High Density QCD, Saturation and Diffractive DIS

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⋆ Selected issues in diffractive DIS

⋆ Measuring the pion WF in hard diffraction into dijets off nuclei

⋆ Why diffraction is crucial for understanding the saturation of nuclear partons?

⋆ (Vanishing ?) impact of intranuclear rescattering on nuclear parton densities

⋆ Signatures of saturation in diffractive DIS

⋆ Antishadowing in gluon-sea fusion vs. gluon-gluon fusion

From incluisve to diffractive DIS

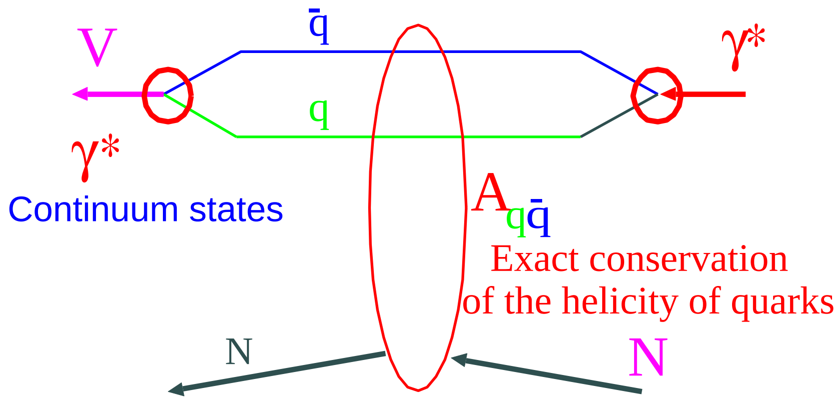
The unitarity relates the inclusive DIS structure function to the forward, $Q_f^2 = Q_{in}^2 = Q^2$ Compton scattering (CS)

$$\gamma_\mu^*(Q_{in}^2)p \to \gamma_\nu^*(Q_f^2)p'$$

Analytic continuation:
$Q_f^2 = 0 \implies$ DVCS
$Q_f^2 = -m_V^2 \implies$ diffractive vector meson (VM) production

$$\gamma_\mu^*(Q^2)p \to V_\nu(\mathbf{\Delta})p'(-\mathbf{\Delta})$$

Self-analyzing decays of V: the full set of helicity amplitudes $A_{\nu\mu}$ probes the mechanism of diffractive DIS to a full complexity.



Color dipole approach to diffractive DIS and diffractive VM production: NNN, B.G. Zakharov (1991), Kopeliovich, B.G. Zakharov (1991), NNN (1992), Kopeliovich et al. (1993), Nemchik, NNN, B.G. Zakharov (1994).

Color dipole (CD) factorization, shrinking photons and $(Q^2 + m_V^2)$ scaling

The CD X-section/amplitude is related to the gluon SF of the target (Barone et al. (1993), NNN, B.G. Zakharov (1993))

$$\sigma(x,\boldsymbol{r}) \approx \frac{\pi^2}{3} r^2 \alpha_S(\frac{A}{r^2}) G(x, \frac{A}{r^2}) \,, \qquad A \approx 10$$

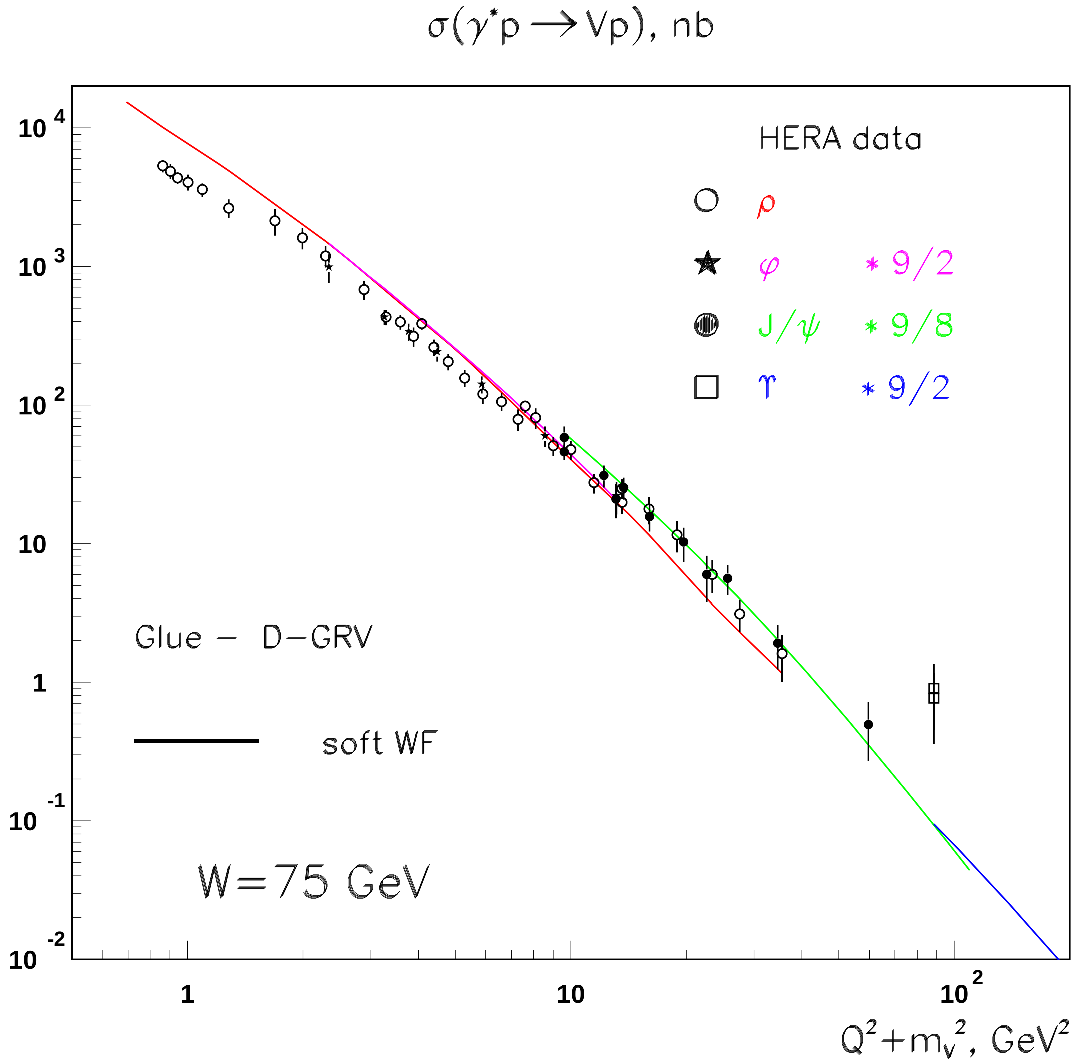
Photons shrink with Q^2 and diffractive VM production probes the CD X-section at a scanning radius (NNN (1992))

$$r_S = \frac{6}{\sqrt{Q^2 + m_V^2}} \,,$$

$$A(\gamma^* \to V) \propto r_S^2 \sigma(x, r_S) \propto \frac{1}{(Q^2 + m_V^2)} \cdot \frac{G(x, \tau \cdot (Q^2 + m_V^2))}{(Q^2 + m_V^2)}$$

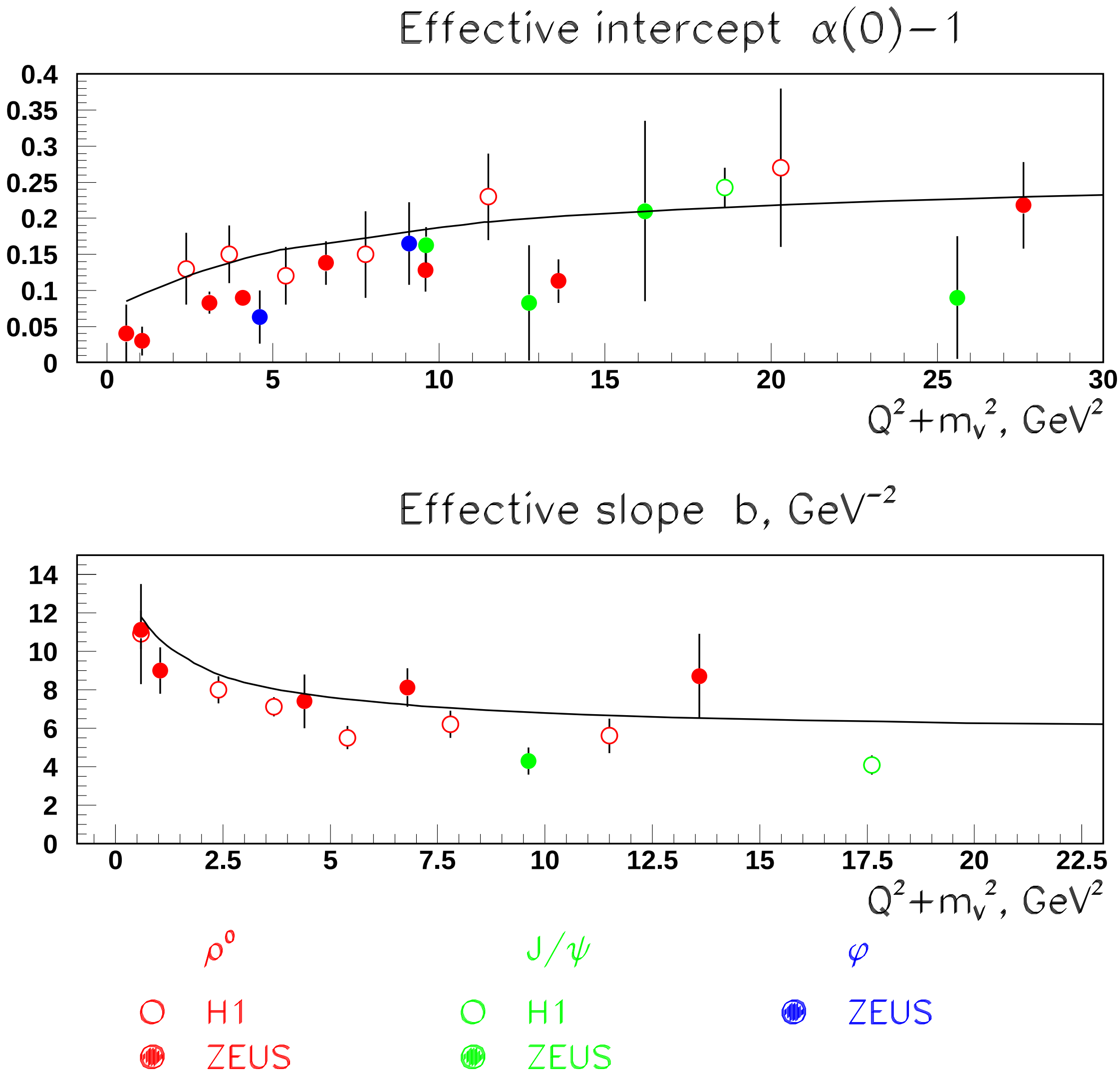
The three fundamental consequences (Nemchik, NNN, Predazzi, Zakharov (1994,1997), NNN, Zakharov, Zoller (1995)):
i) the V production probes the gluon SF of the target
ii) the $(Q^2 + m_V^2)$ scaling modulo the charge-isospin factors and slight flavour dependence of the scale τ.
iii) the contribution to the diffraction slope B from the $\gamma^* \to V$ transition vertex decreases $\propto r_S^2$.

The $(Q^2 + m_V^2)$ scaling of the total cross section $\sigma_L + \sigma_T$:



The calculations by I. Ivanov based on the unintegrated glue of the proton determined from the DIS SF's (Ivanov, NNN (2000)), includes the slight flavor dependence of the hard scale $\tau \cdot (Q^2 + m_V^2)$ (Nemchik, NNN, Zakharov (1994)):
$\tau(\rho) \approx 0.07 - 0.1 \to \tau(J/\Psi) \approx 0.2.$

The $(Q^2 + m_V^2)$ scaling of the effective intercept and diffraction slope:



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$\tau(\rho) \approx 0.07 - 0.1 \rightarrow \tau(J/\Psi) \approx 0.20.$

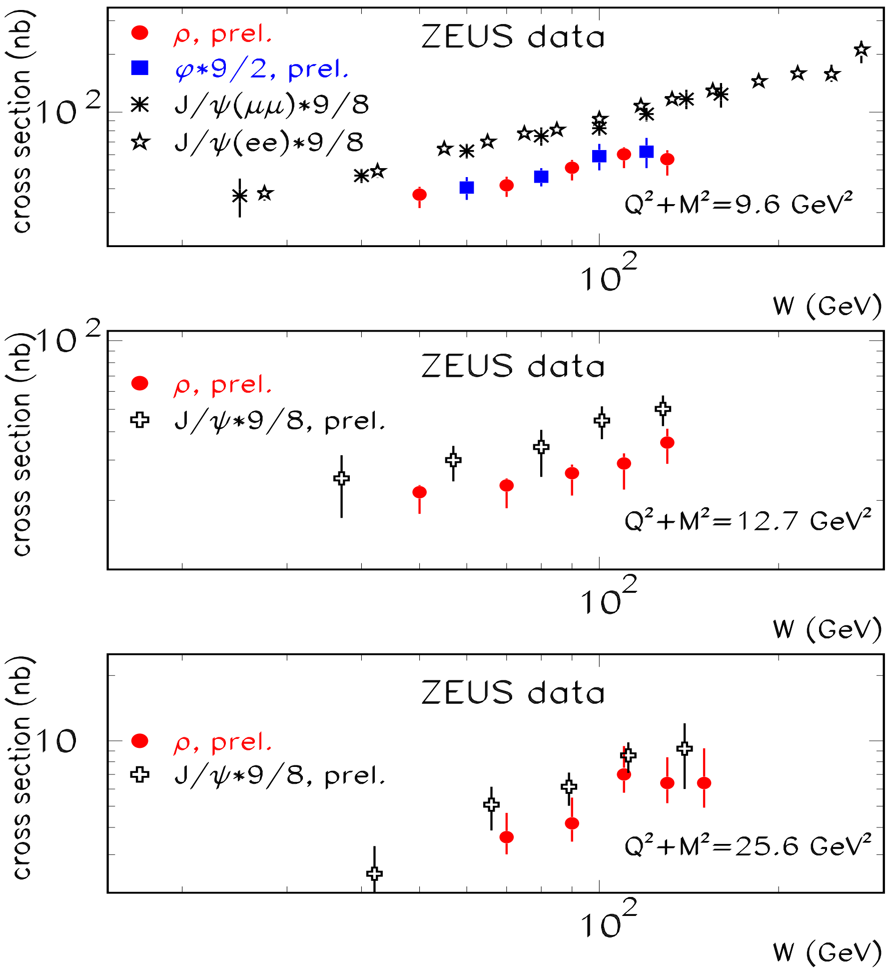
The restoration of flavour symmetry in the variable $(Q^2 + m_V^2)$ is not exact because of the flavour-dependent Fermi motion: nonrelativistic in the J/Ψ vs. relativistic in the ρ

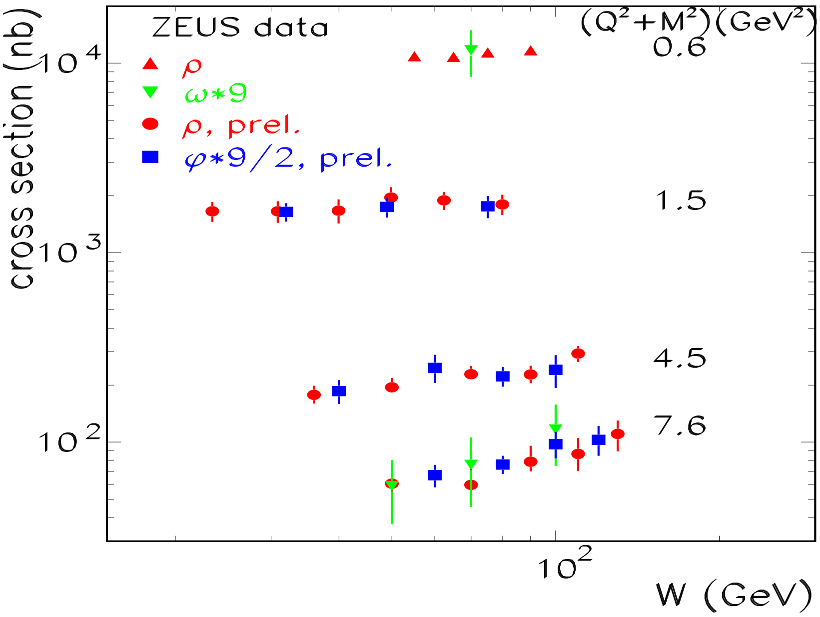
⋆ The hard scales depend on flavour (theory: Nemchik, NNN, Zakharov (1994)):

$$(Q^2+M_V^2)_\rho = \frac{\tau(\rho)}{\tau(J/\Psi)}(Q^2+M_V^2)_{J/\Psi} \approx 0.5\cdot(Q^2+M_V^2)_{J/\Psi}$$

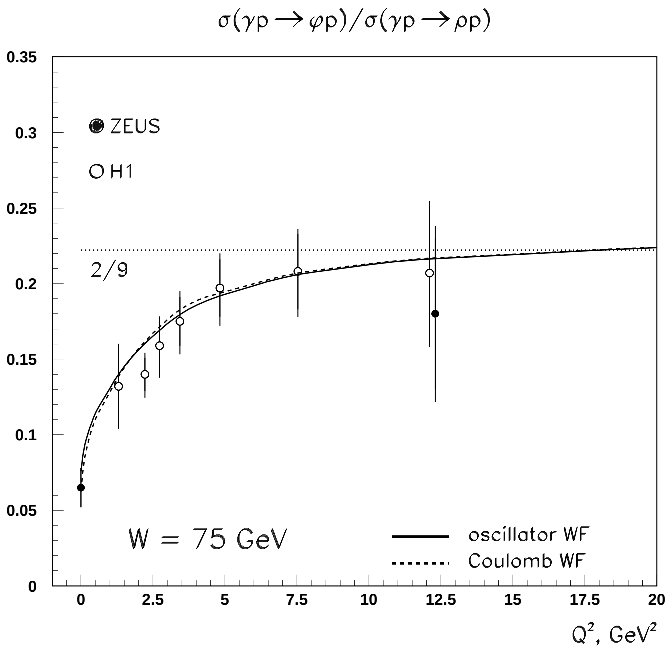
⋆ The absolute normalization is sensitive to the Fermi motion of quarks

⋆ The experimental evidence from ZEUS:





An example of the theoretical prediction for the onset of the $(Q^2 + m_V^2)$ scaling (I.Ivanov (2001): the ϕ/ρ ratio



Shrinkage of the diffraction cone

The moving Regge pole with the trajectory

$$\alpha_{\rm I\!P}(t) = \alpha_{\rm I\!P}(0) + \alpha'_{\rm I\!P} t$$

Gribov's rise of the diffraction slope with energy

$$B(W^2) = B_0 + 2\alpha'_{\rm I\!P} \log(W^2/W_0^2)$$

⋆The common prejudice based on scaling α_S =const approximation without the length scale: $\alpha'_{\rm I\!P} = 0$.

⋆ Fadin, Kuraev and Lipatov (1975): asymptotic freedom splits the fixed branching point into the sequence of moving poles.

⋆ Subleading poles have multinodal color dipole eigen-X-sections (Lipatov (1986), NNN, Zakharov, Zoller (1995)).

⋆ The direct calculation for hard BFKL pomeron from the color dipole BFKL equation (NNN, Zakharov, Zoller (1995)):
$\alpha'_{\rm I\!P} \approx 0.07 {\rm GeV}^{-2}$

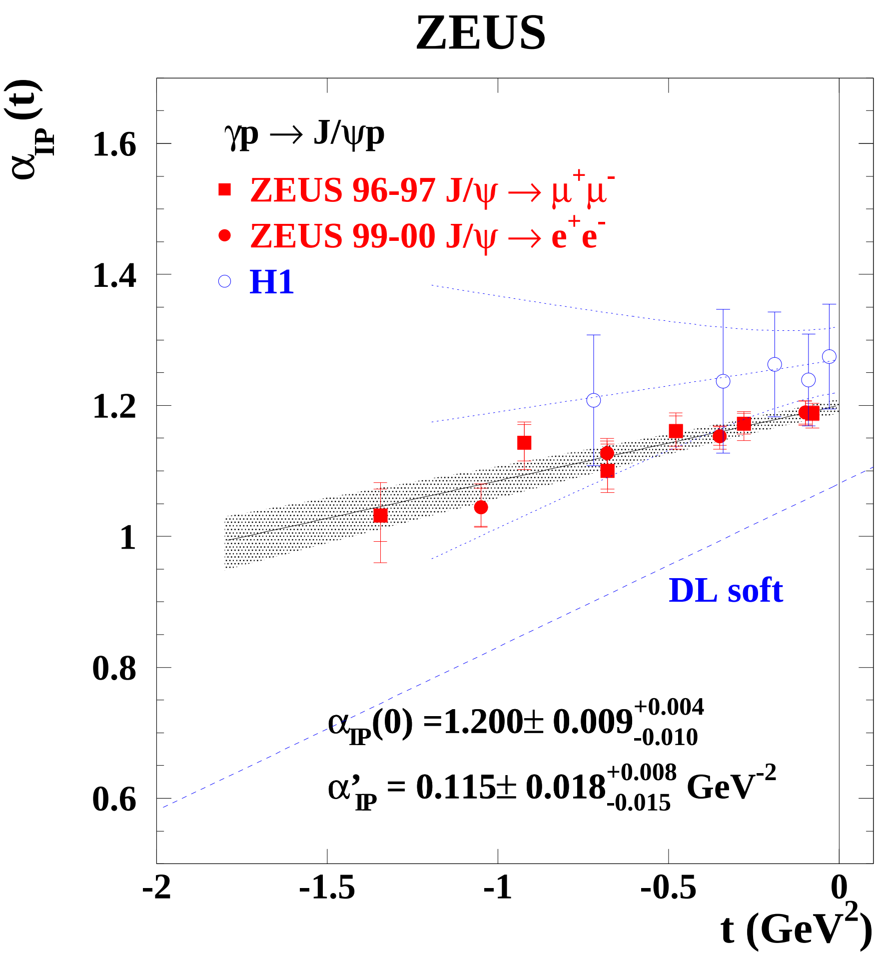
⋆ The scale set by the propagation radius of perturbative gluons, $R_c \sim 0.2 - 0.3$ fm (instanton vacuum, lattice QCD...).

The effect of subleading BFKL poles in the HERA energy range (NNN, Zakharov, Zoller (1995))

$$\alpha'_{eff} \sim 0.15\mathrm{GeV}^{-2}$$

The fundamental confirmation in the hard diffractive photoproduction of the J/Ψ:

$$\mathrm{ZEUS(2001)}: \quad \alpha'_{eff} = 0.115 \pm 0.018^{+0.008}_{-0.015}\ \mathrm{GeV}^{-2}$$



$\Psi(2S)$: probing the radial structure of vector mesons

The radius of the radial excitation of charmonium $\Psi(2S)$ is large, as large as the radius of the light vector meson ϕ:

$$R(\Psi(2S)) \sim R(\phi) \sim 2 \cdot R(J/\Psi)$$

The naive expectation: the larger the size of the $\gamma^* \to V$ transition vertex the larger is the diffraction slope:

$$B(\Psi(2S)) \sim B(\phi) > B(J/\Psi)$$

But the radial wave function of the $\Psi(2S)$ has a node at

$$r_{node} \sim R(J/\Psi)$$

The two effects of the oscillating wave function:

$\star$ suppression of the diffractive $\Psi(2S)$ production compared to the J/Ψ production (Kopeliovich, Zakharov (191), NNN(1992)):

$$\frac{\sigma(\gamma^* \to \Psi(2S))}{\sigma((\gamma^* \to J/\Psi)} \sim 0.2$$

$\star$ The counterintuitive inequality of diffraction slopes (Nemchik, NNN, Predazzi, Zakharov, Zoller (1997)):

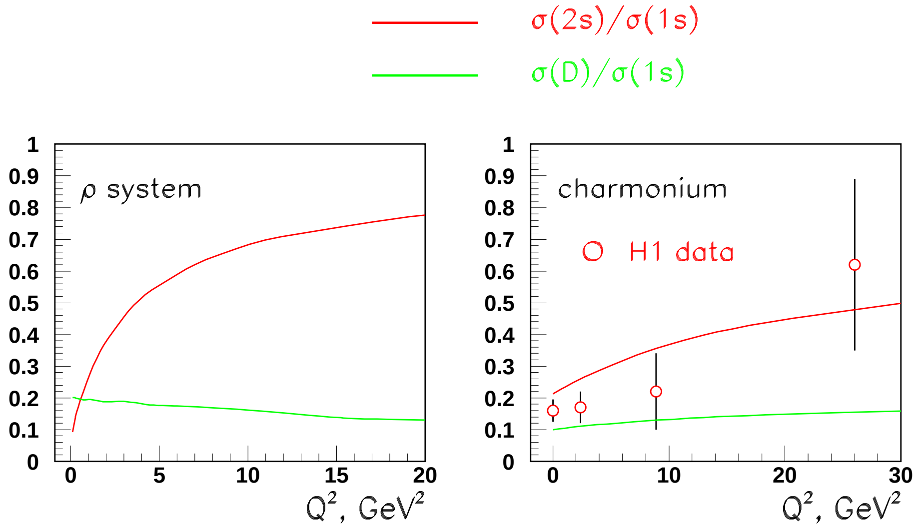
$$B(\Psi(2S)) \lesssim B(J/\Psi)$$

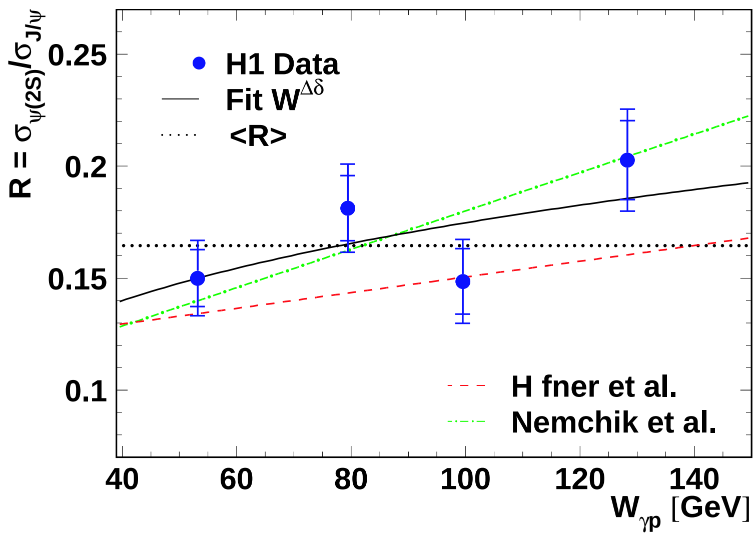
The both node effects go away:

⋆ with Q^2 as soon as $r_S \ll r_{node}$.

⋆ with energy because of the faster rise of the dipole X-section for small dipoles.

Diffractive production of the node-free orbitally excited D-wave state is similar to that of the J/Ψ.





The inequality of the diffraction slopes $B(\Psi(2S)) \lesssim B(J/\Psi)$ (Nemchik, NNN, Predazzi, Zakharov, Zoller (1997)):

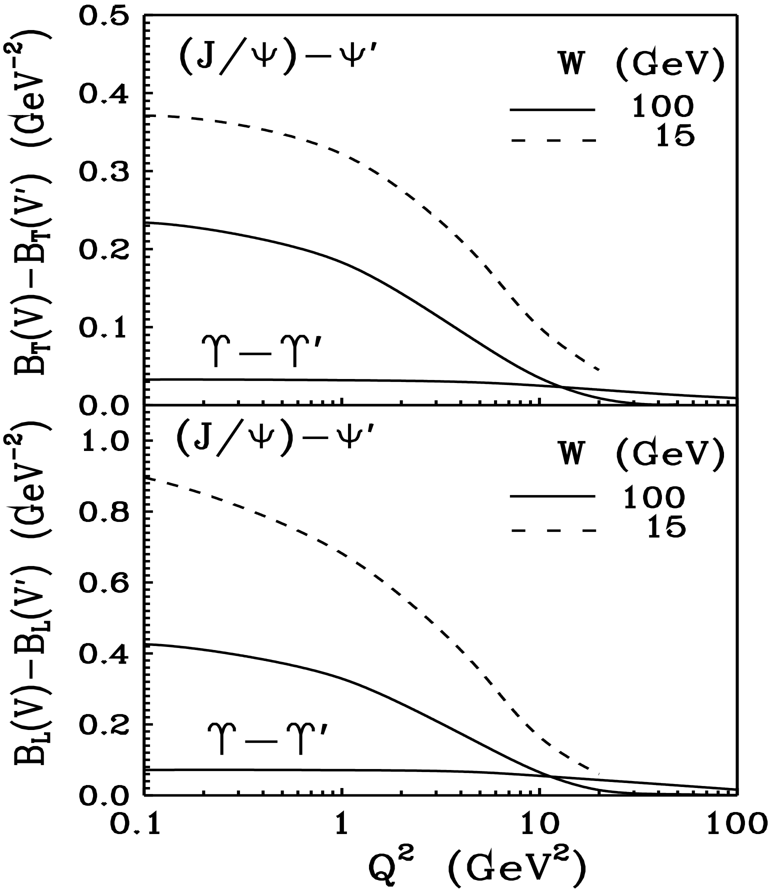


Fig.17

The experimental results from H1 (2002):

⋆ Elastic production:

$B(J/\Psi) = 4.99 \pm 0.13 \pm 0.39 \mathrm{GeV}^{-2}$

$B(\Psi(2S)) = 4.31 \pm 0.57 \pm 0.46 \mathrm{GeV}^{-2}$

Compare with $B(\phi) = 7.3 \pm 1.0 \pm 1.8 \mathrm{GeV}^{-2}$ ZEUS(1996)

⋆ Proton dissociative production:

$B(J/\Psi) = 1.07 \pm 0.03 \pm 0.11 \mathrm{GeV}^{-2}$

$B(\Psi(2S)) = 0.59 \pm 0.13 \pm 0.12 \mathrm{GeV}^{-2}$

Diffraction of pions into hard dijets and the pion wave function

Diffraction on nucleons and nuclei (NNN, W.Schäfer, Schwiete (2000)):

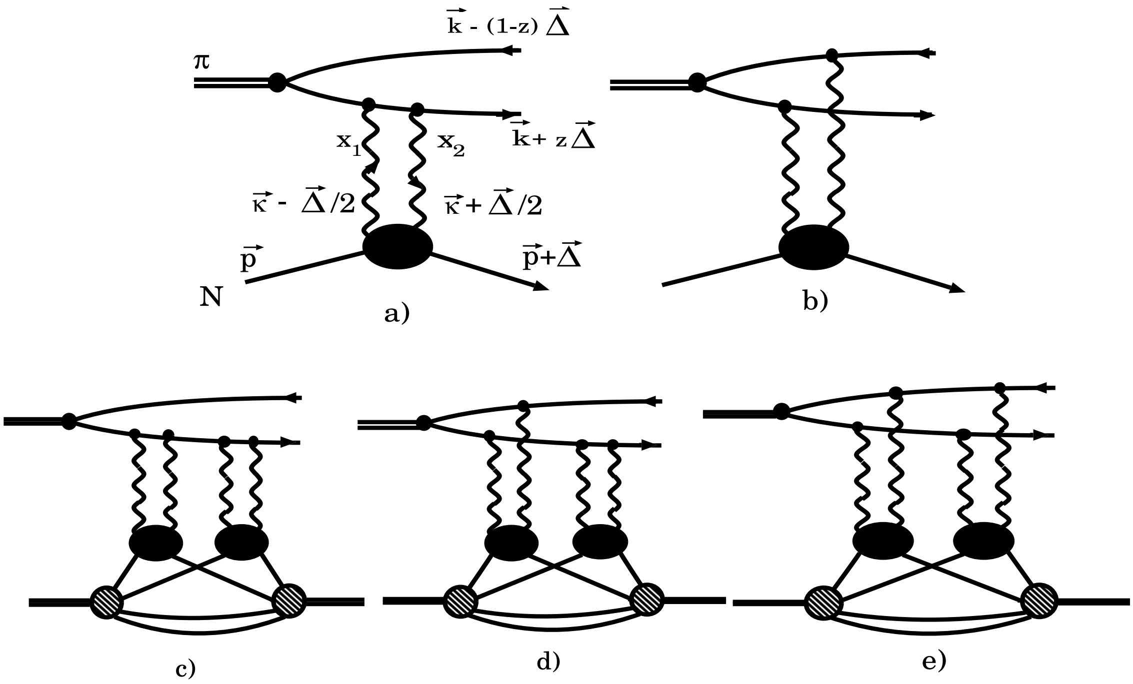


Fig. 1a – a counterpart of the classic Landau, Pomeranchuk, Feinberg, and Glauber (1953,1955) mechanism. Probes the limited intrinsic transverse momentum of q & $\bar{q}$ in the pion.

Fig 1b: the pomeron splitting, the transverse momentum of hard jets comes from gluons in the Pomeron (NNN, Zakharov (1992,1994)), probes the unintegrated gluon SF of the target; the longitudinal momentum distribution probes the one of q & $\bar{q}$ in the pion (the so called pion lightcone distribution amplitude).

⋆ The fundamental point (NNN, Zakharov (1992,1994)):

$$Amplitude(\pi + A \to A + Jet_1 + Jet_2) \propto \frac{dG(x, \mathbf{k}^2)}{d \log \mathbf{k}^2}$$

⋆ Holds also for nuclear targets!

⋆ Make use of diffractive DIS off nuclei for the definition of the nuclear gluon SF !.

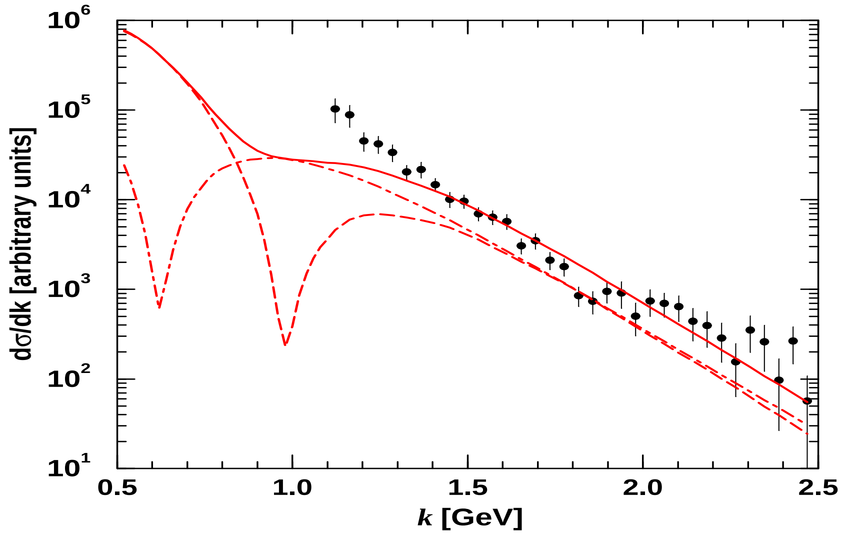
Has been erroneously challenged by Frankfurt, Miller, Strikman (2001), the dominance of the pomeron splitting mechanism was vindicated by Chernyak (2001, 2002) and Braun, Ivanov, A.Schäfer, Szymanowski (2001,2002). The issue is closed.

⋆ Multiple pomeron splitting - jets receive transverse momentum from several gluons, nuclear broadening and nuclear antishadowing for hard jets.

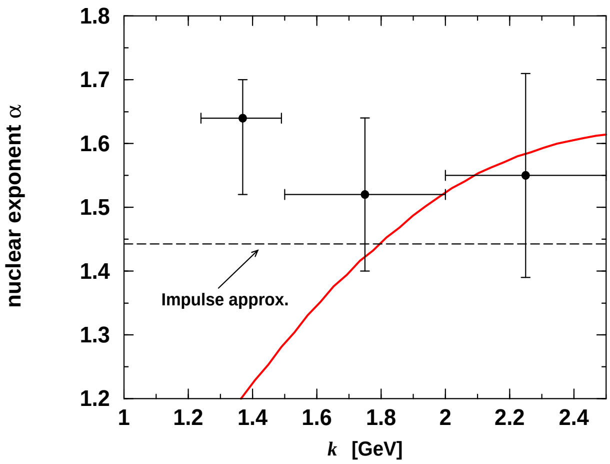
⋆ The E791 (SELEX) experiment at Fermilab: the experimental separation of coherent diffraction off a nucleus
$\pi + A \to A(\text{ground state}) + Jet_1 + Jet_2$
by a forward peak in the total transverse momentum of the dijet.
⋆ The experimental finding by E791: the pion lightcone distribution amplitude is close to the asymptotic one .

The parton model calculation is arguably applicable at the transverse momentum of a jet $k \gtrsim 1.5 - 2$ GeV.

The E791 data for the ^{196}Pt target vs. the theoretical calculations (NNN, Schäfer, Schwiete (2000)). The solid curve is the sum for the two helicity states of jets.



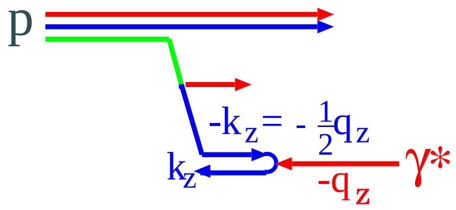
The exponent α of the A-dependence $\sigma \sim A^{\alpha}$:



The role of diffraction in the saturation of nuclear partons

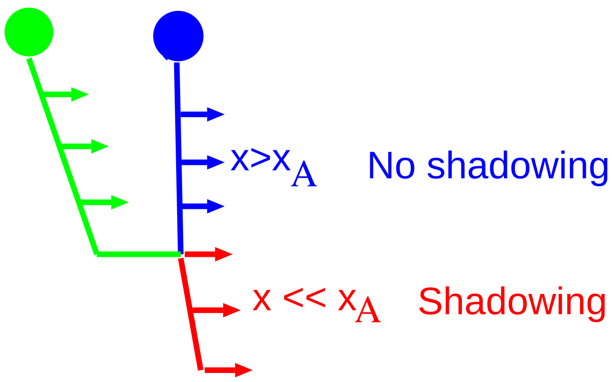
- V.N.Gribov (1969): the relationship between diffraction dissociation and nuclear shadowing.
- N.N.N. & V.I.Zakharov (1975): Nuclear shadowing as a result of fusion of partons in a Lorentz-contracted ultrarelativistic nucleus. Early ideas on fusion by Gribov & Kancheli (1973).
- N.N.N. & B.G..Zakharov (1991,1992) Diffractive DIS and nuclear shadowing in the color dipole approach
- N.N.N. & B.G..Zakharov, V.R. Zoller (1993,1994); A. Mueller (1994): Higher Fock states in the color dipole approach, the color dipole BFKL.
- N.N.N., Lectures at DESY Mini-School on Diffraction, May 1994; N.N.N. & B.G.Zakharov and V.R.Zoller (1995): In the saturation regime coherent diffractive DIS makes 50% of the total DIS rate.
- N.N.N. & B.G.Zakharov (1994); N.N.N. & W.Schäfer and G.Schwiete (2000): Diffractive DIS as a direct probe of unintegrated glue.
- A. Mueller (1990, 1999); L.McLerran & R. Venugopalan (1994): The pQCD/quasiclassical approach to the fusion of partons.
- I.P. Ivanov, N.N.N, W. Schäfer, B.G.Zakharov, V.R. Zoller, hep-ph/0207045; N.N.N, W. Schäfer, B.G.Zakharov, V.R. Zoller, *JETP Lett.* **76** (2002) 231. The consistent theory of nuclear glue and IS & FS nuclear parton densities.

Nuclear partons in the Breit (brick wall) frame:



The Lorentz contraction of a nucleus entails a spatial overlap and fusion of partons at

$$x \lesssim x_A = \frac{1}{R_A m_N}$$



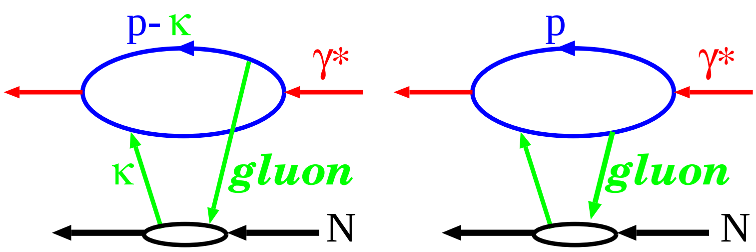
The shadowing is a scaling effect and does not vanish with Q^2. (N.N.N. & V.I.Zakharov, *Yadernaya Fizika* **21** (1975) 434; *Phys. Lett.* **B55** (1975) 397.)

Color dipole approach to DIS

DIS at $x \lesssim x_A$ as an interaction of frozen Fock states of the photon:

$$|\gamma^*\rangle = |\gamma_0^*\rangle + \Psi_{\gamma^*}(z, \boldsymbol{r})|q\bar{q}\rangle + \dots .$$

$$\sigma_T(x, Q^2) = \langle\gamma^*|\sigma(\boldsymbol{r})|\gamma^*\rangle$$



The color dipole X-section (NNN, Zakharov (1993))

$$\sigma(\boldsymbol{r}) = \alpha_S(r)\sigma_0 \int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})[1 - \exp(i\boldsymbol{\kappa}\boldsymbol{r})]$$

The unintegrated gluon SF

$$f(\boldsymbol{\kappa}) = \frac{4\pi}{N_c\sigma_0} \cdot \frac{1}{\kappa^4} \cdot \frac{\partial G_N}{\partial \log \kappa^2},$$

$$\int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa}) = 1$$

Free nucleons: the initial and final state quark densities

The Fourier transform from dipoles to the momentum space:

$$\sigma_N(x,Q^2) = \langle\gamma^*|\sigma(\boldsymbol{r})|\gamma^*\rangle =$$

$$= \frac{1}{2}\sigma_0 \int_0^1 dz \int \frac{d^2\mathbf{p}}{(2\pi)^2}\alpha_S(\mathbf{p}^2)\int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})$$

$$\left(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right)\left(\langle\mathbf{p}|\gamma^*\rangle - \langle\mathbf{p}-\boldsymbol{\kappa}|\gamma^*\rangle\right)$$

$\mathbf{p}$ is the transverse momentum of the observed FS quark.

The final state sea quark density

$$\frac{d\bar{q}_{FS}}{d^2\mathbf{p}} = \frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\cdot\frac{d\sigma_T(x,Q^2)}{d^2\mathbf{p}}$$

$$= \frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\frac{1}{2(2\pi)^2}\sigma_0\int_0^1 dz\,\alpha_S(\mathbf{p}^2)\int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})$$

$$\left(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right)\left(\langle\mathbf{p}|\gamma^*\rangle - \langle\mathbf{p}-\boldsymbol{\kappa}|\gamma^*\rangle\right)$$

coincides with the conventional initial state quark density in the target proton.

The helicity content of the photon:

Transverse photons: $\lambda_\gamma = \pm 1$

$$\left(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right)\cdot\left(\langle\mathbf{p}|\gamma^*\rangle - \langle\mathbf{p}-\boldsymbol{\kappa}|\gamma^*\rangle\right)\Big|_{\lambda_\gamma=\pm 1} =$$

$$2N_c\alpha_{em}\left\{[z^2+(1-z)^2]\left(\frac{\mathbf{p}}{\mathbf{p}^2+\varepsilon^2}-\frac{\mathbf{p}-\boldsymbol{\kappa}}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)\right.$$

$$\times\left(\frac{\mathbf{p}}{\mathbf{p}^2+\varepsilon^2}-\frac{\mathbf{p}-\boldsymbol{\kappa}}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)\Bigg|_{\lambda+\overline{\lambda}=0}$$

$$+m_f^2\left(\frac{1}{\mathbf{p}^2+\varepsilon^2}-\frac{1}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)$$

$$\left.\times\left(\frac{1}{\mathbf{p}^2+\varepsilon^2}-\frac{1}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)\Bigg|_{\lambda+\overline{\lambda}=\lambda_\gamma}\right\}$$

Longitudinal photons: $\lambda_\gamma = \pm 0$

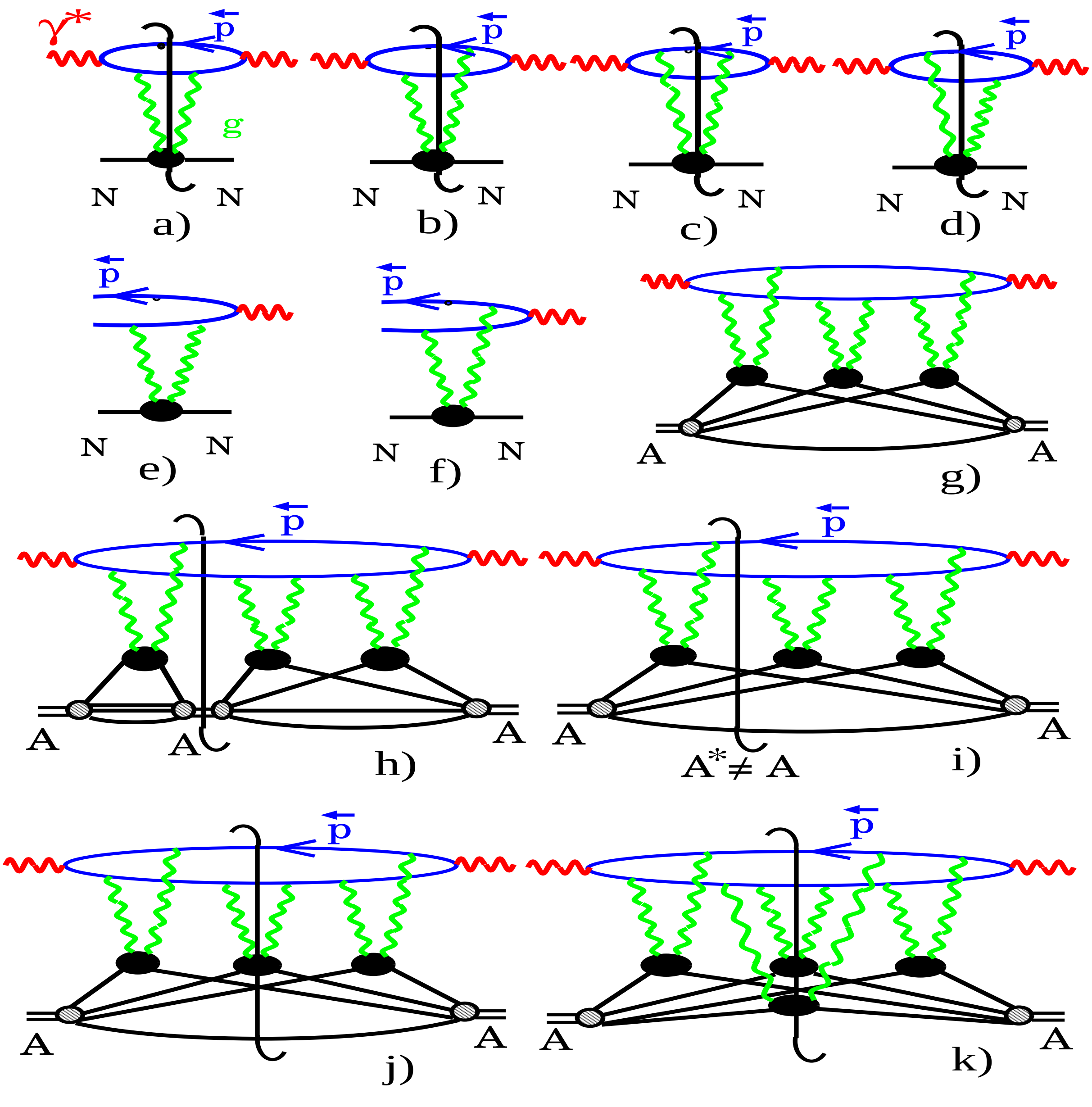
$$\left(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right)\cdot\left(\langle\mathbf{p}|\gamma^*\rangle - \langle\mathbf{p}-\boldsymbol{\kappa}|\gamma^*\rangle\right)\Big|_{\lambda_\gamma=0} =$$

$$2N_c\alpha_{em}4Q^2z^2(1-z)^2\left(\frac{1}{\mathbf{p}^2+\varepsilon^2}-\frac{1}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)$$

$$\times\left(\frac{1}{\mathbf{p}^2+\varepsilon^2}-\frac{1}{(\mathbf{p}-\boldsymbol{\kappa})^2+\varepsilon^2}\right)\Bigg|_{\lambda+\overline{\lambda}=\lambda_\gamma=0}$$

$$\varepsilon^2 = z(1-z)Q^2 + m_f^2$$

Nuclear target: Diffractive and truly inelastic unitarity cuts



Diffractive DIS = a rapidity gap

Truly inelastic DIS = color excited nucleus, no rapidity gap.

Isolation of color excitations in DIS off nuclei

(NNN, Schäfer, Zakharov, Zoller (2002)
The leading order S-matrix for the color dipole-nucleon interaction in terms of the quark-nucleon QCD eikonal $\delta(\mathbf{b})$:

$$S(\mathbf{b}_+,\mathbf{b}_-) = 1+2i[\Delta(\mathbf{b}_+) - \Delta(\mathbf{b}_-)]-2\langle[\delta(\mathbf{b}_+) - \delta(\mathbf{b}_-)]^2\rangle_0$$

$\langle\ldots\rangle_0 \Longrightarrow$ the color singlet component of the two-gluon exchange,
$\Delta(\mathbf{b}) = \delta(\mathbf{b}) \Longrightarrow$ the color-excitation of the target nucleon

The connection between eikonal and unintegrated glue:

$$\sigma(\boldsymbol{r}) = 2\int d^2\mathbf{b}\,\{1-\langle S(\mathbf{b}+\boldsymbol{r},\mathbf{b})\rangle_0\}$$

$$\int d^2\mathbf{b}\langle\delta(\mathbf{b}+\boldsymbol{r})\delta(\mathbf{b})\rangle_0 = \frac{1}{8}\alpha_S(\boldsymbol{r})\sigma_0\int d^2\boldsymbol{\kappa} f(\boldsymbol{\kappa})\exp[i\boldsymbol{\kappa}\boldsymbol{r}]$$

The nuclear S-matrix for a dilute gas nucleus:

$$S_A(\{\Delta\},\{\delta\};\mathbf{b}_+,\mathbf{b}_-) = \prod_{j=1}^{A} S(\mathbf{b}_+ - \mathbf{b}_j, \mathbf{b}_- - \mathbf{b}_j)$$

$S_A(0,\{\delta\};\mathbf{b}_+,\mathbf{b}_-) \Longrightarrow$ the pure diffractive DIS without color excitations in a nucleus.

Apply closure to total DIS and isolate color excitations subtracting diffractive final states:

$$\frac{d\sigma_{in}}{dz d^2\mathbf{p}_+ d^2\mathbf{p}_-} = \frac{1}{(2\pi)^4}\int d^2\mathbf{b}'_+ d^2\mathbf{b}'_- d^2\mathbf{b}_+ d^2\mathbf{b}_-$$
$$\exp[-i\mathbf{p}_+(\mathbf{b}_+ - \mathbf{b}'_+) - i\mathbf{p}_-(\mathbf{b}_- - \mathbf{b}'_-)]$$
$$\Psi^*(\mathbf{b}'_+ - \mathbf{b}'_-)\Psi(\mathbf{b}_+ - \mathbf{b}_-)$$
$$\{\langle A|S_A^*(\{\Delta\},\{\delta\};\mathbf{b}'_+,\mathbf{b}'_-)S_A(\{\Delta\},\{\delta\};\mathbf{b}_+,\mathbf{b}_-)|A\rangle$$
$$-\langle A|S_A^*(0,\{\delta\};\mathbf{b}'_+,\mathbf{b}'_-)S_A(0,\{\delta\};\mathbf{b}_+,\mathbf{b}_-)|A\rangle\}$$

A dilute nucleon gas nucleus:

$$\langle A|S_A^*(\{\Delta\},\{\delta\};\mathbf{b}'_+,\mathbf{b}'_-)S_A(\{\Delta\},\{\delta\};\mathbf{b}_+,\mathbf{b}_-)|A\rangle$$
$$= \exp\{-\frac{1}{2}T(\mathbf{b})\left[\Sigma(\mathbf{b}'_+ - \mathbf{b}_+) + \Sigma(\mathbf{b}'_- - \mathbf{b}_-) - \Sigma(\mathbf{b}'_+ - \mathbf{b}_-)\right.$$
$$\left. -\Sigma(\mathbf{b}_+ - \mathbf{b}'_-) + \sigma(\mathbf{b}'_+ - \mathbf{b}'_-) + \sigma(\mathbf{b}_+ - \mathbf{b}_-)\right]\},$$

$$T(\mathbf{b}) = \int dz n_A(z,\mathbf{b})$$

$\Sigma(\boldsymbol{r}) = \sigma(\boldsymbol{r})$ originates from the color excitation processes. The single particle FS spectrum is distorted by a nucleus (NNN, Schäfer, Zakharov, Zoller (2002)

$$\frac{d\sigma_{in}}{d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2}\int d^2\mathbf{b}\int d^2\boldsymbol{r}' d^2\boldsymbol{r}\exp[i\mathbf{p}(\boldsymbol{r}'-\boldsymbol{r})]\Psi^*(\boldsymbol{r}')\Psi(\boldsymbol{r})$$
$$\left\{\exp[-\frac{1}{2}\sigma(\boldsymbol{r}-\boldsymbol{r}')T(\mathbf{b})] - \exp[-\frac{1}{2}[\sigma(\boldsymbol{r})+\sigma(\boldsymbol{r}')]T(\mathbf{b})]\right\}.$$

NSS-defined WW unintegrated glue of a nucleus

The NNN-Schäfer-Schwiete (2002) representation for a color-triplet dipole starting from a nuclear diffractive DIS:

$$\Gamma_{3A}(\mathbf{b},\boldsymbol{r}) = 1-\exp[-\frac{1}{2}\sigma(\boldsymbol{r})T(\mathbf{b})]$$

$$= \int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\{1-\exp[i\boldsymbol{\kappa}\boldsymbol{r}]\}$$

$$\phi_{3WW}(\boldsymbol{\kappa}) = \sum_{j=1}^{\infty}\nu_{3A}^{j}(\mathbf{b})\cdot\frac{1}{j!}f^{(j)}(\boldsymbol{\kappa})\exp\left[-\nu_{3A}(\mathbf{b})\right]$$

The j-fold convolutions

$$f^{(j)}(\boldsymbol{\kappa}) = \int\prod_{i=1}^{j}d^2\boldsymbol{\kappa}_i f(\boldsymbol{\kappa}_i)\delta(\boldsymbol{\kappa}-\sum_{i=1}^{j}\boldsymbol{\kappa}_i).$$

The nuclear thickness:

$$\nu_{3A}(\mathbf{b}) = \frac{1}{2}\alpha_S(r)\sigma_0 T(\mathbf{b}) \propto A^{1/3}$$

A profound similarity to the color dipole X-section on a free nucleon

$$\sigma(\boldsymbol{r}) = \alpha_S(r)\sigma_0\int d^2\boldsymbol{\kappa}f(\boldsymbol{\kappa})[1-\exp(i\boldsymbol{\kappa}\boldsymbol{r})]$$

The interpretation: $\phi_{3WW}(\boldsymbol{\kappa})$ is an unintegrated Weizsäcker-Williams (WW) nuclear glue per unit area in the impact parameter, $\mathbf{b}$, space.

Properties of the NSS defined WW glue

Convolutions = random walk in the $\boldsymbol{\kappa}$-space.

In the hard regime (NNN, Schäfer, Schwiete (2000))

$$f^{(j)}(\boldsymbol{\kappa}) = j \cdot f(\boldsymbol{\kappa}) \left[1 + \frac{4\pi^2\gamma^2}{N_c\sigma_0\boldsymbol{\kappa}^2} \cdot (j-1) G_N(\boldsymbol{\kappa}^2)\right]$$

The higher twist depends on the exponent γ of

$$f(\boldsymbol{\kappa}) \propto (\boldsymbol{\kappa}^2)^{-\gamma} .$$

The model independent hard WW glue per bound nucleon:

$$f_{3WW} = \frac{\phi_{3WW}}{\nu_{3A}(\mathbf{b})} = f(\boldsymbol{\kappa}) \left[1 + \frac{2C_A\pi^2\gamma^2\alpha_S(r)T(\mathbf{b})G_N(\boldsymbol{\kappa}^2)}{C_F N_c \boldsymbol{\kappa}^2}\right]$$

To the leading twist hard WW glue is not shadowed.

The nuclear higher twist (HT) correction is positive valued and calculable parameter free.

We predict nuclear antishadowing of diffraction off nuclei in the Pomeron splitting regime.

A scaling soft WW glue (I.Ivanov, NNN, Schäfer, Zakharov, Zoller (2002))

$$\phi_{3WW}(\boldsymbol{\kappa}) \approx \frac{1}{\pi}\frac{Q_{3A}^2}{(\boldsymbol{\kappa}^2 + Q_{3A}^2)^2}\,,$$

where the saturation scale

$$Q_{3A}^2 = \nu_{3A}(\mathbf{b})Q_0^2 \propto A^{1/3}\,.$$

The two soft parameters - the saturation scale and saturated dipole X-section - are interrelated:

$$Q_0^2\sigma_0 \sim \frac{2\pi^2}{N_c}G_{soft}\,, \quad G_{soft} \sim 1\,.$$

Strong nuclear dilution of WW glue in a heavy nucleus:

$$\phi_{3WW}(\boldsymbol{\kappa}) \approx \frac{2f^{(2)}(0)}{\nu_{3A}(\mathbf{b})} \propto \frac{1}{A^{1/3}}$$

Still stronger dilution of soft WW glue per bound nucleon

$$f_{3A}(\boldsymbol{\kappa}) \propto \frac{1}{\nu_{3A}^2(\mathbf{b})} \propto \frac{1}{A^{2/3}}$$

DIS off nuclei in terms of the NSS-defined WW glue

The formal NSS definition of the IS sea quark density in a nucleus based on an exact Glauber-Gribov nuclear total X-section

$$\sigma_A(x,Q^2) = \int d^2\mathbf{b} \int dz \int \frac{d^2\mathbf{p}}{(2\pi)^2}$$

$$\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\left|\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right|^2$$

$$\frac{d\bar{q}_{IS}}{d^2\mathbf{b}d^2\mathbf{p}} = \frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\cdot\frac{d\sigma_A}{d^2\mathbf{b}d^2\mathbf{p}}\,.$$

What is a meaning of the Fourier parameter $\mathbf{p}$?
Pursue the NSS representation in terms of the nuclear WW glue:

$$\exp[-\frac{1}{2}\sigma(\boldsymbol{r}-\boldsymbol{r}')T(\mathbf{b})] - \exp[-\frac{1}{2}[\sigma(\boldsymbol{r})+\sigma(\boldsymbol{r}')]T(\mathbf{b})] =$$

$$\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\Big\{e^{i\boldsymbol{\kappa}(\boldsymbol{r}-\boldsymbol{r}')}-1)$$

$$+(1-e^{i\boldsymbol{\kappa}\boldsymbol{r}})+(1-e^{i\boldsymbol{\kappa}\boldsymbol{r}'}])\Big\}$$

$$-\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})(1-e^{i\boldsymbol{\kappa}\boldsymbol{r}})\int d^2\boldsymbol{\kappa}'\phi_{3WW}(\boldsymbol{\kappa}')(1-e^{i\boldsymbol{\kappa}'\boldsymbol{r}'})$$

$$\frac{d\sigma_{in}}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2}$$

$$\times\left\{\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\left|(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle)\right|^2\right.$$

$$\left.-\left|\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle)\right|^2\right\}$$

⋆ A striking dissimilarity of FS spectra in color excitation of a nucleus and free nucleon!

$$\frac{d\sigma_D}{d^2\mathbf{b}d^2\mathbf{p}dz} = \frac{1}{(2\pi)^2}$$

$$\times\left|\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})(\langle\gamma^*|\mathbf{p}\rangle - \langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle)\right|^2 .$$

⋆ A prominent, $\eta_D^A \sim 50\%$, nuclear diffractive DIS has no counterpart in DIS off free nucleons, where $\eta_D^N \lesssim$ 6-10 % is negligible small.

$$d[\sigma_D + \sigma_{in}]/d^2\mathbf{b}d^2\mathbf{p}dz = d\sigma_A/d^2\mathbf{b}d^2\mathbf{p}dz$$

$\Longrightarrow$ an exact equality of the IS and FS parton densities! The Fourier parameter $\mathbf{p}$ of σ_A is the transverse momentum of the observed sea quark! (NNN, Schäfer, Zakharov, Zoller (2002))

⋆ In terms of the WW glue all multiple scattering diagrams for DIS off a nucleus do sum to precisely the same 4 two-gluon exchange diagrams as in DIS off a free nucleon !

Diffractive component of saturated IS quark density

⋆ The saturation domain $\mathbf{p}^2 \lesssim Q^2 \lesssim Q_{3A}^2$: a nucleus is opaque for all color dipoles in the photon.

⋆ Assume that Q_{3A}^2 is so large that $\mathbf{p}^2, Q^2$ are in the pQCD domain and neglect the quark masses.

⋆ In the opacity/saturation regime the contributions to diffractive amplitudes from the crossing diagrams can be neglected.

(NNN, Schäfer, Zakharov, Zoller (2002)):

$$\left.\frac{d\bar{q}_{FS}}{d^2\mathbf{b}d^2\mathbf{p}}\right|_D = \frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\cdot\frac{d\sigma_D}{d^2\mathbf{b}d^2\mathbf{p}} \approx$$

$$\frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\cdot\int dz\left|\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\right|^2 |\langle\gamma^*|\mathbf{p}\rangle|^2 \approx \frac{N_c}{4\pi^4}\,.$$

⋆ Diffraction measures the momentum distribution in the $q\bar{q}$ Fock state of the photon - a feature of the dominance of the Landau-Pomeranchuk mechanism of diffraction in the saturation regime.
⋆ Only the last result uses the specific QCD/QED wave function of the photon and conventional spin-$\frac{1}{2}$ partons.
⋆ Can be identified with the spectator component (a) of DIS.

Inelastic component of saturated IS quark density

(NNN, Schäfer, Zakharov, Zoller (2002)):

⋆ The saturation/opacity domain of $\mathbf{p}^2 \lesssim Q^2 \lesssim Q_{3A}^2$ gives

$$\left.\frac{d\bar{q}_{FS}}{d^2\mathbf{b}d^2\mathbf{p}}\right|_{in} =$$

$$\frac{1}{2}\cdot\frac{Q^2}{4\pi^2\alpha_{em}}\cdot\int dz\int d^2\boldsymbol{\kappa}\phi_{3WW}(\boldsymbol{\kappa})\left|\langle\gamma^*|\mathbf{p}-\boldsymbol{\kappa}\rangle\right|^2$$

$$= \frac{Q^2}{8\pi^2\alpha_{em}}\phi_{3WW}(0)\int^{Q^2} d^2\boldsymbol{\kappa}\int dz\left|\langle\gamma^*|\boldsymbol{\kappa}\rangle\right|^2 = \frac{N_c}{4\pi^4}\cdot\frac{Q^2}{Q_{3A}^2}.$$

A dramatic distinction between diffractive and inelastic DIS:

⋆ The inelastic plateau extends way beyond Q^2, up to $\mathbf{p}^2 \lesssim Q_{3A}^2$.

⋆ The height of the inelastic plateau depends strongly on Q^2.

⋆ At $Q^2 \ll {Q_{3A}}^2$ the inelastic plateau contributes little to the spectrum of soft quarks, $\mathbf{p}^2 \lesssim Q^2$, which is entirely diffraction dominated

⋆ But integral inelastic contribution to the production of FS quarks is exactly equal to that from diffractive DIS - recall

$$\sigma_D = \sigma_{in} = \tfrac{1}{2}\sigma_{tot} \quad !$$

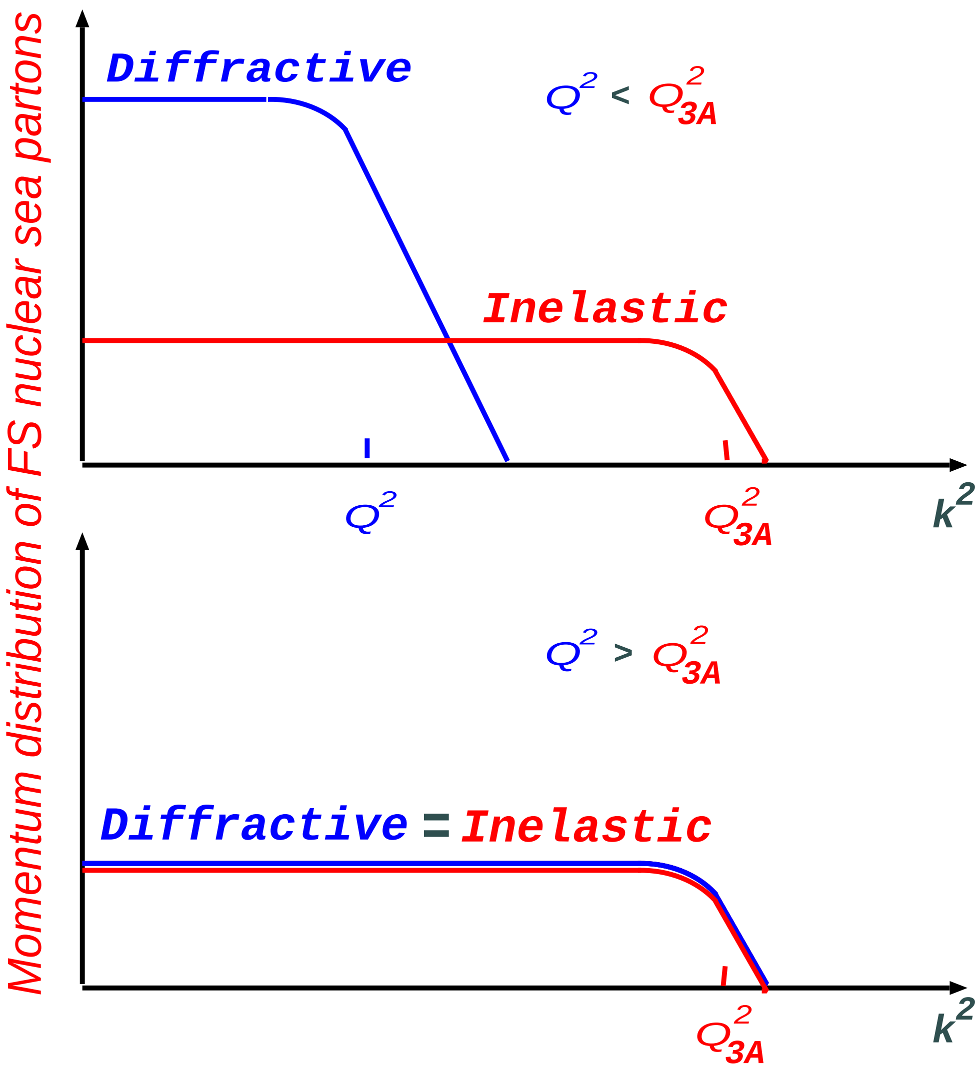
⋆ Inelastic plateau can be identified with the rescattering component (c) of DIS.

The two-plateau momentum distribution of FS quarks

⋆ The two-plateau structure of the FS quark spectrum has not been considered before.

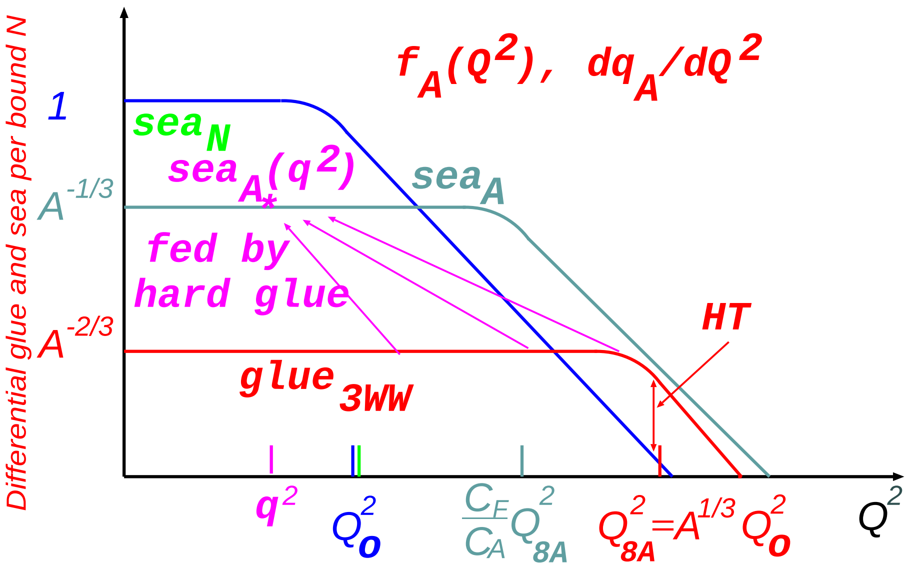
⋆ $Q^2 \lesssim Q^2_{3A}$: The inelastic plateau is much broader than the diffractive one.

⋆ $Q^2 \gtrsim Q^2_{3A}$: inelastic plateau is identical to diffractive one.



Saturated sea from nuclear-diluted WW glue

⋆ The origin of saturation: anti-DGLAP evolution in the plateau region: the sea at a scale $q^2 \lesssim Q_A^2$ is fed by hard gluons with the transverse momentum $\boldsymbol{\kappa}^2 > q^2$ (NNN, Schäfer, Zakharov, Zoller (2002)):



⋆ A nuclear dilution of WW glue is compensated for by the width of the plateau $Q^2_{3A} \propto A^{1/3}$.

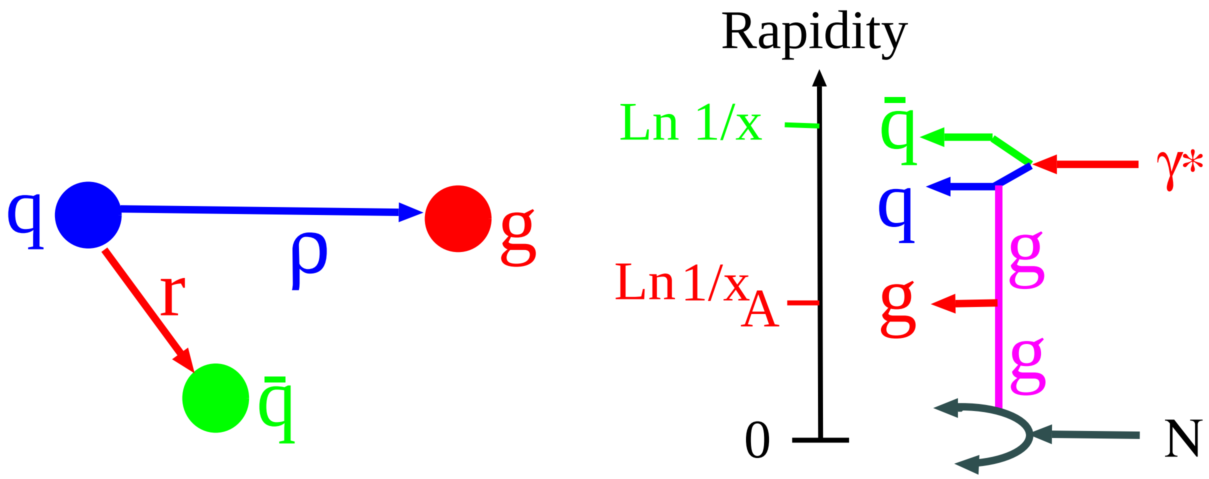
⋆ Compare with the DGLAP collinear splitting

$$\frac{d\bar{q}(x,\mathbf{p}^2)}{d\mathbf{p}^2} = \frac{\alpha_S(\mathbf{p}^2)}{6\pi\mathbf{p}^2} G(x,\mathbf{p}^2)$$

⋆ Saturation removes from the spectrum one factor $\frac{1}{\mathbf{p}^2}$.

A related derivation: A. Mueller, *Nucl. Phys.* **B558** (1999) 285, he didn't discuss the two-plateau structure and anticollinear splitting.

Leading gluons come from interactions of the $q\bar{q}g$ Fock states of the photon (the technique from N.N.N. & B. Zakharov (93)):



$$|\Phi_{q\bar{q}g}(\boldsymbol{r},\boldsymbol{\rho})|^2 = \frac{C_F}{z_g}\cdot|\Psi_{\gamma^*}(z,\boldsymbol{r})|^2\cdot\frac{\alpha_S(r)}{\pi^2}$$

$$\times\left(\frac{\boldsymbol{\rho}}{\boldsymbol{\rho}^2}-\frac{\boldsymbol{\rho}-\boldsymbol{r}}{(\boldsymbol{\rho}-\boldsymbol{r})^2}\right)\cdot\left(\frac{\boldsymbol{\rho}}{\boldsymbol{\rho}^2}-\frac{\boldsymbol{\rho}-\boldsymbol{r}}{(\boldsymbol{\rho}-\boldsymbol{r})^2}\right)$$

DLLA: $\boldsymbol{\rho}^2 \gg \boldsymbol{r}^2$, the effective color-octet dipole

$$\sigma_8(\boldsymbol{\rho},\boldsymbol{r}) = \frac{C_A}{C_F}\sigma(\boldsymbol{\rho})$$

Free nucleon target: collinear splitting of soft gluons into hard sea $\boldsymbol{\kappa}^2 \lessapprox \mathbf{p}^2$,

$$x\frac{dG_L(x,\mathbf{p}^2)}{dx\,d\mathbf{p}^2} = \frac{C_A\alpha_S(\mathbf{p}^2)}{\pi\mathbf{p}^2}G_N(x,\mathbf{p}^2)$$

The LL$\frac{1}{x}$ evolution of glue is driven entirely by glue: an unique definition of the proper WW gluon SF of the nucleus.

To DLLA the color-octet nuclear profile function for the $q\bar{q}g$ state

$$\Gamma_{8A}(\mathbf{b},\boldsymbol{r},\boldsymbol{\rho}) = 1 - \exp[-\frac{1}{2}\sigma_8(\boldsymbol{\rho})T(\mathbf{b})]$$

defines the gluon-gluon fusion driven nuclear WW glue $\phi_{8WW}(\boldsymbol{\kappa})$ with the plateau of width $Q^2_{8A} = \frac{C_A}{C_F}Q^2_{3A}$. In the plateau region

$$\phi_{8WW}(\boldsymbol{\kappa}) = \frac{C_F}{C_A}\phi_{3WW}(\boldsymbol{\kappa})$$

In the hard region the color-triplet and color-octet defined nuclear WW glue per nucleon are identical $f_{8WW}(\boldsymbol{\kappa}) = f_{3WW}(\boldsymbol{\kappa})$

In the saturation domain of $\mathbf{p}^2 \lesssim Q^2_{8A}$

$$x\frac{dG_A(x,\mathbf{p}^2)}{d^2\mathbf{b}dx\,d^2\mathbf{p}}\Bigg|_{x=x_A} = \frac{(N_c^2-1)}{4\pi^4}$$

Saturation is driven entirely by the anti-DGLAP splitting of hard WW gluons with $\boldsymbol{\kappa}^2 \gtrsim \mathbf{p}^2$. It removes the factor $1/\mathbf{p}^2$ from the gluon spectrum.

Signals of saturation in diffractive DIS

⋆ Inclusive DIS: after interaction with the target $q\bar{q}$ dipole of the photon is projected back onto the photon.
⋆ Diffractive DIS: after interaction with the target $q\bar{q}$ dipole of the photon is projected onto the vector meson or continuum states.

Simple rules for saturation signals: identify the hard scale $\overline{Q}^2$

⋆ $\overline{Q}^2 \approx \frac{1}{4}(Q^2 + m_V^2)$ for diffractive vector mesons (Nemchik, NNN, Zakharov 94)
⋆ $\overline{Q}^2 = \mathbf{p}^2$ for diffractive dijets (NNN, Zakharov 92,94)

The effect of saturation at $\overline{Q}^2 \lesssim Q_{3A}^2$ is an enhancement of diffractive amplitude by the factor $\overline{Q}^2$.
An example: a flat $\mathbf{p}^2$ distribution of dijets for a fixed diffractive mass

$$\frac{d\sigma_D}{d^2\mathbf{b}d\beta d\mathbf{p}^2} = \frac{\alpha_{em}N_c}{Q^2 J\pi^3}\left(1 - 2\frac{\mathbf{p}^2}{M^2}\right)$$

in contrast to the free nucleon target result (NNN, Zakharov 92,94)

$$\frac{d\sigma_D}{d\beta d\mathbf{p}^2} \propto \left(\frac{G_N(\mathbf{p}^2)}{\mathbf{p}^2}\right)^2$$

Signal of saturation for diffractive vector mesons

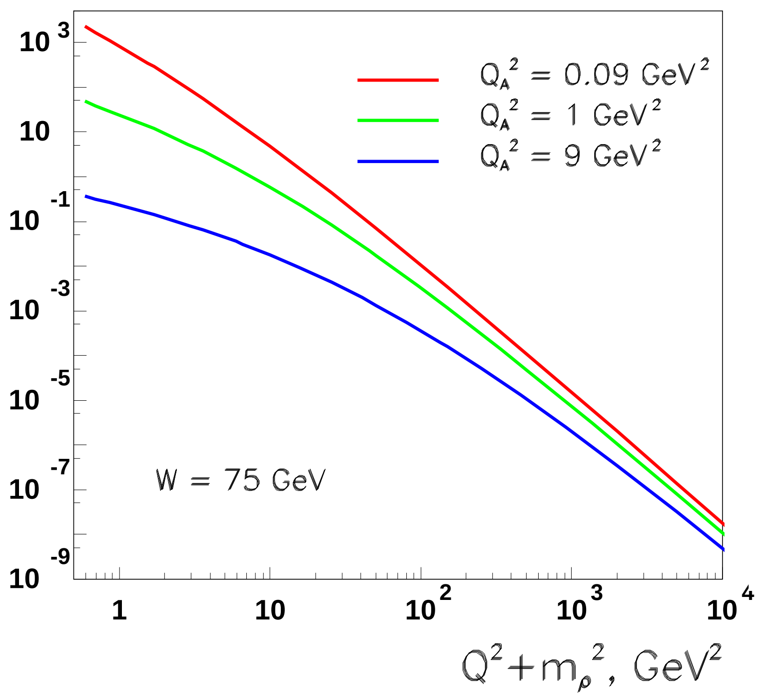
A free nucleon target and nuclei at $\frac{1}{4}(Q^2 + m_V^2) \gtrsim Q_A^2$:

$$\sigma_T \propto \left(\frac{G_N(\frac{1}{4}(Q^2 + m_V^2))}{(Q^2 + m_V^2)^2} \right)^2$$

Nuclei in the saturation regime, $\frac{1}{4}(Q^2 + m_V^2) \lesssim Q_A^2$:

$$\sigma_T \propto \frac{1}{(Q^2 + m_V^2)^2}$$

$\star$ The modification of the diffractive ρ production off a nucleus with the change of the saturation scale (I.Ivanov (2002)):



Saturation of glue in the nuclear pomeron

High mass diffractive DIS is driven by excitation of the $q\bar{q}g$ states (NNN, Zakharov 92, 94). A free nucleon target:

$$M^2 \frac{d\sigma_D}{dM^2 dt}\Big|_{t=0} = \frac{1}{16\pi} \langle \gamma^* | \left[\sigma_3(\boldsymbol{r}, \boldsymbol{\rho}) - \sigma(\boldsymbol{r})\right]^2 | \gamma^* \rangle$$

$$= \int_0^1 dz \int d^2\boldsymbol{r} |\Psi_{\gamma^*}(z, \boldsymbol{r})|^2 \frac{\pi^2}{N_c} \alpha_S(r) \boldsymbol{r}^2 \mathcal{G}_{\mathbb{P}}(x_{\mathbb{P}}, q^2 = \frac{A_\sigma}{\boldsymbol{r}^2})$$

where the 'glue in the pomeron'

$$\frac{d\mathcal{G}_{\mathbb{P}}(x, \mathbf{p}^2)}{d\mathbf{p}^2} = \frac{C_A^2}{4C_F N_c (\mathbf{p}^2)^2} \alpha_S^2(\mathbf{p}^2) G_N^2(x_{\mathbb{P}}, \mathbf{p}^2)$$

describes the forward diffractive gluon jet.

An unintegrated glue in the nuclear pomeron is saturated (I.Ivanov, NNN, Schäfer, Zakharov, Zoller (2002))

$$\frac{d\mathcal{G}_{\mathbb{P}A}}{d^2\mathbf{b} d\mathbf{p}^2} = \frac{C_A^2 N_c}{256 C_F \pi^4} \alpha_S^2(\mathbf{p}^2)$$

Saturation removes the factor $\propto \frac{1}{(\mathbf{p}^2)^2}$ from the spectrum of diffractive leading gluons.

Why $f_{3WW}(\boldsymbol{\kappa}) \neq f_{8WW}(\boldsymbol{\kappa})$, or nuclear antishadowing of sea quarks

On a free nucleon target the small x sea and glue are driven by one and the same glue of the nucleon.

The color-octet WW glue $f_{8WW}(\boldsymbol{\kappa})$ describes the fusion of gluons from different nucleons into glue, enters the LL$\frac{1}{x}$ evolution of glue from the glue, and defines the true nuclear glue.

The color triplet $f_{3WW}(\boldsymbol{\kappa})$ describes the fusion of a sea quark from one of the nucleons with gluons from other nucleons.

Decompose $f_{3WW}(\boldsymbol{\kappa}) = f_{8WW}(\boldsymbol{\kappa}) + \Delta f_{WW}(\boldsymbol{\kappa})$. Because

$$f_{8WW}(0) = \left(\frac{C_F}{C_A}\right)^2 f_{3WW}(0) = \frac{16}{81} f_{3WW}(0)$$

the $\Delta f_{WW}(\boldsymbol{\kappa})$ is a manifestly positive valued quantity and describes the antishadowing component in the quark-gluon fusion as compared to the gluon-gluon fusion. The saturated density for quarks receives $\frac{16}{81}$ from the standard evolution from the nuclear WW glue, and $\frac{65}{81}$ is of the antishadowing origin.

Is saturation of any practical significance?

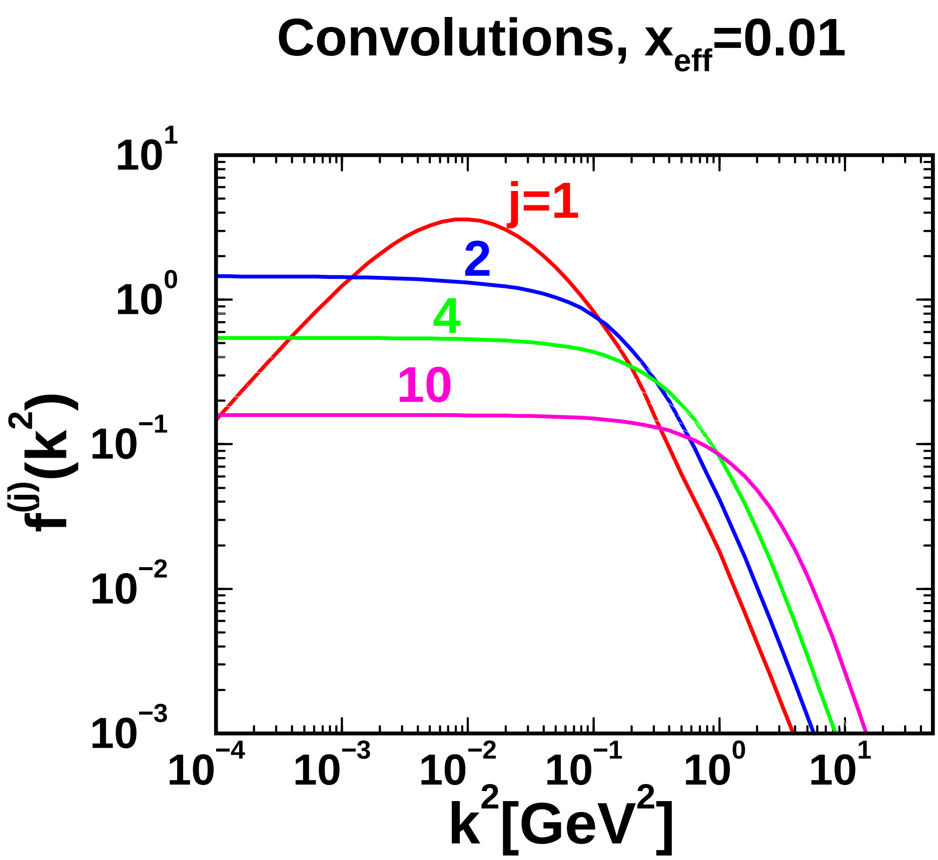
The platinum target, $A = 192$. Nuclear shadowing of $q\bar{q}$ states sets in at

$$x \sim x_A \approx 10^{-2}$$

The average optical thickness for color-triplet dipole

$$\nu_{3A} \approx 4$$

Unintegrated glue of the proton from I.Ivanov & N.N.N., *Phys. Rev.* **D65** (2002) 054004



$$Q^2_{3A}(x=.01) \approx 0.8 \mathrm{GeV}^2$$

I.Ivanov & N.N.N.: at such Q^2 the differential glue has a notoriously large nonperturbative component.

The conclusion: nuclear shadowing of sea quarks is not under full control of pQCD.

Nuclear shadowing of $q\bar{q}g$ states sets in at

$$x \sim 10^{-3}$$

but the shadowed glue has still $x_g \sim x_A \approx 10^{-2}$. The average optical thickness for color-octet dipole

$$\nu_{8A} = \frac{C_A}{C_F}\nu_{3A} \approx 10$$

and

$$Q^2_{8A}(x=.01) \approx 2.2 \mathrm{GeV}^2$$

The conclusion: the DGLAP evolution of nuclear SF of ^{192}Pt is justified only at $Q^2 \gtrsim 2.2$ GeV^2.

The quantitative treatment of the impact of saturation on nuclear SF requires an educated guess for soft glue constrained by a transition from real photoabsorption to DIS on a free nucleon. Not an easy task.....

Conclusions:

- Hard scales in diffractive DIS are well understood, the quantitative theory requires some soft input: wave functions, the size of nucleons, the infrared cutoff of pQCD.....
- $f(\boldsymbol{\kappa})$ for a nucleon $\Longleftrightarrow$ nuclear WW glue $\phi_{WW}(\boldsymbol{\kappa})$. The straightforward & unambiguous calculation of $\phi_{WW}(\boldsymbol{\kappa})$
- A solution of the unitarity problem: an exact equality of the initial state and final state quark densities.
- A nuclear dilution, $\propto \frac{1}{\nu_A(\mathbf{b})}$, of WW glue in the soft plateau region.
- Nuclear saturation of soft sea and leading gluon density is driven by anti-DGLAP splitting of hard WW glue into soft partons.
- The two-plateau structure and the equality of diffractive plateau to the momentum distribution of quarks in the photon show that saturation is driven by opacity of nuclei and has nothing to do with the Fermi statistics.
- Saturation of glue in a nuclear pomeron.
- The saturation gain: $\frac{1}{\mathbf{p}^2}$ off from the leading particle spectra in inclusive DIS and $\frac{1}{(\mathbf{p}^2)^2}$ off in diffractive DIS.
- The difference between gluon-gluon and sea-gluon fusion as an antishadowing component of nuclear sea.
- Q^2_{3A} and Q^2_{8A} are unlikely to prove perturbative large, but are expected to rise towards small x : a subject for the future phenomenology.