FORWARD - BACKWARD MULTIPLICITY CORRELATIONS IN SYMMETRIC & ASYMMETRIC HIGH ENERGY COLLISIONS

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- Definitions and summary of data
- The symmetric case: formalism and applications
- The asymmetric case: formalism and examples
- Conclusions

work done in collaboration with A. Giovannini

see A. Giovannini & R.U. Phys Rev D 66, 034001 (2002)
" " hep-ph/0207217, J. Phys G.

DEFINITIONS

Symmetric definitions of Forward and Backward regions

pp $F \equiv \eta_{c.m.} > 0$ (collision axis)

$p\bar{p}$ $F \equiv$ half-space of outgoing p

e^+e^- $F \equiv$ chosen randomly btw. the two regions defined by a plane $\perp$ thrust axis

B is the symmetric region.

Correlation strength

$$b_{FB} \equiv \frac{\langle (n_F - \bar{n}_F)(n_B - \bar{n}_B) \rangle}{\left[\langle (n_F - \bar{n}_F)^2 \rangle \langle (n_B - \bar{n}_B)^2 \rangle \right]^{1/2}}$$

when doing a linear fit

$$\bar{n}_B(n_F) = a + b_{FB}\, n_F$$

SUMMARY OF DATA

$\boxed{hh}$ ISR $b_{FB} = 0.156 \pm 0.013$ (63 GeV)

UA5 $b_{FB} = 0.58 \pm 0.01$ (546 GeV, $|\eta|<4$)

b_{FB} grows linearly with $\sqrt{s}$ and is rather large

$\boxed{e^+e^-}$ TASSO $b_{FB} = 0.080 \pm 0.016$ (22 GeV)

OPAL $b_{FB} = 0.103 \pm 0.007$ (91 GeV)

" $b_{FB} \simeq 0$ (2-jet and 3-jet samples, separately)

b_{FB} (grows with $\sqrt{s}$) and is rather small

FRAMEWORK:

weighted superposition of different classes of events

$$P(n) = \alpha P_1(n) + (1-\alpha) P_2(n)$$

α = fraction of class 1 events

This approach describes

- shoulder in multiplicity distributions
- H_q vs q oscillations

with the classes

- e^+e^-: 2- & 3-jet events
- hh : events with & without mini-jets

For the joint distribution of F & B particles

$$P(n_F, n_B) = \alpha P_1(n_F, n_B) + (1-\alpha) P_2(n_F, n_B)$$

of course

$$P(n) = \sum_{n_F + n_B = n} P(n_F, n_B)$$

General formulae:

$$b_{FB} = \frac{\alpha b_1 (1+b_2) D^2_{n,1} + (1-\alpha) b_2 (1+b_1) D^2_{n,2} + \frac{\alpha(1-\alpha)}{2} (\bar{n}_2 - \bar{n}_1)^2 (1+b_1)(1+b_2)}{\alpha (1+b_2) D^2_{n,1} + (1-\alpha)(1+b_1) D^2_{n,2} + \frac{\alpha(1-\alpha)}{2} (\bar{n}_2 - \bar{n}_1)^2 (1+b_1)(1+b_2)}$$

where

b_i = correlation strength of class i events

$D_{n,i}$ = dispersion of m.d. " " i "

$\bar{n}_i$ = average multipliaty " " i "

The case $b_1 = b_2 = 0$

$$b_{12} = \frac{1/2\, \alpha(1-\alpha)(\bar{n}_2 - \bar{n}_1)^2}{\alpha D^2_{n,1} + (1-\alpha) D^2_{n,2} + \frac{1}{2}\alpha(1-\alpha)(\bar{n}_2 - \bar{n}_1)^2}$$

- Independent from any specific form of P_1 and P_2
- Depend only on $\alpha, \bar{n}_1, \bar{n}_2, D_{n,1}, D_{n,2}$

APPLICATION I: e^+e^- annihilation at LEP

OPAL result: separate 2- & 3-jet events

$$b_1 \simeq b_2 \simeq 0$$

weak overall correlations

$$b^{(exp)}_{FB} = 0.103 \pm 0.007$$

Using $\alpha_{2\text{-jet}} = 0.46$, the shoulder in m.d. data was described with

$\bar{n}_1 = 18.4 \quad D^2_{n,1} = 25.6 \quad \bar{n}_2 = 24.0 \quad D^2_{n,2} = 44.6$

Now we obtain

$$b_{12} = 0.101$$

in agreement with data.

The weighted superposition mechanism describes FB correlations in e^+e^- annihilation.

APPLICATION II: $\bar{p}p$ collisions (546 GeV)

UA5: $\underline{b_{FB}^{(exp)} = 0.58 \pm 0.01}$

(classes of events not considered)

Shoulder described by $\alpha = 0.75$
(fraction of events without minijets)

$\bar{n}_1 = 24.0 \quad D_{n,1}^2 = 106 \quad \bar{n}_2 = 47.6 \quad D_{n,2}^2 = 209$

The formula with $b_1 = b_2 = 0$ gives here too small a value

$$\underline{b_{12} = 0.28}$$

Non-zero FB correlations are needed in each substructure → full formula

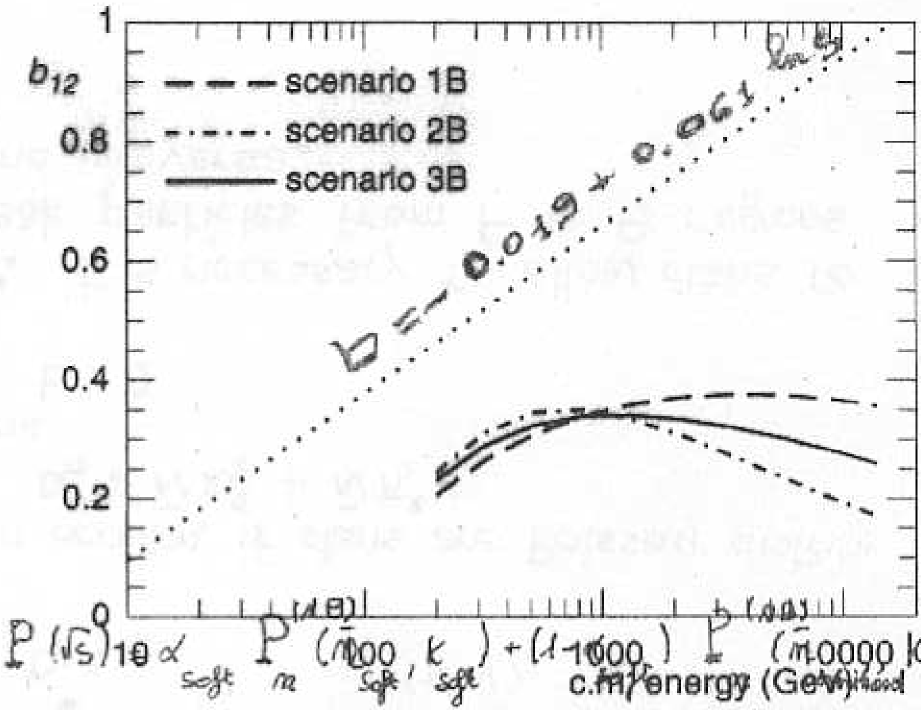
This need is confirmed by our extrapolations to the TeV region (fig)

On the other hand, if particles in each substructure were uncorrelated

$$b_i = \frac{D_{n,i}^2 - \bar{n}_i}{D_{n,i}^2 + \bar{n}_i}$$

and we have too large a value

$$b_{FB} = 0.78$$



$$P(\sqrt{s}) = \alpha_{soft} P_n^{(NB)}(\bar{n}_{soft}, k_{soft}) + (1-\alpha_{soft}) P_n^{(NB)}(\bar{n}_{semihard}, k_{semihard})$$

$$\bar{n}_{TOTAL} = \alpha_{soft}\, \bar{n}_{soft} + (1-\alpha_{soft})\, \bar{n}_{semihard}$$

$$\bar{n}_{soft}(\sqrt{s}) = -5.54 + 4.72 \ln(\sqrt{s})$$

$$\bar{n}_{semihard}(\sqrt{s}) \simeq 2\bar{n}_{soft}(\sqrt{s}) + 0.1 \ln^2(\sqrt{s})$$

$$\alpha_{soft} = 1 + \frac{\bar{n}_{soft} - \bar{n}_{total}}{\bar{n}_{soft} + 0.1 \ln^2(\sqrt{s})}$$

Scenario 1 KNO scaling is assumed for both comp

Scenario 2 strong KNO scaling violation for s. h. comp

Scenario 3 QCD inspired

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TWO-STEP PRODUCTION PROCESS

Generalization of negative binomial clan structure:

I. Production of N independent objects (clans) according to a m.d. $\mathcal{P}(N)$.

II. Particle production within each clan according to m.d. $Q(n_c)$

(when $\mathcal{P}(N)$ is Poisson, the total m.d. is infinitely divisible; when in addition $Q(n_c)$ is the logarithmic distribution, the total m.d. is NB.)

Two-step process is applied to each component (1 & 2) separately as it has been shown the NBD is a good parameterization for a single component.

independent clans (FB-binomial)

+

particles produced by F (B) clans remain all in the F(B) region

=

$$b = \frac{D_n^2/\bar{n} - D_c^2/\bar{n}_c - 4\bar{n}_c}{D_n^2/\bar{n} + D_c^2/\bar{n}_c + 4\bar{n}_c}$$

where

$\bar{n}_c$ = avg. num. particles per clan

$\bar{N}$ = " " clans

$\bar{n}$ = avg. num. particles = $\bar{N}\bar{n}_c$

D_c = dispersion within a clan

D_n = " (total)

in addition, if clans are Poisson distrib:

$$D_n^2 = \bar{N} D_c^2 + \bar{N}\bar{n}_c^2$$

and

$$b = 0$$

$\therefore$ it is necessary to allow clans to leak particles from F to B regions and viceversa.

Define leakage parameters

p = average fraction of particles in a clan which remain in the same (F or B) region

$q = 1 - p$

a clan is classified F or B according to where majority of particles fall $(0.5 \le p \le 1)$.

General result

$$b = \frac{D_n^2/\bar{n} - D_c^2/\bar{n}_c - (p-q)^2\,\bar{n}_c + 4\gamma/\bar{n}_c}{D_n^2/\bar{n} + D_c^2/\bar{n}_c + (p-q)^2\,\bar{n}_c - 4\gamma/\bar{n}_c}$$

where γ = covariance between the F and B multiplicities within a clan

For the NBD and independently distributed particles in a clan:

$$b = \frac{2\beta pq}{1 - 2\beta pq} \quad \text{with} \quad \bar{n}_c = \frac{\beta}{(\beta-1)\ln(1-\beta)} \qquad 0<\beta<1$$

Application to hh collisions

- at 63 GeV, the semi-hard component is negligible

$$b_{FB} = b_{soft} = \frac{2\beta_{soft}\, p_{soft}\, q_{soft}}{1 - 2\beta_{soft}\, p_{soft}\, q_{soft}}$$

from measured values of b_{FB}, $\bar{n}$, D_n obtain

$$p_{soft} = 0.78$$

- $\bar{n}_{c,soft}(63) \simeq 2$, $\bar{n}_{c,soft}(900) \simeq 2.4$

  $\Rightarrow$ $p_{soft} \simeq$ constant in the GeV region

  $\Rightarrow$ calculate $b_{soft}(900)$

- from measured $b_{FB}(900)$ obtain $b_{semi\text{-}hard}(900)$ and consequently

$$p_{semi\text{-}hard} = 0.77$$

- $\bar{n}_{c,semihard}(200) \simeq 1.6$ $\bar{n}_{c,semihard}(900) \simeq 2.6$

  $\Rightarrow$ $p_{semi\text{-}hard} \simeq$ constant in the GeV region

  $\Rightarrow$ extrapolations to LHC energy

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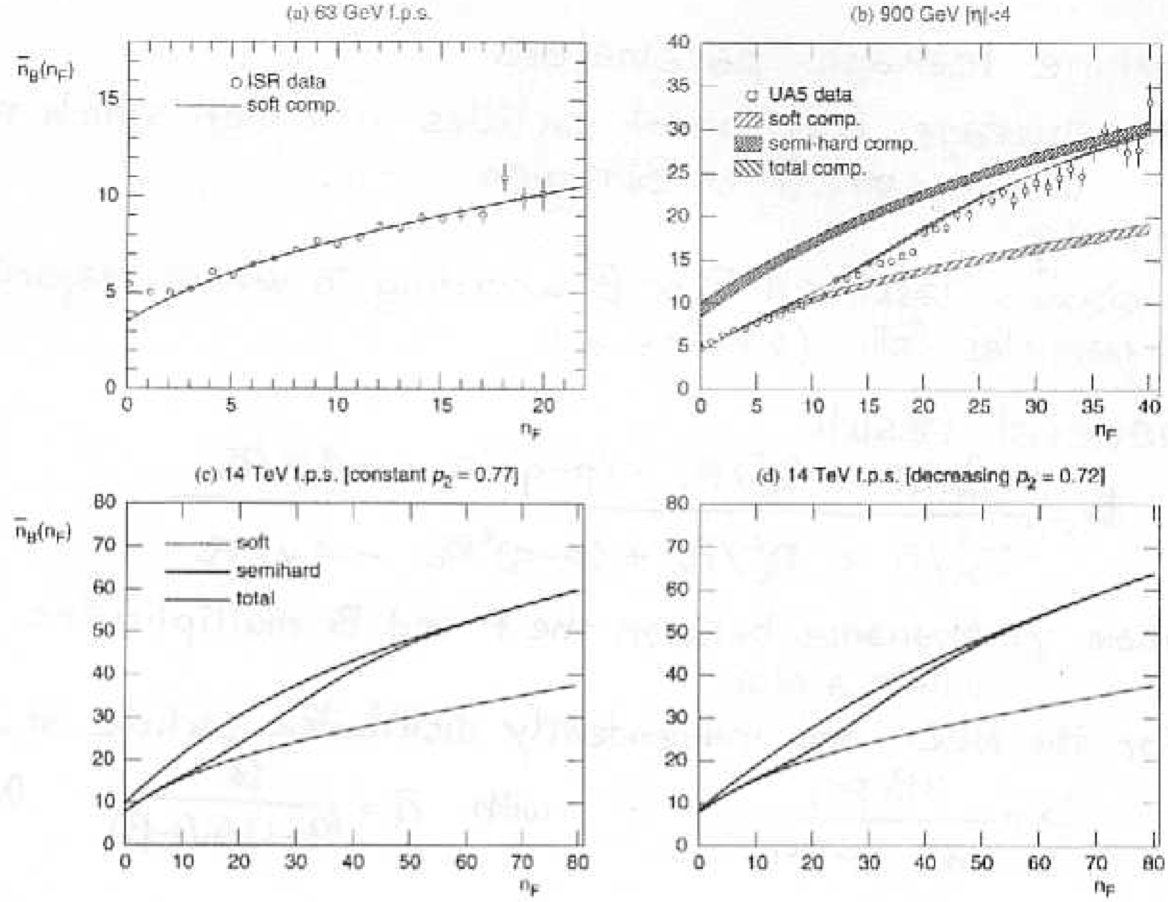

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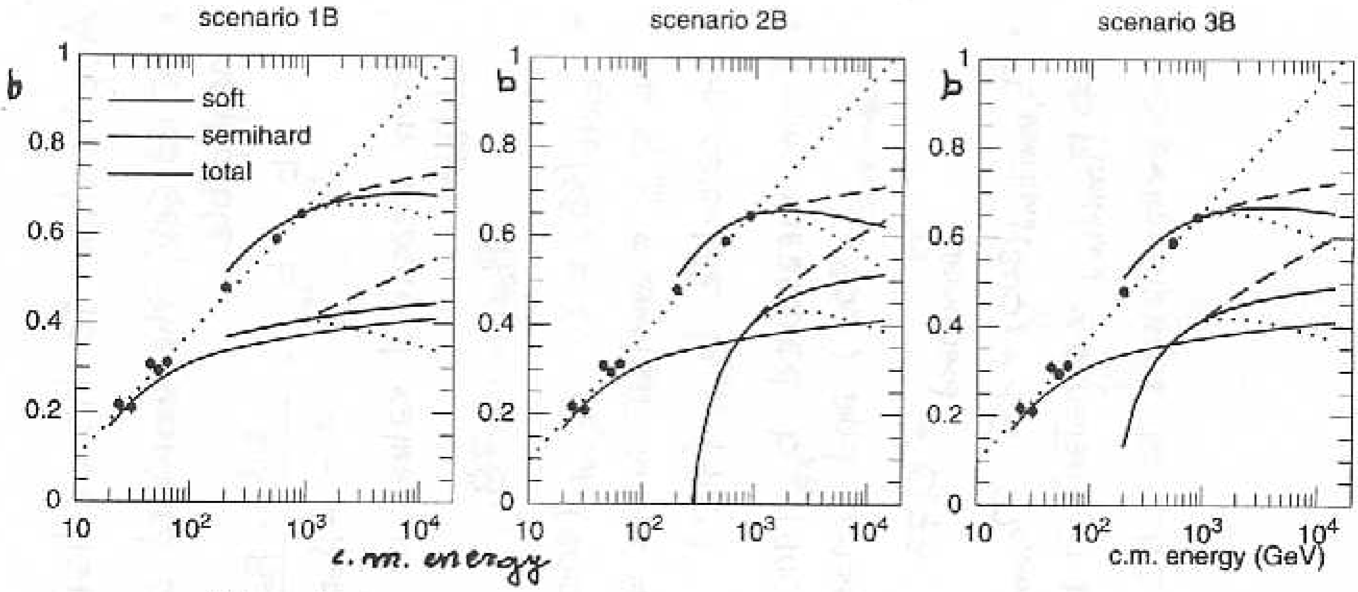

solid : constant p_2
dashed : p_2 decreasing after 1 TeV
dotted : p_2 increasing after 1 TeV

The ASYMMETRIC CASE

- Asymmetric definition of the F & B regions
- Asymmetric reaction (e.g. pA, AB)

Within the two-step process framework, we consider

- leakage from F to B may be different than from B to F:

$$p_F \neq p_B$$

(we still consider binomial distr. in a clan)

- clans are asymmetrically, but still binomially, distributed, say with probability

$$r \neq 1/2$$

(Symmetric case reobtained for $p_F = p_B$ & $r = \frac{1}{2}$).

The following applies to each component separately.

GENERATING FUNCTIONS

for the joint distribution

$$G(z_F, z_B) = \sum_{n_F, n_B} z_F^{n_F} z_B^{n_B} P(n_F, n_B)$$

$\therefore$

$$\bar{n}_F = \left.\frac{\partial G}{\partial z_F}\right|_{z_F = z_B = 1} \quad \text{etc}$$

$$\langle n_F n_B \rangle = \left.\frac{\partial^2 G}{\partial z_F \partial z_B}\right|_{z_F = z_B = 1} \quad \text{etc}$$

$$\bar{n}_F(n_B) = \left[\frac{\partial^{n_B}}{\partial z_B^{n_B}} \frac{\partial}{\partial z_F} G\right] \left[\frac{\partial^{n_B}}{\partial z_B^{n_B}} G\right]^{-1}_{z_B = 0,\, z_F = 1}$$

a) within a clan:

let $f(z) = \sum_{n_c} z^{n_c} Q(n_c)$

then

$$g_{c,F}(z_F, z_B) = f(z_F p_F + z_B q_F) \quad \text{F-clan}$$

$$g_{c,B}(z_F, z_B) = f(z_F q_B + z_B p_B) \quad \text{B-clan}$$

b) given N_F and N_B clans:

$$g(z_F, z_B | N_F, N_B) = [g_{c,F}(z_F, z_B)]^{N_F} \times$$
$$\times [g_{c,B}(z_F, z_B)]^{N_B} .$$

c) summing over the clan m.d., $\mathcal{P}(N_F, N_B)$

let $F(z_F, z_B) = \sum_{N_F, N_B} z_F^{N_F} z_B^{N_B} \mathcal{P}(N_F, N_B)$

then

$$\underline{G(z_F, z_B) = F(g_{c,F}(z_F, z_B), g_{c,B}(z_F, z_B))}$$

d) if the total m.d. is NBD:

$$G(z_F, z_B) = \exp\{ r\bar{N} [f_{log}(z_F p_F + z_B q_B) - 1] \} \times$$
$$\times \exp\{ (1-r)\bar{N} [f_{log}(z_F q_B + z_B p_F) - 1] \}$$

where
$$f_{log}(z) = \frac{\ln(1 - z\beta)}{\ln(1-\beta)}$$

Interesting results for the NBD case

- The marginal (one region) distr. is the convolution of two NBD, thus not a NBD but still infinitely divisible

$$g(z,1) = \left[1 + \frac{\bar{n} r p_F}{k r}(1-z)\right]^{-kr} \times$$
$$\times \left[1 + \frac{\bar{n}(1-r) q_B}{k(1-r)}(1-z)\right]^{-k(1-r)}$$

- When $r = 1/2$ one obtains

$$g(z,1) = \left[1 + \frac{\bar{n}}{k}(1-z) + \frac{\bar{n}^2}{k^2} p_F q_B (1-z)^2\right]^{-k/2}$$

- $\bar{n}_B(n_F)$ is not linear function of n_F (and v.v → fig.)

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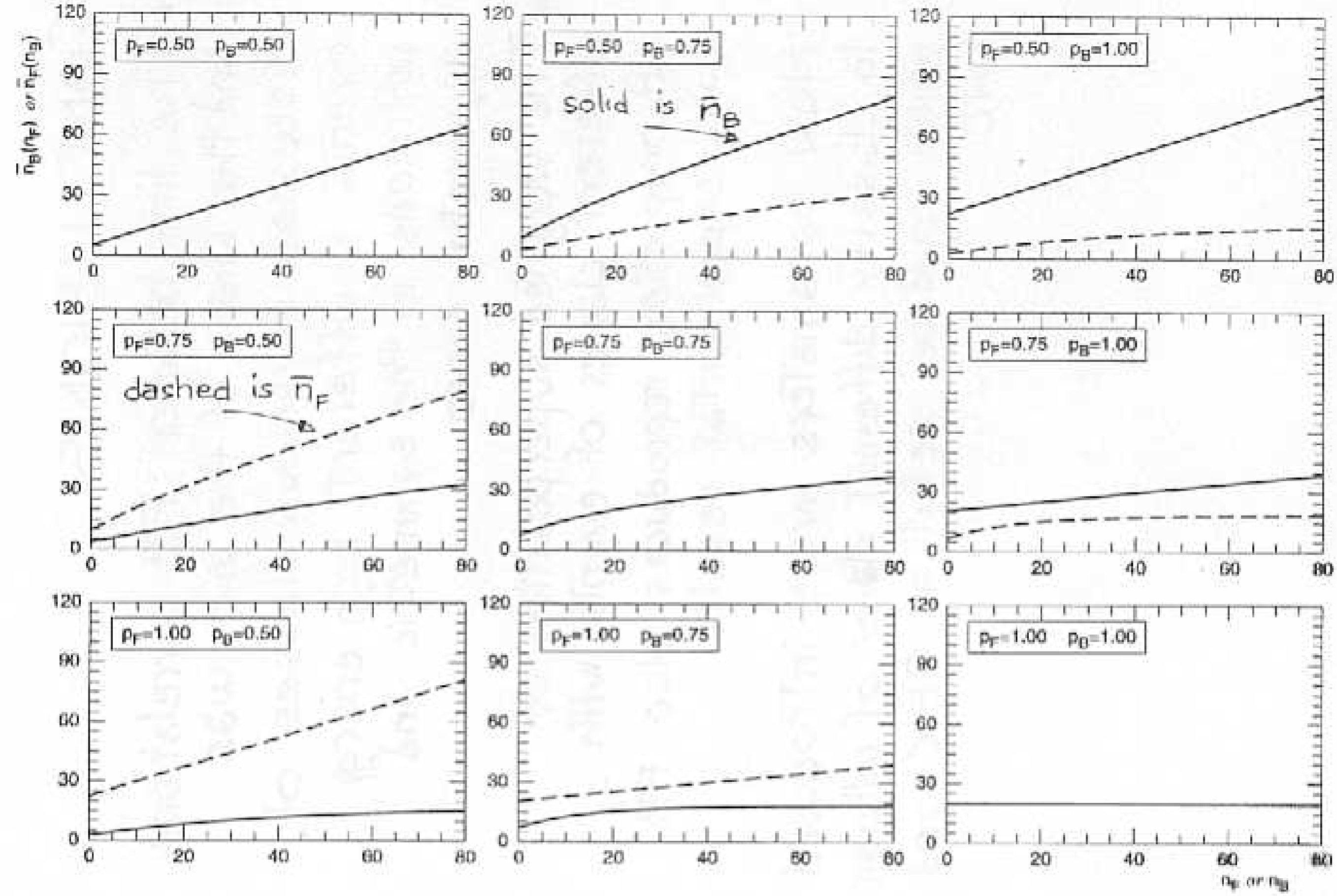
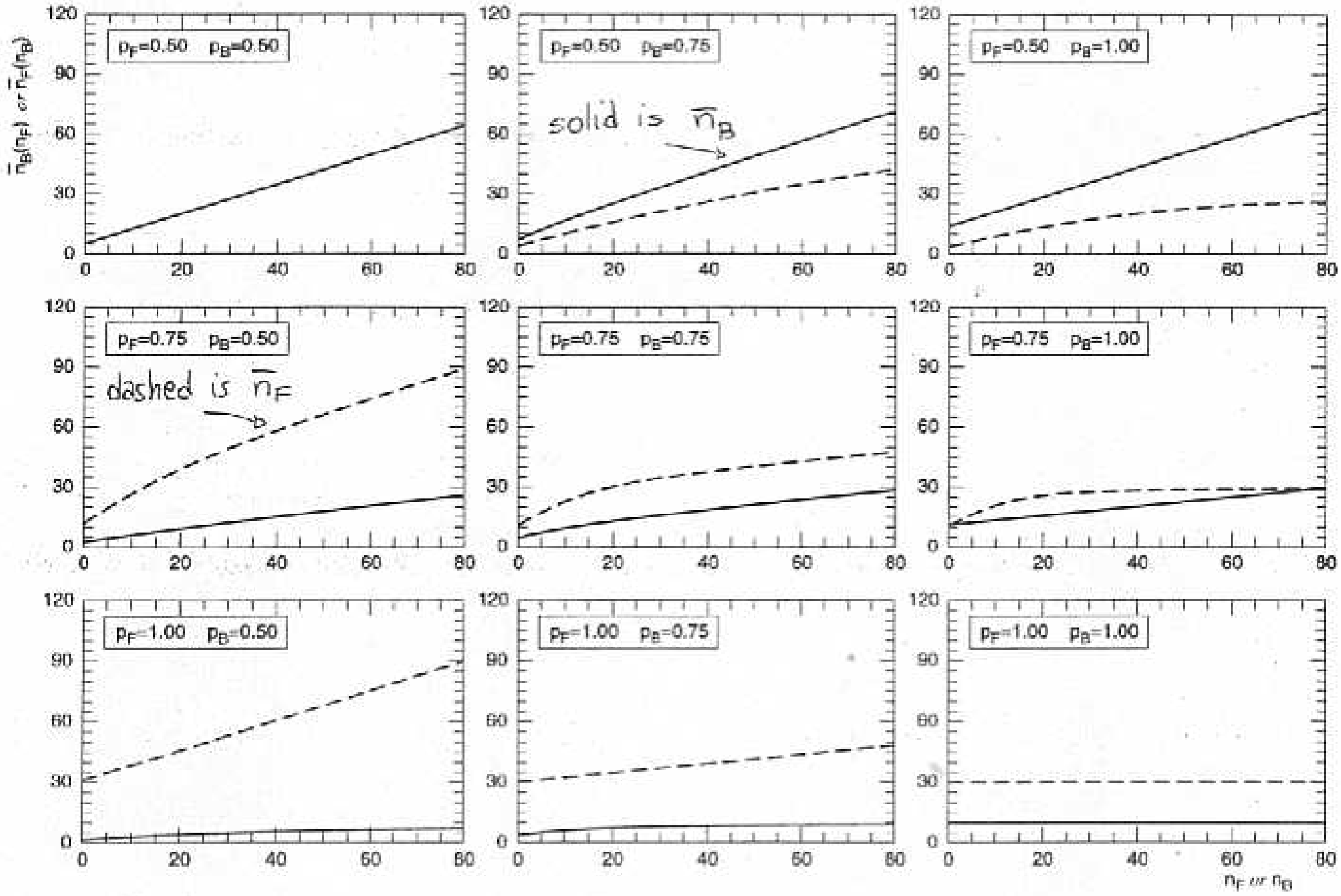
r = 1/2



$\bar{n}_B(n_F)$ or $\bar{n}_F(n_B)$
p_F=0.50 p_B=0.50
p_F=0.50 p_B=0.75
solid is $\bar{n}_B$
p_F=0.50 p_B=1.00
p_F=0.75 p_B=0.50
dashed is $\bar{n}_F$
p_F=0.75 p_B=0.75
p_F=0.75 p_B=1.00
p_F=1.00 p_B=0.50
p_F=1.00 p_B=0.75
p_F=1.00 p_B=1.00
n_F or n_B

r = 3/4



$\bar{n}_B(n_F)$ or $\bar{n}_F(n_B)$
p_F=0.50 p_B=0.50
p_F=0.50 p_B=0.75
solid is $\bar{n}_B$
p_F=0.50 p_B=1.00
p_F=0.75 p_B=0.50
dashed is $\bar{n}_F$
p_F=0.75 p_B=0.75
p_F=0.75 p_B=1.00
p_F=1.00 p_B=0.50
p_F=1.00 p_B=0.75
p_F=1.00 p_B=1.00
n_F or n_B

CONCLUSIONS

- The link between FB correlations and the shape of the m.d. was discussed in various classes of events in different high energy collisions in the symmetric and asymmetric cases
- The weighted superposition of different classes of events with NB properties reproduces also FB corr. experimental results
- New parameters were introduced to classify different classes of collisions which could be useful at RHIC and LHC

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