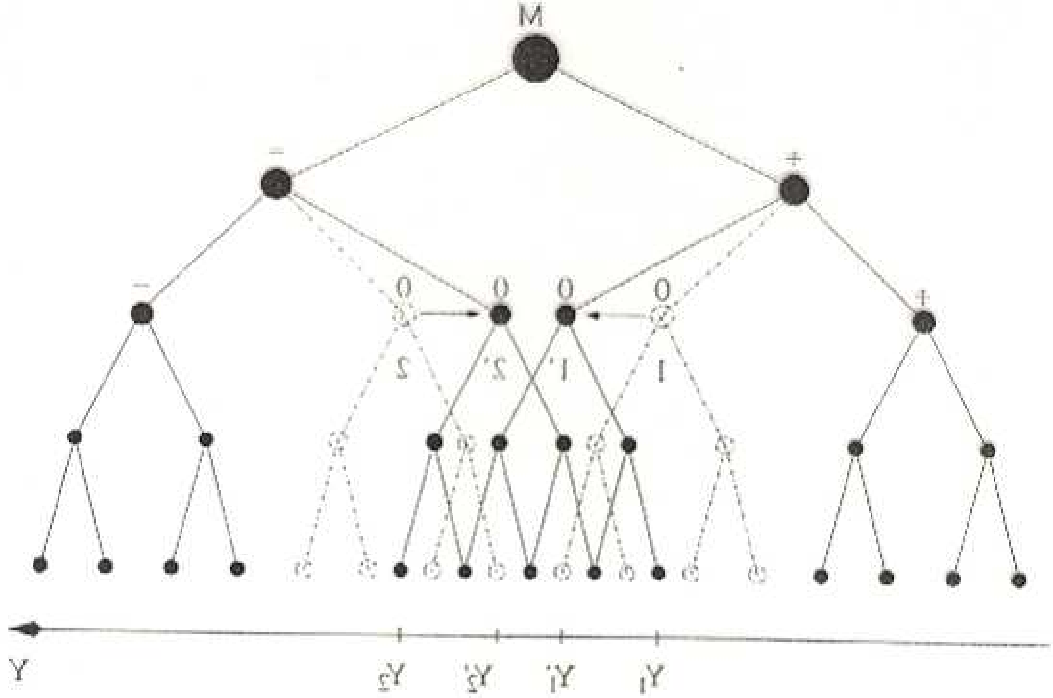


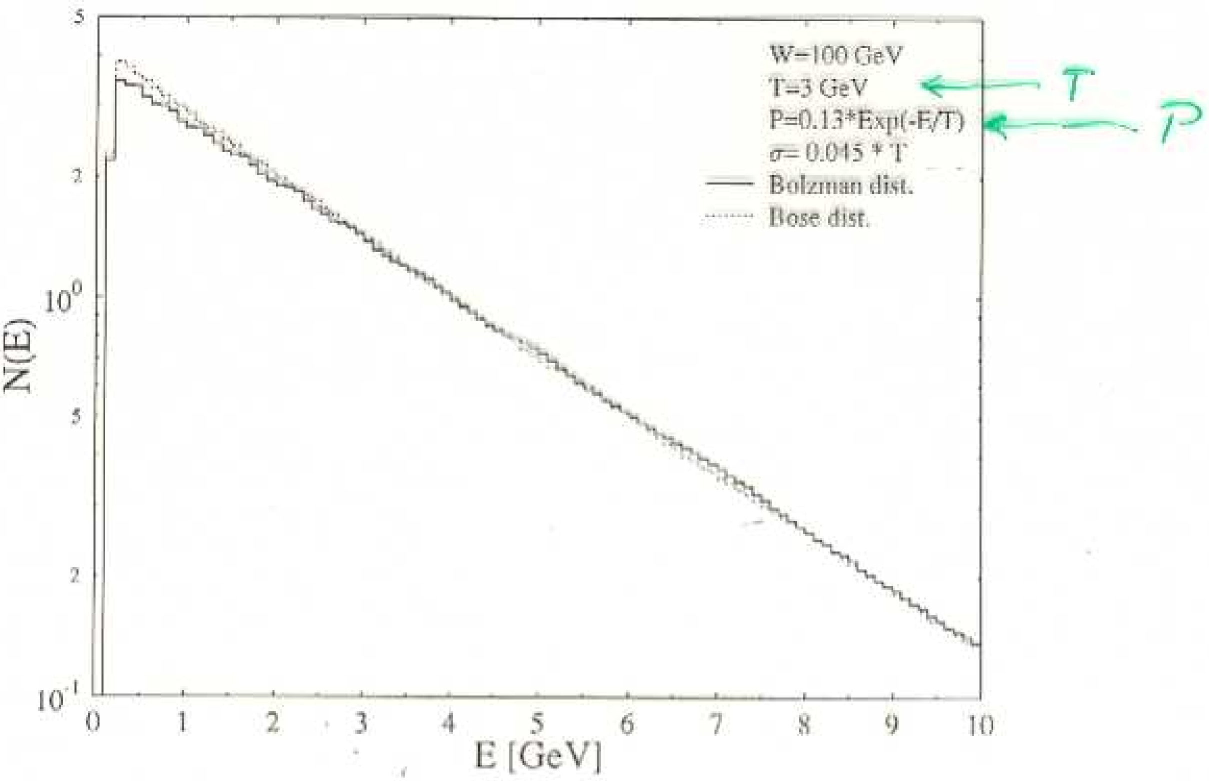

**Fig. IV.2.** Schematic presentation of momenta shifting procedure in the case of CAS (for $D = 1$ case). The originally neutral mass M produces at the second level of cascade two neutral masses $M_1$ and $M_2$, which are now subjected to shifting procedure described in text. The same procedure is repeated on every further step of cascade and involves masses of the same charge.

Version of the cascade model leading to some BEC and positive (+−) correlations at the same time

(notice: energy-momenta are conserved!)

Back to "bosonization algorithm" (*)

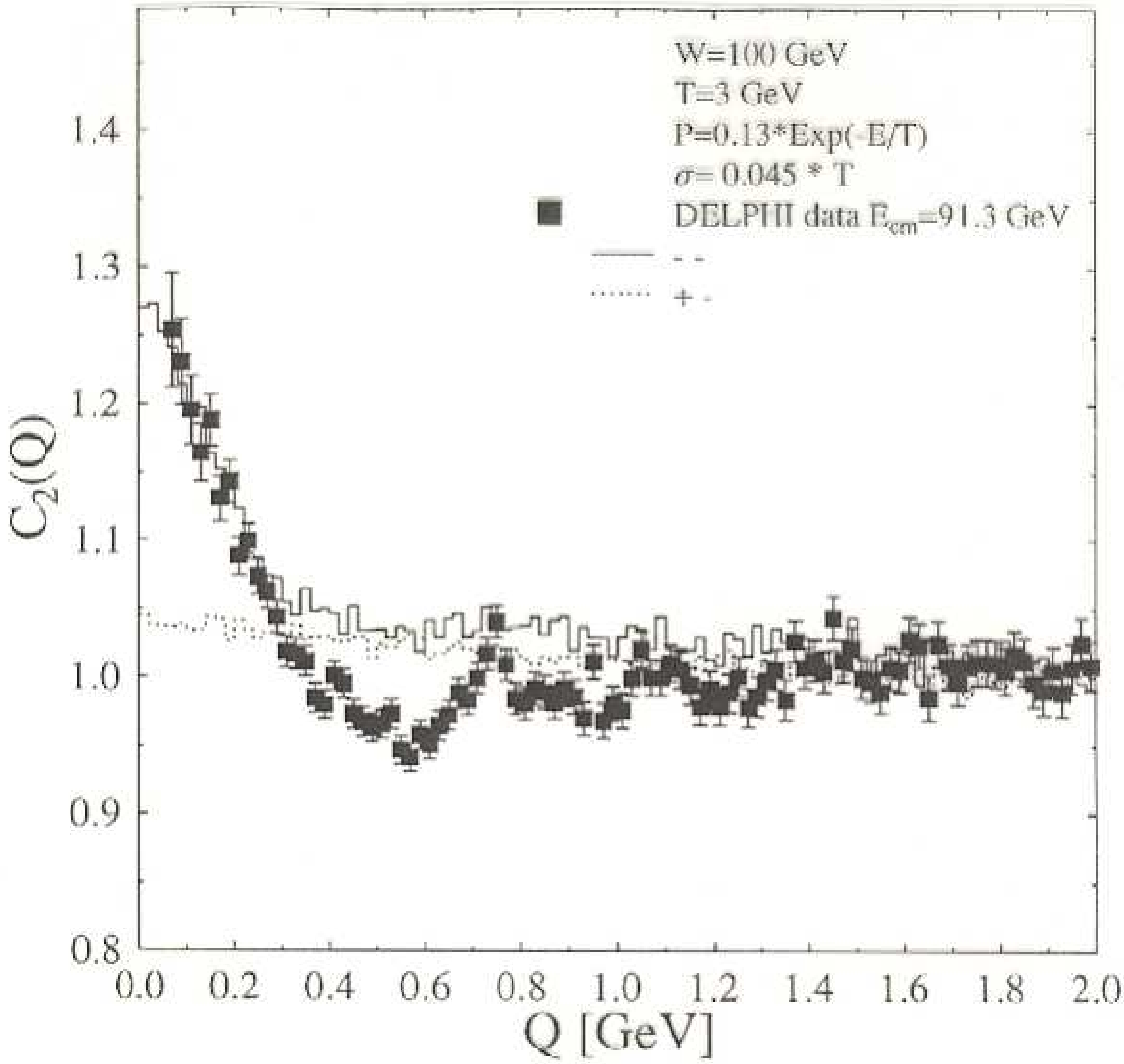
Preliminary results (RW)+(UW)...

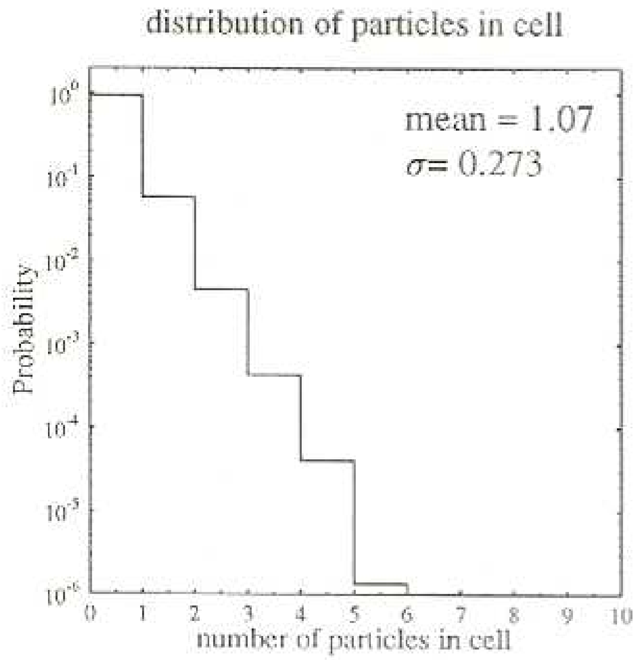


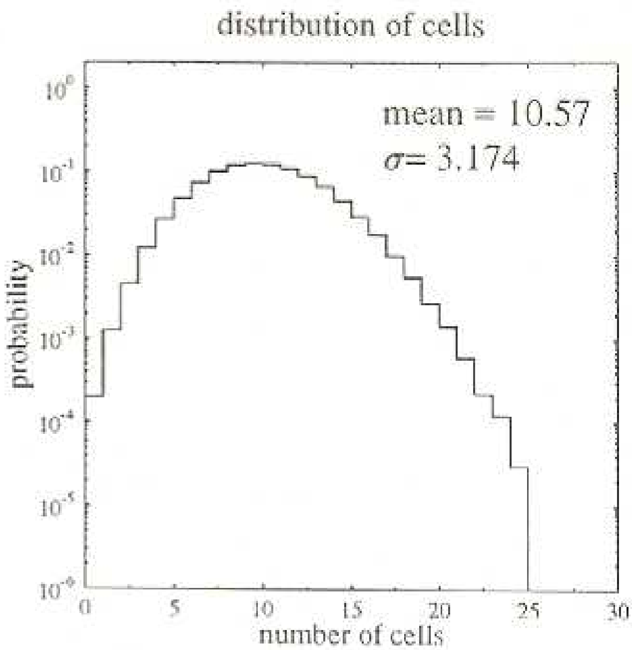

- From pool of energy $W$ select particle $(+/-/0)$ from $e^{-E/T}$
- With $P$ add to it particles of the same charge and $E_i \in (E \pm \sigma)$ from the gaussian distribution
  - do this to first failure
- Start new initial particle $(+/-/0)$ from $e^{-E/T}$

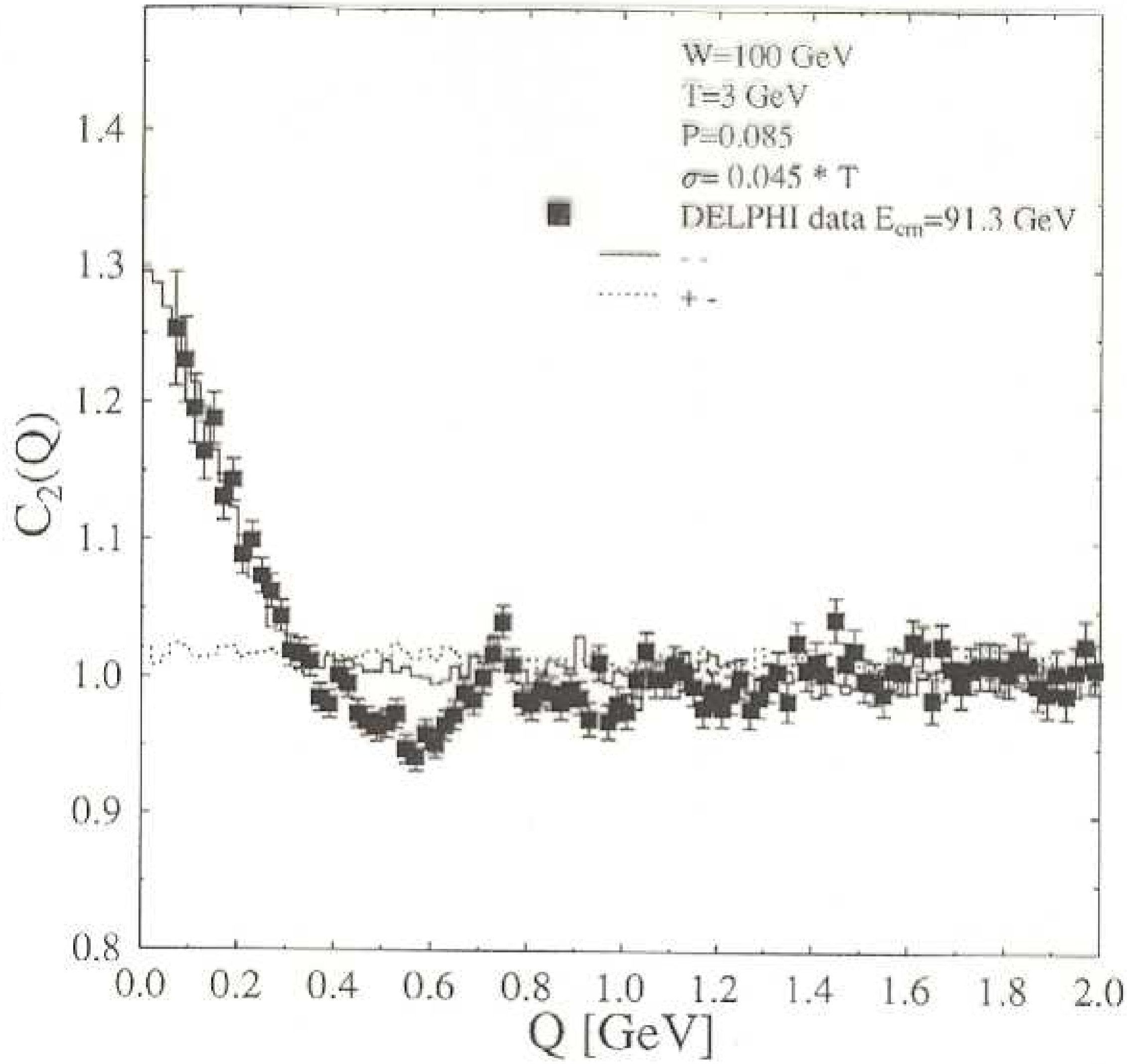
(*) should be regarded as further development of original ideas provided by Osada et al. PR D59 (1999) 014024 and BSWW PL B386 (1996) 297

Notice: here in principle one gets BEC to ALL orders!
- depending on the elementary cell occupancy

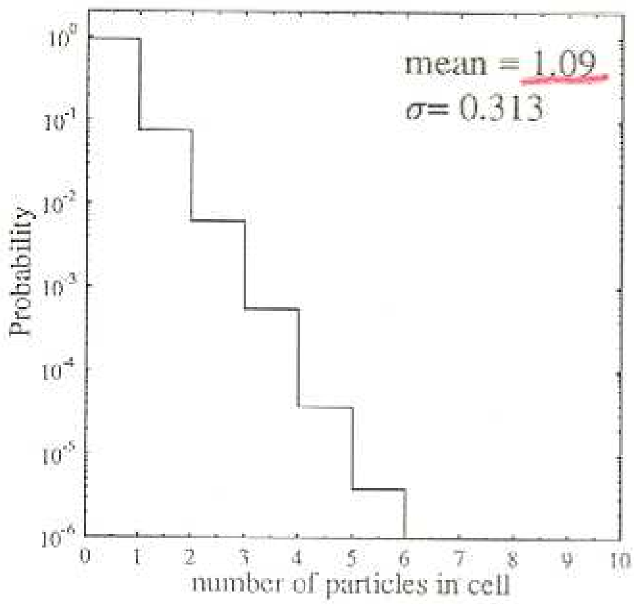


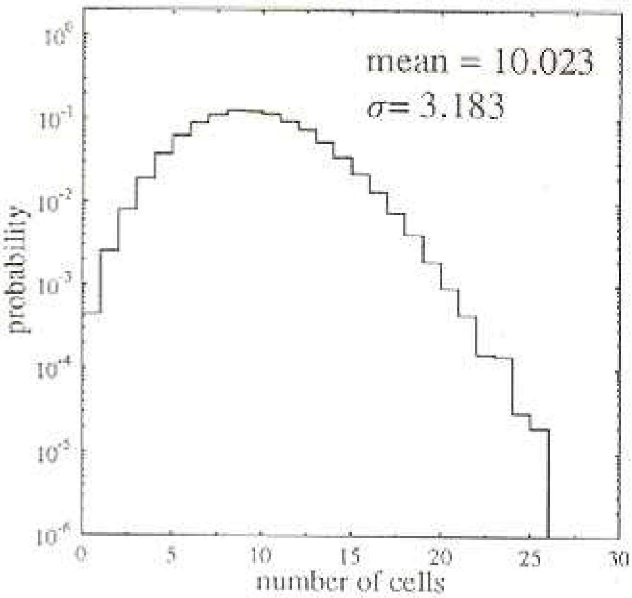






distribution of particles in cell




distribution of cells




Notice: excess over 1 leading to observed
$C_2(Q)$ is very small! ⇐ admixture of high order BEC small!

# Summary

(*) one can directly model Bose character of particles

(*) one can do it even for any outcome of MCEG (the price to pay: forgot about initial charge assignement -> make event more "uncertain")

(*) however: together with (++) correlations the (+-) anti-correlations do occur!

⇒ one has to refit model parameters!

... this can be done, but the original flavor of the approach is lost ... (?)

(*) one can come back to old bosonisation ideas (still to be worked out)

=> R is not a "source dimension" but rather dimension of the elementary emitting cell

2