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On the long–range and IR behavior of the QCD coupling

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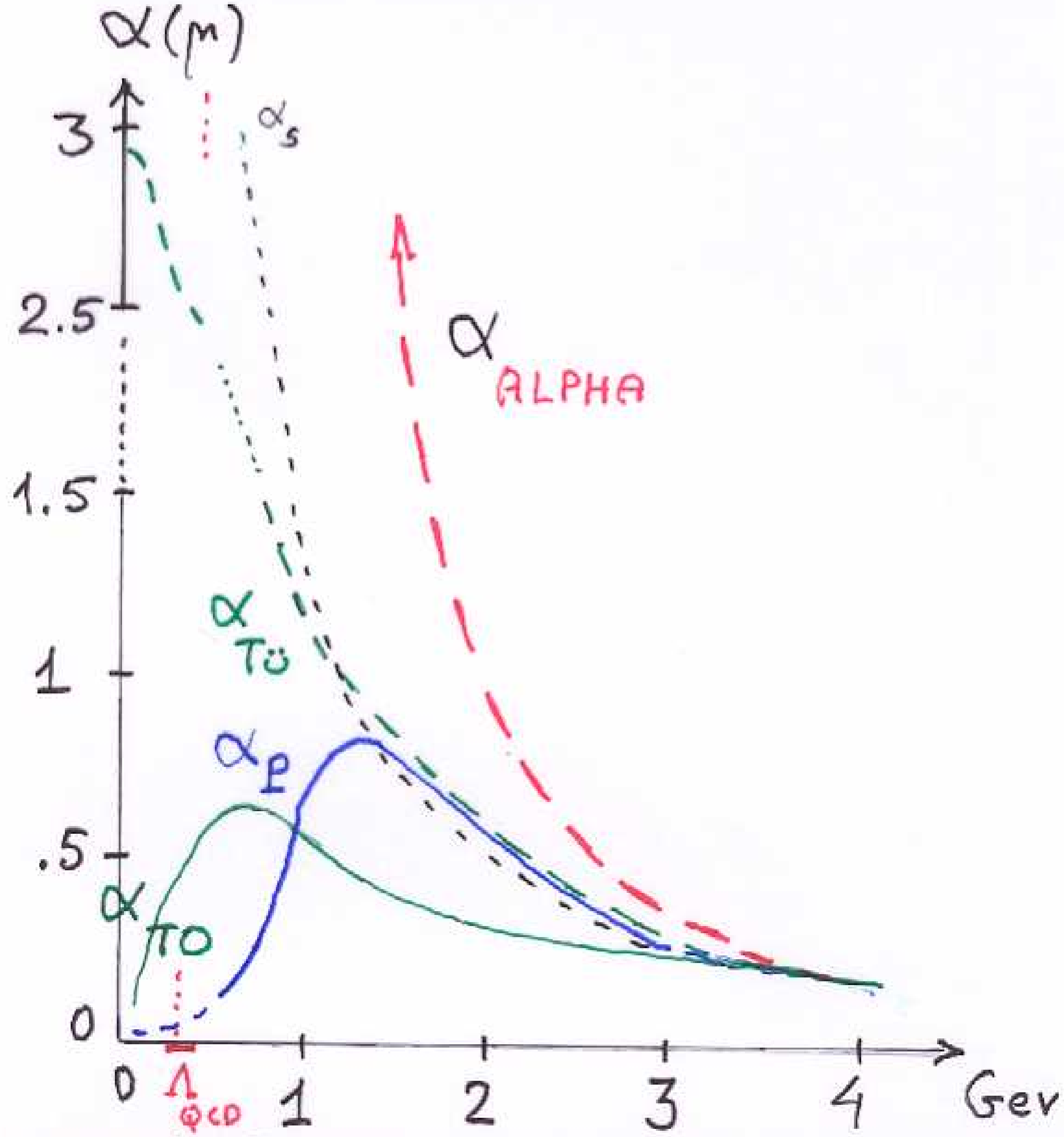
Non-perturbative results for QCD effective coupling by lattice simulations and solution of approxim. Schwinger–Dyson eqs reveal a **wide variety** of IR behaviors for $\bar{\alpha}_s(Q^2)$ rising a question of correlation of these different results.

D.V. Sh. Theor. Mat. Phys, **132** (2002) 484; hep-ph/0208082,

D.V.Sh.– in preparation

Summary of nonperturbative Lattice simulation & SDE solving results



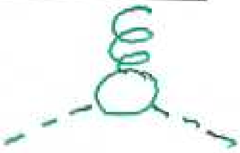
In analysis we use **mass-dependent coupling transform–ns** and conclude that it is possible to correlate various results on QCD $\bar{\alpha}_s(Q^2)$ IR behavior.

I. Lattice simulation and SDE results in the IR

Tübingen group studies only gluon–ghost sector of QCD in the Landau gauge. Here, in the truncated Schwinger–Dyson eqs (SDE) the gluon–ghost vertex Γ is expressible via ren. functions of gluon Z and ghost G propagators and drops out from $\bar{\alpha}_s$

$$\bar{\alpha}_T(Q^2) = \alpha_s Z(Q^2)\, G^2(Q^2)\,. \qquad (1)$$

Tübingen group: Lattice simulations



show specific IR behavior:
– The gluon function $Z(Q^2)$ has a maximum at $Q = \sqrt{Q^2} \simeq 1$ GeV and then goes to zero like $\sim Q$,
– the ghost one $G(Q^2)$ is monotonous and has a power IR singularity close to $Q^{-1/2}$, while
– the product $\bar{\alpha}_T$ **tends to finite value.**
This picture is supported by

Tübingen group: SDE solving

yields power IR behavior

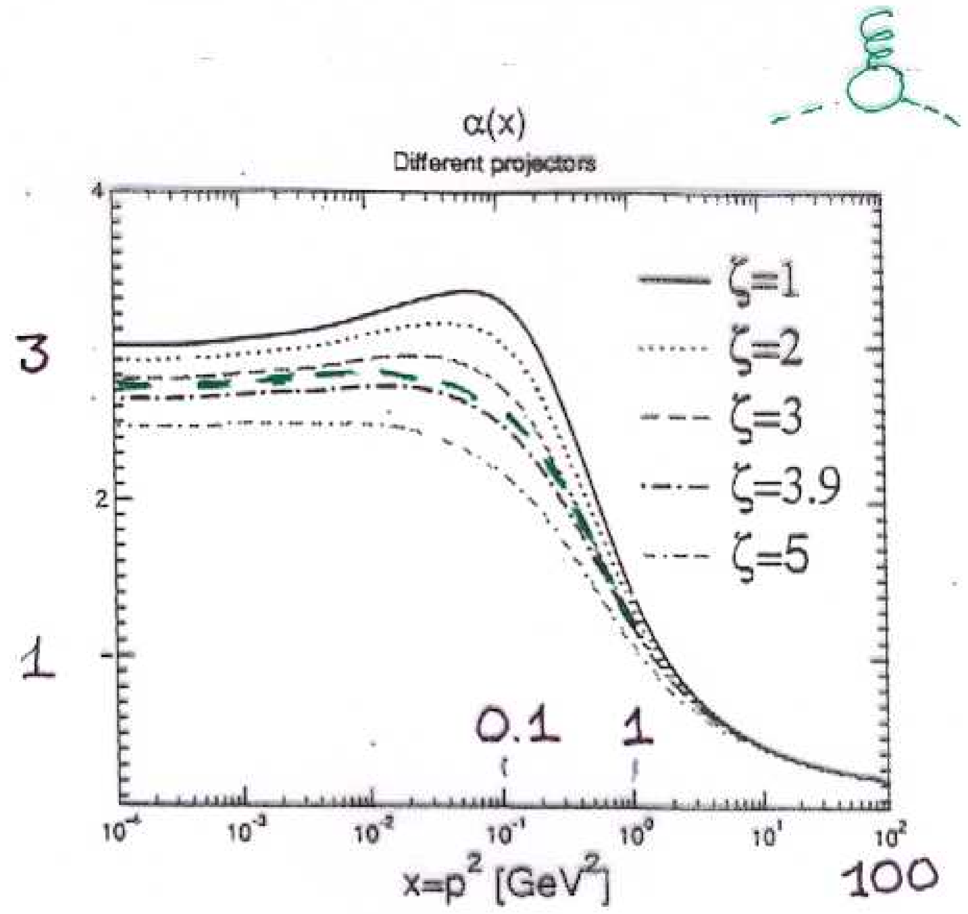
$$G(Q^2) \simeq (Q)^{-\kappa}; \quad Z(Q^2) \simeq (Q^2)^{\kappa}$$

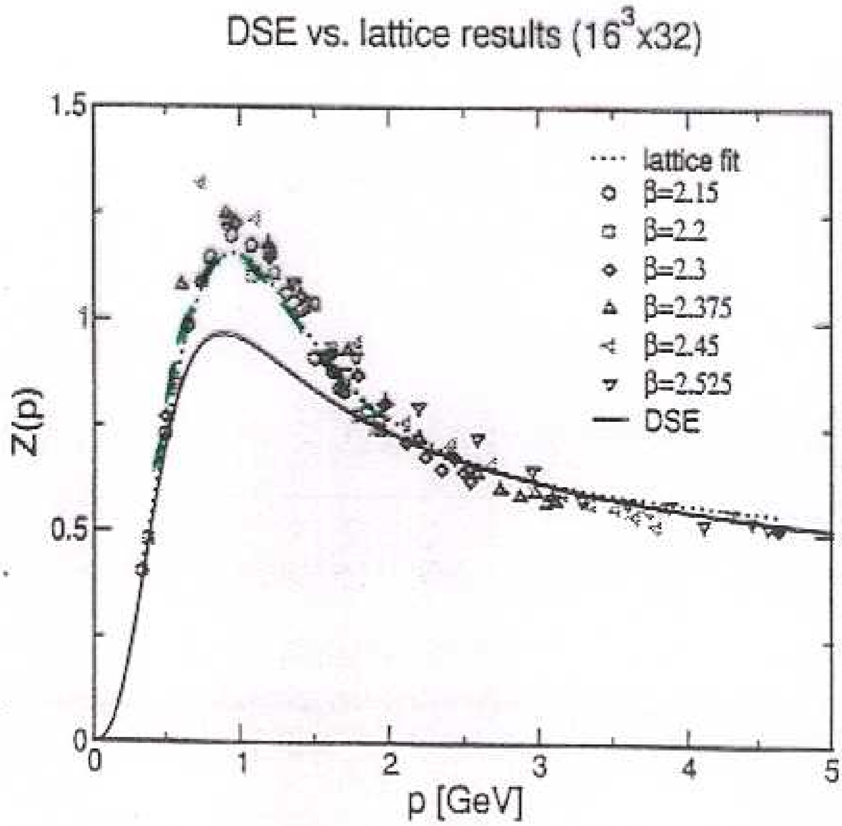
with

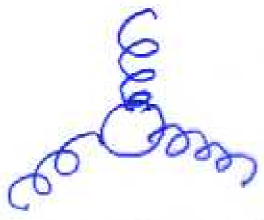
$$\kappa = 0.595 \quad \text{and} \quad \bar{\alpha}_s(0) \simeq 2.973$$

– see Figs 1 and 2.

Tübingen results







Paris group, lattice simulations

Invariant QCD coupling defined with **3–gluon vertex** & non-symm MOM subtraction $\tilde{\Gamma}(Q^2) \equiv \Gamma_{3gl}(Q^2, 0, Q^2)$:

$$\bar{\alpha}_P(Q^2) = \alpha_s \tilde{\Gamma}^2(Q^2) Z^3(Q^2) \qquad (2)$$

in the Landau gauge with **two quark flavors**. According to lattice simul. it has a maximum at $Q \simeq 1.25$ GeV and then quickly approaches zero – see Fig.3

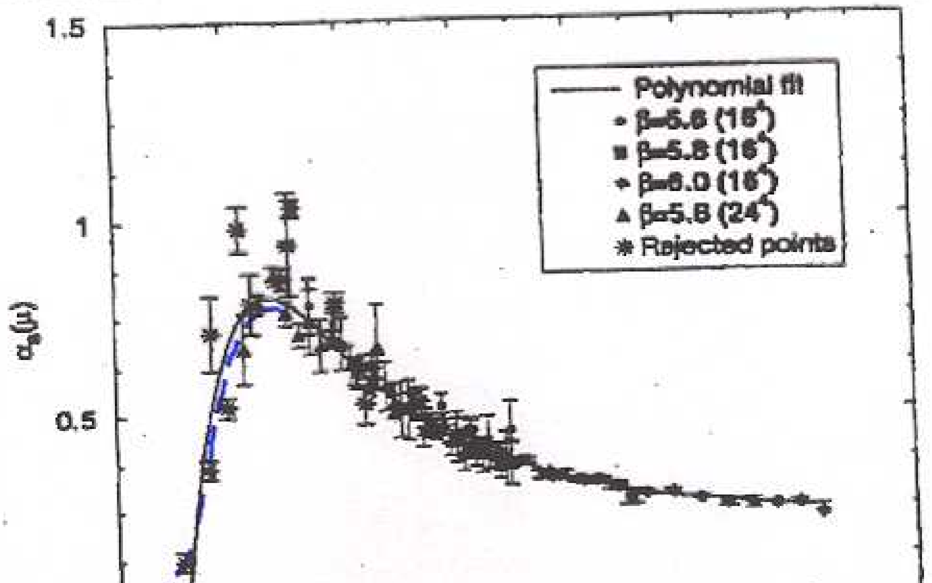


Figure 3: The IR behavior of "Paris" $\bar{\alpha}_s$.

Power decreasing ? $\bar{\alpha}_P(Q^2) \simeq (Q^2)^{\nu}$; $\nu \gtrsim 2$

"Transoceanic", lattice results.

Here, the invariant coupling is defined via **gluon–quark vertex** in the special MOM scheme

$$\bar{\alpha}_{\mathrm{TO}}(Q^2) \sim \Gamma^2_{gl-q}(0 : Q^2, Q^2)$$

with gluon momentum equal to zero. The results for $\bar{\alpha}_{\mathrm{TO}}(Q^2)$ obtained by lattice simulation, qualitatively, are close to the "Paris" ones – see Fig. 4

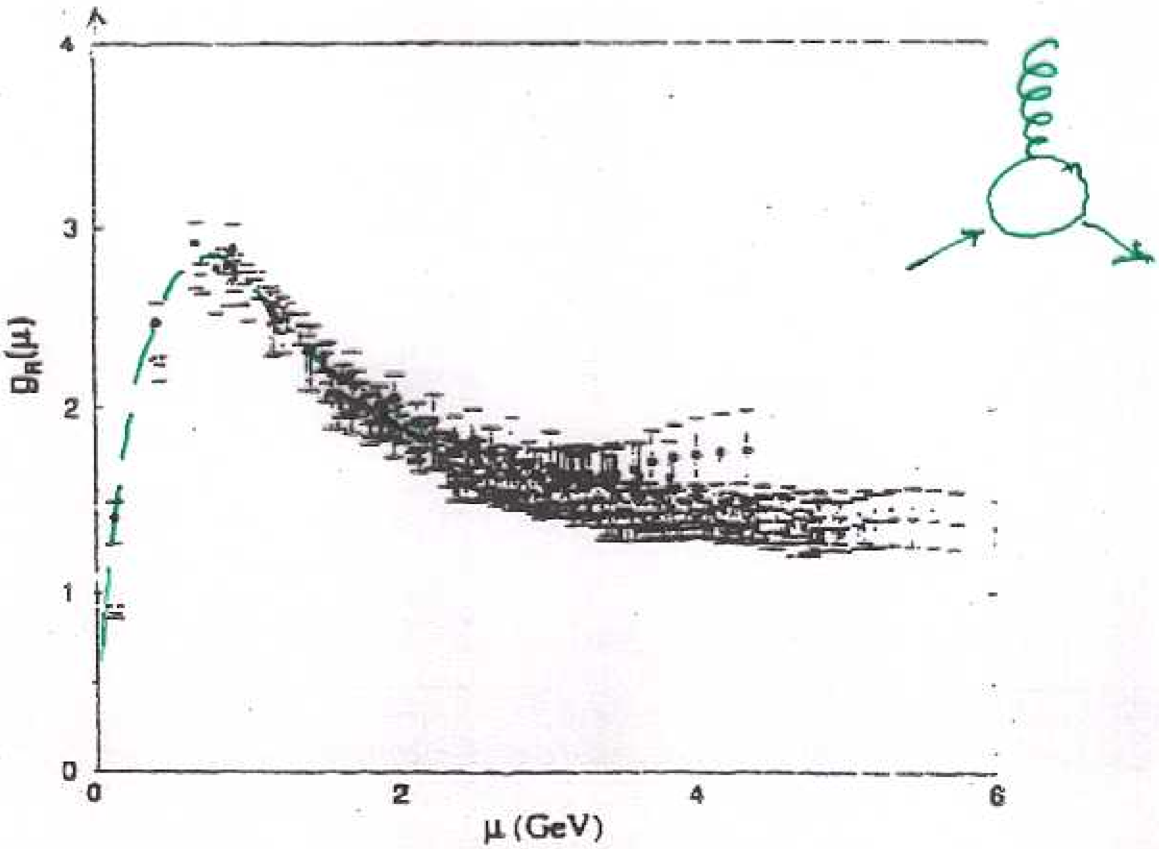


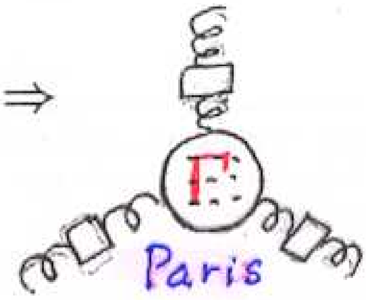
Figure 4: Running of the quark–gluon coupling $\bar{g}$.

J. L. Skullerud et al. hep-lat/0109027

The "vertex" dependence

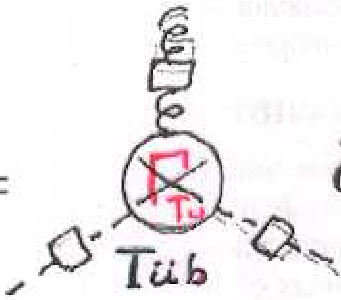
In QCD Lagrangian there are several structures with the same coupling constant. Accordingly, several possibilities for defining effective coupling, e.g. :

3–gluon vertex $\bar{\alpha}_s(Q^2) \Rightarrow$
$\alpha_s\, \Gamma^2_{3gl}(Q^2) Z^3(Q^2) =$



$\bar{\alpha}_{3\mathrm{gl}}$

gluon–ghost —
$\alpha_s \Gamma^2_{gh,gl}(Q^2)\, Z(Q^2) D^2(Q^2) =$



$\bar{\alpha}_{\mathrm{gl,gh}}$

gluon–quark —
$\alpha_s\, \Gamma^2_{gl,q\bar{q}}(Q^2) Z(Q^2) S^2(Q^2) =$

$\bar{\alpha}_{\mathrm{gl},q\bar{q}}$

In the mass–dependent case they are different !
— IR singularities at light-quark threshold

The "APT induced" massless coupling constant transformations

$$\alpha_s \to \alpha_M(\alpha_s) = \frac{1}{\pi\beta_0} \arccos \frac{1}{\sqrt{1+\pi^2\beta_0^2\alpha_s^2}}$$

$$\alpha_s \to \alpha_E(\alpha_s) = \alpha_s + \frac{1}{\beta_0} \cdot \frac{1}{1-e^{1/\beta_0\alpha_s}}$$

with effective coupling transformations

$$\bar{\alpha}_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} = \frac{1}{\beta_0 L} \to \tilde{\alpha}(Q^2) =$$

$$= \frac{1}{\pi\beta_0} \arccos \frac{L}{\sqrt{L^2+\pi^2}}\bigg|_{L>0} = \frac{\arctan(\pi/L)}{\pi\beta_0};$$

$$\bar{\alpha}_s(Q^2) \to \alpha_{\mathrm{an}}(Q^2) = \frac{1}{\beta_0}\left[\frac{1}{L} - \frac{\Lambda^2}{Q^2-\Lambda^2}\right]$$

— without the ghost singularitiesand

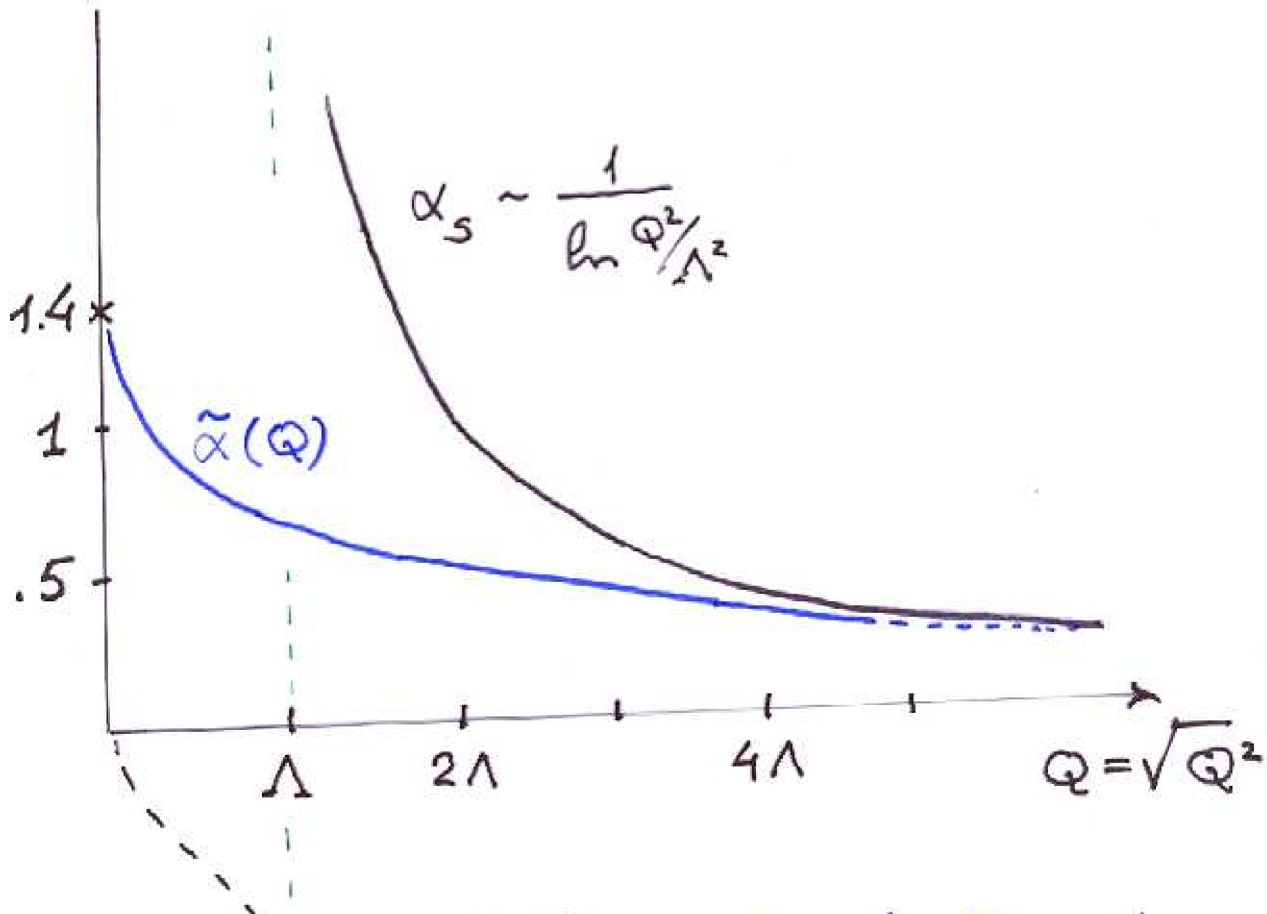
— with smooth IR behavior:

$$\tilde{\alpha}(0) = \alpha_{\mathrm{an}}(0) = \frac{1}{\beta_0} \simeq 1.4$$

$$\alpha_{[S]} \Rightarrow \alpha_M(\alpha) = \frac{1}{\pi\beta_0} \arccos \frac{1}{\sqrt{1+\pi^2\beta_0^2\alpha^2}}$$

$$\simeq \frac{1}{\pi\beta_0} \arctan(\pi\beta_0\alpha) \simeq \alpha - \frac{(\pi\beta_0)^2}{3}\alpha^3 + O(\alpha^5)$$

$$\alpha_s(Q) = \frac{1}{\beta_0 L} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2} \Rightarrow \underbrace{\frac{1}{\pi\beta_0} \arctan \frac{\pi}{L}}_{\tilde{\alpha}(Q^2)} \quad (L>0)$$



$$\alpha_M(\alpha \to \pm\infty) \to \frac{1}{\pi\beta_0} \cdot \frac{\pi}{2} = \frac{1}{2\beta_0} \sim 0.7$$

$$\alpha_M(\alpha \to -0) \to \frac{1}{\beta_0} \sim 1.4$$

The mass–dependent coupling transformations

Here, the IC transformation looks like

$$\bar{\alpha}_s \to \bar{\alpha}^*(x, y; \alpha^*) = N\left\{\frac{y}{x}, \bar{\alpha}_s\left[x, y; n\left(\frac{y}{x}, \alpha_s\right)\right]\right\}.$$

It contains the variable $y/x = m^2/Q^2$ that influences the IR behavior. To illustrate, take a model expression

$$\alpha_s \to \quad N(y, \alpha) = \alpha_M(\alpha_s)\left(\frac{1-y}{1+y}\right)^{c\,\alpha_M(\alpha_s)}$$

that yields

$$\bar{\alpha}^*(Q^2) = \tilde{\alpha}(Q^2)\left(\frac{Q^2 - m^2}{Q^2 + m^2}\right)^{c\,\tilde{\alpha}(Q^2)}$$

with smooth $\tilde{\alpha}(Q^2)$ in the exponent.

“ALPHA” – Schrödinger functional

The "Schrödinger functional" defined in the Euclidean space–"time" manifold in a specific way — all 3 space dimensions are cyclic, while the "time" one is singled out — the gauge fields on "upper" and "bottom" edges differs by phase factor $\exp(-\eta)$.

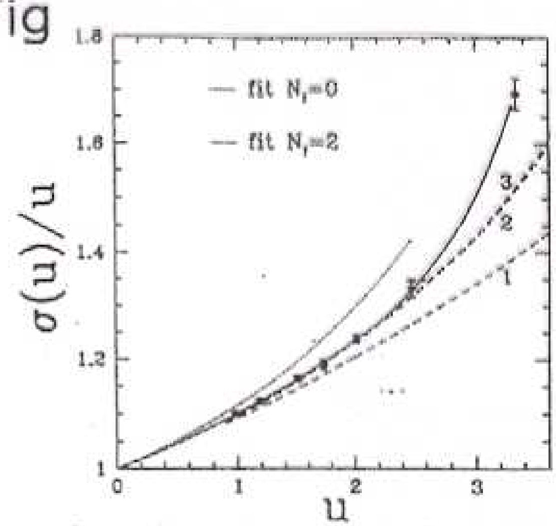
The effective coupling is defined via derivative $\Gamma' = \partial\Gamma/\partial\eta$ of effective action $\Gamma = \alpha^{-1}\Gamma_0 + \Gamma_1 + \alpha_s\Gamma_2 + \ldots$ as

$$\bar{\alpha}_{SF}(L) \equiv \Gamma_0'/\Gamma' ,$$

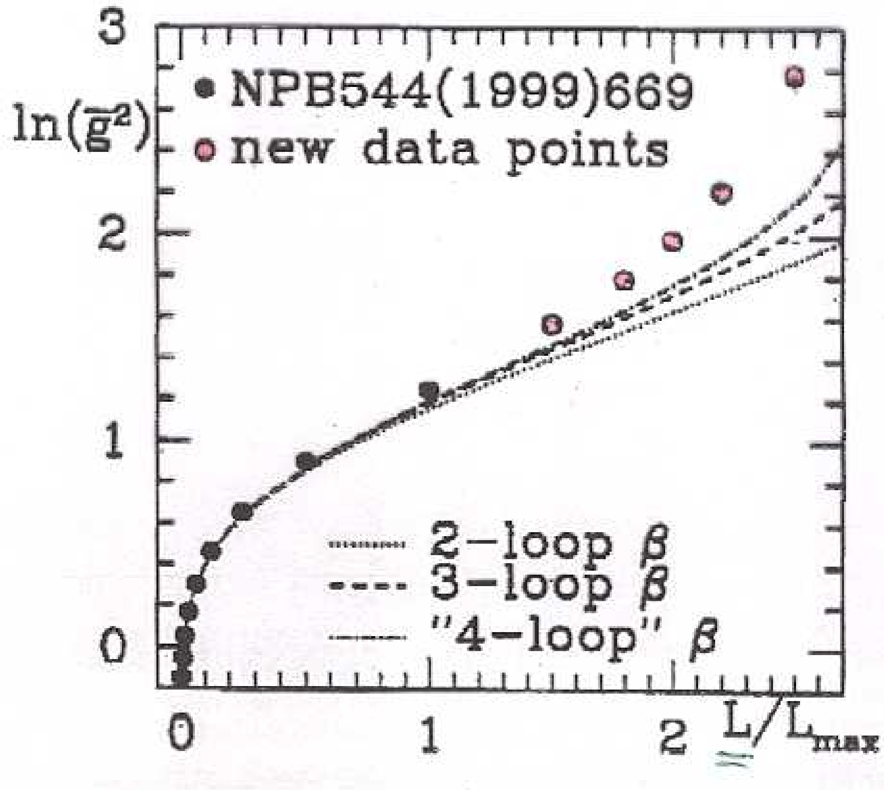
as a function of L, the spatial size of the mentioned manifold.

To follow the $\bar{\alpha}_{SF}(L)$ evolution, a special trick with "step scaling function" $\sigma(\bar{\alpha}_{SF}(L)) = \bar{\alpha}_{SF}(2L)$ was used.

Numerically, $\sigma(\alpha)$ was defined by comparing simulations results for lattice L with $2L$. This gives $\sigma(\alpha) \simeq \alpha^2$ — see Fig



Correspondingly, the steep rise of the SF coupling in the region $\bar{\alpha}_{SF} \simeq 1$.



Function of Lattice size L
(in cm)

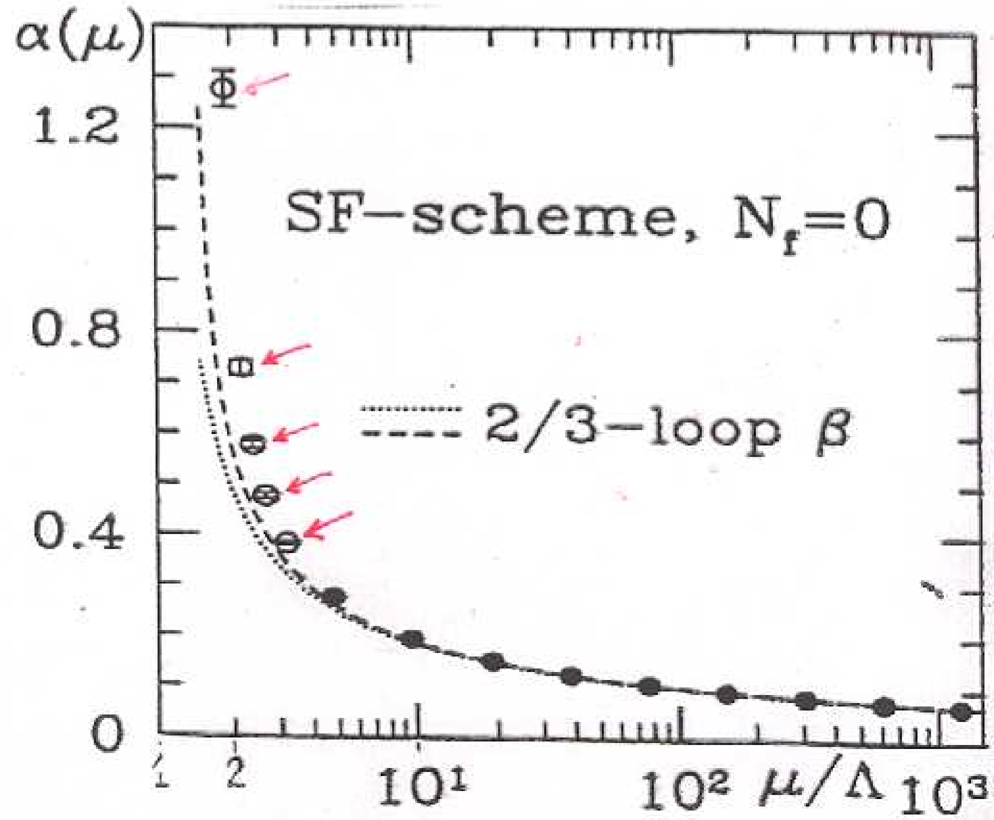
Analytic approximation for numerically calculated $\bar{\alpha}_{SF}$ is of exponential (!) form

$$\bar{\alpha}_{SF}(L) \simeq e^{mL} \quad \text{with} \quad m \simeq 2.3/L_{\max}$$

and $L_{\max}$, is close to $1/m_T$.

Transition to momentum repr.

Simple–minded common "quantum mechanical correspondence" $\boxed{L \to \frac{1}{Q}}$:



Johen Heitger et al. hep-lat/0110201

→ **exponential** $\exp\left(\frac{m}{Q}\right)$ **IR asymptotics**

Correlation between results in

"position Long-Range" $\mathrm{L} \to \infty$ and **"momentum IR"** $\mathrm{Q} \to 0$

Usual "quantum–mechanical correspondence" $\mathrm{L} \to 1/\mathrm{Q}$ **is valid only for a special class** of functions (Tauber's conditions).

E.g., Fourier sine-transformation :

$$\bar{f}(Q^2) = \frac{2}{\pi} \int_0^\infty \frac{d\,L}{L} \sin(QL)\, f(L^2) \equiv F_{\sin}[f](Q^2)$$

that follows from the 3-dimensional one

$$\tilde{\varphi}(Q^2) = (2\pi)^{-2} \int d\mathbf{L} \varphi(L^2) e^{i\mathbf{QL}}\,;$$

$$\tilde{\varphi}(Q^2) = Q^2 \bar{f}\,(Q^2 \equiv \mathbf{Q}^2)\,, \quad \varphi = L\, f\,(L^2)\,.$$

$$\left.\begin{aligned} &F_{\sin}[1] = 1\,; \quad F_{\sin}[\ln L] = -\ln Q + const.\\ &F_{\sin}[L^{-\mu}] = C(\mu)\, Q^{\mu}\,. \end{aligned}\right\} Q = \frac{1}{L}$$

This is not the case for $f \sim e^{mL}$!

Reciprocality of Fourier images

$$F(Q) \sim \int_{(0,-\infty)}^{(\infty)} e^{iQL} f(L) \frac{dL}{L} \qquad \boxed{F(Q) \sim f(1/Q)}$$

as $L \to \infty$, $Q \to 0$

Large-distance IR

$$\underset{\text{as}}{F}(Q) \sim \int e^{ix} \frac{dx}{x} \cdot \underset{\text{as}}{f}\left(\frac{x}{Q}\right)$$
?

is valid for a class of functions f !

Admissible

$f_{as}(L)$	$F_{as}(Q) = f_{as}(1/Q)$
C	C
L^{-K}	Q^{K}
$\ln L$	$\ln \frac{1}{Q}$

Forbidden

$f_{as}(L)$	$F_{as}(Q)$
Le^{-mL}	$\Rightarrow \dfrac{Q}{m^2+Q^2}$
Le^{mL}	$\Rightarrow \dfrac{Q}{(m-K)^2+Q^2}$

Generalized Fourier Transformation $\quad F(Q) \to F(Q \pm iK)$

Equivalent to cutoff e^{-KL}

This is not the case for $f \sim e^{mL}$!
Here, it is possible to use
generalized Fourier transformation.
That is equivalent, in a sense to
introduction of exponential cutoff factor

$$F(L) = exp\{\theta(L - L^*)\,\mu\,(L^* - L)\}\,; \qquad (3)$$

$$L^* = \xi L_m\,; \quad \xi = 2.5 \div 3\,, \quad \mu \geq m\,.$$

$$\alpha_{\mathrm{AL}}(L) \simeq \alpha_{\mathrm{AL,k}}(L) = D_k \frac{\pi}{2} \left(\frac{L}{L_m}\right)^k e^{mL} \qquad (4)$$

with $mL_m \simeq 2.0$ and $D_1 \simeq 0.0013$.

Now, the integration performing of Fourier sine-transformation technically is very simple as the exponent is linear in L!
The non-perturbative part of coupling function in the momentum representation is representable as a sum of two terms

$$\bar{\alpha}_k(Q) = \bar{\alpha}_{\mathrm{reg,k}}(Q) + \bar{\alpha}_{\mathrm{sing,k}}(Q)\,;$$

$$\bar{\alpha}_k(Q) = \bar{\alpha}_{\mathrm{reg,k}}(Q) + \bar{\alpha}_{\mathrm{sing,k}}(Q)\,;$$

$$\bar{\alpha}_{\mathrm{reg,k}}(Q) = \frac{D_k}{L_m^k}\int_0^{L^*} dL\, L^k \sin(QL) e^{mL} =$$

$$= \frac{D_k}{L_m^k}\left[f_k(Q, L^*) - f_k(Q, 0)\right]\,;$$

$$\bar{\alpha}_{\mathrm{sing,k}}(Q) = e^{\mu L^*}\frac{D_k}{L_m^k}\int_{L^*}^{\infty} dL\, L^k \sin(QL) e^{(m-\mu)L}\,,$$

$$\bar{\alpha}_{\mathrm{sing,k}}(Q) = \frac{D_k}{L_m^k}\varphi_k(Q, L^*; \mu)\,.$$

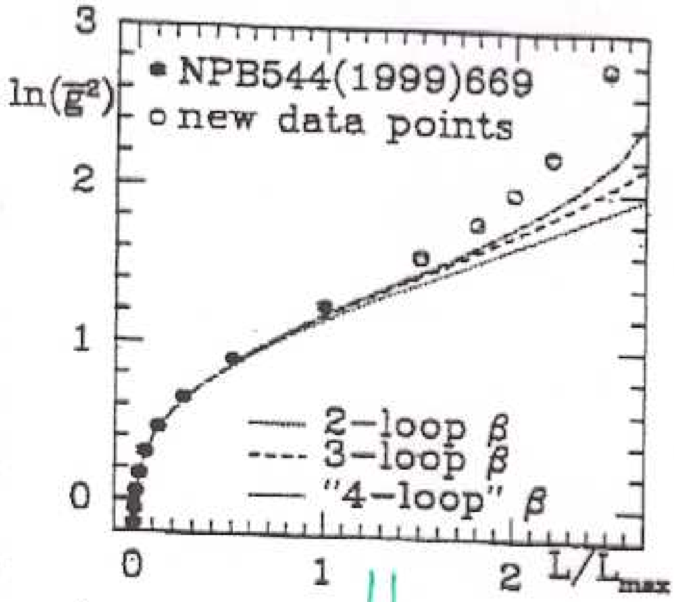
$$f_1(Q, L) = e^{mL}\,\frac{m\sin(QL) - Q\cos(QL)}{m^2 + Q^2}\,,$$

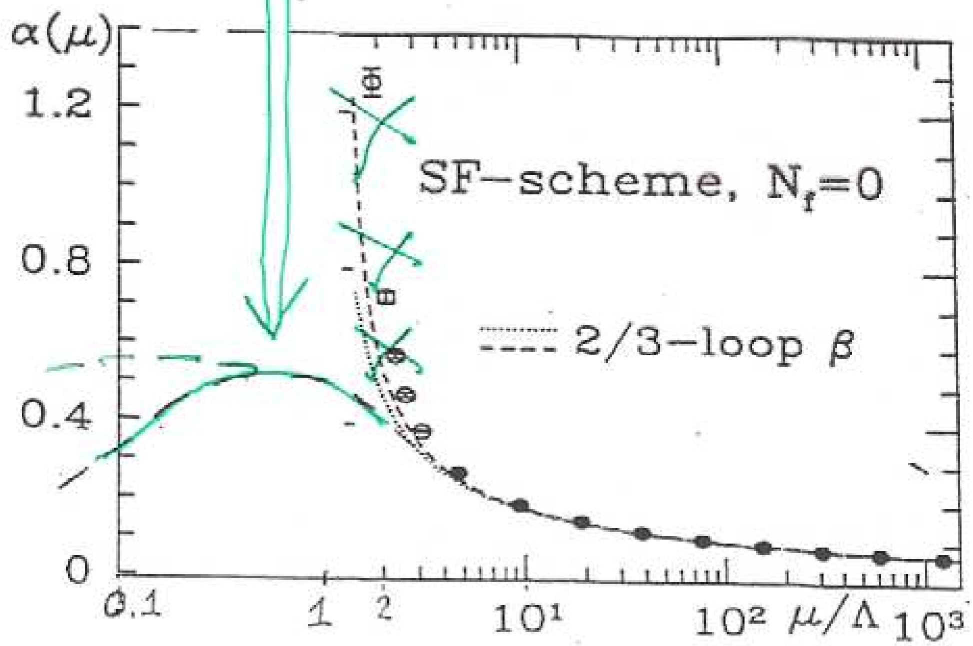
$$\varphi_1(Q, L; \mu) = e^{mL}\,\frac{Q\cos(QL) + (\mu - m)\sin(QL)}{(m-\mu)^2 + Q^2}$$

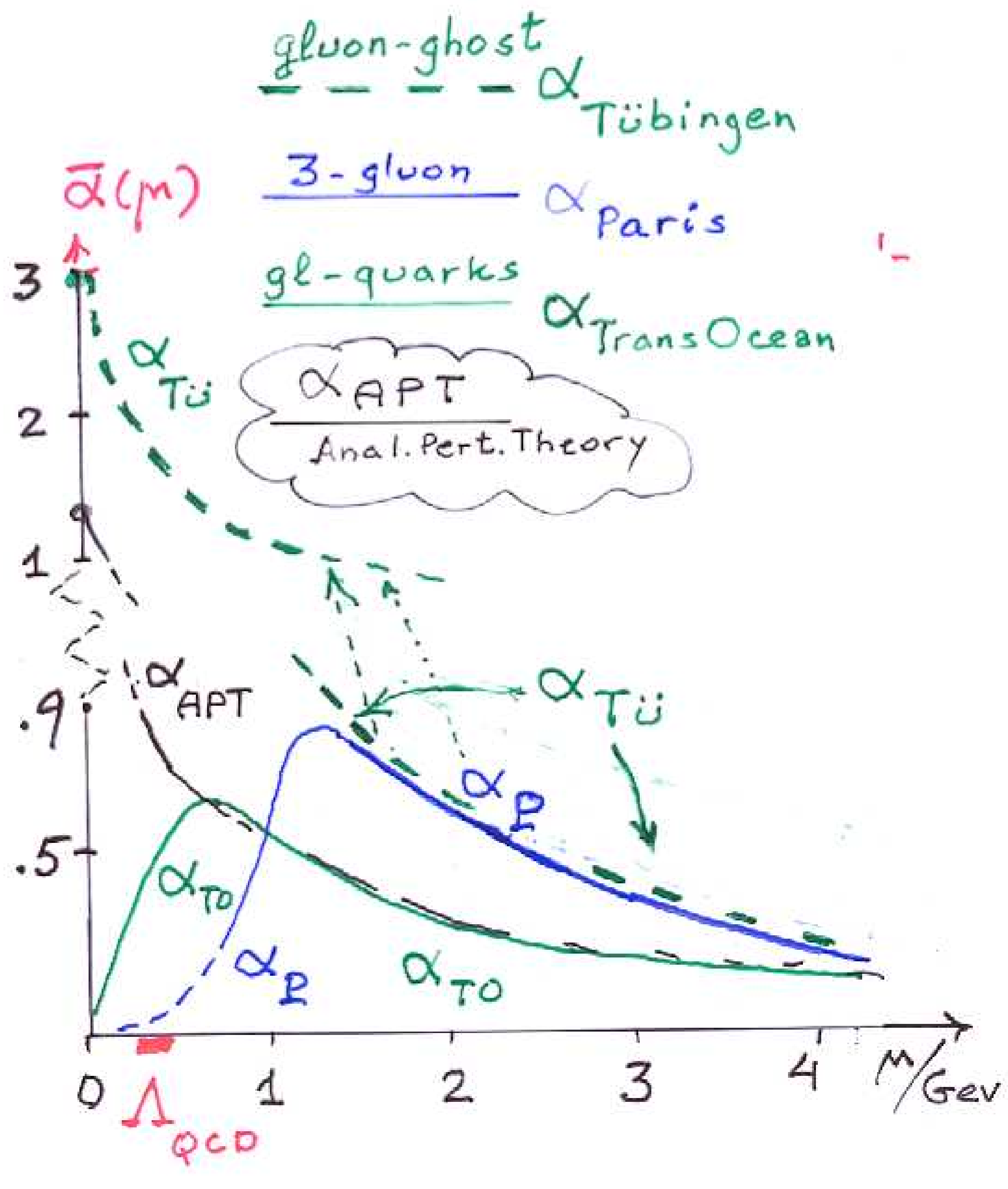
$$f_1(Q, L) \simeq \varphi_1(Q, L) \to c\,Q \quad \text{as} \quad Q \to 0\,,$$

Hence, $\bar{\alpha}_{SF}(Q=0) = C \geq 0$;

with finite C contrary to the exp. growth $\sim \exp(m/Q)$ of ALPHA group obtained via quant.-mech. relation $L \to 1/Q$.






gluon-ghost
$\alpha_{Tübingen}$
3-gluon
α_{Paris}
gl-quarks
$\alpha_{TransOcean}$
α_{APT}
Anal. Pert. Theory
$\bar{\alpha}(\mu)$
$\alpha_{Tü}$
α_{APT}
$\alpha_{Tü}$
α_{P}
α_{TO}
α_{P}
α_{TO}
Λ_{QCD}
μ/Gev

Conclusion

- All non-perturbative calculations of $\bar{\alpha}_s(Q^2)$ — lattice and SDE — reveal smooth behavior in the IR :
  - No unphysical singularity;
  - Finite or vanishing $\bar{\alpha}(Q = 0)$.
- The ALPHA results in the IR could be understood in a similar way.
- Diversity of results is compatible with **vertex and mass dependence.**
- The IR property of the QCD objects is not the physical question, unless it is not related to confinement and hadronization.

The End of Shirkov's talk